

Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.7-d-hyper-^m-a+b-c-
sech-ⁿ-^p

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Contents

1	Introduction	3
1.1	Listing of CAS systems tested	3
1.2	Results	3
1.3	Performance	7
1.4	list of integrals that has no closed form antiderivative	8
1.5	list of integrals solved by CAS but has no known antiderivative	8
1.6	list of integrals solved by CAS but failed verification	8
1.7	Timing	9
1.8	Verification	9
1.9	Important notes about some of the results	9
1.9.1	Important note about Maxima results	9
1.9.2	Important note about FriCAS and Giac/XCAS results	10
1.9.3	Important note about finding leaf size of antiderivative	10
1.9.4	Important note about Mupad results	11
1.10	Design of the test system	11
2	detailed summary tables of results	13
2.1	List of integrals sorted by grade for each CAS	13
2.1.1	Rubi	13
2.1.2	Mathematica	13
2.1.3	Maple	13
2.1.4	Maxima	14
2.1.5	FriCAS	14
2.1.6	Sympy	14
2.1.7	Giac	15
2.1.8	Mupad	15
2.2	Detailed conclusion table per each integral for all CAS systems	16
2.3	Detailed conclusion table specific for Rubi results	52
3	Listing of integrals	61
3.1	$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^4(c + dx) dx$	61
3.2	$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^3(c + dx) dx$	64
3.3	$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^2(c + dx) dx$	67
3.4	$\int (a + b \operatorname{sech}^2(c + dx)) \sinh(c + dx) dx$	70
3.5	$\int \operatorname{csch}(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$	72

3.6	$\int \operatorname{csch}^2(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	75
3.7	$\int \operatorname{csch}^3(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	77
3.8	$\int \operatorname{csch}^4(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	81
3.9	$\int (a+b\operatorname{sech}^2(c+dx))^2 \sinh^4(c+dx) dx$	84
3.10	$\int (a+b\operatorname{sech}^2(c+dx))^2 \sinh^3(c+dx) dx$	88
3.11	$\int (a+b\operatorname{sech}^2(c+dx))^2 \sinh^2(c+dx) dx$	91
3.12	$\int (a+b\operatorname{sech}^2(c+dx))^2 \sinh(c+dx) dx$	94
3.13	$\int \operatorname{csch}(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	97
3.14	$\int \operatorname{csch}^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	100
3.15	$\int \operatorname{csch}^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	103
3.16	$\int \operatorname{csch}^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	108
3.17	$\int (a+b\operatorname{sech}^2(c+dx))^3 \sinh^4(c+dx) dx$	111
3.18	$\int (a+b\operatorname{sech}^2(c+dx))^3 \sinh^3(c+dx) dx$	116
3.19	$\int (a+b\operatorname{sech}^2(c+dx))^3 \sinh^2(c+dx) dx$	119
3.20	$\int (a+b\operatorname{sech}^2(c+dx))^3 \sinh(c+dx) dx$	123
3.21	$\int \operatorname{csch}(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	126
3.22	$\int \operatorname{csch}^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	131
3.23	$\int \operatorname{csch}^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	134
3.24	$\int \operatorname{csch}^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	141
3.25	$\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	145
3.26	$\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	150
3.27	$\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	154
3.28	$\int \frac{\sinh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	158
3.29	$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	161
3.30	$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	165
3.31	$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	169
3.32	$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	174
3.33	$\int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	178
3.34	$\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	185
3.35	$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	191
3.36	$\int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	197
3.37	$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	201
3.38	$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	206
3.39	$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	210
3.40	$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	217

3.41	$\int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	223
3.42	$\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	235
3.43	$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	243
3.44	$\int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	253
3.45	$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	259
3.46	$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	267
3.47	$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	274
3.48	$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	279
3.49	$\int \cosh^4(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	284
3.50	$\int \cosh^3(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	287
3.51	$\int \cosh^2(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	289
3.52	$\int \cosh(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	291
3.53	$\int \operatorname{sech}(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	293
3.54	$\int \operatorname{sech}^2(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	296
3.55	$\int \operatorname{sech}^3(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	299
3.56	$\int \operatorname{sech}^4(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	302
3.57	$\int \cosh^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	305
3.58	$\int \cosh^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	308
3.59	$\int \cosh^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	311
3.60	$\int \cosh(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	314
3.61	$\int \operatorname{sech}(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	317
3.62	$\int \operatorname{sech}^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	321
3.63	$\int \operatorname{sech}^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	324
3.64	$\int \operatorname{sech}^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	329
3.65	$\int \cosh^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	333
3.66	$\int \cosh^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	336
3.67	$\int \cosh^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	340
3.68	$\int \cosh(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	343
3.69	$\int \operatorname{sech}(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	348
3.70	$\int \operatorname{sech}^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	354
3.71	$\int \operatorname{sech}^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	358
3.72	$\int \operatorname{sech}^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	365
3.73	$\int \frac{\cosh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	369
3.74	$\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	374
3.75	$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	378
3.76	$\int \frac{\cosh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	382

3.77	$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	385
3.78	$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	388
3.79	$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	391
3.80	$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	395
3.81	$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	398
3.82	$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	402
3.83	$\int \frac{\cosh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	406
3.84	$\int \frac{\cosh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	412
3.85	$\int \frac{\cosh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	417
3.86	$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	422
3.87	$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	426
3.88	$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	430
3.89	$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	434
3.90	$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	438
3.91	$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	443
3.92	$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	448
3.93	$\int \frac{\cosh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	455
3.94	$\int \frac{\cosh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	464
3.95	$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	472
3.96	$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	479
3.97	$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	485
3.98	$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	491
3.99	$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	497
3.100	$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	503
3.101	$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	509
3.102	$\int (a+b\operatorname{sech}^2(c+dx)) \tanh^4(c+dx) dx$	517
3.103	$\int (a+b\operatorname{sech}^2(c+dx)) \tanh^3(c+dx) dx$	520
3.104	$\int (a+b\operatorname{sech}^2(c+dx)) \tanh^2(c+dx) dx$	524
3.105	$\int (a+b\operatorname{sech}^2(c+dx)) \tanh(c+dx) dx$	527
3.106	$\int (a+b\operatorname{sech}^2(c+dx)) dx$	530

3.107	$\int \coth(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	532
3.108	$\int \coth^2(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	535
3.109	$\int \coth^3(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	538
3.110	$\int \coth^4(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	541
3.111	$\int \coth^5(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx$	544
3.112	$\int (a+b\operatorname{sech}^2(c+dx))^2 \tanh^4(c+dx) dx$	548
3.113	$\int (a+b\operatorname{sech}^2(c+dx))^2 \tanh^3(c+dx) dx$	552
3.114	$\int (a+b\operatorname{sech}^2(c+dx))^2 \tanh^2(c+dx) dx$	557
3.115	$\int (a+b\operatorname{sech}^2(c+dx))^2 \tanh(c+dx) dx$	560
3.116	$\int (a+b\operatorname{sech}^2(c+dx))^2 dx$	563
3.117	$\int \coth(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	566
3.118	$\int \coth^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	569
3.119	$\int \coth^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	572
3.120	$\int \coth^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	575
3.121	$\int \coth^5(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	578
3.122	$\int \coth^6(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	581
3.123	$\int \coth^7(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx$	585
3.124	$\int (a+b\operatorname{sech}^2(c+dx))^3 \tanh^4(c+dx) dx$	590
3.125	$\int (a+b\operatorname{sech}^2(c+dx))^3 \tanh^3(c+dx) dx$	595
3.126	$\int (a+b\operatorname{sech}^2(c+dx))^3 \tanh^2(c+dx) dx$	601
3.127	$\int (a+b\operatorname{sech}^2(c+dx))^3 \tanh(c+dx) dx$	605
3.128	$\int (a+b\operatorname{sech}^2(c+dx))^3 dx$	609
3.129	$\int \coth(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	612
3.130	$\int \coth^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	616
3.131	$\int \coth^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	619
3.132	$\int \coth^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	623
3.133	$\int \coth^5(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	626
3.134	$\int \coth^6(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	630
3.135	$\int \coth^7(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx$	634
3.136	$\int (a+b\operatorname{sech}^2(c+dx))^4 dx$	638
3.137	$\int (a+b\operatorname{sech}^2(c+dx))^5 dx$	642
3.138	$\int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	648
3.139	$\int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	652
3.140	$\int \frac{\tanh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	656
3.141	$\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	659
3.142	$\int \frac{\tanh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	663
3.143	$\int \frac{1}{a+b\operatorname{sech}^2(c+dx)} dx$	666
3.144	$\int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	669
3.145	$\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	672

3.146	$\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	676
3.147	$\int \frac{\coth^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$	680
3.148	$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	686
3.149	$\int \frac{\tanh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	690
3.150	$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	695
3.151	$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	698
3.152	$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	703
3.153	$\int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	706
3.154	$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	710
3.155	$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	714
3.156	$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	720
3.157	$\int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$	725
3.158	$\int \frac{\tanh^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	734
3.159	$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	742
3.160	$\int \frac{\tanh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	746
3.161	$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	754
3.162	$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	758
3.163	$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	766
3.164	$\int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	770
3.165	$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	777
3.166	$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	782
3.167	$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	793
3.168	$\int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$	801
3.169	$\int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^4} dx$	809
3.170	$\int (1 - \operatorname{sech}^2(x))^{3/2} dx$	815
3.171	$\int \sqrt{1 - \operatorname{sech}^2(x)} dx$	818
3.172	$\int \frac{1}{\sqrt{1 - \operatorname{sech}^2(x)}} dx$	820
3.173	$\int (-1 + \operatorname{sech}^2(x))^{3/2} dx$	823
3.174	$\int \sqrt{-1 + \operatorname{sech}^2(x)} dx$	826

3.175	$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}} dx$	828
3.176	$\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh^5(x) dx$	831
3.177	$\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh^4(x) dx$	836
3.178	$\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh^3(x) dx$	844
3.179	$\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh^2(x) dx$	848
3.180	$\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh(x) dx$	854
3.181	$\int \sqrt{a+b\operatorname{sech}^2(x)} dx$	858
3.182	$\int \coth(x) \sqrt{a+b\operatorname{sech}^2(x)} dx$	863
3.183	$\int \coth^2(x) \sqrt{a+b\operatorname{sech}^2(x)} dx$	868
3.184	$\int \coth^3(x) \sqrt{a+b\operatorname{sech}^2(x)} dx$	872
3.185	$\int \coth^4(x) \sqrt{a+b\operatorname{sech}^2(x)} dx$	878
3.186	$\int \coth^5(x) \sqrt{a+b\operatorname{sech}^2(x)} dx$	883
3.187	$\int (a+b\operatorname{sech}^2(x))^{3/2} \tanh^3(x) dx$	887
3.188	$\int (a+b\operatorname{sech}^2(x))^{3/2} \tanh^2(x) dx$	892
3.189	$\int (a+b\operatorname{sech}^2(x))^{3/2} \tanh(x) dx$	900
3.190	$\int (a+b\operatorname{sech}^2(x))^{3/2} dx$	904
3.191	$\int \coth(x) (a+b\operatorname{sech}^2(x))^{3/2} dx$	909
3.192	$\int \coth^2(x) (a+b\operatorname{sech}^2(x))^{3/2} dx$	914
3.193	$\int (a+b\operatorname{sech}^2(c+dx))^{5/2} dx$	919
3.194	$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	923
3.195	$\int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	927
3.196	$\int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	933
3.197	$\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	937
3.198	$\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	942
3.199	$\int \frac{1}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	946
3.200	$\int \frac{\coth(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	949
3.201	$\int \frac{\coth^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	954
3.202	$\int \frac{\coth^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$	958
3.203	$\int \frac{\tanh^5(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	964
3.204	$\int \frac{\tanh^4(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	969
3.205	$\int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	975

3.206	$\int \frac{\tanh^2(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	979
3.207	$\int \frac{\tanh(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	983
3.208	$\int \frac{1}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	987
3.209	$\int \frac{\coth(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	991
3.210	$\int \frac{\coth^2(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$	998
3.211	$\int \frac{\tanh^6(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1003
3.212	$\int \frac{\tanh^5(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1007
3.213	$\int \frac{\tanh^4(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1012
3.214	$\int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1018
3.215	$\int \frac{\tanh^2(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1023
3.216	$\int \frac{\tanh(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1029
3.217	$\int \frac{1}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1034
3.218	$\int \frac{\coth(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1040
3.219	$\int \frac{\coth^2(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$	1044
3.220	$\int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^{7/2}} dx$	1053

4	Listing of Grading functions	1057
4.0.1	Mathematica and Rubi grading function	1057
4.0.2	Maple grading function	1059
4.0.3	Sympy grading function	1062
4.0.4	SageMath grading function	1064

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [220]. This is test number [180].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (220)	% 0.00 (0)
Mathematica	% 100.00 (220)	% 0.00 (0)
Maple	% 81.82 (180)	% 18.18 (40)
Maxima	% 66.82 (147)	% 33.18 (73)
Fricas	% 95.91 (211)	% 4.09 (9)
Sympy	% 4.55 (10)	% 95.45 (210)
Giac	% 64.09 (141)	% 35.91 (79)
Mupad	% 55.00 (121)	% 45.00 (99)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

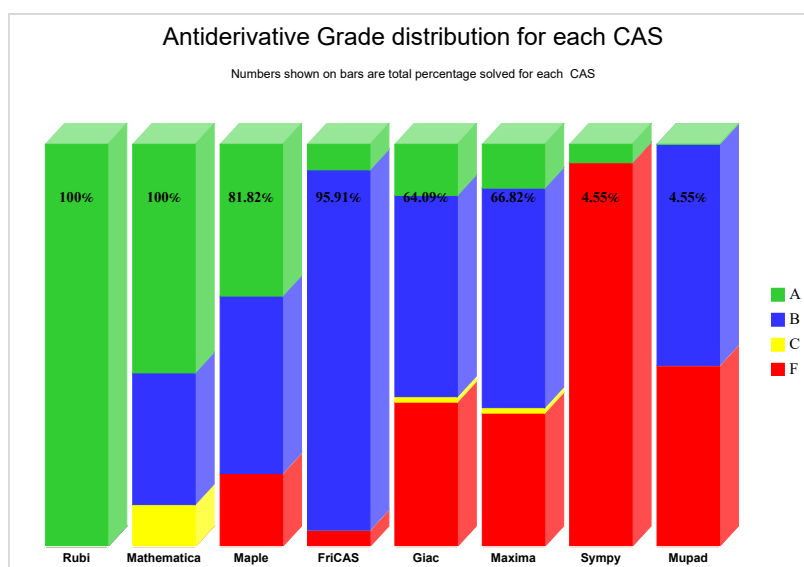
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

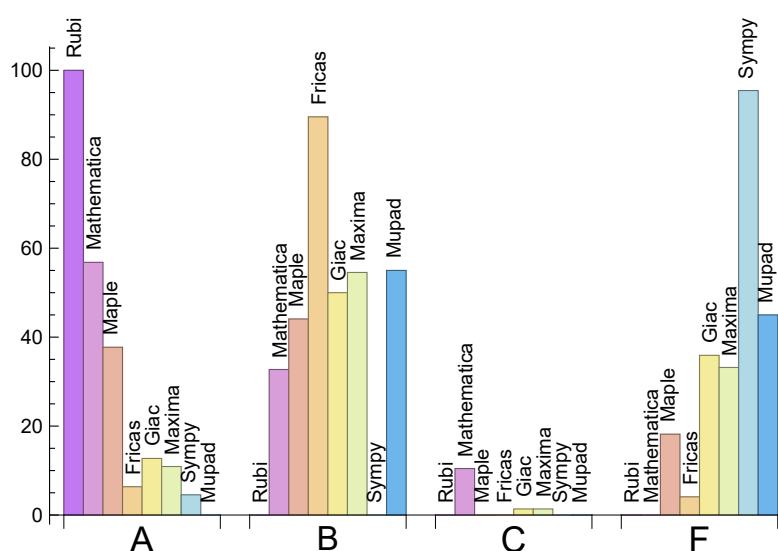
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	56.82	32.73	10.45	0.00
Maple	37.73	44.09	0.00	18.18
Maxima	10.91	54.55	1.36	33.18
Fricas	6.36	89.55	0.00	4.09
Sympy	4.55	0.00	0.00	95.45
Giac	12.73	50.00	1.36	35.91
Mupad	0.00	55.00	0.00	45.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	40	100.00 %	0.00 %	0.00 %
Maxima	73	98.63 %	0.00 %	1.37 %
Fricas	9	0.00 %	100.00 %	0.00 %
Sympy	210	79.52 %	20.48 %	0.00 %
Giac	79	0.00 %	0.00 %	100.00 %
Mupad	99	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

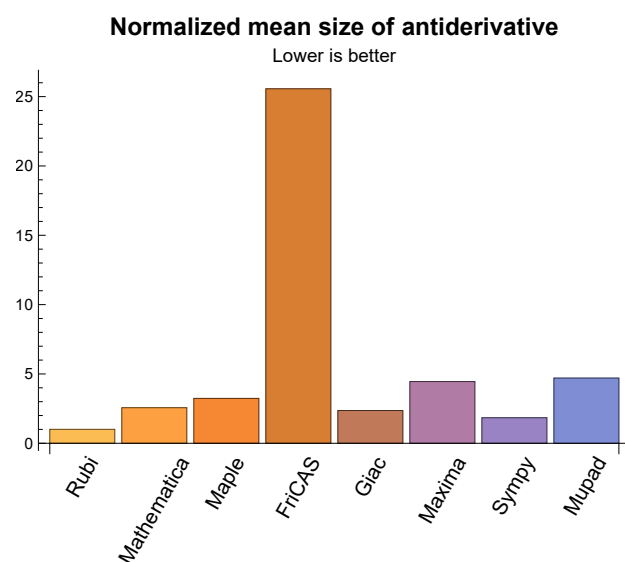
1.3 Performance

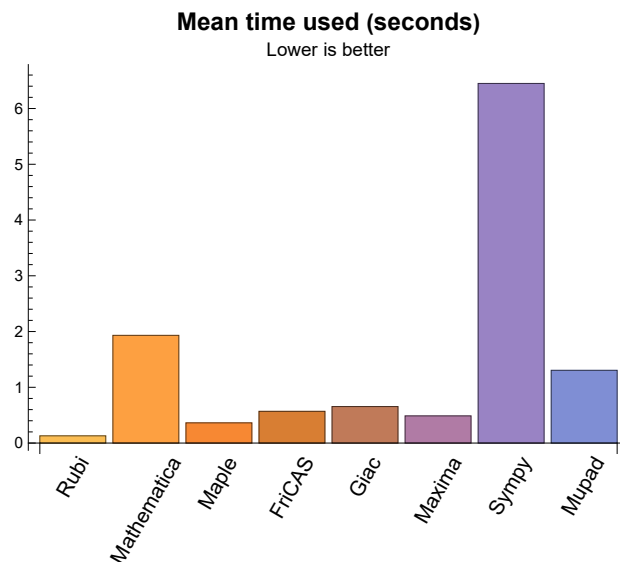
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	84.68	1.00	76.00	1.00
Mathematica	1.93	262.51	2.56	130.50	1.77
Maple	0.36	341.99	3.24	146.00	2.07
Maxima	0.49	450.10	4.45	244.00	3.00
Fricas	0.57	2489.24	25.57	1569.00	21.42
Sympy	6.45	87.20	1.84	72.50	1.54
Giac	0.65	200.89	2.36	167.00	2.19
Mupad	1.30	347.68	4.70	238.00	3.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {33, 34, 35, 41, 42, 43, 44, 46, 48, 66, 68, 69, 151, 155, 157, 158, 160, 162, 164, 166, 168, 169}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))
```



```
except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

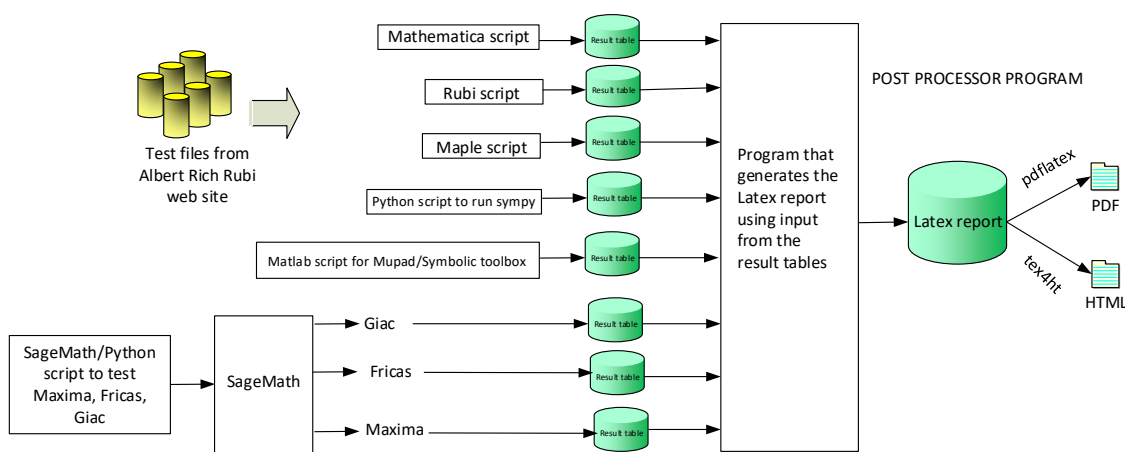
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 15, 18, 20, 21, 23, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 71, 73, 74, 75, 76, 77, 78, 83, 84, 86, 88, 89, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 197, 201, 202, 203, 204, 208, 209, 210, 211, 212, 214, 216, 217, 219, 220 }

B grade: { 5, 7, 13, 14, 16, 17, 19, 22, 24, 25, 27, 30, 32, 35, 38, 40, 41, 43, 48, 70, 72, 79, 80, 81, 82, 85, 87, 90, 91, 92, 96, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 180, 181, 191, 196, 198, 199, 200, 205, 206, 207, 213, 215, 218 }

C grade: { 26, 28, 29, 31, 33, 34, 36, 37, 39, 42, 44, 45, 46, 47, 66, 68, 69, 108, 110, 166, 168, 169, 189 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28, 29, 31, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 116, 117, 118, 119, 123, 125, 127, 128, 129, 130, 131, 133, 136, 137, 142, 152, 163, 180, 189, 198, 207, 216 }

B grade: { 22, 24, 25, 26, 27, 30, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 110, 112, 114, 120, 121, 122, 124, 126, 132, 134, 135, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

C grade: { }

F grade: { 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.4 Maxima

A grade: { 3, 4, 6, 12, 20, 49, 51, 52, 57, 59, 60, 65, 103, 104, 105, 106, 115, 118, 127, 138, 140, 170, 171, 172 }

B grade: { 1, 2, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 50, 53, 54, 55, 56, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 82, 84, 87, 89, 91, 93, 96, 98, 100, 102, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 49, 50, 51, 57, 59, 65, 171, 172, 173, 174, 175 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 219 }

C grade: { }

F grade: { 47, 48, 168, 169, 186, 193, 211, 218, 220 }

2.1.6 Sympy

A grade: { 51, 103, 105, 113, 115, 125, 127, 142, 207, 216 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158,

159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.7 Giac

A grade: { 4, 6, 8, 12, 16, 17, 20, 30, 32, 33, 35, 52, 56, 57, 58, 75, 78, 80, 82, 87, 89, 106, 107, 108, 118, 143, 151, 153 }

B grade: { 1, 2, 3, 5, 7, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 27, 38, 40, 41, 43, 46, 48, 49, 50, 51, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 84, 91, 93, 96, 98, 100, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 144, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 138, 139, 140, 141, 142, 145, 146, 147, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 163, 180, 189, 198, 207, 216 }

C grade: { }

F grade: { 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	78	129	114	0	130	73
normalized size	1	1.00	0.77	1.11	1.84	1.63	0.00	1.86	1.04
time (sec)	N/A	0.081	0.333	0.327	0.313	0.400	0.000	1.103	1.609
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	56	111	85	0	85	44
normalized size	1	1.00	1.20	1.27	2.52	1.93	0.00	1.93	1.00
time (sec)	N/A	0.056	0.052	0.312	0.338	0.441	0.000	0.133	0.177
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	45	62	67	0	92	55
normalized size	1	1.00	1.33	1.05	1.44	1.56	0.00	2.14	1.28
time (sec)	N/A	0.056	0.194	0.176	0.327	0.405	0.000	0.164	1.428
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	36	38	0	45	26
normalized size	1	1.00	1.46	1.08	1.50	1.58	0.00	1.88	1.08
time (sec)	N/A	0.033	0.022	0.090	0.321	0.428	0.000	0.125	0.083
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	67	36	80	180	0	72	79
normalized size	1	1.00	2.48	1.33	2.96	6.67	0.00	2.67	2.93
time (sec)	N/A	0.045	0.048	0.196	0.325	0.418	0.000	0.126	1.412

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	44	39	91	0	34	34
normalized size	1	1.00	1.37	1.63	1.44	3.37	0.00	1.26	1.26
time (sec)	N/A	0.046	0.073	0.381	0.333	0.400	0.000	0.143	1.376
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	131	70	198	924	0	142	160
normalized size	1	1.00	2.43	1.30	3.67	17.11	0.00	2.63	2.96
time (sec)	N/A	0.071	0.049	0.382	0.342	0.431	0.000	0.154	0.166
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	84	73	187	246	0	80	172
normalized size	1	1.00	1.87	1.62	4.16	5.47	0.00	1.78	3.82
time (sec)	N/A	0.058	0.054	0.437	0.339	0.384	0.000	0.135	1.401
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	153	109	211	342	0	231	269
normalized size	1	1.00	1.34	0.96	1.85	3.00	0.00	2.03	2.36
time (sec)	N/A	0.138	1.663	0.343	0.338	0.417	0.000	0.177	0.270
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	93	266	212	0	140	201
normalized size	1	1.00	1.15	1.29	3.69	2.94	0.00	1.94	2.79
time (sec)	N/A	0.087	0.526	0.329	0.340	0.394	0.000	0.189	1.496
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	126	90	160	252	0	144	236
normalized size	1	1.00	1.73	1.23	2.19	3.45	0.00	1.97	3.23
time (sec)	N/A	0.107	0.952	0.336	0.332	0.409	0.000	0.177	0.159

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	59	43	65	133	0	75	45
normalized size	1	1.00	1.31	0.96	1.44	2.96	0.00	1.67	1.00
time (sec)	N/A	0.046	0.173	0.096	0.321	0.403	0.000	0.142	1.466
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	108	72	197	1148	0	139	232
normalized size	1	1.00	2.08	1.38	3.79	22.08	0.00	2.67	4.46
time (sec)	N/A	0.077	0.577	0.215	0.337	0.425	0.000	0.140	1.503
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	109	91	140	284	0	111	215
normalized size	1	1.00	2.18	1.82	2.80	5.68	0.00	2.22	4.30
time (sec)	N/A	0.062	1.767	0.490	0.337	0.416	0.000	0.151	1.487
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	126	354	2930	0	228	316
normalized size	1	1.00	1.38	1.21	3.40	28.17	0.00	2.19	3.04
time (sec)	N/A	0.136	1.645	0.425	0.339	0.453	0.000	0.157	1.655
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	151	138	285	408	0	115	115
normalized size	1	1.00	2.01	1.84	3.80	5.44	0.00	1.53	1.53
time (sec)	N/A	0.083	1.320	0.549	0.343	0.400	0.000	0.175	0.213
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	651	182	422	727	0	339	686
normalized size	1	1.00	3.58	1.00	2.32	3.99	0.00	1.86	3.77
time (sec)	N/A	0.232	2.515	0.440	0.345	0.453	0.000	0.218	0.367

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	119	130	489	403	0	193	348
normalized size	1	1.00	1.20	1.31	4.94	4.07	0.00	1.95	3.52
time (sec)	N/A	0.111	1.253	0.371	0.342	0.415	0.000	0.209	0.330
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	143	480	145	443	595	0	278	592
normalized size	1	1.28	4.29	1.29	3.96	5.31	0.00	2.48	5.29
time (sec)	N/A	0.193	1.916	0.418	0.339	0.429	0.000	0.202	0.230
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	58	94	276	0	101	288
normalized size	1	1.00	1.45	0.91	1.47	4.31	0.00	1.58	4.50
time (sec)	N/A	0.059	0.257	0.130	0.322	0.394	0.000	0.191	1.497
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	134	118	358	3443	0	228	434
normalized size	1	1.00	1.61	1.42	4.31	41.48	0.00	2.75	5.23
time (sec)	N/A	0.100	1.281	0.245	0.329	0.445	0.000	0.168	1.532
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	380	148	358	622	0	249	644
normalized size	1	1.00	5.43	2.11	5.11	8.89	0.00	3.56	9.20
time (sec)	N/A	0.073	2.793	0.591	0.335	0.470	0.000	0.181	1.479
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	224	192	556	6717	0	341	536
normalized size	1	1.00	1.56	1.33	3.86	46.65	0.00	2.37	3.72
time (sec)	N/A	0.178	3.887	0.443	0.338	0.596	0.000	0.197	1.644

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	213	213	664	955	0	355	745
normalized size	1	1.00	2.05	2.05	6.38	9.18	0.00	3.41	7.16
time (sec)	N/A	0.105	2.466	0.659	0.344	0.587	0.000	0.189	1.602
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	294	708	526	1681	0	220	328
normalized size	1	1.00	2.51	6.05	4.50	14.37	0.00	1.88	2.80
time (sec)	N/A	0.196	2.483	0.411	0.462	0.593	0.000	2.883	2.426
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	372	261	0	1246	0	0	473
normalized size	1	1.00	5.24	3.68	0.00	17.55	0.00	0.00	6.66
time (sec)	N/A	0.105	2.226	0.336	0.000	0.492	0.000	0.000	1.841
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	236	383	352	805	0	132	276
normalized size	1	1.00	3.15	5.11	4.69	10.73	0.00	1.76	3.68
time (sec)	N/A	0.110	0.997	0.319	0.424	0.483	0.000	1.603	2.029
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	328	44	0	595	0	0	42
normalized size	1	1.00	6.98	0.94	0.00	12.66	0.00	0.00	0.89
time (sec)	N/A	0.048	1.015	0.079	0.000	0.512	0.000	0.000	0.145
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	232	67	0	533	0	0	616
normalized size	1	1.00	4.22	1.22	0.00	9.69	0.00	0.00	11.20
time (sec)	N/A	0.082	0.928	0.250	0.000	0.507	0.000	0.000	2.181

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	179	147	100	588	0	75	847
normalized size	1	1.00	3.38	2.77	1.89	11.09	0.00	1.42	15.98
time (sec)	N/A	0.071	0.752	0.300	0.426	0.557	0.000	0.633	2.467
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	338	134	0	1881	0	0	1586
normalized size	1	1.00	3.89	1.54	0.00	21.62	0.00	0.00	18.23
time (sec)	N/A	0.118	2.050	0.325	0.000	0.464	0.000	0.000	3.070
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	216	258	195	1753	0	123	248
normalized size	1	1.00	2.88	3.44	2.60	23.37	0.00	1.64	3.31
time (sec)	N/A	0.099	2.082	0.357	0.456	0.524	0.000	0.669	2.170
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1330	1034	1299	5169	0	323	-1
normalized size	1	1.00	6.86	5.33	6.70	26.64	0.00	1.66	-0.01
time (sec)	N/A	0.273	14.252	0.395	0.514	0.562	0.000	3.728	0.000
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	861	561	0	3804	0	0	-1
normalized size	1	1.00	7.55	4.92	0.00	33.37	0.00	0.00	-0.01
time (sec)	N/A	0.144	4.819	0.368	0.000	0.486	0.000	0.000	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	791	543	696	2925	0	234	-1
normalized size	1	1.00	6.04	4.15	5.31	22.33	0.00	1.79	-0.01
time (sec)	N/A	0.196	11.332	0.371	0.459	0.502	0.000	2.051	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	479	74	0	1780	0	0	71
normalized size	1	1.00	5.70	0.88	0.00	21.19	0.00	0.00	0.85
time (sec)	N/A	0.065	2.763	0.172	0.000	0.544	0.000	0.000	1.645
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	377	431	0	2376	0	0	-1
normalized size	1	1.00	3.81	4.35	0.00	24.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.251	0.336	0.000	0.580	0.000	0.000	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	220	313	262	2407	0	239	-1
normalized size	1	1.00	2.39	3.40	2.85	26.16	0.00	2.60	-0.01
time (sec)	N/A	0.084	2.625	0.394	0.461	0.623	0.000	0.732	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	462	496	0	6878	0	0	-1
normalized size	1	1.00	3.14	3.37	0.00	46.79	0.00	0.00	-0.01
time (sec)	N/A	0.206	2.227	0.423	0.000	0.733	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	295	577	430	6143	0	253	-1
normalized size	1	1.00	2.40	4.69	3.50	49.94	0.00	2.06	-0.01
time (sec)	N/A	0.195	6.078	0.504	0.520	0.525	0.000	0.772	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	3080	1668	2468	12353	0	518	-1
normalized size	1	1.00	12.73	6.89	10.20	51.05	0.00	2.14	-0.00
time (sec)	N/A	0.399	27.316	0.426	0.603	0.637	0.000	6.128	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1217	1296	0	8667	0	0	-1
normalized size	1	1.00	7.90	8.42	0.00	56.28	0.00	0.00	-0.01
time (sec)	N/A	0.218	10.382	0.381	0.000	0.564	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	2544	1329	1373	9730	0	370	-1
normalized size	1	1.00	13.60	7.11	7.34	52.03	0.00	1.98	-0.01
time (sec)	N/A	0.292	17.845	0.389	0.533	0.596	0.000	3.702	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	453	107	0	4829	0	0	103
normalized size	1	1.00	3.91	0.92	0.00	41.63	0.00	0.00	0.89
time (sec)	N/A	0.084	9.604	0.168	0.000	0.514	0.000	0.000	1.607
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	440	1476	0	8742	0	0	-1
normalized size	1	1.00	2.86	9.58	0.00	56.77	0.00	0.00	-0.01
time (sec)	N/A	0.223	2.498	0.359	0.000	0.637	0.000	0.000	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	981	816	533	7275	0	347	-1
normalized size	1	1.00	7.79	6.48	4.23	57.74	0.00	2.75	-0.01
time (sec)	N/A	0.100	6.920	0.406	0.528	0.549	0.000	1.761	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	524	1555	0	0	0	0	-1
normalized size	1	1.00	2.46	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	4.117	0.447	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F(-1)	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	985	1443	782	0	0	406	-1
normalized size	1	1.00	5.97	8.75	4.74	0.00	0.00	2.46	-0.01
time (sec)	N/A	0.273	5.530	0.483	0.595	0.000	0.000	1.664	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	66	97	61	0	116	50
normalized size	1	1.00	0.74	1.08	1.59	1.00	0.00	1.90	0.82
time (sec)	N/A	0.046	0.093	0.457	0.308	0.397	0.000	0.137	0.129
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	34	85	41	0	72	34
normalized size	1	1.00	1.67	1.13	2.83	1.37	0.00	2.40	1.13
time (sec)	N/A	0.050	0.018	0.426	0.309	0.394	0.000	0.157	0.085
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	38	28	60	66	23
normalized size	1	1.00	1.06	1.19	1.23	0.90	1.94	2.13	0.74
time (sec)	N/A	0.031	0.030	0.306	0.310	0.405	11.245	0.146	0.077
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	28	93	0	36	62
normalized size	1	1.00	1.46	1.08	1.17	3.88	0.00	1.50	2.58
time (sec)	N/A	0.030	0.018	0.286	0.412	0.450	0.000	0.146	1.436
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	45	81	321	0	84	124
normalized size	1	1.00	1.20	1.12	2.02	8.02	0.00	2.10	3.10
time (sec)	N/A	0.027	0.024	0.300	0.403	0.409	0.000	0.124	0.159

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	43	39	34	112	158	0	61	61
normalized size	1	1.43	1.30	1.13	3.73	5.27	0.00	2.03	2.03
time (sec)	N/A	0.040	0.012	0.336	0.317	0.383	0.000	0.124	1.382
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	83	184	1112	0	156	283
normalized size	1	1.00	0.86	1.19	2.63	15.89	0.00	2.23	4.04
time (sec)	N/A	0.050	0.105	0.417	0.410	0.422	0.000	0.158	1.383
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	65	71	56	300	343	0	85	292
normalized size	1	1.30	1.42	1.12	6.00	6.86	0.00	1.70	5.84
time (sec)	N/A	0.046	0.014	0.366	0.321	0.384	0.000	0.145	1.435
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	58	79	105	78	0	151	66
normalized size	1	1.00	0.71	0.96	1.28	0.95	0.00	1.84	0.80
time (sec)	N/A	0.093	0.131	0.397	0.306	0.399	0.000	0.155	1.371
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	105	414	0	94	114
normalized size	1	1.00	1.47	1.35	2.14	8.45	0.00	1.92	2.33
time (sec)	N/A	0.059	0.024	0.364	0.412	0.409	0.000	0.151	0.166
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	63	80	0	128	65
normalized size	1	1.00	1.11	1.09	1.34	1.70	0.00	2.72	1.38
time (sec)	N/A	0.078	0.149	0.355	0.322	0.392	0.000	0.173	0.164

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	63	101	653	0	112	172
normalized size	1	1.00	1.43	1.12	1.80	11.66	0.00	2.00	3.07
time (sec)	N/A	0.068	0.041	0.365	0.420	0.418	0.000	0.139	1.439
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	106	201	1372	0	170	303
normalized size	1	1.00	0.79	1.18	2.23	15.24	0.00	1.89	3.37
time (sec)	N/A	0.084	0.133	0.385	0.423	0.429	0.000	0.141	1.518
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	93	70	324	404	0	156	452
normalized size	1	1.00	1.75	1.32	6.11	7.62	0.00	2.94	8.53
time (sec)	N/A	0.066	0.031	0.418	0.335	0.406	0.000	0.173	1.465
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	169	348	2946	0	293	569
normalized size	1	1.00	0.81	1.32	2.72	23.02	0.00	2.29	4.45
time (sec)	N/A	0.149	0.244	0.460	0.420	0.432	0.000	0.160	1.570
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	144	102	671	677	0	197	692
normalized size	1	1.00	1.80	1.28	8.39	8.46	0.00	2.46	8.65
time (sec)	N/A	0.076	0.024	0.445	0.329	0.390	0.000	0.159	1.416
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	93	130	153	0	177	117
normalized size	1	1.00	0.83	1.11	1.55	1.82	0.00	2.11	1.39
time (sec)	N/A	0.111	0.427	0.479	0.324	0.395	0.000	0.214	1.529

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	483	103	179	1409	0	163	218
normalized size	1	1.00	5.96	1.27	2.21	17.40	0.00	2.01	2.69
time (sec)	N/A	0.093	6.882	0.454	0.415	0.423	0.000	0.213	0.223
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	77	160	270	0	152	221
normalized size	1	1.00	0.89	1.07	2.22	3.75	0.00	2.11	3.07
time (sec)	N/A	0.092	0.508	0.461	0.319	0.427	0.000	0.195	0.167
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	575	125	221	1992	0	199	344
normalized size	1	1.00	6.18	1.34	2.38	21.42	0.00	2.14	3.70
time (sec)	N/A	0.103	7.875	0.507	0.412	0.466	0.000	0.191	0.191
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	1430	193	365	3465	0	310	535
normalized size	1	1.00	9.73	1.31	2.48	23.57	0.00	2.11	3.64
time (sec)	N/A	0.143	9.836	0.460	0.414	0.473	0.000	0.189	1.472
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	319	116	695	816	0	302	978
normalized size	1	1.00	4.31	1.57	9.39	11.03	0.00	4.08	13.22
time (sec)	N/A	0.080	1.486	0.571	0.329	0.437	0.000	0.191	1.495
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	297	280	556	6114	0	485	931
normalized size	1	1.00	1.52	1.43	2.84	31.19	0.00	2.47	4.75
time (sec)	N/A	0.232	9.396	0.472	0.421	0.505	0.000	0.177	1.599

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	348	158	1245	1190	0	360	1333
normalized size	1	1.00	3.22	1.46	11.53	11.02	0.00	3.33	12.34
time (sec)	N/A	0.094	1.756	0.520	0.343	0.417	0.000	0.182	1.473
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	493	526	1713	0	208	260
normalized size	1	1.00	0.81	4.21	4.50	14.64	0.00	1.78	2.22
time (sec)	N/A	0.189	0.495	0.541	0.457	0.458	0.000	2.623	1.998
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	79	256	0	1616	0	0	332
normalized size	1	1.00	1.04	3.37	0.00	21.26	0.00	0.00	4.37
time (sec)	N/A	0.090	0.274	0.489	0.000	0.465	0.000	0.000	2.213
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	278	352	829	0	125	206
normalized size	1	1.00	0.89	3.71	4.69	11.05	0.00	1.67	2.75
time (sec)	N/A	0.116	0.235	0.464	0.424	0.457	0.000	1.574	1.968
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	128	0	718	0	0	277
normalized size	1	1.00	1.00	2.46	0.00	13.81	0.00	0.00	5.33
time (sec)	N/A	0.061	0.124	0.402	0.000	0.459	0.000	0.000	1.739
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	82	0	487	0	0	108
normalized size	1	1.00	1.00	2.28	0.00	13.53	0.00	0.00	3.00
time (sec)	N/A	0.042	0.066	0.278	0.000	0.423	0.000	0.000	1.896

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	104	66	411	0	47	125
normalized size	1	1.00	1.00	2.89	1.83	11.42	0.00	1.31	3.47
time (sec)	N/A	0.061	0.080	0.252	0.417	0.429	0.000	0.639	0.570
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	194	108	0	526	0	0	307
normalized size	1	1.00	3.53	1.96	0.00	9.56	0.00	0.00	5.58
time (sec)	N/A	0.069	0.712	0.252	0.000	0.437	0.000	0.000	1.815
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	182	141	91	645	0	72	166
normalized size	1	1.00	3.50	2.71	1.75	12.40	0.00	1.38	3.19
time (sec)	N/A	0.070	0.632	0.217	0.426	0.426	0.000	0.622	0.475
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	213	189	0	1518	0	0	946
normalized size	1	1.00	2.48	2.20	0.00	17.65	0.00	0.00	11.00
time (sec)	N/A	0.103	1.836	0.281	0.000	0.489	0.000	0.000	3.500
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	214	316	160	1905	0	118	334
normalized size	1	1.00	2.78	4.10	2.08	24.74	0.00	1.53	4.34
time (sec)	N/A	0.089	2.178	0.282	0.461	0.488	0.000	0.639	1.990
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	517	0	5842	0	0	-1
normalized size	1	1.00	0.90	4.14	0.00	46.74	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.919	0.533	0.000	0.539	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	103	557	696	3739	0	323	-1
normalized size	1	1.00	0.72	3.87	4.83	25.97	0.00	2.24	-0.01
time (sec)	N/A	0.235	1.379	0.513	0.466	0.492	0.000	2.018	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	234	385	0	3154	0	0	-1
normalized size	1	1.00	2.34	3.85	0.00	31.54	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.812	0.514	0.000	0.487	0.000	0.000	0.000
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	124	331	0	1856	0	0	-1
normalized size	1	1.00	1.51	4.04	0.00	22.63	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.314	0.345	0.000	0.453	0.000	0.000	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	187	263	150	1489	0	130	-1
normalized size	1	1.00	2.53	3.55	2.03	20.12	0.00	1.76	-0.01
time (sec)	N/A	0.074	1.015	0.253	0.456	0.444	0.000	0.744	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	108	241	0	1570	0	0	-1
normalized size	1	1.00	1.48	3.30	0.00	21.51	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.228	0.270	0.000	0.475	0.000	0.000	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	374	165	1569	0	139	-1
normalized size	1	1.00	1.06	4.51	1.99	18.90	0.00	1.67	-0.01
time (sec)	N/A	0.090	0.216	0.294	0.478	0.451	0.000	0.769	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	282	361	0	2069	0	0	-1
normalized size	1	1.00	2.79	3.57	0.00	20.49	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.529	0.272	0.000	0.498	0.000	0.000	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	229	419	244	2958	0	225	-1
normalized size	1	1.00	2.27	4.15	2.42	29.29	0.00	2.23	-0.01
time (sec)	N/A	0.142	3.813	0.276	0.554	0.480	0.000	0.789	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	489	448	0	6499	0	0	-1
normalized size	1	1.00	3.20	2.93	0.00	42.48	0.00	0.00	-0.01
time (sec)	N/A	0.200	5.193	0.288	0.000	0.581	0.000	0.000	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	156	1435	1373	11740	0	395	-1
normalized size	1	1.00	0.76	7.03	6.73	57.55	0.00	1.94	-0.00
time (sec)	N/A	0.377	4.229	0.591	0.546	0.655	0.000	3.671	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	292	1238	0	9856	0	0	-1
normalized size	1	1.00	1.90	8.04	0.00	64.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	3.653	0.559	0.000	0.639	0.000	0.000	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	214	1172	0	6806	0	0	-1
normalized size	1	1.00	1.51	8.25	0.00	47.93	0.00	0.00	-0.01
time (sec)	N/A	0.137	2.936	0.398	0.000	0.541	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	258	615	353	5109	0	282	-1
normalized size	1	1.00	2.39	5.69	3.27	47.31	0.00	2.61	-0.01
time (sec)	N/A	0.090	2.169	0.344	0.514	0.508	0.000	1.754	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	159	1038	0	6037	0	0	-1
normalized size	1	1.00	1.29	8.44	0.00	49.08	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.789	0.342	0.000	0.526	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	250	1084	369	5447	0	274	-1
normalized size	1	1.00	2.00	8.67	2.95	43.58	0.00	2.19	-0.01
time (sec)	N/A	0.107	3.386	0.284	0.535	0.519	0.000	1.734	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	125	592	0	5006	0	0	-1
normalized size	1	1.00	1.18	5.58	0.00	47.23	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.287	0.320	0.000	0.512	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	1245	395	5887	0	302	-1
normalized size	1	1.00	0.87	8.65	2.74	40.88	0.00	2.10	-0.01
time (sec)	N/A	0.142	1.012	0.345	1.367	0.528	0.000	1.751	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	247	1201	0	7993	0	0	-1
normalized size	1	1.00	1.61	7.85	0.00	52.24	0.00	0.00	-0.01
time (sec)	N/A	0.205	4.721	0.299	0.000	0.624	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	98	92	327	0	105	433
normalized size	1	1.00	1.19	2.04	1.92	6.81	0.00	2.19	9.02
time (sec)	N/A	0.064	0.030	0.329	0.615	0.421	0.000	0.202	1.545
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	64	78	1072	80	116	173
normalized size	1	1.00	0.92	1.31	1.59	21.88	1.63	2.37	3.53
time (sec)	N/A	0.056	0.022	0.201	0.747	0.430	1.662	0.179	0.125
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	60	42	155	0	69	163
normalized size	1	1.00	1.28	1.88	1.31	4.84	0.00	2.16	5.09
time (sec)	N/A	0.057	0.017	0.328	0.405	0.398	0.000	0.161	1.503
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	27	359	42	80	72
normalized size	1	1.00	1.00	1.00	0.93	12.38	1.45	2.76	2.48
time (sec)	N/A	0.028	0.021	0.138	0.331	0.419	0.487	0.134	1.470
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	36	0	23	23
normalized size	1	1.00	1.00	1.07	1.53	2.40	0.00	1.53	1.53
time (sec)	N/A	0.013	0.003	0.269	0.370	0.403	0.000	0.121	1.395
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	65	69	0	56	167
normalized size	1	1.00	1.57	0.93	2.32	2.46	0.00	2.00	5.96
time (sec)	N/A	0.051	0.041	0.256	0.534	0.420	0.000	0.153	0.172

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	30	47	39	0	27	25
normalized size	1	1.00	2.28	1.67	2.61	2.17	0.00	1.50	1.39
time (sec)	N/A	0.059	0.031	0.333	0.336	0.414	0.000	0.146	0.111
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	42	108	378	0	81	76
normalized size	1	1.00	1.68	1.35	3.48	12.19	0.00	2.61	2.45
time (sec)	N/A	0.057	0.179	0.329	0.333	0.419	0.000	0.208	1.414
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	49	70	170	140	0	67	161
normalized size	1	1.00	1.44	2.06	5.00	4.12	0.00	1.97	4.74
time (sec)	N/A	0.063	0.023	0.389	0.344	0.414	0.000	0.214	1.492
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	78	251	1099	0	117	179
normalized size	1	1.00	1.22	1.53	4.92	21.55	0.00	2.29	3.51
time (sec)	N/A	0.078	0.263	0.313	0.512	0.426	0.000	0.232	0.110
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	395	181	649	721	0	275	1022
normalized size	1	1.00	5.13	2.35	8.43	9.36	0.00	3.57	13.27
time (sec)	N/A	0.105	1.185	0.437	0.548	0.417	0.000	0.249	0.183
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	107	110	333	2591	129	241	349
normalized size	1	1.00	1.39	1.43	4.32	33.65	1.68	3.13	4.53
time (sec)	N/A	0.095	0.290	0.242	0.439	0.450	5.089	0.202	1.522

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	281	115	325	435	0	193	513
normalized size	1	1.00	4.76	1.95	5.51	7.37	0.00	3.27	8.69
time (sec)	N/A	0.098	0.864	0.405	0.524	0.423	0.000	0.171	0.142
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	81	48	55	1180	63	159	182
normalized size	1	1.00	1.69	1.00	1.15	24.58	1.31	3.31	3.79
time (sec)	N/A	0.050	0.145	0.158	0.482	0.418	1.665	0.142	1.689
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	47	120	176	0	79	163
normalized size	1	1.00	2.65	1.18	3.00	4.40	0.00	1.98	4.08
time (sec)	N/A	0.034	0.377	0.365	0.457	0.396	0.000	0.144	1.458
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	60	161	665	0	171	308
normalized size	1	1.00	1.58	1.13	3.04	12.55	0.00	3.23	5.81
time (sec)	N/A	0.080	0.246	0.262	0.698	0.458	0.000	0.171	0.296
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	64	71	106	0	65	60
normalized size	1	1.00	2.28	1.78	1.97	2.94	0.00	1.81	1.67
time (sec)	N/A	0.085	0.741	0.388	0.545	0.438	0.000	0.201	1.422
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	82	78	206	637	0	161	240
normalized size	1	1.00	1.49	1.42	3.75	11.58	0.00	2.93	4.36
time (sec)	N/A	0.089	0.200	0.311	0.480	0.449	0.000	0.252	0.220

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	160	96	268	201	0	97	183
normalized size	1	1.00	3.48	2.09	5.83	4.37	0.00	2.11	3.98
time (sec)	N/A	0.092	0.810	0.360	0.427	0.427	0.000	0.272	1.424
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	77	102	282	1252	0	147	207
normalized size	1	1.00	1.48	1.96	5.42	24.08	0.00	2.83	3.98
time (sec)	N/A	0.094	0.232	0.321	0.449	0.444	0.000	0.337	1.466
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	256	163	613	425	0	167	511
normalized size	1	1.00	4.00	2.55	9.58	6.64	0.00	2.61	7.98
time (sec)	N/A	0.097	1.101	0.445	0.542	0.422	0.000	0.374	1.439
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	107	164	696	2548	0	216	377
normalized size	1	1.00	1.24	1.91	8.09	29.63	0.00	2.51	4.38
time (sec)	N/A	0.125	0.516	0.337	0.408	0.478	0.000	0.459	1.530
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	301	274	1453	1323	0	472	1834
normalized size	1	1.00	2.74	2.49	13.21	12.03	0.00	4.29	16.67
time (sec)	N/A	0.118	5.936	0.554	0.385	0.435	0.000	0.305	1.623
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	128	156	652	4658	178	384	573
normalized size	1	1.00	1.24	1.51	6.33	45.22	1.73	3.73	5.56
time (sec)	N/A	0.098	0.754	0.298	0.462	0.513	13.795	0.273	1.638

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	479	180	788	881	0	356	1133
normalized size	1	1.00	5.21	1.96	8.57	9.58	0.00	3.87	12.32
time (sec)	N/A	0.110	1.790	0.515	0.442	0.417	0.000	0.197	0.213
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	100	67	85	2519	87	268	347
normalized size	1	1.00	1.41	0.94	1.20	35.48	1.23	3.77	4.89
time (sec)	N/A	0.058	0.301	0.161	0.431	0.441	4.911	0.186	0.215
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	268	83	332	470	0	182	502
normalized size	1	1.00	3.67	1.14	4.55	6.44	0.00	2.49	6.88
time (sec)	N/A	0.047	0.928	0.420	0.411	0.425	0.000	0.154	1.442
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	111	300	2376	0	325	360
normalized size	1	1.00	1.36	1.32	3.57	28.29	0.00	3.87	4.29
time (sec)	N/A	0.107	0.663	0.308	0.469	0.457	0.000	0.183	1.638
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	126	111	172	359	0	132	234
normalized size	1	1.00	2.07	1.82	2.82	5.89	0.00	2.16	3.84
time (sec)	N/A	0.103	1.799	0.573	0.413	0.413	0.000	0.211	0.142
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	110	137	314	1701	0	292	324
normalized size	1	1.00	1.36	1.69	3.88	21.00	0.00	3.60	4.00
time (sec)	N/A	0.116	1.242	0.359	0.470	0.443	0.000	0.289	1.723

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	343	149	366	354	0	155	260
normalized size	1	1.00	5.72	2.48	6.10	5.90	0.00	2.58	4.33
time (sec)	N/A	0.101	1.726	0.483	0.521	0.401	0.000	0.343	1.479
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	101	153	422	1830	0	248	384
normalized size	1	1.00	1.25	1.89	5.21	22.59	0.00	3.06	4.74
time (sec)	N/A	0.121	0.907	0.318	0.497	0.449	0.000	0.410	1.636
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	303	199	826	521	0	210	547
normalized size	1	1.00	4.39	2.88	11.97	7.55	0.00	3.04	7.93
time (sec)	N/A	0.109	1.087	0.425	0.361	0.408	0.000	0.427	1.603
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	189	727	2632	0	239	411
normalized size	1	1.00	1.27	2.45	9.44	34.18	0.00	3.10	5.34
time (sec)	N/A	0.121	0.797	0.363	0.420	0.464	0.000	0.586	1.614
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	455	129	703	941	0	334	1083
normalized size	1	1.00	4.10	1.16	6.33	8.48	0.00	3.01	9.76
time (sec)	N/A	0.070	1.666	0.525	0.567	0.424	0.000	0.151	0.196
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	724	185	1277	1652	0	537	1952
normalized size	1	1.00	4.44	1.13	7.83	10.13	0.00	3.29	11.98
time (sec)	N/A	0.095	6.567	0.606	0.349	0.427	0.000	0.167	0.317

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	331	131	736	0	0	421
normalized size	1	1.00	1.40	4.73	1.87	10.51	0.00	0.00	6.01
time (sec)	N/A	0.112	0.307	0.316	0.459	0.485	0.000	0.000	1.812
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	196	386	637	683	0	0	183
normalized size	1	1.00	3.32	6.54	10.80	11.58	0.00	0.00	3.10
time (sec)	N/A	0.178	1.086	0.349	0.636	0.449	0.000	0.000	1.873
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	196	77	112	0	0	238
normalized size	1	1.00	0.91	4.36	1.71	2.49	0.00	0.00	5.29
time (sec)	N/A	0.088	0.117	0.333	0.474	0.460	0.000	0.000	1.650
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	174	251	291	419	0	0	105
normalized size	1	1.00	3.78	5.46	6.33	9.11	0.00	0.00	2.28
time (sec)	N/A	0.140	0.304	0.358	0.497	0.455	0.000	0.000	0.426
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	38	51	76	124	0	51
normalized size	1	1.00	1.13	1.65	2.22	3.30	5.39	0.00	2.22
time (sec)	N/A	0.035	0.183	0.146	0.378	0.421	7.035	0.000	0.345
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	172	149	83	436	0	64	470
normalized size	1	1.00	3.74	3.24	1.80	9.48	0.00	1.39	10.22
time (sec)	N/A	0.046	0.250	0.277	0.467	0.441	0.000	0.390	2.161

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	133	100	115	0	97	228
normalized size	1	1.00	0.91	2.89	2.17	2.50	0.00	2.11	4.96
time (sec)	N/A	0.087	0.092	0.375	0.421	0.461	0.000	0.334	1.817
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	193	189	429	749	0	0	977
normalized size	1	1.00	3.11	3.05	6.92	12.08	0.00	0.00	15.76
time (sec)	N/A	0.180	1.161	0.457	0.477	0.457	0.000	0.000	3.389
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	100	199	187	862	0	0	523
normalized size	1	1.00	1.37	2.73	2.56	11.81	0.00	0.00	7.16
time (sec)	N/A	0.122	0.236	0.476	0.435	0.524	0.000	0.000	2.069
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	380	298	1435	2705	0	0	779
normalized size	1	1.00	4.37	3.43	16.49	31.09	0.00	0.00	8.95
time (sec)	N/A	0.286	3.325	0.493	0.627	0.465	0.000	0.000	4.146
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	109	351	154	853	0	208	-1
normalized size	1	1.00	1.43	4.62	2.03	11.22	0.00	2.74	-0.01
time (sec)	N/A	0.117	0.454	0.372	0.446	0.495	0.000	1.376	0.000
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	228	676	1053	1479	0	187	-1
normalized size	1	1.00	2.51	7.43	11.57	16.25	0.00	2.05	-0.01
time (sec)	N/A	0.194	2.133	0.402	0.837	0.450	0.000	1.162	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	186	108	485	0	121	-1
normalized size	1	1.00	1.59	3.65	2.12	9.51	0.00	2.37	-0.02
time (sec)	N/A	0.090	0.760	0.351	0.375	0.416	0.000	0.889	0.000
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	326	411	597	1846	0	147	-1
normalized size	1	1.00	3.84	4.84	7.02	21.72	0.00	1.73	-0.01
time (sec)	N/A	0.169	4.629	0.351	0.547	0.473	0.000	0.718	0.000
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	79	60	106	476	0	121	53
normalized size	1	1.00	1.61	1.22	2.16	9.71	0.00	2.47	1.08
time (sec)	N/A	0.062	0.593	0.185	0.423	0.418	0.000	0.423	1.730
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	221	423	187	1690	0	163	-1
normalized size	1	1.00	2.38	4.55	2.01	18.17	0.00	1.75	-0.01
time (sec)	N/A	0.096	2.042	0.382	0.694	0.461	0.000	0.435	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	115	292	209	1031	0	246	-1
normalized size	1	1.00	1.39	3.52	2.52	12.42	0.00	2.96	-0.01
time (sec)	N/A	0.129	0.295	0.423	0.351	0.551	0.000	0.556	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	268	481	1070	3624	0	282	-1
normalized size	1	1.00	2.21	3.98	8.84	29.95	0.00	2.33	-0.01
time (sec)	N/A	0.284	2.824	0.457	0.917	0.508	0.000	0.955	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	130	367	384	3624	0	383	-1
normalized size	1	1.00	1.18	3.34	3.49	32.95	0.00	3.48	-0.01
time (sec)	N/A	0.175	1.276	0.530	0.473	0.716	0.000	1.318	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	350	634	2961	9849	0	302	-1
normalized size	1	1.00	2.17	3.94	18.39	61.17	0.00	1.88	-0.01
time (sec)	N/A	0.411	5.093	0.520	1.414	0.612	0.000	1.604	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	515	1713	3239	5463	0	371	-1
normalized size	1	1.00	3.48	11.57	21.89	36.91	0.00	2.51	-0.01
time (sec)	N/A	0.320	5.782	0.446	1.993	0.524	0.000	2.251	0.000
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	136	579	206	1741	0	187	-1
normalized size	1	1.00	1.77	7.52	2.68	22.61	0.00	2.43	-0.01
time (sec)	N/A	0.120	2.168	0.417	0.420	0.459	0.000	1.883	0.000
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1457	1306	2201	6464	0	295	-1
normalized size	1	1.00	10.48	9.40	15.83	46.50	0.00	2.12	-0.01
time (sec)	N/A	0.294	13.087	0.481	1.020	0.573	0.000	1.614	0.000
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	131	672	209	1753	0	175	-1
normalized size	1	1.00	1.62	8.30	2.58	21.64	0.00	2.16	-0.01
time (sec)	N/A	0.126	1.568	0.405	0.336	0.447	0.000	1.221	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1457	1173	1255	7158	0	309	-1
normalized size	1	1.00	10.48	8.44	9.03	51.50	0.00	2.22	-0.01
time (sec)	N/A	0.246	12.056	0.430	0.927	0.568	0.000	0.966	0.000
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	129	82	193	1666	0	187	94
normalized size	1	1.00	1.77	1.12	2.64	22.82	0.00	2.56	1.29
time (sec)	N/A	0.082	2.010	0.207	0.420	0.454	0.000	0.526	1.604
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	301	1283	402	6538	0	327	-1
normalized size	1	1.00	2.06	8.79	2.75	44.78	0.00	2.24	-0.01
time (sec)	N/A	0.181	6.209	0.411	0.528	0.570	0.000	0.856	0.000
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	155	1046	419	4132	0	475	-1
normalized size	1	1.00	1.19	8.05	3.22	31.78	0.00	3.65	-0.01
time (sec)	N/A	0.186	1.056	0.473	0.721	0.847	0.000	0.716	0.000
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	2083	1433	1971	11606	0	402	-1
normalized size	1	1.00	11.45	7.87	10.83	63.77	0.00	2.21	-0.01
time (sec)	N/A	0.403	7.228	0.507	0.941	0.630	0.000	1.272	0.000
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	172	1128	692	10255	0	766	-1
normalized size	1	1.00	1.13	7.42	4.55	67.47	0.00	5.04	-0.01
time (sec)	N/A	0.243	1.922	0.570	0.538	1.341	0.000	2.475	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F(-1)	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	3334	1610	4920	0	0	482	-1
normalized size	1	1.00	14.37	6.94	21.21	0.00	0.00	2.08	-0.00
time (sec)	N/A	0.514	7.738	0.567	1.838	0.000	0.000	2.587	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	1405	2880	718	0	0	594	-1
normalized size	1	1.00	6.79	13.91	3.47	0.00	0.00	2.87	-0.00
time (sec)	N/A	0.346	6.926	0.446	0.840	0.000	0.000	0.853	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	120	33	183	0	72	-1
normalized size	1	1.00	0.86	4.14	1.14	6.31	0.00	2.48	-0.03
time (sec)	N/A	0.031	0.023	0.284	0.460	0.671	0.000	0.126	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	13	18	0	26	-1
normalized size	1	1.00	1.00	5.64	0.93	1.29	0.00	1.86	-0.07
time (sec)	N/A	0.022	0.006	0.299	0.606	0.773	0.000	0.117	0.000
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	22	18	0	31	-1
normalized size	1	1.00	1.00	5.64	1.57	1.29	0.00	2.21	-0.07
time (sec)	N/A	0.024	0.010	0.258	0.670	0.534	0.000	0.134	0.000
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	123	33	1	0	83	-1
normalized size	1	1.00	0.79	3.62	0.97	0.03	0.00	2.44	-0.03
time (sec)	N/A	0.029	0.014	0.316	0.489	0.679	0.000	0.142	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	13	1	0	31	-1
normalized size	1	1.00	1.00	5.06	0.81	0.06	0.00	1.94	-0.06
time (sec)	N/A	0.020	0.006	0.321	0.545	0.513	0.000	0.125	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	22	1	0	37	-1
normalized size	1	1.00	1.00	5.06	1.38	0.06	0.00	2.31	-0.06
time (sec)	N/A	0.022	0.010	0.299	0.439	0.492	0.000	0.141	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	114	0	0	4594	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	55.35	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.749	0.453	0.000	1.326	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	192	0	0	8852	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	70.82	0.00	0.00	-0.01
time (sec)	N/A	0.313	0.470	0.397	0.000	1.218	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	90	0	0	2394	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.107	0.397	0.319	0.000	0.730	0.000	0.000	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	0	0	4316	0	0	-1
normalized size	1	1.00	1.72	0.00	0.00	49.61	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.287	0.273	0.000	0.889	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	90	43	0	1608	0	0	32
normalized size	1	1.00	2.25	1.08	0.00	40.20	0.00	0.00	0.80
time (sec)	N/A	0.063	0.274	0.067	0.000	0.486	0.000	0.000	1.710
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	134	0	0	2949	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	49.98	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.741	0.308	0.000	0.584	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	111	0	0	3597	0	0	-1
normalized size	1	1.00	1.98	0.00	0.00	64.23	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.130	0.381	0.000	0.616	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	75	0	0	1303	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	27.15	0.00	0.00	-0.02
time (sec)	N/A	0.178	0.521	0.388	0.000	0.523	0.000	0.000	0.000
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	156	0	0	5247	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	63.22	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.594	0.411	0.000	0.634	0.000	0.000	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	149	0	0	2341	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	27.87	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.625	0.412	0.000	0.622	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	191	0	0	0	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	1.022	0.419	0.000	0.000	0.000	0.000	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	129	0	0	4226	0	0	-1
normalized size	1	1.00	1.70	0.00	0.00	55.61	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.971	0.302	0.000	2.146	0.000	0.000	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	197	0	0	8582	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	68.66	0.00	0.00	-0.01
time (sec)	N/A	0.361	0.853	0.293	0.000	1.153	0.000	0.000	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	56	0	2312	0	0	45
normalized size	1	1.00	1.14	0.98	0.00	40.56	0.00	0.00	0.79
time (sec)	N/A	0.084	0.116	0.049	0.000	0.842	0.000	0.000	3.359
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	152	0	0	4140	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	47.05	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.275	0.283	0.000	1.851	0.000	0.000	0.000
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	159	0	0	4123	0	134	-1
normalized size	1	1.00	2.27	0.00	0.00	58.90	0.00	1.91	-0.01
time (sec)	N/A	0.132	0.517	0.365	0.000	1.307	0.000	0.473	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	3349	0	222	-1
normalized size	1	1.00	1.78	0.00	0.00	41.35	0.00	2.74	-0.01
time (sec)	N/A	0.232	0.350	0.375	0.000	4.041	0.000	0.523	0.000
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	280	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	9.281	0.620	0.000	0.000	0.000	0.000	0.000
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	109	0	0	2678	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.131	0.520	0.414	0.000	0.613	0.000	0.000	0.000
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	169	0	0	4569	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	50.77	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.530	0.401	0.000	0.658	0.000	0.000	0.000
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	105	0	0	1650	0	0	-1
normalized size	1	1.00	2.50	0.00	0.00	39.29	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.225	0.358	0.000	0.524	0.000	0.000	0.000
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	107	0	0	2856	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	47.60	0.00	0.00	-0.02
time (sec)	N/A	0.193	0.181	0.352	0.000	0.562	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	70	30	0	1430	0	0	19
normalized size	1	1.00	2.80	1.20	0.00	57.20	0.00	0.00	0.76
time (sec)	N/A	0.053	0.083	0.118	0.000	0.465	0.000	0.000	1.667
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	62	0	0	1059	0	0	-1
normalized size	1	1.00	2.14	0.00	0.00	36.52	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.041	0.369	0.000	0.453	0.000	0.000	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	124	0	0	3663	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	65.41	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.261	0.402	0.000	0.574	0.000	0.000	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	94	0	0	1365	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	25.75	0.00	0.00	-0.02
time (sec)	N/A	0.188	0.112	0.470	0.000	0.510	0.000	0.000	0.000
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	159	0	0	6475	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	71.94	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.848	0.411	0.000	0.734	0.000	0.000	0.000
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	129	0	0	3360	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	49.41	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.531	0.382	0.000	0.676	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	169	0	0	5170	0	0	-1
normalized size	1	1.00	1.97	0.00	0.00	60.12	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.409	0.378	0.000	0.700	0.000	0.000	0.000
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	103	0	0	2194	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	44.78	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.234	0.335	0.000	0.537	0.000	0.000	0.000
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	128	0	0	1733	0	0	-1
normalized size	1	1.00	2.51	0.00	0.00	33.98	0.00	0.00	-0.02
time (sec)	N/A	0.218	0.755	0.334	0.000	0.542	0.000	0.000	0.000
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	98	46	0	2034	44	0	35
normalized size	1	1.00	2.28	1.07	0.00	47.30	1.02	0.00	0.81
time (sec)	N/A	0.073	0.396	0.099	0.000	0.519	5.925	0.000	1.741
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	107	0	0	2095	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	36.75	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.841	0.280	0.000	0.550	0.000	0.000	0.000
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	155	0	0	6939	0	0	-1
normalized size	1	1.00	1.96	0.00	0.00	87.84	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.686	0.393	0.000	0.738	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	120	0	0	3941	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	44.78	0.00	0.00	-0.01
time (sec)	N/A	0.274	0.458	0.399	0.000	0.670	0.000	0.000	0.000
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	178	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.747	0.395	0.000	0.000	0.000	0.000	0.000
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	126	0	0	5184	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	68.21	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.564	0.377	0.000	0.764	0.000	0.000	0.000
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	290	0	0	3559	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	39.54	0.00	0.00	-0.01
time (sec)	N/A	0.262	2.168	0.382	0.000	0.696	0.000	0.000	0.000
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	124	0	0	4644	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	68.29	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.999	0.385	0.000	0.726	0.000	0.000	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	290	0	0	4989	0	0	-1
normalized size	1	1.00	3.30	0.00	0.00	56.69	0.00	0.00	-0.01
time (sec)	N/A	0.248	1.199	0.334	0.000	0.770	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	112	61	0	3994	65	0	50
normalized size	1	1.00	1.81	0.98	0.00	64.42	1.05	0.00	0.81
time (sec)	N/A	0.087	0.718	0.106	0.000	0.703	12.680	0.000	3.069

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	130	0	0	6299	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	66.31	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.451	0.285	0.000	0.801	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	242	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	1.114	0.381	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	155	0	0	11205	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	84.25	0.00	0.00	-0.01
time (sec)	N/A	0.376	0.829	0.401	0.000	4.182	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	330	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	7.985	0.604	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [190] had the largest ratio of [.5833]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.28	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	5	4	1.00	23	0.174
24	A	3	2	1.00	23	0.087
25	A	6	6	1.00	23	0.261
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	7	1.00	23	0.304
48	A	6	5	1.00	23	0.217
49	A	3	3	1.00	21	0.143
50	A	3	2	1.00	21	0.095
51	A	2	2	1.00	21	0.095
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105
54	A	3	3	1.43	21	0.143
55	A	3	3	1.00	21	0.143
56	A	3	2	1.30	21	0.095
57	A	4	4	1.00	23	0.174
58	A	4	3	1.00	23	0.130
59	A	5	4	1.00	23	0.174
60	A	5	4	1.00	21	0.190
61	A	4	4	1.00	21	0.190
62	A	3	2	1.00	23	0.087
63	A	5	5	1.00	23	0.217
64	A	3	2	1.00	23	0.087
65	A	6	5	1.00	23	0.217
66	A	5	4	1.00	23	0.174
67	A	5	4	1.00	23	0.174
68	A	6	5	1.00	21	0.238
69	A	5	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	3	2	1.00	23	0.087
73	A	6	6	1.00	23	0.261
74	A	4	3	1.00	23	0.130
75	A	5	5	1.00	23	0.217
76	A	3	3	1.00	21	0.143
77	A	2	2	1.00	21	0.095
78	A	2	2	1.00	23	0.087
79	A	4	4	1.00	23	0.174
80	A	3	3	1.00	23	0.130
81	A	5	5	1.00	23	0.217
82	A	4	3	1.00	23	0.130
83	A	5	4	1.00	23	0.174
84	A	6	6	1.00	23	0.261
85	A	5	4	1.00	21	0.190
86	A	3	3	1.00	21	0.143
87	A	3	3	1.00	23	0.130
88	A	3	3	1.00	23	0.130
89	A	3	3	1.00	23	0.130
90	A	5	5	1.00	23	0.217
91	A	5	4	1.00	23	0.174
92	A	6	6	1.00	23	0.261
93	A	7	6	1.00	23	0.261
94	A	6	5	1.00	21	0.238
95	A	4	4	1.00	21	0.190
96	A	4	3	1.00	23	0.130
97	A	4	4	1.00	23	0.174
98	A	4	4	1.00	23	0.174
99	A	4	3	1.00	23	0.130
100	A	4	4	1.00	23	0.174
101	A	6	6	1.00	23	0.261
102	A	4	3	1.00	21	0.143
103	A	4	3	1.00	21	0.143
104	A	4	3	1.00	21	0.143
105	A	3	2	1.00	19	0.105
106	A	3	2	1.00	12	0.167
107	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	4	3	1.00	21	0.143
109	A	4	3	1.00	21	0.143
110	A	4	3	1.00	21	0.143
111	A	4	3	1.00	21	0.143
112	A	4	3	1.00	23	0.130
113	A	4	3	1.00	23	0.130
114	A	4	3	1.00	23	0.130
115	A	4	3	1.00	21	0.143
116	A	4	3	1.00	14	0.214
117	A	4	3	1.00	21	0.143
118	A	4	3	1.00	23	0.130
119	A	4	3	1.00	23	0.130
120	A	4	3	1.00	23	0.130
121	A	4	3	1.00	23	0.130
122	A	4	3	1.00	23	0.130
123	A	5	4	1.00	23	0.174
124	A	4	3	1.00	23	0.130
125	A	5	4	1.00	23	0.174
126	A	4	3	1.00	23	0.130
127	A	4	3	1.00	21	0.143
128	A	4	3	1.00	14	0.214
129	A	4	3	1.00	21	0.143
130	A	4	3	1.00	23	0.130
131	A	4	3	1.00	23	0.130
132	A	4	3	1.00	23	0.130
133	A	4	3	1.00	23	0.130
134	A	4	3	1.00	23	0.130
135	A	4	3	1.00	23	0.130
136	A	4	3	1.00	14	0.214
137	A	4	3	1.00	14	0.214
138	A	4	3	1.00	23	0.130
139	A	6	6	1.00	23	0.261
140	A	4	3	1.00	23	0.130
141	A	5	5	1.00	23	0.217
142	A	2	2	1.00	21	0.095
143	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	4	3	1.00	21	0.143
145	A	6	6	1.00	23	0.261
146	A	4	3	1.00	23	0.130
147	A	7	7	1.00	23	0.304
148	A	4	3	1.00	23	0.130
149	A	6	6	1.00	23	0.261
150	A	4	3	1.00	23	0.130
151	A	6	6	1.00	23	0.261
152	A	4	3	1.00	21	0.143
153	A	5	5	1.00	14	0.357
154	A	4	3	1.00	21	0.143
155	A	7	7	1.00	23	0.304
156	A	4	3	1.00	23	0.130
157	A	8	7	1.00	23	0.304
158	A	7	7	1.00	23	0.304
159	A	4	3	1.00	23	0.130
160	A	7	7	1.00	23	0.304
161	A	4	3	1.00	23	0.130
162	A	7	7	1.00	23	0.304
163	A	4	3	1.00	21	0.143
164	A	6	6	1.00	14	0.429
165	A	4	3	1.00	21	0.143
166	A	8	8	1.00	23	0.348
167	A	4	3	1.00	23	0.130
168	A	9	8	1.00	23	0.348
169	A	7	6	1.00	14	0.429
170	A	4	4	1.00	12	0.333
171	A	3	3	1.00	12	0.250
172	A	3	3	1.00	12	0.250
173	A	4	4	1.00	10	0.400
174	A	3	3	1.00	10	0.300
175	A	3	3	1.00	10	0.300
176	A	7	6	1.00	17	0.353
177	A	9	9	1.00	17	0.529
178	A	6	6	1.00	17	0.353
179	A	8	8	1.00	17	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	12	0.500
182	A	7	5	1.00	15	0.333
183	A	6	6	1.00	17	0.353
184	A	8	6	1.00	17	0.353
185	A	7	7	1.00	17	0.412
186	A	9	7	1.00	17	0.412
187	A	7	6	1.00	17	0.353
188	A	9	9	1.00	17	0.529
189	A	6	5	1.00	15	0.333
190	A	7	7	1.00	12	0.583
191	A	8	6	1.00	15	0.400
192	A	8	8	1.00	17	0.471
193	A	8	8	1.00	16	0.500
194	A	6	5	1.00	17	0.294
195	A	8	8	1.00	17	0.471
196	A	5	5	1.00	17	0.294
197	A	7	7	1.00	17	0.412
198	A	4	4	1.00	15	0.267
199	A	3	3	1.00	12	0.250
200	A	7	5	1.00	15	0.333
201	A	6	6	1.00	17	0.353
202	A	8	6	1.00	17	0.353
203	A	6	5	1.00	17	0.294
204	A	8	8	1.00	17	0.471
205	A	5	5	1.00	17	0.294
206	A	5	5	1.00	17	0.294
207	A	5	5	1.00	15	0.333
208	A	4	4	1.00	12	0.333
209	A	8	6	1.00	15	0.400
210	A	7	7	1.00	17	0.412
211	A	9	9	1.00	17	0.529
212	A	6	5	1.00	17	0.294
213	A	7	7	1.00	17	0.412
214	A	6	6	1.00	17	0.353
215	A	7	7	1.00	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	6	5	1.00	15	0.333
217	A	6	6	1.00	12	0.500
218	A	9	7	1.00	15	0.467
219	A	8	8	1.00	17	0.471
220	A	7	6	1.00	16	0.375

Chapter 3

Listing of integrals

3.1 $\int (a + b \operatorname{sech}^2(c + dx)) \sinh^4(c + dx) dx$

Optimal. Leaf size=70

$$\frac{(5a - 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3}{8}x(a-4b) + \frac{a \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{b \tanh(c + dx)}{d}$$

[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3*sinh(d*x+c)/d+b*tanh(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4132, 455, 1157, 388, 206}

$$\frac{(5a - 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3}{8}x(a-4b) + \frac{a \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]

[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b*Tanh[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&

(IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1157

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x,
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4132

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)])^(n_)^(p_)*sin[(e_) + (f_)*(x_)
]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \sinh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4(a+b-bx^2)}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-a-4ax^2+4bx^4}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{4d} \\ &= -\frac{(5a - 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} \\ &= -\frac{(5a - 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} \\ &= \frac{3}{8}(a - 4b)x - \frac{(5a - 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.33, size = 54, normalized size = 0.77

$$\frac{12(a - 4b)(c + dx) - 8(a - b) \sinh(2(c + dx)) + a \sinh(4(c + dx)) + 32b \tanh(c + dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(c + d*x) - 8*(a - b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)]
+ 32*b*Tanh[c + d*x])/(32*d)
```

fricas [A] time = 0.40, size = 114, normalized size = 1.63

$$\frac{a \sinh(dx + c)^5 + (10a \cosh(dx + c)^2 - 7a + 8b) \sinh(dx + c)^3 + 8(3(a - 4b)dx - 8b) \cosh(dx + c) + (5a \cosh(dx + c)^4 - 4a \sinh(dx + c)^2)}{64d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="fricas")
```

[Out] $1/64*(a*\sinh(dx + c)^5 + (10*a*\cosh(dx + c)^2 - 7*a + 8*b)*\sinh(dx + c)^3 + 8*(3*(a - 4*b)*dx - 8*b)*\cosh(dx + c) + (5*a*\cosh(dx + c)^4 - 3*(7*a - 8*b)*\cosh(dx + c)^2 - 8*a + 72*b)*\sinh(dx + c))/(d*\cosh(dx + c))$

giac [B] time = 1.10, size = 130, normalized size = 1.86

$$\frac{24(dx + c)(a - 4b) + ae^{(4dx+4c)} - 8ae^{(2dx+2c)} + 8be^{(2dx+2c)} - (18ae^{(4dx+4c)} - 72be^{(4dx+4c)} - 8ae^{(2dx+2c)} + 8b)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="giac")

[Out] $1/64*(24*(dx + c)*(a - 4*b) + a*e^{(4*d*x + 4*c)} - 8*a*e^{(2*d*x + 2*c)} + 8*b*e^{(2*d*x + 2*c)} - (18*a*e^{(4*d*x + 4*c)} - 72*b*e^{(4*d*x + 4*c)} - 8*a*e^{(2*d*x + 2*c)} + 8*b*e^{(2*d*x + 2*c)} + a)*e^{(-4*d*x - 4*c)} - 128*b/(e^{(2*d*x + 2*c)} + 1))/d$

maple [A] time = 0.33, size = 78, normalized size = 1.11

$$\frac{a \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3 \sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + b \left(\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3 \tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x)

[Out] $1/d*(a*((1/4*\sinh(dx+c)^3-3/8*\sinh(dx+c))*\cosh(dx+c)+3/8*d*x+3/8*c)+b*(1/2*\sinh(dx+c)^3/\cosh(dx+c)-3/2*d*x-3/2*c+3/2*\tanh(dx+c)))$

maxima [B] time = 0.31, size = 129, normalized size = 1.84

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{8} b \left(\frac{12(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)}}{d(e^{(-2dx-2c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="maxima")

[Out] $1/64*a*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) - 1/8*b*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)})))$

mupad [B] time = 1.61, size = 73, normalized size = 1.04

$$\frac{3ax}{8} - \frac{3bx}{2} - \frac{a \sinh(2c + 2dx)}{4d} + \frac{a \sinh(4c + 4dx)}{32d} + \frac{b \sinh(2c + 2dx)}{4d} + \frac{b \sinh(c + dx)}{d \cosh(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)

[Out] $(3*a*x)/8 - (3*b*x)/2 - (a*\sinh(2*c + 2*d*x))/(4*d) + (a*\sinh(4*c + 4*d*x))/(32*d) + (b*\sinh(2*c + 2*d*x))/(4*d) + (b*\sinh(c + d*x))/(d*\cosh(c + d*x))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**4,x)

[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**4, x)

3.2 $\int (a + b \operatorname{sech}^2(c + dx)) \sinh^3(c + dx) dx$

Optimal. Leaf size=44

$$-\frac{(a-b) \cosh(c+dx)}{d} + \frac{a \cosh^3(c+dx)}{3d} + \frac{b \operatorname{sech}(c+dx)}{d}$$

[Out] $-(a-b)*\cosh(d*x+c)/d+1/3*a*\cosh(d*x+c)^3/d+b*\operatorname{sech}(d*x+c)/d$

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4133, 448}

$$-\frac{(a-b) \cosh(c+dx)}{d} + \frac{a \cosh^3(c+dx)}{3d} + \frac{b \operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]`

[Out] $-\left(\frac{(a-b)*\operatorname{Cosh}[c+d*x]}{d}\right) + \frac{a*\operatorname{Cosh}[c+d*x]^3}{(3*d)} + \frac{b*\operatorname{Sech}[c+d*x]}{d}$

Rule 448

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]`

Rule 4133

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^(m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \sinh^3(c + dx) dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(a\left(1 - \frac{b}{a}\right) + \frac{b}{x^2} - ax^2\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{(a-b) \cosh(c+dx)}{d} + \frac{a \cosh^3(c+dx)}{3d} + \frac{b \operatorname{sech}(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.20

$$-\frac{3a \cosh(c+dx)}{4d} + \frac{a \cosh(3(c+dx))}{12d} + \frac{b \cosh(c+dx)}{d} + \frac{b \operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]`

[Out] $(-3*a*\text{Cosh}[c + d*x])/(4*d) + (b*\text{Cosh}[c + d*x])/d + (a*\text{Cosh}[3*(c + d*x)])/(12*d) + (b*\text{Sech}[c + d*x])/d$

fricas [B] time = 0.44, size = 85, normalized size = 1.93

$$\frac{a \cosh(dx + c)^4 + a \sinh(dx + c)^4 - 4(2a - 3b) \cosh(dx + c)^2 + 2(3a \cosh(dx + c)^2 - 4a + 6b) \sinh(dx + c)}{24d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] $1/24*(a*\cosh(d*x + c)^4 + a*\sinh(d*x + c)^4 - 4*(2*a - 3*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 - 4*a + 6*b)*\sinh(d*x + c)^2 - 9*a + 36*b)/(d*\cosh(d*x + c))$

giac [B] time = 0.13, size = 85, normalized size = 1.93

$$\frac{a(e^{(dx+c)} + e^{(-dx-c)})^3 - 12a(e^{(dx+c)} + e^{(-dx-c)}) + 12b(e^{(dx+c)} + e^{(-dx-c)}) + \frac{48b}{e^{(dx+c)} + e^{(-dx-c)}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="giac")

[Out] $1/24*(a*(e^{(d*x + c)} + e^{(-d*x - c)})^3 - 12*a*(e^{(d*x + c)} + e^{(-d*x - c)}) + 12*b*(e^{(d*x + c)} + e^{(-d*x - c)}) + 48*b/(e^{(d*x + c)} + e^{(-d*x - c)}))/d$

maple [A] time = 0.31, size = 56, normalized size = 1.27

$$\frac{a\left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3}\right)\cosh(dx+c) + b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x)

[Out] $1/d*(a*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)+b*(\sinh(d*x+c)^2/\cosh(d*x+c)+2/\cosh(d*x+c)))$

maxima [B] time = 0.34, size = 111, normalized size = 2.52

$$\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right) + \frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d} + \frac{5e^{(-2dx-2c)} + 1}{d(e^{(-dx-c)} + e^{(-3dx-3c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="maxima")

[Out] $1/24*a*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + 1/2*b*(e^{(-d*x - c)}/d + (5*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)})))$

mupad [B] time = 0.18, size = 44, normalized size = 1.00

$$\frac{a \cosh(c + dx)^3}{3d} - \frac{\cosh(c + dx)(a - b)}{d} + \frac{b}{d \cosh(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)

[Out] $(a \cosh(c + dx)^3)/(3d) - (\cosh(c + dx)(a - b))/d + b/(d \cosh(c + dx))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**3,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**3, x)`

3.3 $\int (a + b \operatorname{sech}^2(c + dx)) \sinh^2(c + dx) dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x(a-2b) + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} - \frac{b \tanh(c+dx)}{d}$$

[Out] $-1/2*(a-2*b)*x+1/2*a*\cosh(d*x+c)*\sinh(d*x+c)/d-b*\tanh(d*x+c)/d$

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4132, 455, 388, 206}

$$-\frac{1}{2}x(a-2b) + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} - \frac{b \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]

[Out] $-((a - 2*b)*x)/2 + (a*\cosh[c + d*x]*\sinh[c + d*x])/(2*d) - (b*\tanh[c + d*x])/d$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p+1))/(2*b^(m/2 + 1)*(p+1)), x] + Dist[1/(2*b^(m/2 + 1)*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*x^2*Together[(b^(m/2)*x^(m-2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_))*sin[(e_.) + (f_.)*(x_)^(m_)], x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m+1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2)], x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^2(c + dx)) \sinh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{a-2bx^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\
&= \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{b \tanh(c + dx)}{d} - \frac{(a - 2b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\
&= -\frac{1}{2}(a - 2b)x + \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{b \tanh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 57, normalized size = 1.33

$$\frac{a(-c - dx)}{2d} + \frac{a \sinh(2(c + dx))}{4d} + \frac{b \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]

[Out] (a*(-c - d*x))/(2*d) + (b*ArcTanh[Tanh[c + d*x]])/d + (a*Sinh[2*(c + d*x)])/(4*d) - (b*Tanh[c + d*x])/d

fricas [A] time = 0.41, size = 67, normalized size = 1.56

$$\frac{a \sinh(dx + c)^3 - 4((a - 2b)dx - 2b) \cosh(dx + c) + (3a \cosh(dx + c)^2 + a - 8b) \sinh(dx + c)}{8d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/8*(a*sinh(d*x + c)^3 - 4*((a - 2*b)*d*x - 2*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a - 8*b)*sinh(d*x + c))/(d*cosh(d*x + c))

giac [B] time = 0.16, size = 92, normalized size = 2.14

$$\frac{4(dx + c)(a - 2b) - ae^{(2dx+2c)} - \frac{ae^{(4dx+4c)} - 2be^{(4dx+4c)} + 14be^{(2dx+2c)} - a}{e^{(4dx+4c)} + e^{(2dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="giac")

[Out] -1/8*(4*(d*x + c)*(a - 2*b) - a*e^(2*d*x + 2*c) - (a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + 14*b*e^(2*d*x + 2*c) - a)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d

maple [A] time = 0.18, size = 45, normalized size = 1.05

$$\frac{a\left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b(dx + c - \tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x)

[Out] $1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(d*x+c-tanh(d*x+c)))$

maxima [A] time = 0.33, size = 62, normalized size = 1.44

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="maxima")`

[Out] $-1/8*a*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + b*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1)))$

mupad [B] time = 1.43, size = 55, normalized size = 1.28

$$\frac{a \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\frac{adx}{2} - bdx}{d} - \frac{b \sinh(c + dx)}{d \cosh(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)`

[Out] $(a*\cosh(c + d*x)*\sinh(c + d*x))/(2*d) - ((a*d*x)/2 - b*d*x)/d - (b*\sinh(c + d*x))/(d*\cosh(c + d*x))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**2, x)`

3.4 $\int (a + b \operatorname{sech}^2(c + dx)) \sinh(c + dx) dx$

Optimal. Leaf size=24

$$\frac{a \cosh(c + dx)}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

[Out] a*cosh(d*x+c)/d-b*sech(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4133, 14}

$$\frac{a \cosh(c + dx)}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x], x]

[Out] (a*Cosh[c + d*x])/d - (b*Sech[c + d*x])/d

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4133

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m-1)/2)*(b + a*(ff*x)^n)^p/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \sinh(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a + \frac{b}{x^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{a \cosh(c + dx)}{d} - \frac{b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a \sinh(c) \sinh(dx)}{d} + \frac{a \cosh(c) \cosh(dx)}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x], x]

[Out] (a*Cosh[c]*Cosh[d*x])/d - (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d

fricas [A] time = 0.43, size = 38, normalized size = 1.58

$$\frac{a \cosh(dx + c)^2 + a \sinh(dx + c)^2 + a - 2b}{2d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="fricas")

[Out] 1/2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a - 2*b)/(d*cosh(d*x + c))

giac [A] time = 0.12, size = 45, normalized size = 1.88

$$\frac{a(e^{(dx+c)} + e^{(-dx-c)}) - \frac{4b}{e^{(dx+c)}+e^{(-dx-c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="giac")

[Out] 1/2*(a*(e^(d*x + c) + e^(-d*x - c)) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d

maple [A] time = 0.09, size = 26, normalized size = 1.08

$$-\frac{b \operatorname{sech}(dx + c) - \frac{a}{\operatorname{sech}(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c),x)

[Out] -1/d*(b*sech(d*x+c)-a/sech(d*x+c))

maxima [A] time = 0.32, size = 36, normalized size = 1.50

$$\frac{a \cosh(dx + c)}{d} - \frac{2b}{d(e^{(dx+c)} + e^{(-dx-c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="maxima")

[Out] a*cosh(d*x + c)/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))

mupad [B] time = 0.08, size = 26, normalized size = 1.08

$$-\frac{b - a \cosh(c + dx)^2}{d \cosh(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2),x)

[Out] -(b - a*cosh(c + d*x)^2)/(d*cosh(c + d*x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c),x)

[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x), x)

3.5 $\int \operatorname{csch}(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=27

$$\frac{b \operatorname{sech}(c + dx)}{d} - \frac{(a + b) \tanh^{-1}(\cosh(c + dx))}{d}$$

[Out] $-(a+b)*\operatorname{arctanh}(\cosh(d*x+c))/d+b*\operatorname{sech}(d*x+c)/d$

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4133, 453, 206}

$$\frac{b \operatorname{sech}(c + dx)}{d} - \frac{(a + b) \tanh^{-1}(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sech}[c + d*x]^2), x]$

[Out] $-(((a + b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d) + (b*\operatorname{Sech}[c + d*x])/d$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2]]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 453

$\operatorname{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(m_)}), x_Symbol] :> \operatorname{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*e^{(m+1)}), x] + \operatorname{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1) + 1))/(a*e^n*(m+1)), \operatorname{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& (\operatorname{IntegerQ}[n] \ || \ \operatorname{GtQ}[e, 0]) \ \&\& ((\operatorname{GtQ}[n, 0] \ \&\& \operatorname{LtQ}[m, -1]) \ || \ (\operatorname{LtQ}[n, 0] \ \&\& \operatorname{GtQ}[m+n, -1])) \ \&\& \ !\operatorname{LtQ}[p, -1]$

Rule 4133

$\operatorname{Int}[(a_ + (b_)*\sec[(e_ + (f_)*(x_)]^{(n_)})^{(p_)}*\sin[(e_ + (f_)*(x_)]^{(m_)}), x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(b + a*(ff*x)^n)^p]/(ff*x)^{(n*p)}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}[\{a, b, e, f\}, x] \ \&\& \operatorname{IntegerQ}[(m-1)/2] \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{b \operatorname{sech}(c + dx)}{d} - \frac{(a + b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{(a + b) \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.05, size = 67, normalized size = 2.48

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b \operatorname{sech}(c + dx)}{d} + \frac{b \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Log[Tanh[(c + d*x)/2]])/d + (b*Sech[c + d*x])/d

fricas [B] time = 0.42, size = 180, normalized size = 6.67

$$\frac{2b \cosh(dx + c) - ((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a + b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] (2*b*cosh(d*x + c) - ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)

giac [B] time = 0.13, size = 72, normalized size = 2.67

$$\frac{(a + b) \log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - (a + b) \log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) - \frac{4b}{e^{(dx+c)} + e^{(-dx-c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*((a + b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d

maple [A] time = 0.20, size = 36, normalized size = 1.33

$$\frac{-2a \operatorname{arctanh}\left(e^{dx+c}\right) + b\left(\frac{1}{\cosh(dx+c)} - 2 \operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2), x)

[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))

maxima [B] time = 0.32, size = 80, normalized size = 2.96

$$-b\left(\frac{\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} - \frac{2e^{(-dx-c)}}{d\left(e^{(-2dx-2c)} + 1\right)}\right) + \frac{a \log\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $-b \cdot (\log(e^{-d \cdot x - c}) + 1) / d - \log(e^{-d \cdot x - c}) - 1 / d - 2 \cdot e^{-d \cdot x - c} / (d \cdot (e^{-2 \cdot d \cdot x - 2 \cdot c} + 1)) + a \cdot \log(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)) / d$

mupad [B] time = 1.41, size = 79, normalized size = 2.93

$$\frac{b}{d \cosh(c + d x)} - \frac{2 \operatorname{atan}\left(\frac{e^{d x} e^c (a \sqrt{-d^2} + b \sqrt{-d^2})}{d \sqrt{a^2 + 2 a b + b^2}}\right) \sqrt{a^2 + 2 a b + b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)/sinh(c + d*x), x)`

[Out] $b / (d \cdot \cosh(c + d \cdot x)) - (2 \cdot \operatorname{atan}((\exp(d \cdot x) \cdot \exp(c) \cdot (a \cdot (-d^2)^{1/2} + b \cdot (-d^2)^{1/2}))) / (d \cdot (2 \cdot a \cdot b + a^2 + b^2)^{1/2})) \cdot (2 \cdot a \cdot b + a^2 + b^2)^{1/2} / (-d^2)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2), x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*csch(c + d*x), x)`

3.6 $\int \operatorname{csch}^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=27

$$-\frac{(a+b) \operatorname{coth}(c+dx)}{d} - \frac{b \operatorname{tanh}(c+dx)}{d}$$

[Out] $-(a+b)*\operatorname{coth}(d*x+c)/d-b*\operatorname{tanh}(d*x+c)/d$

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4132, 14}

$$-\frac{(a+b) \operatorname{coth}(c+dx)}{d} - \frac{b \operatorname{tanh}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] $-(((a + b)*\operatorname{Coth}[c + d*x])/d) - (b*\operatorname{Tanh}[c + d*x])/d$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{x^2} dx, x, \operatorname{tanh}(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-b + \frac{a+b}{x^2}\right) dx, x, \operatorname{tanh}(c + dx)\right)}{d} \\ &= -\frac{(a+b) \operatorname{coth}(c + dx)}{d} - \frac{b \operatorname{tanh}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 1.37

$$-\frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \operatorname{tanh}(c + dx)}{d} - \frac{b \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] $-((a*\operatorname{Coth}[c + d*x])/d) - (b*\operatorname{Coth}[c + d*x])/d - (b*\operatorname{Tanh}[c + d*x])/d$

fricas [B] time = 0.40, size = 91, normalized size = 3.37

$$\frac{4((a+b)\cosh(dx+c) - b\sinh(dx+c))}{d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + d\sinh(dx+c)^3 - d\cosh(dx+c) + (3d\cosh(dx+c)^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] -4*((a+b)*cosh(d*x+c) - b*sinh(d*x+c))/(d*cosh(d*x+c)^3 + 3*d*cosh(d*x+c)*sinh(d*x+c)^2 + d*sinh(d*x+c)^3 - d*cosh(d*x+c) + (3*d*cosh(d*x+c)^2 + d)*sinh(d*x+c))

giac [A] time = 0.14, size = 34, normalized size = 1.26

$$\frac{2(ae^{2dx+2c} + a + 2b)}{d(e^{4dx+4c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] -2*(a*e^(2*d*x + 2*c) + a + 2*b)/(d*(e^(4*d*x + 4*c) - 1))

maple [A] time = 0.38, size = 44, normalized size = 1.63

$$\frac{-\coth(dx+c)a + b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2\tanh(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x)

[Out] 1/d*(-coth(d*x+c)*a+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))

maxima [A] time = 0.33, size = 39, normalized size = 1.44

$$\frac{2a}{d(e^{-2dx-2c} - 1)} + \frac{4b}{d(e^{-4dx-4c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))

mupad [B] time = 1.38, size = 34, normalized size = 1.26

$$\frac{2(a + 2b + ae^{2c+2dx})}{d(e^{4c+4dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^2,x)

[Out] -(2*(a + 2*b + a*exp(2*c + 2*d*x)))/(d*(exp(4*c + 4*d*x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2),x)

[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**2, x)

3.7 $\int \operatorname{csch}^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=54

$$\frac{(a + 3b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{(a + b) \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

[Out] 1/2*(a+3*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)*coth(d*x+c)*csch(d*x+c)/d-b*sech(d*x+c)/d

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4133, 456, 453, 206}

$$\frac{(a + 3b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{(a + b) \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 3*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)*Coth[c + d*x]*Csch[c + d*x])/(2*d) - (b*Sech[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2))]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx) (a+b\operatorname{sech}^2(c+dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)} dx, x, \cosh(c+dx)\right)}{2d} \\
&= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b\operatorname{sech}(c+dx)}{d} + \frac{(a+3b)\operatorname{Subst}\left(\int \frac{1}{x} dx, x, \cosh(c+dx)\right)}{2d} \\
&= \frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}
\end{aligned}$$

Mathematica [B] time = 0.05, size = 131, normalized size = 2.43

$$\frac{\operatorname{acsch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{\operatorname{asech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (3*b*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[c + d*x])/d

fricas [B] time = 0.43, size = 924, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*(a + 3*b)*cosh(d*x + c)^5 + 10*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^4 + 2*(a + 3*b)*sinh(d*x + c)^5 + 4*(a - b)*cosh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c) - ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(5*(a + 3*b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 - d

) $\sinh(dx + c)^2 + 2(3d\cosh(dx + c)^5 - 2d\cosh(dx + c)^3 - d\cosh(dx + c))\sinh(dx + c) + d$

giac [B] time = 0.15, size = 142, normalized size = 2.63

$$\frac{(a + 3b) \log(e^{(dx+c)} + e^{(-dx-c)} + 2) - (a + 3b) \log(e^{(dx+c)} + e^{(-dx-c)} - 2) - \frac{4(a(e^{(dx+c)} + e^{(-dx-c)})^2 + 3b(e^{(dx+c)} + e^{(-dx-c)})^2)}{(e^{(dx+c)} + e^{(-dx-c)})^3 - 4e^{(dx+c)} - 4e^{(-dx-c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sech(dx+c)^2),x, algorithm="giac")

[Out] $\frac{1}{4}((a + 3b) \log(e^{(dx+c)} + e^{(-dx-c)} + 2) - (a + 3b) \log(e^{(dx+c)} + e^{(-dx-c)} - 2) - 4(a(e^{(dx+c)} + e^{(-dx-c)})^2 + 3b(e^{(dx+c)} + e^{(-dx-c)})^2) / ((e^{(dx+c)} + e^{(-dx-c)})^3 - 4e^{(dx+c)} - 4e^{(-dx-c)}) / d$

maple [A] time = 0.38, size = 70, normalized size = 1.30

$$\frac{a \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{arctanh}(e^{dx+c}) \right) + b \left(-\frac{1}{2 \sinh(dx+c)^2 \cosh(dx+c)} - \frac{3}{2 \cosh(dx+c)} + 3 \operatorname{arctanh}(e^{dx+c}) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(dx+c)^3*(a+b*sech(dx+c)^2),x)

[Out] $\frac{1}{d} (a(-1/2 \operatorname{csch}(dx+c) \operatorname{coth}(dx+c) + \operatorname{arctanh}(\exp(dx+c))) + b(-1/2 / \sinh(dx+c)^2 / \cosh(dx+c) - 3/2 / \cosh(dx+c) + 3 \operatorname{arctanh}(\exp(dx+c))))$

maxima [B] time = 0.34, size = 198, normalized size = 3.67

$$\frac{1}{2} b \left(\frac{3 \log(e^{(-dx-c)} + 1)}{d} - \frac{3 \log(e^{(-dx-c)} - 1)}{d} + \frac{2(3e^{(-dx-c)} - 2e^{(-3dx-3c)} + 3e^{(-5dx-5c)})}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)} - e^{(-6dx-6c)} - 1)} \right) + \frac{1}{2} a \left(\frac{\log(e^{(-dx-c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sech(dx+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{2} b (3 \log(e^{(-dx-c)} + 1) / d - 3 \log(e^{(-dx-c)} - 1) / d + 2(3e^{(-dx-c)} - 2e^{(-3dx-3c)} + 3e^{(-5dx-5c)}) / (d(e^{(-2dx-2c)} + e^{(-4dx-4c)} - e^{(-6dx-6c)} - 1))) + \frac{1}{2} a (\log(e^{(-dx-c)} + 1) / d - \log(e^{(-dx-c)} - 1) / d + 2(e^{(-dx-c)} + e^{(-3dx-3c)}) / (d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)))$

mupad [B] time = 0.17, size = 160, normalized size = 2.96

$$\frac{\operatorname{atan} \left(\frac{e^{dx} e^c (a \sqrt{-d^2} + 3b \sqrt{-d^2})}{d \sqrt{a^2 + 6ab + 9b^2}} \right) \sqrt{a^2 + 6ab + 9b^2}}{\sqrt{-d^2}} - \frac{e^{c+dx} (a+b)}{d (e^{2c+2dx} - 1)} - \frac{2e^{c+dx} (a+b)}{d (e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{2be^{c+dx}}{d (e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + dx)^2)/sinh(c + dx)^3,x)

[Out] $(\operatorname{atan}((\exp(dx) \exp(c) (a(-d^2)^{(1/2)} + 3b(-d^2)^{(1/2)})) / (d(6ab + a^2 + 9b^2)^{(1/2)})) * (6ab + a^2 + 9b^2)^{(1/2)}) / (-d^2)^{(1/2)} - (\exp(c + dx) * (a + b)) / (d(\exp(2c + 2dx) - 1)) - (2 \exp(c + dx) * (a + b)) / (d(\exp(4c + 4dx) - 2 \exp(2c + 2dx) + 1)) - (2b \exp(c + dx)) / (d(\exp(2c + 2dx) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**3, x)
```

3.8 $\int \operatorname{csch}^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=45

$$-\frac{(a+b)\operatorname{coth}^3(c+dx)}{3d} + \frac{(a+2b)\operatorname{coth}(c+dx)}{d} + \frac{b\tanh(c+dx)}{d}$$

[Out] (a+2*b)*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d+b*tanh(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4132, 448}

$$-\frac{(a+b)\operatorname{coth}^3(c+dx)}{3d} + \frac{(a+2b)\operatorname{coth}(c+dx)}{d} + \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b)*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x])/d

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)^(n_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f*ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^{(a+b-bx^2)}}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(b + \frac{a+b}{x^4} + \frac{-a-2b}{x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a+2b)\operatorname{coth}(c+dx)}{d} - \frac{(a+b)\operatorname{coth}^3(c+dx)}{3d} + \frac{b\tanh(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 1.87

$$\frac{2a\operatorname{coth}(c+dx)}{3d} - \frac{a\operatorname{coth}(c+dx)\operatorname{csch}^2(c+dx)}{3d} + \frac{b\tanh(c+dx)}{d} + \frac{5b\operatorname{coth}(c+dx)}{3d} - \frac{b\operatorname{coth}(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] $(2*a*\text{Coth}[c + d*x])/(3*d) + (5*b*\text{Coth}[c + d*x])/(3*d) - (a*\text{Coth}[c + d*x]*\text{Csch}[c + d*x]^2)/(3*d) - (b*\text{Coth}[c + d*x]*\text{Csch}[c + d*x]^2)/(3*d) + (b*\text{Tanh}[c + d*x])/d$

fricas [B] time = 0.38, size = 246, normalized size = 5.47

$$3(d \cosh(dx + c)^6 + 6d \cosh(dx + c) \sinh(dx + c)^5 + d \sinh(dx + c)^6 - 2d \cosh(dx + c)^4 + (15d \cosh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")`

[Out] $-8/3*((a - 2*b)*\cosh(d*x + c)^2 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a - 2*b)*\sinh(d*x + c)^2 + a + 4*b)/(d*\cosh(d*x + c)^6 + 6*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + d*\sinh(d*x + c)^6 - 2*d*\cosh(d*x + c)^4 + (15*d*\cosh(d*x + c)^2 - 2*d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 - 2*d*\cosh(d*x + c))*\sinh(d*x + c)^3 - d*\cosh(d*x + c)^2 + (15*d*\cosh(d*x + c)^4 - 12*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^5 - 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + 2*d)$

giac [A] time = 0.13, size = 80, normalized size = 1.78

$$\frac{2\left(\frac{3b}{e^{2dx+2c}+1} - \frac{3be^{4dx+4c}-6ae^{2dx+2c}-12be^{2dx+2c}+2a+5b}{(e^{2dx+2c}-1)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")`

[Out] $-2/3*(3*b/(e^{2*d*x + 2*c} + 1) - (3*b*e^{4*d*x + 4*c} - 6*a*e^{2*d*x + 2*c}) - 12*b*e^{2*d*x + 2*c} + 2*a + 5*b)/(e^{2*d*x + 2*c} - 1)^3)/d$

maple [A] time = 0.44, size = 73, normalized size = 1.62

$$\frac{a\left(\frac{2}{3} - \frac{\text{csch}(dx+c)^2}{3}\right)\text{coth}(dx + c) + b\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)} + \frac{4}{3\sinh(dx+c)\cosh(dx+c)} + \frac{8\tanh(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x)`

[Out] $1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c)))$

maxima [B] time = 0.34, size = 187, normalized size = 4.16

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + \frac{16}{3}b\left(\frac{1}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} - 1)} - \frac{1}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $4/3*a*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + 16/3*b*(2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - 2*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} - 1)) - 1/(d*(2*e^{(-2*d*x - 2*c)} - 2*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} - 1)))$

mupad [B] time = 1.40, size = 172, normalized size = 3.82

$$\frac{\frac{2b}{3d} + \frac{2be^{4c+4dx}}{3d} - \frac{4e^{2c+2dx}(2a+3b)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} - \frac{\frac{2(2a+3b)}{3d} - \frac{2be^{2c+2dx}}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} + \frac{2b}{3d(e^{2c+2dx} - 1)} - \frac{2b}{d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^4,x)

[Out] ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) - (4*exp(2*c + 2*d*x)*(2*a + 3*b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*(2*a + 3*b))/(3*d) - (2*b*exp(2*c + 2*d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + (2*b)/(3*d*(exp(2*c + 2*d*x) - 1)) - (2*b)/(d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2), x)

[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**4, x)

3.9 $\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^4(c + dx) dx$

Optimal. Leaf size=114

$$-\frac{(a^2 - 8ab + 4b^2) \tanh(c + dx)}{4d} + \frac{1}{8}x(3a^2 - 24ab + 8b^2) + \frac{a^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} - \frac{a(a - 8b) \sinh(c + dx)}{8d}$$

[Out] $1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*\cosh(d*x+c)*\sinh(d*x+c)/d-1/4*(a^2-8*a*b+4*b^2)*\tanh(d*x+c)/d+1/4*a^2*\sinh(d*x+c)^4*\tanh(d*x+c)/d-1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 463, 455, 1153, 206}

$$-\frac{(a^2 - 8ab + 4b^2) \tanh(c + dx)}{4d} + \frac{1}{8}x(3a^2 - 24ab + 8b^2) + \frac{a^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} - \frac{a(a - 8b) \sinh(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]

[Out] $((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*\cosh[c + d*x]*\sinh[c + d*x])/(8*d) - ((a^2 - 8*a*b + 4*b^2)*\tanh[c + d*x])/(4*d) + (a^2*\sinh[c + d*x]^4*\tanh[c + d*x])/(4*d) - (b^2*\tanh[c + d*x]^3)/(3*d)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4132

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2)], x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
 \int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4(a+b-bx^2)^2}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{a^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{x^4(5a^2 - 4(a+b)^2 + 4b^2x^2)}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{4d} \\
 &= -\frac{a(a-8b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} \\
 &= -\frac{a(a-8b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} \\
 &= -\frac{a(a-8b) \cosh(c + dx) \sinh(c + dx)}{8d} - \frac{(a^2 - 8ab + 4b^2) \tanh(c + dx)}{4d} \\
 &= \frac{1}{8} (3a^2 - 24ab + 8b^2) x - \frac{a(a-8b) \cosh(c + dx) \sinh(c + dx)}{8d} - \frac{(a^2 - 8ab + 4b^2) \tanh(c + dx)}{4d}
 \end{aligned}$$

Mathematica [A] time = 1.66, size = 153, normalized size = 1.34

$$\frac{\operatorname{sech}^3(c + dx) (a \cosh^2(c + dx) + b)^2 (3 \cosh^3(c + dx) (4dx (3a^2 - 24ab + 8b^2) + a^2 \sinh(4(c + dx))) - 8a(a - 2b) \cosh(c + dx) \sinh(c + dx) + 3a^2 \sinh^4(c + dx))}{24d(a \cosh(c + dx) \sinh(c + dx) + b \cosh^2(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]

[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(32*b^2*Sech[c]*Sinh[d*x] + 64*(3*a - 2*b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*Cosh[c + d*x]^3*(4*(3*a^2 - 24*a*b + 8*b^2)*d*x - 8*a*(a - 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c + d*x)]) + 32*b^2*Cosh[c + d*x]*Tanh[c]))/(24*d*(a + 2*b + a*Cosh[2*(c + d*x)]))^2)

fricas [B] time = 0.42, size = 342, normalized size = 3.00

$$\frac{3a^2 \sinh(dx + c)^7 + 3(21a^2 \cosh(dx + c)^2 - 5a^2 + 16ab) \sinh(dx + c)^5 + 8(3(3a^2 - 24ab + 8b^2)dx - 48a^2) \cosh(dx + c) \sinh(dx + c)^3 + 24(3(3a^2 - 24ab + 8b^2)d^2x - 48ab + 32b^2) \cosh(dx + c)^2 \sinh(dx + c)^2 + (105a^2 \cosh(dx + c)^4 - 30(5a^2 - 16ab) \cosh(dx + c)^2 - 63a^2 + 528ab - 256b^2) \sinh(dx + c)^3 + 24(3(3a^2 - 24ab + 8b^2)d^2x - 48ab + 32b^2) \cosh(dx + c) + 3(7a^2 \cosh(dx + c) \sinh(dx + c)^2 - 14ab \cosh(dx + c) \sinh(dx + c) + 7b^2 \sinh(dx + c)^2)}{24d^2(a + 2b + a \cosh(2(dx + c)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="fricas")

[Out] 1/192*(3*a^2*sinh(d*x + c)^7 + 3*(21*a^2*cosh(d*x + c)^2 - 5*a^2 + 16*a*b)*sinh(d*x + c)^5 + 8*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + (105*a^2*cosh(d*x + c)^4 - 30*(5*a^2 - 16*a*b)*cosh(d*x + c)^2 - 63*a^2 + 528*a*b - 256*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c) + 3*(7*a^2*cosh(d*x + c)sinh(d*x + c)^2 - 14*a*b*cosh(d*x + c)sinh(d*x + c) + 7*b^2*sinh(d*x + c)^2)

$c^6 - 5(5a^2 - 16ab)\cosh(dx + c)^4 - (63a^2 - 528ab + 256b^2)\cosh(dx + c)^2 - 15a^2 + 160ab)\sinh(dx + c)/(d\cosh(dx + c)^3 + 3d\cosh(dx + c)\sinh(dx + c)^2 + 3d\cosh(dx + c))$

giac [B] time = 0.18, size = 231, normalized size = 2.03

$$3a^2e^{4dx+4c} - 24a^2e^{2dx+2c} + 48abe^{2dx+2c} + 24(3a^2 - 24ab + 8b^2)(dx + c) - 3(18a^2e^{4dx+4c} - 144abe^{4dx+4c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="giac")

[Out] $\frac{1}{192}(3a^2e^{4dx+4c} - 24a^2e^{2dx+2c} + 48ab e^{2dx+2c} + 24(3a^2 - 24ab + 8b^2)(dx + c) - 3(18a^2e^{4dx+4c} - 144ab e^{4dx+4c}) + 48b^2e^{4dx+4c} - 8a^2e^{2dx+2c} + 16ab e^{2dx+2c} + a^2)e^{-4dx-4c} - 256(3ab e^{4dx+4c} - 3b^2e^{4dx+4c} + 6ab e^{2dx+2c} - 3b^2e^{2dx+2c} + 3ab - 2b^2)/(e^{2dx+2c} + 1)^3/d$

maple [A] time = 0.34, size = 109, normalized size = 0.96

$$\frac{a^2 \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3\sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 2ab \left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3\tanh(dx+c)}{2} \right) + b^2 (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x)

[Out] $\frac{1}{d}(a^2((1/4\sinh(dx+c)^3 - 3/8\sinh(dx+c))\cosh(dx+c) + 3/8dx + 3/8c) + 2ab(1/2\sinh(dx+c)^3/\cosh(dx+c) - 3/2dx - 3/2c + 3/2\tanh(dx+c)) + b^2(dx+c - \tanh(dx+c) - 1/3\tanh(dx+c)^3))$

maxima [B] time = 0.34, size = 211, normalized size = 1.85

$$\frac{1}{64}a^2\left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d}\right) + \frac{1}{3}b^2\left(3x + \frac{3c}{d} - \frac{4(3e^{-2dx-2c} + 3e^{-4dx-4c})}{d(3e^{-2dx-2c} + 3e^{-4dx-4c})} + \frac{e^{2c+2dx}}{3d(e^{2c+2dx} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="maxima")

[Out] $\frac{1}{64}a^2(24x + e^{4dx+4c}/d - 8e^{2dx+2c}/d + 8e^{-2dx-2c}/d - e^{-4dx-4c}/d) + \frac{1}{3}b^2(3x + 3c/d - 4(3e^{-2dx-2c} + 3e^{-4dx-4c})/(d(3e^{-2dx-2c} + 3e^{-4dx-4c}) + e^{-6dx-6c} + 1)) - \frac{1}{4}ab(12(dx+c)/d + e^{-2dx-2c}/d - (17e^{-2dx-2c} + 1)/(d(e^{-2dx-2c} + e^{-4dx-4c})))$

mupad [B] time = 0.27, size = 269, normalized size = 2.36

$$x \left(\frac{3a^2}{8} - 3ab + b^2 \right) - \frac{\frac{4(ab-b^2)}{3d} + \frac{4e^{4c+4dx}(ab-b^2)}{3d} + \frac{8abe^{2c+2dx}}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{4e^{2c+2dx}(ab-b^2)}{3d} + \frac{4ab}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{4(ab-b^2)}{3d(e^{2c+2dx} + 1)} + \frac{e^{2c+2dx}}{3d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $x((3a^2)/8 - 3ab + b^2) - ((4(ab - b^2))/(3d) + (4\exp(4c + 4d*x)(ab - b^2))/(3d) + (8ab\exp(2c + 2d*x))/(3d))/(3\exp(2c + 2d*x) + 3\exp(4c + 4d*x) + 3\exp(6c + 6d*x) + 1)$

$$3\exp(4c + 4d*x) + \exp(6c + 6d*x) + 1) - ((4\exp(2c + 2d*x)*(a*b - b^2))/(3*d) + (4*a*b)/(3*d))/(2*\exp(2c + 2d*x) + \exp(4c + 4d*x) + 1) - (4*(a*b - b^2))/(3*d*(\exp(2c + 2d*x) + 1)) + (\exp(2c + 2d*x)*(2*a*b - a^2))/(8*d) - (a^2*\exp(-4c - 4d*x))/(64*d) + (a^2*\exp(4c + 4d*x))/(64*d) + (a*\exp(-2c - 2d*x)*(a - 2*b))/(8*d)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**4, x)

[Out] Timed out

3.10 $\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^3(c + dx) dx$

Optimal. Leaf size=72

$$\frac{a^2 \cosh^3(c + dx)}{3d} - \frac{a(a - 2b) \cosh(c + dx)}{d} + \frac{b(2a - b) \operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] $-a*(a-2*b)*\cosh(d*x+c)/d+1/3*a^2*\cosh(d*x+c)^3/d+(2*a-b)*b*\operatorname{sech}(d*x+c)/d+1/3*b^2*\operatorname{sech}(d*x+c)^3/d$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4133, 448}

$$\frac{a^2 \cosh^3(c + dx)}{3d} - \frac{a(a - 2b) \cosh(c + dx)}{d} + \frac{b(2a - b) \operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\operatorname{Sech}[c + d*x]^2)^2*\operatorname{Sinh}[c + d*x]^3, x]$

[Out] $-((a*(a - 2*b)*\operatorname{Cosh}[c + d*x])/d) + (a^2*\operatorname{Cosh}[c + d*x]^3)/(3*d) + ((2*a - b)*b*\operatorname{Sech}[c + d*x])/d + (b^2*\operatorname{Sech}[c + d*x]^3)/(3*d)$

Rule 448

$\text{Int}[(e_.*x_*)^{m_*}((a_*) + (b_*)x_*)^{n_*})^{p_*}((c_*) + (d_*)x_*)^{q_*}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0]$

Rule 4133

$\text{Int}[(a_*) + (b_*)\operatorname{sec}[(e_*) + (f_*)x_*)^{n_*}]^{p_*}\sin[(e_*) + (f_*)x_*)^{m_*}, x_Symbol] :> \text{With}\{ff = \text{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\text{Dist}[ff/ff, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(b + a*(ff*x)^n)^p]/(ff*x)^{n*p}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, e, f\}, x] \&\& \text{IntegerQ}[(m-1)/2] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^3(c + dx) dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(a(a-2b) + \frac{b^2}{x^4} + \frac{(2a-b)b}{x^2} - a^2x^2\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a(a-2b) \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} + \frac{(2a-b)b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.53, size = 83, normalized size = 1.15

$$\frac{\operatorname{sech}^3(c + dx) \left(-3(11a^2 - 64ab + 16b^2) \cosh(2(c + dx)) + a^2 \cosh(6(c + dx)) - 26a^2 - 6a(a - 4b) \cosh(4(c + dx))\right)}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]

[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cosh[2*(c + d*x)] - 6*a*(a - 4*b)*Cosh[4*(c + d*x)] + a^2*Cosh[6*(c + d*x)])*Sech[c + d*x]^3)/(96*d)

fricas [B] time = 0.39, size = 212, normalized size = 2.94

$$\frac{a^2 \cosh(dx + c)^6 + a^2 \sinh(dx + c)^6 - 6(a^2 - 4ab) \cosh(dx + c)^4 + 3(5a^2 \cosh(dx + c)^2 - 2a^2 + 8ab) \sinh(dx + c)^2}{24(d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] 1/24*(a^2*cosh(d*x + c)^6 + a^2*sinh(d*x + c)^6 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 - 2*a^2 + 8*a*b)*sinh(d*x + c)^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 - 11*a^2 + 64*a*b - 16*b^2)*sinh(d*x + c)^2 - 26*a^2 + 168*a*b - 16*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.19, size = 140, normalized size = 1.94

$$\frac{a^2(e^{dx+c} + e^{-dx-c})^3 - 12a^2(e^{dx+c} + e^{-dx-c}) + 24ab(e^{dx+c} + e^{-dx-c}) + \frac{16(6ab(e^{dx+c} + e^{-dx-c})^2 - 3b^2(e^{dx+c} + e^{-dx-c}))}{(e^{dx+c} + e^{-dx-c})^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="giac")

[Out] 1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x - c)) + 24*a*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d

maple [A] time = 0.33, size = 93, normalized size = 1.29

$$\frac{a^2 \left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3} \right) \cosh(dx+c) + 2ab \left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)} \right) + b^2 \left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3} - \frac{2}{3 \cosh(dx+c)^3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x)

[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+b^2*(-sinh(d*x+c)^2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3))

maxima [B] time = 0.34, size = 266, normalized size = 3.69

$$\frac{1}{24} a^2 \left(\frac{e^{3dx+3c}}{d} - \frac{9e^{dx+c}}{d} - \frac{9e^{-dx-c}}{d} + \frac{e^{-3dx-3c}}{d} \right) + ab \left(\frac{e^{-dx-c}}{d} + \frac{5e^{-2dx-2c} + 1}{d(e^{-dx-c} + e^{-3dx-3c})} \right) - \frac{2}{3} b^2 \left(\frac{1}{d(3e^{-2dx-2c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="maxima")

[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) - 2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 1)))

$$3e^{(-4dx - 4c) + e^{(-6dx - 6c) + 1}} + 2e^{(-3dx - 3c)}/(d(3e^{(-2dx - 2c) + 3e^{(-4dx - 4c) + e^{(-6dx - 6c) + 1}} + 3e^{(-5dx - 5c)}/(d(3e^{(-2dx - 2c) + 3e^{(-4dx - 4c) + e^{(-6dx - 6c) + 1})))$$

mupad [B] time = 1.50, size = 201, normalized size = 2.79

$$\frac{e^{c+dx} (8ab - 3a^2)}{8d} + \frac{e^{-c-dx} (8ab - 3a^2)}{8d} + \frac{a^2 e^{-3c-3dx}}{24d} + \frac{a^2 e^{3c+3dx}}{24d} - \frac{8b^2 e^{c+dx}}{3d (3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)

[Out] (exp(c + d*x)*(8*a*b - 3*a^2))/(8*d) + (exp(-c - d*x)*(8*a*b - 3*a^2))/(8*d) + (a^2*exp(-3*c - 3*d*x))/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) - (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*exp(c + d*x)*(2*a*b - b^2))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**3,x)

[Out] Timed out

3.11 $\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^2(c + dx) dx$

Optimal. Leaf size=73

$$\frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} + \frac{a(a - 4b) \tanh(c + dx)}{2d} - \frac{1}{2}ax(a - 4b) + \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] $-1/2*a*(a-4*b)*x+1/2*a*(a-4*b)*\tanh(d*x+c)/d+1/2*a^2*\sinh(d*x+c)^2*\tanh(d*x+c)/d+1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 463, 459, 321, 206}

$$\frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} + \frac{a(a - 4b) \tanh(c + dx)}{2d} - \frac{1}{2}ax(a - 4b) + \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]

[Out] $-(a*(a - 4*b)*x)/2 + (a*(a - 4*b)*\operatorname{Tanh}[c + d*x])/(2*d) + (a^2*\operatorname{Sinh}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(2*d) + (b^2*\operatorname{Tanh}[c + d*x]^3)/(3*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] := -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f

$f^{2x^2(m/2 + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n/2]$

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-x^2)^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3a^2-2(a+b)^2+2b^2x^2)}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= \frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} + \frac{b^2 \tanh^3(c + dx)}{3d} - \frac{(a(a-4b)) \operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, \tanh(c + dx)\right)}{3d} \\ &= \frac{a(a-4b) \tanh(c + dx)}{2d} + \frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} + \frac{b^2 \tanh^3(c + dx)}{3d} \\ &= -\frac{1}{2}a(a-4b)x + \frac{a(a-4b) \tanh(c + dx)}{2d} + \frac{a^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.95, size = 126, normalized size = 1.73

$$\frac{\operatorname{sech}^3(c + dx) (a \cosh^2(c + dx) + b)^2 (3a \cosh^3(c + dx)(a \sinh(2(c + dx)) - 2dx(a - 4b)) - 4b(6a - b) \operatorname{sech}(c) \sinh(2(c + dx)))}{3d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]

[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-4*b^2*Sech[c]*Sinh[d*x] - 4*(6*a - b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*a*Cosh[c + d*x]^3*(-2*(a - 4*b)*d*x + a*Sinh[2*(c + d*x)]) - 4*b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.41, size = 252, normalized size = 3.45

$$\frac{3a^2 \sinh(dx + c)^5 - 4(3(a^2 - 4ab)dx - 12ab + 2b^2) \cosh(dx + c)^3 - 12(3(a^2 - 4ab)dx - 12ab + 2b^2) \cosh(dx + c)}{3d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/24*(3*a^2*sinh(d*x + c)^5 - 4*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*a^2*cosh(d*x + c)^2 + 9*a^2 - 48*a*b + 8*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c) + 3*(5*a^2*cosh(d*x + c)^4 + (9*a^2 - 48*a*b + 8*b^2)*cosh(d*x + c)^2 + 2*a^2 - 16*a*b - 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.18, size = 144, normalized size = 1.97

$$\frac{3a^2 e^{2(dx+c)} - 12(a^2 - 4ab)(dx + c) + 3(2a^2 e^{2(dx+c)} - 8abe^{2(dx+c)} - a^2)e^{(-2dx-2c)} + \frac{16(6abe^{4(dx+c)} - 3b^2e^{4(dx+c)} + e^{2(dx+c)})}{(e^{2(dx+c)} + 1)^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="giac")

[Out] $\frac{1}{24}*(3*a^2*e^{(2*d*x + 2*c)} - 12*(a^2 - 4*a*b)*(d*x + c) + 3*(2*a^2*e^{(2*d*x + 2*c)} - 8*a*b*e^{(2*d*x + 2*c)} - a^2)*e^{(-2*d*x - 2*c)} + 16*(6*a*b*e^{(4*d*x + 4*c)} - 3*b^2*e^{(4*d*x + 4*c)} + 12*a*b*e^{(2*d*x + 2*c)} + 6*a*b - b^2)/(e^{(2*d*x + 2*c)} + 1)^3)/d$

maple [A] time = 0.34, size = 90, normalized size = 1.23

$$\frac{a^2 \left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + 2ab(dx+c - \tanh(dx+c)) + b^2 \left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x)

[Out] $\frac{1}{d}*(a^2*(\frac{1}{2}*\cosh(d*x+c)*\sinh(d*x+c)-\frac{1}{2}*d*x-\frac{1}{2}*c)+2*a*b*(d*x+c-\tanh(d*x+c))+b^2*(-\frac{1}{2}*\sinh(d*x+c)/\cosh(d*x+c)^3+\frac{1}{2}*(\frac{2}{3}+\frac{1}{3}*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.33, size = 160, normalized size = 2.19

$$-\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + 2ab\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + \frac{2}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="maxima")

[Out] $-\frac{1}{8}a^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 2*a*b*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1))) + 2/3*b^2*(3*e^{(-4*d*x - 4*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))$

mupad [B] time = 0.16, size = 236, normalized size = 3.23

$$\frac{\frac{2(b^2+2ab)}{3d} + \frac{2e^{2c+2dx}(2ab-b^2)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} + x\left(2ab - \frac{a^2}{2}\right) + \frac{\frac{2(2ab-b^2)}{3d} + \frac{4e^{2c+2dx}(b^2+2ab)}{3d} + \frac{2e^{4c+4dx}(2ab-b^2)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} + \frac{2(2ab-b^2)}{3d(e^{2c+2dx} + e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $((2*(2*a*b + b^2))/(3*d) + (2*\exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) + x*(2*a*b - a^2/2) + ((2*(2*a*b - b^2))/(3*d) + (4*\exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*\exp(4*c + 4*d*x)*(2*a*b - b^2))/(3*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) + (2*(2*a*b - b^2))/(3*d*(\exp(2*c + 2*d*x) + 1)) - (a^2*\exp(-2*c - 2*d*x))/(8*d) + (a^2*\exp(2*c + 2*d*x))/(8*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**2, x)

3.12 $\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh(c + dx) dx$

Optimal. Leaf size=45

$$\frac{a^2 \cosh(c + dx)}{d} - \frac{2ab \operatorname{sech}(c + dx)}{d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] $a^2 \cosh(d*x+c)/d - 2*a*b*\operatorname{sech}(d*x+c)/d - 1/3*b^2*\operatorname{sech}(d*x+c)^3/d$

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4133, 270}

$$\frac{a^2 \cosh(c + dx)}{d} - \frac{2ab \operatorname{sech}(c + dx)}{d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x], x]

[Out] $(a^2*\operatorname{Cosh}[c + d*x])/d - (2*a*b*\operatorname{Sech}[c + d*x])/d - (b^2*\operatorname{Sech}[c + d*x]^3)/(3*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \sinh(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^4} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a^2 + \frac{b^2}{x^4} + \frac{2ab}{x^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{a^2 \cosh(c + dx)}{d} - \frac{2ab \operatorname{sech}(c + dx)}{d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.17, size = 59, normalized size = 1.31

$$\frac{\operatorname{sech}^3(c + dx) (3a^2 \cosh(4(c + dx)) + 9a^2 + 12a(a - 2b) \cosh(2(c + dx)) - 24ab - 8b^2)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x], x]

[Out] $((9a^2 - 24ab - 8b^2 + 12a(a - 2b)\cosh[2(c + dx)] + 3a^2\cosh[4(c + dx)])\operatorname{sech}[c + dx]^3)/(24d)$

fricas [B] time = 0.40, size = 133, normalized size = 2.96

$$\frac{3a^2 \cosh(dx + c)^4 + 3a^2 \sinh(dx + c)^4 + 12(a^2 - 2ab) \cosh(dx + c)^2 + 6(3a^2 \cosh(dx + c)^2 + 2a^2 - 4ab)}{6(d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + 3d \cosh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="fricas")`

[Out] $1/6*(3a^2*\cosh(d*x + c)^4 + 3a^2*\sinh(d*x + c)^4 + 12*(a^2 - 2*a*b)*\cosh(d*x + c)^2 + 6*(3a^2*\cosh(d*x + c)^2 + 2*a^2 - 4*a*b)*\sinh(d*x + c)^2 + 9*a^2 - 24*a*b - 8*b^2)/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + 3*d*\cosh(d*x + c))$

giac [A] time = 0.14, size = 75, normalized size = 1.67

$$\frac{3a^2(e^{(dx+c)} + e^{(-dx-c)}) - \frac{8(3ab(e^{(dx+c)} + e^{(-dx-c)})^2 + 2b^2)}{(e^{(dx+c)} + e^{(-dx-c)})^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="giac")`

[Out] $1/6*(3a^2*(e^{(d*x + c)} + e^{(-d*x - c)}) - 8*(3a*b*(e^{(d*x + c)} + e^{(-d*x - c)})^2 + 2*b^2)/(e^{(d*x + c)} + e^{(-d*x - c)})^3)/d$

maple [A] time = 0.10, size = 43, normalized size = 0.96

$$\frac{\frac{b^2 \operatorname{sech}(dx+c)^3}{3} + 2ab \operatorname{sech}(dx+c) - \frac{a^2}{\operatorname{sech}(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x)`

[Out] $-1/d*(1/3*b^2*\operatorname{sech}(d*x+c)^3+2*a*b*\operatorname{sech}(d*x+c)-a^2/\operatorname{sech}(d*x+c))$

maxima [A] time = 0.32, size = 65, normalized size = 1.44

$$\frac{a^2 \cosh(dx + c)}{d} - \frac{4ab}{d(e^{(dx+c)} + e^{(-dx-c)})} - \frac{8b^2}{3d(e^{(dx+c)} + e^{(-dx-c)})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="maxima")`

[Out] $a^2*\cosh(d*x + c)/d - 4*a*b/(d*(e^{(d*x + c)} + e^{(-d*x - c)})) - 8/3*b^2/(d*(e^{(d*x + c)} + e^{(-d*x - c)})^3)$

mupad [B] time = 1.47, size = 45, normalized size = 1.00

$$\frac{a^2 \cosh(c + dx)}{d} - \frac{\frac{b^2}{3} + 2ab \cosh(c + dx)^2}{d \cosh(c + dx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*cosh(c + d*x))/d - (b^2/3 + 2*a*b*cosh(c + d*x)^2)/(d*cosh(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x), x)
```

3.13 $\int \operatorname{csch}(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=52

$$\frac{b(2a + b)\operatorname{sech}(c + dx)}{d} - \frac{(a + b)^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] $-(a+b)^2 \operatorname{arctanh}(\cosh(d*x+c))/d + b*(2*a+b)*\operatorname{sech}(d*x+c)/d + 1/3*b^2*\operatorname{sech}(d*x+c)^3/d$

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4133, 461, 207}

$$\frac{b(2a + b)\operatorname{sech}(c + dx)}{d} - \frac{(a + b)^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2, x]

[Out] $-\left(\frac{(a + b)^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{b*(2*a + b)*\operatorname{Sech}[c + d*x]}{d} + \frac{b^2*\operatorname{Sech}[c + d*x]^3}{(3*d)}$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_.))*sin[(e_.) + (f_.)*(x_)^(m_.)], x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^4} + \frac{b(2a+b)}{x^2} - \frac{(a+b)^2}{-1+x^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} + \frac{(a + b)^2 \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{(a + b)^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 0.58, size = 108, normalized size = 2.08

$$\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(-3b(2a+b)\cosh^2(c+dx)+3(a+b)^2\cosh^3(c+dx)\right)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cosh[c + d*x]^2 + 3*(a + b)^2*Cosh[c + d*x]^3*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^3)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.42, size = 1148, normalized size = 22.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + 5*b^2)*cosh(d*x + c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b + 5*b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.14, size = 139, normalized size = 2.67

$$\frac{3(a^2 + 2ab + b^2)\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - 3(a^2 + 2ab + b^2)\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) - \frac{4(6ab(e^{(dx+c)} + e^{(-dx-c)})}{(e^{(dx+c)} + e^{(-dx-c)})^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/6*(3*(a^2 + 2*a*b + b^2)*\log(e^{(d*x + c)} + e^{(-d*x - c)} + 2) - 3*(a^2 + 2*a*b + b^2)*\log(e^{(d*x + c)} + e^{(-d*x - c)} - 2) - 4*(6*a*b*(e^{(d*x + c)} + e^{(-d*x - c)})^2 + 3*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^2 + 4*b^2)/(e^{(d*x + c)} + e^{(-d*x - c)})^3)/d$

maple [A] time = 0.22, size = 72, normalized size = 1.38

$$\frac{-2a^2 \operatorname{arctanh}(e^{dx+c}) + 2ab \left(\frac{1}{\cosh(dx+c)} - 2 \operatorname{arctanh}(e^{dx+c}) \right) + b^2 \left(\frac{1}{3 \cosh(dx+c)^3} + \frac{1}{\cosh(dx+c)} - 2 \operatorname{arctanh}(e^{dx+c}) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)`

[Out] $1/d*(-2*a^2*\operatorname{arctanh}(\exp(d*x+c))+2*a*b*(1/\cosh(d*x+c)-2*\operatorname{arctanh}(\exp(d*x+c)))+b^2*(1/3/\cosh(d*x+c)^3+1/\cosh(d*x+c)-2*\operatorname{arctanh}(\exp(d*x+c))))$

maxima [B] time = 0.34, size = 197, normalized size = 3.79

$$\frac{1}{3} b^2 \left(\frac{3 \log(e^{-dx-c} + 1)}{d} - \frac{3 \log(e^{-dx-c} - 1)}{d} - \frac{2(3e^{-dx-c} + 10e^{-3dx-3c} + 3e^{-5dx-5c})}{d(3e^{-2dx-2c} + 3e^{-4dx-4c} + e^{-6dx-6c} + 1)} \right) - 2ab \left(\frac{\log(e^{dx+c})}{d} - \frac{\log(e^{-dx-c})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-1/3*b^2*(3*\log(e^{-d*x - c}) + 1)/d - 3*\log(e^{-d*x - c} - 1)/d - 2*(3*e^{-d*x - c} + 10*e^{-3*d*x - 3*c} + 3*e^{-5*d*x - 5*c})/(d*(3*e^{-2*d*x - 2*c} + 3*e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c} + 1)) - 2*a*b*(\log(e^{-d*x - c}) + 1)/d - \log(e^{-d*x - c} - 1)/d - 2*e^{-d*x - c}/(d*(e^{-2*d*x - 2*c} + 1)) + a^2*\log(\tanh(1/2*d*x + 1/2*c))/d$

mupad [B] time = 1.50, size = 232, normalized size = 4.46

$$\frac{2e^{c+dx} (b^2 + 2ab)}{d (e^{2c+2dx} + 1)} - \frac{8b^2 e^{c+dx}}{3d (3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{2 \operatorname{atan} \left(\frac{e^{dx} e^c (a^2 \sqrt{-d^2} + b^2 \sqrt{-d^2} + 2ab \sqrt{-d^2})}{d \sqrt{a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + b^4}} \right) \sqrt{a^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2/sinh(c + d*x),x)`

[Out] $(2*\exp(c + d*x)*(2*a*b + b^2))/(d*(\exp(2*c + 2*d*x) + 1)) - (8*b^2*\exp(c + d*x))/(3*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (2*\operatorname{atan}((\exp(d*x)*\exp(c)*(a^2*(-d^2)^{(1/2)} + b^2*(-d^2)^{(1/2)} + 2*a*b*(-d^2)^{(1/2)}))/(d*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^{(1/2)}))*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^{(1/2)})/(-d^2)^{(1/2)} + (8*b^2*\exp(c + d*x))/(3*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x), x)`

3.14 $\int \operatorname{csch}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=50

$$-\frac{2b(a+b)\tanh(c+dx)}{d} - \frac{(a+b)^2 \operatorname{coth}(c+dx)}{d} + \frac{b^2 \tanh^3(c+dx)}{3d}$$

[Out] $-(a+b)^2 \operatorname{coth}(d*x+c)/d - 2*b*(a+b)*\tanh(d*x+c)/d + 1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4132, 270}

$$-\frac{2b(a+b)\tanh(c+dx)}{d} - \frac{(a+b)^2 \operatorname{coth}(c+dx)}{d} + \frac{b^2 \tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $-\left(\frac{(a+b)^2 \operatorname{Coth}[c+d*x]}{d}\right) - \frac{(2*b*(a+b)*\operatorname{Tanh}[c+d*x])}{d} + \frac{(b^2*\operatorname{Tanh}[c+d*x]^3)}{(3*d)}$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 4132

Int[((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^2}{x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-2b(a+b) + \frac{(a+b)^2}{x^2} + b^2x^2\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a+b)^2 \operatorname{coth}(c + dx)}{d} - \frac{2b(a+b)\tanh(c + dx)}{d} + \frac{b^2 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 1.77, size = 109, normalized size = 2.18

$$\frac{4 \operatorname{sech}^3(c + dx) \left(a \cosh^2(c + dx) + b \right)^2 \left(\sinh(dx) \cosh^2(c + dx) \left(b(6a + 5b) \operatorname{sech}(c) - 3(a + b)^2 \operatorname{csch}(c) \operatorname{coth}(c + dx) \right) \right)}{3d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $(-4*(b + a*\cosh[c + d*x]^2)^2*\operatorname{sech}[c + d*x]^3*(b^2*\operatorname{sech}[c]*\sinh[d*x] + \cosh[c + d*x]^2*(-3*(a + b)^2*\operatorname{coth}[c + d*x]*\operatorname{csch}[c] + b*(6*a + 5*b)*\operatorname{sech}[c]))*\sinh[d*x] + b^2*\cosh[c + d*x]*\tanh[c]))/(3*d*(a + 2*b + a*\cosh[2*(c + d*x)])^2)$

fricas [B] time = 0.42, size = 284, normalized size = 5.68

$$\frac{4\left(\left(3a^2 + 6ab + 4b^2\right)\cosh(dx + c)^3 + 3\left(3a^2 + 6ab + 4b^2\right)\cosh(dx + c)\sinh(dx + c)\right)}{3\left(d\cosh(dx + c)^5 + 5d\cosh(dx + c)\sinh(dx + c)^4 + d\sinh(dx + c)^5 + d\cosh(dx + c)^3 + \left(10d\cosh(dx + c)\sinh(dx + c)^2 + 3d\sinh(dx + c)^4 + 3d\cosh(dx + c)^2 + 3d\sinh(dx + c)^2\right)\sinh(dx + c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $-4/3*((3*a^2 + 6*a*b + 4*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*\sinh(d*x + c)^3 + (9*a^2 + 18*a*b + 8*b^2)*\cosh(d*x + c) - 2*(3*(3*a*b + 2*b^2)*\cosh(d*x + c)^2 + 3*a*b + 4*b^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + d*\sinh(d*x + c)^5 + d*\cosh(d*x + c)^3 + (10*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^3 + (10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^2 - 2*d*\cosh(d*x + c) + (5*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c))$

giac [B] time = 0.15, size = 111, normalized size = 2.22

$$\frac{2\left(\frac{3(a^2 + 2ab + b^2)}{e^{2dx+2c}-1} - \frac{6abe^{4dx+4c} + 3b^2e^{4dx+4c} + 12abe^{2dx+2c} + 12b^2e^{2dx+2c} + 6ab + 5b^2}{(e^{2dx+2c} + 1)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $-2/3*(3*(a^2 + 2*a*b + b^2)/(e^{2*d*x + 2*c} - 1) - (6*a*b*e^{4*d*x + 4*c} + 3*b^2*e^{4*d*x + 4*c} + 12*a*b*e^{2*d*x + 2*c} + 12*b^2*e^{2*d*x + 2*c} + 6*a*b + 5*b^2)/(e^{2*d*x + 2*c} + 1)^3)/d$

maple [A] time = 0.49, size = 91, normalized size = 1.82

$$\frac{-a^2 \coth(dx + c) + 2ab \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2 \tanh(dx + c) \right) + b^2 \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3} - 4 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)`

[Out] $1/d*(-a^2*\coth(d*x+c)+2*a*b*(-1/\sinh(d*x+c)/\cosh(d*x+c)-2*\tanh(d*x+c))+b^2*(-1/\sinh(d*x+c)/\cosh(d*x+c)^3-4*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.34, size = 140, normalized size = 2.80

$$-\frac{16}{3}b^2\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} - e^{(-8dx-8c)} + 1)} + \frac{1}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} - e^{(-8dx-8c)} + 1)}\right) + \frac{2}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-16/3*b^2*(2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - 2*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} + 1)) + 1/(d*(2*e^{(-2*d*x - 2*c)} - 2*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} + 1))) + \frac{2}{d(e^{(-2dx-2c)} - 1)}$

$\wedge(-8*d*x - 8*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) + 8*a*b/(d*(e^(-4*d*x - 4*c) - 1))$

mupad [B] time = 1.49, size = 215, normalized size = 4.30

$$\frac{\frac{2(3b^2+2ab)}{3d} + \frac{2e^{2c+2dx}(b^2+2ab)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} + \frac{\frac{2(b^2+2ab)}{3d} + \frac{2e^{4c+4dx}(b^2+2ab)}{3d} + \frac{4e^{2c+2dx}(3b^2+2ab)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{2(a^2 + 2ab + b^2)}{d(e^{2c+2dx} - 1)} + \frac{2(b^2 + 2ab + a^2)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^2,x)

[Out] ((2*(2*a*b + 3*b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**2, x)

3.15 $\int \operatorname{csch}^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=104

$$\frac{(3a^2 + 6ab + 5b^2) \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{6d} - \frac{b(6a + 5b) \operatorname{sech}(c + dx)}{3d} + \frac{(a + b)(a + 5b) \tanh^{-1}(\cosh(c + dx))}{2d}$$

[Out] 1/2*(a+b)*(a+5*b)*arctanh(cosh(d*x+c))/d-1/6*(3*a^2+6*a*b+5*b^2)*coth(d*x+c)*csch(d*x+c)/d-1/3*b*(6*a+5*b)*sech(d*x+c)/d+1/3*b^2*csch(d*x+c)^2*sech(d*x+c)^3/d

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 462, 456, 453, 206}

$$\frac{(3a^2 + 6ab + 5b^2) \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{6d} - \frac{b(6a + 5b) \operatorname{sech}(c + dx)}{3d} + \frac{(a + b)(a + 5b) \tanh^{-1}(\cosh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + b)*(a + 5*b)*ArcTanh[Cosh[c + d*x]]/(2*d) - ((3*a^2 + 6*a*b + 5*b^2)*Coth[c + d*x]*Csch[c + d*x])/(6*d) - (b*(6*a + 5*b)*Sech[c + d*x])/(3*d) + (b^2*Csch[c + d*x]^2*Sech[c + d*x]^3)/(3*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2))]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 462

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] := Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && GtQ[n, 0]

Rule 4133

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)])^(n_)^(p_)*sin[(e_) + (f_)*(x_)
]^(m_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{b^2 \operatorname{csch}^2(c + dx) \operatorname{sech}^3(c + dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{3d} \\ &= -\frac{(3a^2 + 6ab + 5b^2) \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{6d} + \frac{b^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{3d} \\ &= -\frac{(3a^2 + 6ab + 5b^2) \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{6d} - \frac{b(6a + 5b) \operatorname{sech}(c + dx)}{3d} \\ &= \frac{(a + b)(a + 5b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{(3a^2 + 6ab + 5b^2) \operatorname{coth}(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 1.64, size = 144, normalized size = 1.38

$$\frac{\operatorname{sech}^3(c + dx) (a \cosh^2(c + dx) + b)^2 \left(48b(a + b) \cosh^2(c + dx) + 3(a + b) \cosh^3(c + dx)\right) \left((a + b) \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) - \frac{1}{2} \operatorname{csch}(c + dx)\right)}{6d(a \cosh(2(c + dx)) - 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/6*((b + a*Cosh[c + d*x]^2)^2*(8*b^2 + 48*b*(a + b)*Cosh[c + d*x]^2 + 3*(a + b)*Cosh[c + d*x]^3*((a + b)*Csch[(c + d*x)/2]^2 - 4*(a + 5*b)*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]])) + (a + b)*Sech[(c + d*x)/2]^2)*Sech[c + d*x]^3)/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.45, size = 2930, normalized size = 28.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 54*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^9 + 8*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 42*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 9*a^2 + 6*a*b - 11*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 70*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 6*a*b - 11*b^2)*sinh(d*x + c)^4)
```

$$\begin{aligned}
&^2) * \cosh(dx + c) * \sinh(dx + c)^4 + 8 * (3a^2 + 6ab + 5b^2) * \cosh(dx + c) \\
&^3 + 8 * (63(a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 + 35(3a^2 + 6ab + 5b \\
&^2) * \cosh(dx + c)^4 + 5(9a^2 + 6ab - 11b^2) * \cosh(dx + c)^2 + 3a^2 + \\
&6ab + 5b^2) * \sinh(dx + c)^3 + 8 * (27(a^2 + 6ab + 5b^2) * \cosh(dx + c)^7 \\
&+ 21(3a^2 + 6ab + 5b^2) * \cosh(dx + c)^5 + 5(9a^2 + 6ab - 11b^2) \\
&* \cosh(dx + c)^3 + 3(3a^2 + 6ab + 5b^2) * \cosh(dx + c) * \sinh(dx + c)^2 \\
&+ 6(a^2 + 6ab + 5b^2) * \cosh(dx + c) - 3((a^2 + 6ab + 5b^2) * \cosh(dx \\
&x + c)^{10} + 10(a^2 + 6ab + 5b^2) * \cosh(dx + c) * \sinh(dx + c)^9 + (a^2 + \\
&6ab + 5b^2) * \sinh(dx + c)^{10} + (a^2 + 6ab + 5b^2) * \cosh(dx + c)^8 + \\
&(45(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 + a^2 + 6ab + 5b^2) * \sinh(dx + \\
&c)^8 + 8(15(a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 + (a^2 + 6ab + 5b^2) \\
&* \cosh(dx + c)) * \sinh(dx + c)^7 - 2(a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 + \\
&2(105(a^2 + 6ab + 5b^2) * \cosh(dx + c)^4 + 14(a^2 + 6ab + 5b^2) * \cosh \\
&sh(dx + c)^2 - a^2 - 6ab - 5b^2) * \sinh(dx + c)^6 + 4(63(a^2 + 6ab + \\
&5b^2) * \cosh(dx + c)^5 + 14(a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 - 3(a^2 \\
&+ 6ab + 5b^2) * \cosh(dx + c)) * \sinh(dx + c)^5 - 2(a^2 + 6ab + 5b^2) * \\
&cosh(dx + c)^4 + 2(105(a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 + 35(a^2 + \\
&6ab + 5b^2) * \cosh(dx + c)^4 - 15(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 - \\
&a^2 - 6ab - 5b^2) * \sinh(dx + c)^4 + 8(15(a^2 + 6ab + 5b^2) * \cosh(dx \\
&x + c)^7 + 7(a^2 + 6ab + 5b^2) * \cosh(dx + c)^5 - 5(a^2 + 6ab + 5b^2) \\
&) * \cosh(dx + c)^3 - (a^2 + 6ab + 5b^2) * \cosh(dx + c) * \sinh(dx + c)^3 + \\
&(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 + (45(a^2 + 6ab + 5b^2) * \cosh(dx \\
&+ c)^8 + 28(a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 - 30(a^2 + 6ab + 5b^2) \\
&) * \cosh(dx + c)^4 - 12(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 + a^2 + 6ab \\
&+ 5b^2) * \sinh(dx + c)^2 + a^2 + 6ab + 5b^2 + 2(5(a^2 + 6ab + 5b^2) \\
&* \cosh(dx + c)^9 + 4(a^2 + 6ab + 5b^2) * \cosh(dx + c)^7 - 6(a^2 + 6ab \\
&+ 5b^2) * \cosh(dx + c)^5 - 4(a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 + (a^2 \\
&+ 6ab + 5b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx \\
&+ c) + 1) + 3((a^2 + 6ab + 5b^2) * \cosh(dx + c)^{10} + 10(a^2 + 6ab + \\
&5b^2) * \cosh(dx + c) * \sinh(dx + c)^9 + (a^2 + 6ab + 5b^2) * \sinh(dx + c)^{10} \\
&+ (a^2 + 6ab + 5b^2) * \cosh(dx + c)^8 + (45(a^2 + 6ab + 5b^2) * \cosh \\
&(dx + c)^2 + a^2 + 6ab + 5b^2) * \sinh(dx + c)^8 + 8(15(a^2 + 6ab + 5 \\
&b^2) * \cosh(dx + c)^3 + (a^2 + 6ab + 5b^2) * \cosh(dx + c)) * \sinh(dx + c)^7 \\
&- 2(a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 + 2(105(a^2 + 6ab + 5b^2) * \\
&cosh(dx + c)^4 + 14(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 - a^2 - 6ab - \\
&5b^2) * \sinh(dx + c)^6 + 4(63(a^2 + 6ab + 5b^2) * \cosh(dx + c)^5 + 14(\\
&a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 - 3(a^2 + 6ab + 5b^2) * \cosh(dx + c) \\
&)) * \sinh(dx + c)^5 - 2(a^2 + 6ab + 5b^2) * \cosh(dx + c)^4 + 2(105(a^2 \\
&+ 6ab + 5b^2) * \cosh(dx + c)^6 + 35(a^2 + 6ab + 5b^2) * \cosh(dx + c)^4 \\
&- 15(a^2 + 6ab + 5b^2) * \cosh(dx + c)^2 - a^2 - 6ab - 5b^2) * \sinh(dx \\
&+ c)^4 + 8(15(a^2 + 6ab + 5b^2) * \cosh(dx + c)^7 + 7(a^2 + 6ab + 5 \\
&b^2) * \cosh(dx + c)^5 - 5(a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 - (a^2 + 6a \\
&b + 5b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + (a^2 + 6ab + 5b^2) * \cosh(dx \\
&+ c)^2 + (45(a^2 + 6ab + 5b^2) * \cosh(dx + c)^8 + 28(a^2 + 6ab + 5b \\
&^2) * \cosh(dx + c)^6 - 30(a^2 + 6ab + 5b^2) * \cosh(dx + c)^4 - 12(a^2 + \\
&6ab + 5b^2) * \cosh(dx + c)^2 + a^2 + 6ab + 5b^2) * \sinh(dx + c)^2 + a^2 \\
&+ 6ab + 5b^2 + 2(5(a^2 + 6ab + 5b^2) * \cosh(dx + c)^9 + 4(a^2 + 6 \\
&aab + 5b^2) * \cosh(dx + c)^7 - 6(a^2 + 6ab + 5b^2) * \cosh(dx + c)^5 - 4 \\
&(a^2 + 6ab + 5b^2) * \cosh(dx + c)^3 + (a^2 + 6ab + 5b^2) * \cosh(dx + c) \\
&) * \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(27(a^2 + 6ab \\
&b + 5b^2) * \cosh(dx + c)^8 + 28(3a^2 + 6ab + 5b^2) * \cosh(dx + c)^6 + 1 \\
&0(9a^2 + 6ab - 11b^2) * \cosh(dx + c)^4 + 12(3a^2 + 6ab + 5b^2) * \cos \\
&h(dx + c)^2 + 3a^2 + 18ab + 15b^2) * \sinh(dx + c)) / (d * \cosh(dx + c)^{10} \\
&+ 10 * d * \cosh(dx + c) * \sinh(dx + c)^9 + d * \sinh(dx + c)^{10} + d * \cosh(dx + c) \\
&^8 + (45 * d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^8 + 8 * (15 * d * \cosh(dx + c)^3 + \\
&d * \cosh(dx + c)) * \sinh(dx + c)^7 - 2 * d * \cosh(dx + c)^6 + 2 * (105 * d * \cosh(dx \\
&+ c)^4 + 14 * d * \cosh(dx + c)^2 - d) * \sinh(dx + c)^6 + 4 * (63 * d * \cosh(dx + c) \\
&^5 + 14 * d * \cosh(dx + c)^3 - 3 * d * \cosh(dx + c)) * \sinh(dx + c)^5 - 2 * d * \cosh(dx \\
&* x + c)^4 + 2 * (105 * d * \cosh(dx + c)^6 + 35 * d * \cosh(dx + c)^4 - 15 * d * \cosh(dx
\end{aligned}$$

$$+ c)^2 - d) \sinh(dx + c)^4 + 8(15d \cosh(dx + c)^7 + 7d \cosh(dx + c)^5 - 5d \cosh(dx + c)^3 - d \cosh(dx + c)) \sinh(dx + c)^3 + d \cosh(dx + c)^2 + (45d \cosh(dx + c)^8 + 28d \cosh(dx + c)^6 - 30d \cosh(dx + c)^4 - 12d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 2(5d \cosh(dx + c)^9 + 4d \cosh(dx + c)^7 - 6d \cosh(dx + c)^5 - 4d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d$$

giac [B] time = 0.16, size = 228, normalized size = 2.19

$$3(a^2 + 6ab + 5b^2) \log(e^{(dx+c)} + e^{(-dx-c)} + 2) - 3(a^2 + 6ab + 5b^2) \log(e^{(dx+c)} + e^{(-dx-c)} - 2) - \frac{12(a^2(e^{(dx+c)} + e^{(-dx-c)}))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

$$[Out] \frac{1}{12} (3(a^2 + 6ab + 5b^2) \log(e^{(dx+c)} + e^{(-dx-c)} + 2) - 3(a^2 + 6ab + 5b^2) \log(e^{(dx+c)} + e^{(-dx-c)} - 2) - 12(a^2(e^{(dx+c)} + e^{(-dx-c)})) + 2ab(e^{(dx+c)} + e^{(-dx-c)}) + b^2(e^{(dx+c)} + e^{(-dx-c)})) / ((e^{(dx+c)} + e^{(-dx-c)})^2 - 4) - 16(3ab(e^{(dx+c)} + e^{(-dx-c)})^2 + 3b^2(e^{(dx+c)} + e^{(-dx-c)})^2 + 2b^2) / (e^{(dx+c)} + e^{(-dx-c)})^3) / d$$

maple [A] time = 0.42, size = 126, normalized size = 1.21

$$a^2 \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{arctanh}(e^{dx+c}) \right) + 2ab \left(-\frac{1}{2 \sinh(dx+c)^2 \cosh(dx+c)} - \frac{3}{2 \cosh(dx+c)} + 3 \operatorname{arctanh}(e^{dx+c}) \right) + b^2 \left(\right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(dx+c)^3*(a+b*sech(dx+c)^2)^2,x)

$$[Out] \frac{1}{d} (a^2 (-1/2 \operatorname{csch}(dx+c) \operatorname{coth}(dx+c) + \operatorname{arctanh}(\exp(dx+c))) + 2ab (-1/2 \sinh(dx+c)^2 / \cosh(dx+c) - 3/2 / \cosh(dx+c) + 3 \operatorname{arctanh}(\exp(dx+c))) + b^2 (-1/2 \sinh(dx+c)^2 / \cosh(dx+c)^3 - 5/6 / \cosh(dx+c)^3 - 5/2 / \cosh(dx+c) + 5 \operatorname{arctanh}(\exp(dx+c))))$$

maxima [B] time = 0.34, size = 354, normalized size = 3.40

$$\frac{1}{6} b^2 \left(\frac{15 \log(e^{(-dx-c)} + 1)}{d} - \frac{15 \log(e^{(-dx-c)} - 1)}{d} - \frac{2(15e^{(-dx-c)} + 20e^{(-3dx-3c)} - 22e^{(-5dx-5c)} + 20e^{(-7dx-7c)} + 1)}{d(e^{(-2dx-2c)} - 2e^{(-4dx-4c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} + e^{(-10dx-10c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sech(dx+c)^2)^2,x, algorithm="maxima")

$$[Out] \frac{1}{6} b^2 (15 \log(e^{(-dx-c)} + 1) / d - 15 \log(e^{(-dx-c)} - 1) / d - 2(15e^{(-dx-c)} + 20e^{(-3dx-3c)} - 22e^{(-5dx-5c)} + 20e^{(-7dx-7c)} + 1) / (d(e^{(-2dx-2c)} - 2e^{(-4dx-4c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1))) + ab(3 \log(e^{(-dx-c)} + 1) / d - 3 \log(e^{(-dx-c)} - 1) / d + 2(3e^{(-dx-c)} - 2e^{(-3dx-3c)} + 3e^{(-5dx-5c)}) / (d(e^{(-2dx-2c)} + e^{(-4dx-4c)} - e^{(-6dx-6c)} - 1))) + 1/2 a^2 (log(e^{(-dx-c)} + 1) / d - log(e^{(-dx-c)} - 1) / d + 2(e^{(-dx-c)} + e^{(-3dx-3c)}) / (d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)))$$

mupad [B] time = 1.65, size = 316, normalized size = 3.04

$$\operatorname{atan} \left(\frac{e^{dx} e^c (a^2 \sqrt{-d^2} + 5b^2 \sqrt{-d^2} + 6ab \sqrt{-d^2})}{d \sqrt{a^4 + 12a^3b + 46a^2b^2 + 60ab^3 + 25b^4}} \right) \frac{\sqrt{a^4 + 12a^3b + 46a^2b^2 + 60ab^3 + 25b^4}}{\sqrt{-d^2}} - \frac{e^{c+dx} (a^2 + 2ab + b^2)}{d (e^{2c+2dx} - 1)} + \frac{1}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^3,x)`

[Out] $(\operatorname{atan}((\exp(dx)\exp(c)(a^2(-d^2)^{1/2} + 5b^2(-d^2)^{1/2} + 6ab(-d^2)^{1/2}))^{1/2}))/ (d(60ab^3 + 12a^3b + a^4 + 25b^4 + 46a^2b^2)^{1/2})) * (60ab^3 + 12a^3b + a^4 + 25b^4 + 46a^2b^2)^{1/2} / (-d^2)^{1/2} - (\exp(c + dx)(2ab + a^2 + b^2)) / (d(\exp(2c + 2dx) - 1)) + (8b^2\exp(c + dx)) / (3d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)) - (2\exp(c + dx)(2ab + a^2 + b^2)) / (d(\exp(4c + 4dx) - 2\exp(2c + 2dx) + 1)) - (4\exp(c + dx)(ab + b^2)) / (d(\exp(2c + 2dx) + 1)) - (8b^2\exp(c + dx)) / (3d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**3, x)`

3.16 $\int \operatorname{csch}^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=75

$$\frac{b(2a + 3b) \tanh(c + dx)}{d} - \frac{(a + b)^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{(a + b)(a + 3b) \operatorname{coth}(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] (a+b)*(a+3*b)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d+b*(2*a+3*b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4132, 448}

$$\frac{b(2a + 3b) \tanh(c + dx)}{d} - \frac{(a + b)^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{(a + b)(a + 3b) \operatorname{coth}(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + b)*(a + 3*b)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d) + (b*(2*a + 3*b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4132

Int[((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst} \left(\int \frac{(1-x^2)(a+b-bx^2)^2}{x^4} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(b(2a + 3b) + \frac{(a+b)^2}{x^4} + \frac{(-a-3b)(a+b)}{x^2} - b^2x^2 \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{(a + b)(a + 3b) \operatorname{coth}(c + dx)}{d} - \frac{(a + b)^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{b(2a + 3b)}{d} \end{aligned}$$

Mathematica [B] time = 1.32, size = 151, normalized size = 2.01

$$\frac{\operatorname{csch}(2c) \operatorname{csch}^3(2(c + dx)) \left(-3a^2 \sinh(2(c + dx)) + a^2 \sinh(6(c + dx)) + 3a^2 \sinh(4c + 2dx) + a^2 \sinh(4c + 6dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out]
$$\frac{-1/6*(\text{Csch}[2*c]*\text{Csch}[2*(c + d*x)]^3*(8*a*(a + 2*b)*\text{Sinh}[2*c] - 6*(a + 2*b)^2*\text{Sinh}[2*d*x] - 3*a^2*\text{Sinh}[2*(c + d*x)] - 6*a*b*\text{Sinh}[2*(c + d*x)]) + a^2*\text{Sinh}[6*(c + d*x)] + 2*a*b*\text{Sinh}[6*(c + d*x)] + 3*a^2*\text{Sinh}[4*c + 2*d*x] + a^2*\text{Sinh}[4*c + 6*d*x] + 8*a*b*\text{Sinh}[4*c + 6*d*x] + 8*b^2*\text{Sinh}[4*c + 6*d*x])}{d}$$

fricas [B] time = 0.40, size = 408, normalized size = 5.44

$$\frac{8\left((a^2 - 4ab - 4b^2)\cosh(dx + c)^4 + 8(a^2 + 2ab + 2b^2)\cosh(dx + c)\sinh(dx + c)\right)}{3\left(d\cosh(dx + c)^8 + 56d\cosh(dx + c)^3\sinh(dx + c)^5 + 28d\cosh(dx + c)^2\sinh(dx + c)^6 + 8d\cosh(dx + c)\sinh(dx + c)^7 + d\sinh(dx + c)^8 - 4d\cosh(dx + c)^4 + 2(35d\cosh(dx + c)^4 - 2d)\sinh(dx + c)^4 + 8(7d\cosh(dx + c)^5 - d\cosh(dx + c))\sinh(dx + c)^3 + 4(7d\cosh(dx + c)^6 - 6d\cosh(dx + c)^2)\sinh(dx + c)^2 + 8(d\cosh(dx + c)^7 - d\cosh(dx + c)^3)\sinh(dx + c) + 3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\frac{-8/3*((a^2 - 4*a*b - 4*b^2)*\cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + 2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 - 4*a*b - 4*b^2)*\sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*(a^2 - 4*a*b - 4*b^2)*\cosh(d*x + c)^2 + 2*a^2 + 4*a*b)*\sinh(d*x + c)^2 + 3*a^2 + 12*a*b + 12*b^2 + 8*((a^2 + 2*a*b + 2*b^2)*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c))}{(d*\cosh(d*x + c)^8 + 56*d*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*d*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 - 4*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 - 2*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 - d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*d*\cosh(d*x + c)^6 - 6*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 - d*\cosh(d*x + c)^3)*\sinh(d*x + c) + 3*d)}$$

giac [A] time = 0.18, size = 115, normalized size = 1.53

$$\frac{4\left(3a^2e^{(8dx+8c)} + 8a^2e^{(6dx+6c)} + 16abe^{(6dx+6c)} + 6a^2e^{(4dx+4c)} + 24abe^{(4dx+4c)} + 24b^2e^{(4dx+4c)} - a^2 - 8ab - 8b^2\right)}{3d\left(e^{(4dx+4c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out]
$$\frac{-4/3*(3*a^2*e^{(8*d*x + 8*c)} + 8*a^2*e^{(6*d*x + 6*c)} + 16*a*b*e^{(6*d*x + 6*c)} + 6*a^2*e^{(4*d*x + 4*c)} + 24*a*b*e^{(4*d*x + 4*c)} + 24*b^2*e^{(4*d*x + 4*c)} - a^2 - 8*a*b - 8*b^2)}{(d*(e^{(4*d*x + 4*c)} - 1))^3}$$

maple [A] time = 0.55, size = 138, normalized size = 1.84

$$\frac{a^2\left(\frac{2}{3} - \frac{\text{csch}(dx+c)^2}{3}\right)\coth(dx + c) + 2ab\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)} + \frac{4}{3\sinh(dx+c)\cosh(dx+c)} + \frac{8\tanh(dx+c)}{3}\right) + b^2\left(-\frac{1}{3\sinh(dx+c)^3} + \frac{4}{3\sinh(dx+c)\cosh(dx+c)} + \frac{8\tanh(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$\frac{1}{d}\left(a^2\left(\frac{2}{3} - \frac{1}{3}\text{csch}(d*x+c)^2\right)\coth(d*x+c) + 2*a*b\left(-\frac{1}{3}\frac{1}{\sinh(d*x+c)^3\cosh(d*x+c)} + \frac{4}{3}\frac{1}{\sinh(d*x+c)\cosh(d*x+c)} + \frac{8}{3}\tanh(d*x+c)\right) + b^2\left(-\frac{1}{3}\frac{1}{\sinh(d*x+c)^3\cosh(d*x+c)} + \frac{4}{3}\frac{1}{\sinh(d*x+c)\cosh(d*x+c)} + \frac{8}{3}\tanh(d*x+c)\right)\right)$$

maxima [B] time = 0.34, size = 285, normalized size = 3.80

$$\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + \frac{32}{3}ab\left(\frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{4}{3}a^2 \frac{(3e^{-(2dx-2c)} / (d(3e^{-(2dx-2c)} - 3e^{-(4dx-4c)} + e^{-(6dx-6c)} - 1)) - 1 / (d(3e^{-(2dx-2c)} - 3e^{-(4dx-4c)} + e^{-(6dx-6c)} - 1))) + 32/3ab(2e^{-(2dx-2c)} / (d(2e^{-(2dx-2c)} - 2e^{-(6dx-6c)} + e^{-(8dx-8c)} - 1)) - 1 / (d(2e^{-(2dx-2c)} - 2e^{-(6dx-6c)} + e^{-(8dx-8c)} - 1))) + 32/3b^2(3e^{-(4dx-4c)} / (d(3e^{-(4dx-4c)} - 3e^{-(8dx-8c)} + e^{-(12dx-12c)} - 1)) - 1 / (d(3e^{-(4dx-4c)} - 3e^{-(8dx-8c)} + e^{-(12dx-12c)} - 1)))}{3d(e^{4c+4dx} - 1)^3}$

mupad [B] time = 0.21, size = 115, normalized size = 1.53

$$\frac{4 \left(6a^2 e^{4c+4dx} - a^2 - 8b^2 - 8ab + 8a^2 e^{6c+6dx} + 3a^2 e^{8c+8dx} + 24b^2 e^{4c+4dx} + 24abe^{4c+4dx} + 16abe^{6c+6dx} \right)}{3d \left(e^{4c+4dx} - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^4,x)

[Out] $-(4*(6*a^2*\exp(4*c + 4*d*x) - a^2 - 8*b^2 - 8*a*b + 8*a^2*\exp(6*c + 6*d*x) + 3*a^2*\exp(8*c + 8*d*x) + 24*b^2*\exp(4*c + 4*d*x) + 24*a*b*\exp(4*c + 4*d*x) + 16*a*b*\exp(6*c + 6*d*x)))/(3*d*(\exp(4*c + 4*d*x) - 1)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**4, x)

3.17 $\int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^4(c + dx) dx$

Optimal. Leaf size=182

$$\frac{b(6a^2 - 23ab - 8b^2) \tanh^3(c + dx)}{8d} - \frac{3a(a^2 - 12ab + 8b^2) \tanh(c + dx)}{8d} + \frac{3}{8}ax(a^2 - 12ab + 8b^2) - \frac{3b^2(5a - 16b)}{4d}$$

[Out] $\frac{3}{8}a*(a^2-12*a*b+8*b^2)*x-\frac{3}{8}a*(a^2-12*a*b+8*b^2)*\tanh(d*x+c)/d+\frac{1}{8}b*(6*a^2-23*a*b-8*b^2)*\tanh(d*x+c)^3/d-\frac{3}{40}*(5*a-16*b)*b^2*\tanh(d*x+c)^5/d-\frac{3}{8}*(a-2*b)*\sinh(d*x+c)^2*\tanh(d*x+c)*(a+b-b*\tanh(d*x+c)^2)^2/d+\frac{1}{4}*\cosh(d*x+c)*\sinh(d*x+c)^3*(a+b-b*\tanh(d*x+c)^2)^3/d$

Rubi [A] time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 467, 577, 570, 206}

$$\frac{b(6a^2 - 23ab - 8b^2) \tanh^3(c + dx)}{8d} - \frac{3a(a^2 - 12ab + 8b^2) \tanh(c + dx)}{8d} + \frac{3}{8}ax(a^2 - 12ab + 8b^2) - \frac{3b^2(5a - 16b)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]

[Out] $(3*a*(a^2 - 12*a*b + 8*b^2)*x)/8 - (3*a*(a^2 - 12*a*b + 8*b^2)*\operatorname{Tanh}[c + d*x])/8d + (b*(6*a^2 - 23*a*b - 8*b^2)*\operatorname{Tanh}[c + d*x]^3)/(8*d) - (3*(5*a - 16*b)*b^2*\operatorname{Tanh}[c + d*x]^5)/(40*d) - (3*(a - 2*b)*\operatorname{Sinh}[c + d*x]^2*\operatorname{Tanh}[c + d*x]*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^2)/(8*d) + (\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x]^3*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^3)/(4*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 467

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 570

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Int[ExpandIntegrand[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 577

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n

$*q + 1)) * x^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])

Rule 4132

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4(a+b-bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh^3(c + dx) (a + b - b \tanh^2(c + dx))^3}{4d} - \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3(a - 2b) \sinh^2(c + dx) \tanh(c + dx) (a + b - b \tanh^2(c + dx))^2}{8d} + \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3(a - 2b) \sinh^2(c + dx) \tanh(c + dx) (a + b - b \tanh^2(c + dx))^2}{8d} + \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3a(a^2 - 12ab + 8b^2) \tanh(c + dx)}{8d} + \frac{b(6a^2 - 23ab - 8b^2) \tanh^3(c + dx)}{8d} \\ &= \frac{3}{8}a(a^2 - 12ab + 8b^2)x - \frac{3a(a^2 - 12ab + 8b^2) \tanh(c + dx)}{8d} + \frac{b(6a^2 - 23ab - 8b^2) \tanh^3(c + dx)}{8d} \end{aligned}$$

Mathematica [B] time = 2.51, size = 651, normalized size = 3.58

$$\frac{\operatorname{sech}(c) \operatorname{sech}^5(c + dx) (a \cosh^2(c + dx) + b)^3 (-180a^3 \sinh(2c + dx) - 310a^3 \sinh(2c + 3dx) - 310a^3 \sinh(4c + 3dx) + 180a^3 \sinh(4c + 5dx) + 310a^3 \sinh(4c + 7dx) + 310a^3 \sinh(4c + 9dx) - 180a^3 \sinh(4c + 11dx))}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]

[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c]*Sech[c + d*x]^5*(1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[d*x] + 1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[2*c + d*x] + 600*a^3*d*x*Cosh[2*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[2*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[2*c + 3*d*x] + 600*a^3*d*x*Cosh[4*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[4*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[4*c + 3*d*x] + 120*a^3*d*x*Cosh[4*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[4*c + 5*d*x] + 960*a*b^2*d*x*Cosh[4*c + 5*d*x] + 120*a^3*d*x*Cosh[6*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[6*c + 5*d*x] + 960*a*b^2*d*x*Cosh[6*c + 5*d*x] - 180*a^3*Sinh[d*x] + 12120*a^2*b*Sinh[d*x] - 14080*a*b^2*Sinh[d*x] + 1280*b^3*Sinh[d*x] - 180*a^3*Sinh[2*c + d*x] - 7080*a^2*b*Sinh[2*c + d*x] + 11520*a*b^2*Sinh[2*c + d*x] - 310*a^3*Sinh[2*c + 3*d*x] + 8760*a^2*b*Sinh[2*c + 3*d*x] - 8960*a*b^2*Sinh[2*c + 3*d*x] - 310*a^3*Sinh[4*c + 3*d*x] - 840*a^2*b*Sinh[4*c + 3*d*x] + 3840*a*b^2*Sinh[4*c + 3*d*x] - 640*b^3*Sinh[4*c + 3*d*x] - 150*a^3*Sinh[4*c + 5*d*x] + 2520*a^2*b

$*\text{Sinh}[4*c + 5*d*x] - 2560*a*b^2*\text{Sinh}[4*c + 5*d*x] + 128*b^3*\text{Sinh}[4*c + 5*d*x]$
 $- 150*a^3*\text{Sinh}[6*c + 5*d*x] + 600*a^2*b*\text{Sinh}[6*c + 5*d*x] - 15*a^3*\text{Sinh}[$
 $6*c + 7*d*x] + 120*a^2*b*\text{Sinh}[6*c + 7*d*x] - 15*a^3*\text{Sinh}[8*c + 7*d*x] + 120$
 $*a^2*b*\text{Sinh}[8*c + 7*d*x] + 5*a^3*\text{Sinh}[8*c + 9*d*x] + 5*a^3*\text{Sinh}[10*c + 9*d$
 $x])/(1280*d*(a + 2*b + a*\text{Cosh}[2*(c + d*x)])^3)$

fricas [B] time = 0.45, size = 727, normalized size = 3.99

$$5a^3 \sinh(dx + c)^9 + 15(12a^3 \cosh(dx + c)^2 - a^3 + 8a^2b) \sinh(dx + c)^7 - 8(120a^2b - 160ab^2 + 8b^3 - 15(a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c))^2)^3*sinh(d*x+c)^4,x, algorithm="fricas")

[Out] $1/320*(5*a^3*\text{sinh}(d*x + c)^9 + 15*(12*a^3*\text{cosh}(d*x + c)^2 - a^3 + 8*a^2*b)*$
 $\text{sinh}(d*x + c)^7 - 8*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8$
 $*a*b^2)*d*x)*\text{cosh}(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3$
 $- 12*a^2*b + 8*a*b^2)*d*x)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^4 + (630*a^3*\text{cosh}(d$
 $x + c)^4 - 150*a^3 + 1560*a^2*b - 1280*a*b^2 + 64*b^3 - 315*(a^3 - 8*a^2*b)$
 $*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*$
 $(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*\text{cosh}(d*x + c)^3 + 5*(84*a^3*\text{cosh}(d*x + c)^6$
 $- 105*(a^3 - 8*a^2*b)*\text{cosh}(d*x + c)^4 - 62*a^3 + 792*a^2*b - 512*a*b^2 - 6$
 $4*b^3 - 4*(75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*\text{cosh}(d*x + c)^2)*\text{sinh}(d$
 $*x + c)^3 - 40*(2*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a$
 $*b^2)*d*x)*\text{cosh}(d*x + c)^3 + 3*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 1$
 $2*a^2*b + 8*a*b^2)*d*x)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^2 - 80*(120*a^2*b - 16$
 $0*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*\text{cosh}(d*x + c) + 5*(9*a$
 $^3*\text{cosh}(d*x + c)^8 - 21*(a^3 - 8*a^2*b)*\text{cosh}(d*x + c)^6 - 2*(75*a^3 - 780*a$
 $^2*b + 640*a*b^2 - 32*b^3)*\text{cosh}(d*x + c)^4 - 36*a^3 + 504*a^2*b - 256*a*b^2$
 $+ 128*b^3 - 6*(31*a^3 - 396*a^2*b + 256*a*b^2 + 32*b^3)*\text{cosh}(d*x + c)^2)*\text{s}$
 $\text{inh}(d*x + c))/(d*\text{cosh}(d*x + c)^5 + 5*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^4 + 5*d*$
 $\text{cosh}(d*x + c)^3 + 5*(2*d*\text{cosh}(d*x + c)^3 + 3*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)$
 $^2 + 10*d*\text{cosh}(d*x + c))$

giac [A] time = 0.22, size = 339, normalized size = 1.86

$$5a^3e^{(4dx+4c)} - 40a^3e^{(2dx+2c)} + 120a^2be^{(2dx+2c)} + 120(a^3 - 12a^2b + 8ab^2)(dx + c) - 5(18a^3e^{(4dx+4c)} - 216a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c))^2)^3*sinh(d*x+c)^4,x, algorithm="giac")

[Out] $1/320*(5*a^3*e^{(4*d*x + 4*c)} - 40*a^3*e^{(2*d*x + 2*c)} + 120*a^2*b*e^{(2*d*x$
 $+ 2*c)} + 120*(a^3 - 12*a^2*b + 8*a*b^2)*(d*x + c) - 5*(18*a^3*e^{(4*d*x + 4*$
 $c)} - 216*a^2*b*e^{(4*d*x + 4*c)} + 144*a*b^2*e^{(4*d*x + 4*c)} - 8*a^3*e^{(2*d*x$
 $+ 2*c)} + 24*a^2*b*e^{(2*d*x + 2*c)} + a^3)*e^{(-4*d*x - 4*c)} - 128*(15*a^2*b*$
 $e^{(8*d*x + 8*c)} - 30*a*b^2*e^{(8*d*x + 8*c)} + 5*b^3*e^{(8*d*x + 8*c)} + 60*a^2$
 $*b*e^{(6*d*x + 6*c)} - 90*a*b^2*e^{(6*d*x + 6*c)} + 90*a^2*b*e^{(4*d*x + 4*c)} -$
 $110*a*b^2*e^{(4*d*x + 4*c)} + 10*b^3*e^{(4*d*x + 4*c)} + 60*a^2*b*e^{(2*d*x + 2*$
 $c)} - 70*a*b^2*e^{(2*d*x + 2*c)} + 15*a^2*b - 20*a*b^2 + b^3)/(e^{(2*d*x + 2*c)}$
 $+ 1)^5)/d$

maple [A] time = 0.44, size = 182, normalized size = 1.00

$$a^3 \left(\left(\frac{(\sinh^3(dx+c))}{4} - \frac{3\sinh(dx+c)}{8} \right) \cosh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 3a^2b \left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3\tanh(dx+c)}{2} \right) + 3ab^2 \left(a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x)`

[Out] $1/d*(a^3*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*\sinh(d*x+c)^3/\cosh(d*x+c)-3/2*d*x-3/2*c+3/2*\tanh(d*x+c))+3*a*b^2*(d*x+c-\tanh(d*x+c)-1/3*\tanh(d*x+c)^3)+b^3*(-1/2*\sinh(d*x+c)^3/\cosh(d*x+c)^5-3/8*\sinh(d*x+c)/\cosh(d*x+c)^5+3/8*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.34, size = 422, normalized size = 2.32

$$\frac{1}{64} a^3 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + ab^2 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)})}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="maxima")`

[Out] $1/64*a^3*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + a*b^2*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) - 3/8*a^2*b*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)}))) + 2/5*b^3*(10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 5*e^{(-8*d*x - 8*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)))$

mupad [B] time = 0.37, size = 686, normalized size = 3.77

$$\frac{\frac{2(-3a^2b+3ab^2+b^3)}{5d} - \frac{6e^{2c+2dx}(3a^2b-2ab^2+b^3)}{5d} + \frac{6e^{4c+4dx}(-3a^2b+3ab^2+b^3)}{5d} - \frac{2e^{6c+6dx}(3a^2b-6ab^2+b^3)}{5d} - \frac{2(3a^2b-6ab^2+b^3)}{5d} - \frac{8}{5d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)`

[Out] $((2*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*\exp(2*c + 2*d*x)*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) + (6*\exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*\exp(6*c + 6*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - ((2*(3*a^2*b - 6*a*b^2 + b^3))/(5*d) - (8*\exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (12*\exp(4*c + 4*d*x)*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) - (8*\exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*\exp(8*c + 8*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) + ((2*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*\exp(2*c + 2*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) - (4*\exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*\exp(4*c + 4*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) + (3*a*x*(a^2 - 12*a*b + 8*b^2))/8 - (a^3*\exp(-4*c - 4*d*x))/(64*d) + (a^3*\exp(4*c + 4*d*x))/(64*d) - (2*(3*a^2*b - 6*a*b^2 + b^3))/(5*d*(\exp(2*c + 2*d*x) + 1)) + (a^2*\exp(-2*c - 2*d*x)*(a - 3*b))/(8*d) - (a^2*\exp(2*c + 2*d*x)*(a - 3*b))/(8*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.18 $\int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^3(c + dx) dx$

Optimal. Leaf size=99

$$\frac{a^3 \cosh^3(c + dx)}{3d} - \frac{a^2(a - 3b) \cosh(c + dx)}{d} + \frac{b^2(3a - b) \operatorname{sech}^3(c + dx)}{3d} + \frac{3ab(a - b) \operatorname{sech}(c + dx)}{d} + \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

[Out] $-a^2(a-3b)*\cosh(d*x+c)/d+1/3*a^3*\cosh(d*x+c)^3/d+3*a*(a-b)*b*\operatorname{sech}(d*x+c)/d+1/3*(3*a-b)*b^2*\operatorname{sech}(d*x+c)^3/d+1/5*b^3*\operatorname{sech}(d*x+c)^5/d$

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4133, 448}

$$-\frac{a^2(a-3b)\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{b^2(3a-b)\operatorname{sech}^3(c+dx)}{3d} + \frac{3ab(a-b)\operatorname{sech}(c+dx)}{d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]

[Out] $-((a^2(a-3b)*\operatorname{Cosh}[c+d*x])/d) + (a^3*\operatorname{Cosh}[c+d*x]^3)/(3*d) + (3*a*(a-b)*b*\operatorname{Sech}[c+d*x])/d + ((3*a-b)*b^2*\operatorname{Sech}[c+d*x]^3)/(3*d) + (b^3*\operatorname{Sech}[c+d*x]^5)/(5*d)$

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4133

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*sin[(e_) + (f_)*(x_)^(m_)], x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^3(c + dx) dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^6} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(a^2(a-3b) + \frac{b^3}{x^6} + \frac{(3a-b)b^2}{x^4} + \frac{3a(a-b)b}{x^2} - a^3x^2\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a^2(a-3b)\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{3a(a-b)b\operatorname{sech}(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 1.25, size = 119, normalized size = 1.20

$$\frac{4\operatorname{sech}^5(c + dx) \left(a \cosh^2(c + dx) + b\right)^3 \left(5a^2 \cosh^6(c + dx)(a \cosh(2(c + dx)) - 5a + 18b) + 10b^2(3a - b) \cosh^2(c + dx)\right)}{15d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]

[Out] (4*(b + a*Cosh[c + d*x]^2)^3*(6*b^3 + 10*(3*a - b)*b^2*Cosh[c + d*x]^2 + 90*a*(a - b)*b*Cosh[c + d*x]^4 + 5*a^2*Cosh[c + d*x]^6*(-5*a + 18*b + a*Cosh[2*(c + d*x)]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.41, size = 403, normalized size = 4.07

$$\frac{5a^3 \cosh(dx+c)^8 + 5a^3 \sinh(dx+c)^8 - 20(a^3 - 9a^2b) \cosh(dx+c)^6 + 20(7a^3 \cosh(dx+c)^2 - a^3 + 9a^2b)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] 1/120*(5*a^3*cosh(d*x + c)^8 + 5*a^3*sinh(d*x + c)^8 - 20*(a^3 - 9*a^2*b)*cosh(d*x + c)^6 + 20*(7*a^3*cosh(d*x + c)^2 - a^3 + 9*a^2*b)*sinh(d*x + c)^6 - 20*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^4 + 10*(35*a^3*cosh(d*x + c)^4 - 22*a^3 + 180*a^2*b - 72*a*b^2 - 30*(a^3 - 9*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 425*a^3 + 3960*a^2*b - 1200*a*b^2 + 64*b^3 - 20*(31*a^3 - 279*a^2*b + 96*a*b^2 + 16*b^3)*cosh(d*x + c)^2 + 20*(7*a^3*cosh(d*x + c)^6 - 15*(a^3 - 9*a^2*b)*cosh(d*x + c)^4 - 31*a^3 + 279*a^2*b - 96*a*b^2 - 16*b^3 - 6*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.21, size = 193, normalized size = 1.95

$$\frac{5a^3(e^{dx+c} + e^{-dx-c})^3 - 60a^3(e^{dx+c} + e^{-dx-c}) + 180a^2b(e^{dx+c} + e^{-dx-c}) + \frac{16(45a^2b(e^{dx+c} + e^{-dx-c})^4 - 45ab^2(e^{dx+c} + e^{-dx-c})^2 + 16b^3)}{120d}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="giac")

[Out] 1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x - c)) + 180*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 - 45*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d

maple [A] time = 0.37, size = 130, normalized size = 1.31

$$\frac{a^3 \left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3} \right) \cosh(dx+c) + 3a^2b \left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)} \right) + 3ab^2 \left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3} - \frac{2}{3\cosh(dx+c)^3} \right) + b^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x)

[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+3*a*b^2*(-sinh(d*x+c)^2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^5-2/15/cosh(d*x+c)^5))

maxima [B] time = 0.34, size = 489, normalized size = 4.94

$$\frac{1}{24}a^3 \left(\frac{e^{3dx+3c}}{d} - \frac{9e^{dx+c}}{d} - \frac{9e^{-dx-c}}{d} + \frac{e^{-3dx-3c}}{d} \right) + \frac{3}{2}a^2b \left(\frac{e^{-dx-c}}{d} + \frac{5e^{-2dx-2c} + 1}{d(e^{-dx-c} + e^{-3dx-3c})} \right) - 2ab^2 \left(\frac{1}{d(3e^{-dx-c} + e^{-3dx-3c})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="maxima")

[Out] $\frac{1}{24}a^3\left(\frac{e^{3dx+3c}}{d} - 9\frac{e^{dx+c}}{d} - 9\frac{e^{-dx-c}}{d} + e^{-3dx-3c}\right) + \frac{3}{2}a^2b\left(\frac{e^{-dx-c}}{d} + \frac{5e^{-2dx-2c} + 1}{d(e^{-dx-c} + e^{-3dx-3c})}\right) - 2ab^2\left(\frac{3e^{-dx-c}}{d(3e^{-2dx-2c} + 3e^{-4dx-4c} + e^{-6dx-6c} + 1)} + \frac{2e^{-3dx-3c}}{d(3e^{-2dx-2c} + 3e^{-4dx-4c} + e^{-6dx-6c} + 1)} + \frac{3e^{-5dx-5c}}{d(3e^{-2dx-2c} + 3e^{-4dx-4c} + e^{-6dx-6c} + 1)}\right) - \frac{8}{15}b^3\left(\frac{5e^{-3dx-3c}}{d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)} - \frac{2e^{-5dx-5c}}{d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)} + \frac{5e^{-7dx-7c}}{d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)}\right)$

mupad [B] time = 0.33, size = 348, normalized size = 3.52

$$\frac{a^3 e^{-3c-3dx}}{24d} + \frac{a^3 e^{3c+3dx}}{24d} - \frac{3a^2 e^{-c-dx} (a-4b)}{8d} + \frac{8e^{c+dx} (3ab^2 - b^3)}{3d (2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{64b^3 e^{c+dx}}{5d (4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)

[Out] $(a^3 \exp(-3c-3dx))/(24d) + (a^3 \exp(3c+3dx))/(24d) - (3a^2 \exp(-c-dx)(a-4b))/(8d) + (8 \exp(c+dx)(3ab^2 - b^3))/(3d(2 \exp(2c+2dx) + \exp(4c+4dx) + 1)) - (64b^3 \exp(c+dx))/(5d(4 \exp(2c+2dx) + 6 \exp(4c+4dx) + 4 \exp(6c+6dx) + \exp(8c+8dx) + 1)) - (8 \exp(c+dx)(15ab^2 - 17b^3))/(15d(3 \exp(2c+2dx) + 3 \exp(4c+4dx) + \exp(6c+6dx) + 1)) + (32b^3 \exp(c+dx))/(5d(5 \exp(2c+2dx) + 10 \exp(4c+4dx) + 10 \exp(6c+6dx) + 5 \exp(8c+8dx) + \exp(10c+10dx) + 1)) - (6 \exp(c+dx)(ab^2 - a^2b))/(d(\exp(2c+2dx) + 1)) - (3a^2 \exp(c+dx)(a-4b))/(8d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**3,x)

[Out] Timed out

3.19 $\int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^2(c + dx) dx$

Optimal. Leaf size=112

$$\frac{a^3}{4d(1 - \tanh(c + dx))} - \frac{a^3}{4d(\tanh(c + dx) + 1)} - \frac{3a^2b \tanh(c + dx)}{d} - \frac{1}{2}a^2x^{a-6b} + \frac{b^2(3a + b) \tanh^3(c + dx)}{3d} - \frac{b^3 \tanh^4(c + dx)}{4d}$$

[Out] $-1/2*a^2*(a-6*b)*x+1/4*a^3/d/(1-\tanh(d*x+c))-3*a^2*b*\tanh(d*x+c)/d+1/3*b^2*(3*a+b)*\tanh(d*x+c)^3/d-1/5*b^3*\tanh(d*x+c)^5/d-1/4*a^3/d/(1+\tanh(d*x+c))$

Rubi [A] time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 467, 528, 388, 206}

$$-\frac{b(81a^2 - 28ab - 4b^2) \tanh(c + dx)}{30d} - \frac{1}{2}a^2x^{a-6b} - \frac{7b \tanh(c + dx) (a - b \tanh^2(c + dx) + b)^2}{10d} - \frac{b(33a - 2b) \tanh^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]

[Out] $-(a^2*(a - 6*b)*x)/2 - (b*(81*a^2 - 28*a*b - 4*b^2)*\operatorname{Tanh}[c + d*x])/(30*d) - ((33*a - 2*b)*b*\operatorname{Tanh}[c + d*x]*(a + b - b*\operatorname{Tanh}[c + d*x]^2))/(30*d) - (7*b*\operatorname{Tanh}[c + d*x]*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^2)/(10*d) + (\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x]*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^3)/(2*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 467

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 528

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 4132

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \sinh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx) (a + b - b \tanh^2(c + dx))^3}{2d} - \frac{\operatorname{Subst}\left(\int \frac{(a-bx^2)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{7b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^2}{10d} + \frac{\cosh(c + dx) \sinh(c + dx) (a + b - b \tanh^2(c + dx))^3}{2d} \\ &= -\frac{(33a - 2b)b \tanh(c + dx) (a + b - b \tanh^2(c + dx))}{30d} - \frac{7b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^2}{10d} \\ &= -\frac{b(81a^2 - 28ab - 4b^2) \tanh(c + dx)}{30d} - \frac{(33a - 2b)b \tanh(c + dx) (a + b - b \tanh^2(c + dx))}{30d} \\ &= -\frac{1}{2}a^2(a - 6b)x - \frac{b(81a^2 - 28ab - 4b^2) \tanh(c + dx)}{30d} - \frac{(33a - 2b)b \tanh(c + dx) (a + b - b \tanh^2(c + dx))}{30d} \end{aligned}$$

Mathematica [B] time = 1.92, size = 480, normalized size = 4.29

$$\operatorname{sech}(c) \operatorname{sech}^5(c + dx) (75a^3 \sinh(2c + dx) + 135a^3 \sinh(2c + 3dx) + 135a^3 \sinh(4c + 3dx) + 75a^3 \sinh(4c + 5dx))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]

[Out] (Sech[c]*Sech[c + d*x]^5*(-600*a^2*(a - 6*b)*d*x*Cosh[d*x] - 600*a^2*(a - 6*b)*d*x*Cosh[2*c + d*x] - 300*a^3*d*x*Cosh[2*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[2*c + 3*d*x] - 300*a^3*d*x*Cosh[4*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[4*c + 3*d*x] - 60*a^3*d*x*Cosh[4*c + 5*d*x] + 360*a^2*b*d*x*Cosh[4*c + 5*d*x] - 60*a^3*d*x*Cosh[6*c + 5*d*x] + 360*a^2*b*d*x*Cosh[6*c + 5*d*x] + 75*a^3*Sinh[d*x] - 4320*a^2*b*Sinh[d*x] + 960*a*b^2*Sinh[d*x] - 160*b^3*Sinh[d*x] + 75*a^3*Sinh[2*c + d*x] + 2880*a^2*b*Sinh[2*c + d*x] - 1440*a*b^2*Sinh[2*c + d*x] - 480*b^3*Sinh[2*c + d*x] + 135*a^3*Sinh[2*c + 3*d*x] - 2880*a^2*b*Sinh[2*c + 3*d*x] + 480*a*b^2*Sinh[2*c + 3*d*x] + 160*b^3*Sinh[2*c + 3*d*x] + 135*a^3*Sinh[4*c + 3*d*x] + 720*a^2*b*Sinh[4*c + 3*d*x] - 720*a*b^2*Sinh[4*c + 3*d*x] + 75*a^3*Sinh[4*c + 5*d*x] - 720*a^2*b*Sinh[4*c + 5*d*x] + 240*a*b^2*Sinh[4*c + 5*d*x] + 32*b^3*Sinh[4*c + 5*d*x] + 75*a^3*Sinh[6*c + 5*d*x] + 15*a^3*Sinh[6*c + 7*d*x] + 15*a^3*Sinh[8*c + 7*d*x]))/(3840*d)

fricas [B] time = 0.43, size = 595, normalized size = 5.31

$$15a^3 \sinh(dx + c)^7 + 4(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx) \cosh(dx + c)^5 + 20(90a^2b - 30ab^2 - 4b^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="fricas")

[Out] $\frac{1}{120}(15a^3\sinh(dx+c)^7 + 4(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c)^5 + 20(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c)\sinh(dx+c)^4 + (315a^3\cosh(dx+c)^2 + 75a^3 - 360a^2b + 120ab^2 + 16b^3)\sinh(dx+c)^5 + 20(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c)^3 + 5(105a^3\cosh(dx+c)^4 + 27a^3 - 216a^2b - 24ab^2 + 16b^3 + 2(75a^3 - 360a^2b + 120ab^2 + 16b^3)\cosh(dx+c)^2)\sinh(dx+c)^3 + 20(2(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c)^3 + 3(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c))\sinh(dx+c)^2 + 40(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)d)x)\cosh(dx+c) + 5(21a^3\cosh(dx+c)^6 + (75a^3 - 360a^2b + 120ab^2 + 16b^3)\cosh(dx+c)^4 + 15a^3 - 144a^2b - 48ab^2 - 64b^3 + 3(27a^3 - 216a^2b - 24ab^2 + 16b^3)\cosh(dx+c)^2)\sinh(dx+c))/(d\cosh(dx+c)^5 + 5d\cosh(dx+c)\sinh(dx+c)^4 + 5d\cosh(dx+c)^3 + 5(2d\cosh(dx+c)^3 + 3d\cosh(dx+c))\sinh(dx+c)^2 + 10d\cosh(dx+c))$

giac [B] time = 0.20, size = 278, normalized size = 2.48

$$15a^3e^{2dx+2c} - 60(a^3 - 6a^2b)(dx+c) + 15(2a^3e^{2dx+2c} - 12a^2be^{2dx+2c} - a^3)e^{(-2dx-2c)} + \frac{16(45a^2be^{8dx+8c} - 45a^2b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="giac")

[Out] $\frac{1}{120}(15a^3e^{(2dx+2c)} - 60(a^3 - 6a^2b)(dx+c) + 15(2a^3e^{(2dx+2c)} - 12a^2be^{(2dx+2c)} - a^3)e^{(-2dx-2c)} + 16(45a^2be^{(8dx+8c)} - 45a^2b)e^{(8dx+8c)} + 180a^2be^{(6dx+6c)} - 90ab^2e^{(6dx+6c)} - 30b^3e^{(6dx+6c)} + 270a^2be^{(4dx+4c)} - 60ab^2e^{(4dx+4c)} + 10b^3e^{(4dx+4c)} + 180a^2be^{(2dx+2c)} - 30ab^2e^{(2dx+2c)} - 10b^3e^{(2dx+2c)} + 45a^2b - 15ab^2 - 2b^3)/(e^{(2dx+2c)} + 1)^5)/d$

maple [A] time = 0.42, size = 145, normalized size = 1.29

$$a^3 \left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + 3a^2b(dx+c - \tanh(dx+c)) + 3ab^2 \left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x)

[Out] $\frac{1}{d}(a^3(1/2\cosh(dx+c)\sinh(dx+c) - 1/2dx - 1/2c) + 3a^2b((dx+c) - \tanh(dx+c)) + 3ab^2(-1/2\sinh(dx+c)/\cosh(dx+c)^3 + 1/2(2/3 + 1/3\operatorname{sech}(dx+c)^2)\tanh(dx+c)) + b^3(-1/4\sinh(dx+c)/\cosh(dx+c)^5 + 1/4(8/15 + 1/5\operatorname{sech}(dx+c)^4 + 4/15\operatorname{sech}(dx+c)^2)\tanh(dx+c)))$

maxima [B] time = 0.34, size = 443, normalized size = 3.96

$$-\frac{1}{8}a^3\left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + 3a^2b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + \frac{4}{15}b^3\left(\frac{1}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="maxima")

[Out] $-1/8*a^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 3*a^2*b*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1))) + 4/15*b^3*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) - 5*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 15*e^{(-6*d*x - 6*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 2*a*b^2*(3*e^{(-4*d*x - 4*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))$

mupad [B] time = 0.23, size = 592, normalized size = 5.29

$$\frac{\frac{2(9a^2b+3ab^2+4b^3)}{15d} - \frac{6e^{4c+4dx}(ab^2-a^2b)}{5d} + \frac{4e^{2c+2dx}(3a^2b-b^3)}{5d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} + \frac{\frac{2(3a^2b-b^3)}{5d} - \frac{6e^{6c+6dx}(ab^2-a^2b)}{5d} + \frac{2e^{2c+2dx}(9a^2b+3ab^2+4b^3)}{5d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^2*(a + b/cosh(c + d*x))^2)^3,x`

[Out] $((2*(3*a*b^2 + 9*a^2*b + 4*b^3))/(15*d) - (6*\exp(4*c + 4*d*x)*(a*b^2 - a^2*b))/(5*d) + (4*\exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d) - (6*\exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (2*\exp(2*c + 2*d*x)*(3*a*b^2 + 9*a^2*b + 4*b^3))/(5*d) + (6*\exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d) - (6*\exp(2*c + 2*d*x)*(a*b^2 - a^2*b))/(5*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) + ((4*\exp(4*c + 4*d*x)*(3*a*b^2 + 9*a^2*b + 4*b^3))/(5*d) - (6*\exp(8*c + 8*d*x)*(a*b^2 - a^2*b))/(5*d) - (6*(a*b^2 - a^2*b))/(5*d) + (8*\exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d) + (8*\exp(6*c + 6*d*x)*(3*a^2*b - b^3))/(5*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - (6*(a*b^2 - a^2*b))/(5*d*(\exp(2*c + 2*d*x) + 1)) - (a^2*x*(a - 6*b))/2 - (a^3*\exp(-2*c - 2*d*x))/(8*d) + (a^3*\exp(2*c + 2*d*x))/(8*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**2,x)`

[Out] Timed out

3.20 $\int (a + b \operatorname{sech}^2(c + dx))^3 \sinh(c + dx) dx$

Optimal. Leaf size=64

$$\frac{a^3 \cosh(c + dx)}{d} - \frac{3a^2 b \operatorname{sech}(c + dx)}{d} - \frac{ab^2 \operatorname{sech}^3(c + dx)}{d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

[Out] $a^3 \cosh(d*x+c)/d - 3*a^2*b*\operatorname{sech}(d*x+c)/d - a*b^2*\operatorname{sech}(d*x+c)^3/d - 1/5*b^3*\operatorname{sech}(d*x+c)^5/d$

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4133, 270}

$$-\frac{3a^2 b \operatorname{sech}(c + dx)}{d} + \frac{a^3 \cosh(c + dx)}{d} - \frac{ab^2 \operatorname{sech}^3(c + dx)}{d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\operatorname{Sech}[c + d*x]^2)^3*\operatorname{Sinh}[c + d*x], x]$

[Out] $(a^3*\operatorname{Cosh}[c + d*x])/d - (3*a^2*b*\operatorname{Sech}[c + d*x])/d - (a*b^2*\operatorname{Sech}[c + d*x]^3)/d - (b^3*\operatorname{Sech}[c + d*x]^5)/(5*d)$

Rule 270

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

Rule 4133

$\text{Int}[(a_*) + (b_*)*\sec[(e_*) + (f_*)*(x_*)^{(n_*)}]^{(p_*)}*\sin[(e_*) + (f_*)*(x_*)^{(n_*)}]^{(m_*)}, x_Symbol] \rightarrow \text{With}\{\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/ff, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(b + a*(ff*x)^n)^p]/(ff*x)^{(n*p)}, x], x, \text{Cos}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, e, f\}, x] \&\& \text{IntegerQ}[(m-1)/2] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \sinh(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^6} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(a^3 + \frac{b^3}{x^6} + \frac{3ab^2}{x^4} + \frac{3a^2b}{x^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{a^3 \cosh(c + dx)}{d} - \frac{3a^2 b \operatorname{sech}(c + dx)}{d} - \frac{ab^2 \operatorname{sech}^3(c + dx)}{d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.26, size = 93, normalized size = 1.45

$$\frac{8 \operatorname{sech}^5(c + dx) (a \cosh^2(c + dx) + b)^3 (5a^3 \cosh^6(c + dx) - 15a^2 b \cosh^4(c + dx) - 5ab^2 \cosh^2(c + dx) - b^3)}{5d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x], x]

[Out] (8*(b + a*Cosh[c + d*x]^2)^3*(-b^3 - 5*a*b^2*Cosh[c + d*x]^2 - 15*a^2*b*Cosh[c + d*x]^4 + 5*a^3*Cosh[c + d*x]^6)*Sech[c + d*x]^5)/(5*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.39, size = 276, normalized size = 4.31

$$\frac{5 a^3 \cosh (d x+c)^6+5 a^3 \sinh (d x+c)^6+30\left(a^3-2 a^2 b\right) \cosh (d x+c)^4+15\left(5 a^3 \cosh (d x+c)^2+2 a^3-4 a^2 b\right)}{10\left(d \cosh (d x+c)^5+5 d \cosh (d x+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c), x, algorithm="fricas")

[Out] 1/10*(5*a^3*cosh(d*x + c)^6 + 5*a^3*sinh(d*x + c)^6 + 30*(a^3 - 2*a^2*b)*cosh(d*x + c)^4 + 15*(5*a^3*cosh(d*x + c)^2 + 2*a^3 - 4*a^2*b)*sinh(d*x + c)^4 + 50*a^3 - 180*a^2*b - 80*a*b^2 - 32*b^3 + 5*(15*a^3 - 48*a^2*b - 16*a*b^2)*cosh(d*x + c)^2 + 5*(15*a^3*cosh(d*x + c)^4 + 15*a^3 - 48*a^2*b - 16*a*b^2 + 36*(a^3 - 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [A] time = 0.19, size = 101, normalized size = 1.58

$$\frac{5 a^3\left(e^{(d x+c)}+e^{(-d x-c)}\right)-\frac{4\left(15 a^2 b\left(e^{(d x+c)}+e^{(-d x-c)}\right)^4+20 a b^2\left(e^{(d x+c)}+e^{(-d x-c)}\right)^2+16 b^3\right)}{\left(e^{(d x+c)}+e^{(-d x-c)}\right)^5}}{10 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c), x, algorithm="giac")

[Out] 1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) - 4*(15*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 20*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d

maple [A] time = 0.13, size = 58, normalized size = 0.91

$$-\frac{\frac{b^3 \operatorname{sech}(d x+c)^5}{5}+a b^2 \operatorname{sech}(d x+c)^3+3 a^2 b \operatorname{sech}(d x+c)-\frac{a^3}{\operatorname{sech}(d x+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c), x)

[Out] -1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3+3*a^2*b*sech(d*x+c)-a^3/sech(d*x+c))

maxima [A] time = 0.32, size = 94, normalized size = 1.47

$$\frac{a^3 \cosh (d x+c)}{d}-\frac{6 a^2 b}{d\left(e^{(d x+c)}+e^{(-d x-c)}\right)}-\frac{8 a b^2}{d\left(e^{(d x+c)}+e^{(-d x-c)}\right)^3}-\frac{32 b^3}{5 d\left(e^{(d x+c)}+e^{(-d x-c)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c), x, algorithm="maxima")

[Out] a^3*cosh(d*x + c)/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8*a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3) - 32/5*b^3/(d*(e^(d*x + c) + e^(-d*x - c))^5)

mupad [B] time = 1.50, size = 288, normalized size = 4.50

$$\frac{a^3 e^{c+dx}}{2d} + \frac{a^3 e^{-c-dx}}{2d} + \frac{64b^3 e^{c+dx}}{5d (4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1)} + \frac{8e^{c+dx} (5ab^2 - 4b^3)}{5d (3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (a^3*exp(c + d*x))/(2*d) + (a^3*exp(- c - d*x))/(2*d) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 - 4*b^3))/(5*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (6*a^2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (8*a*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c),x)

[Out] Timed out

3.21 $\int \operatorname{csch}(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=83

$$\frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} + \frac{b^2(3a + b) \operatorname{sech}^3(c + dx)}{3d} - \frac{(a + b)^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

[Out] $-(a+b)^3 \operatorname{arctanh}(\cosh(dx+c))/d + b(3a^2+3ab+b^2) \operatorname{sech}(dx+c)/d + 1/3 b^2(3a+b) \operatorname{sech}(dx+c)^3/d + 1/5 b^3 \operatorname{sech}(dx+c)^5/d$

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4133, 461, 207}

$$\frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} + \frac{b^2(3a + b) \operatorname{sech}^3(c + dx)}{3d} - \frac{(a + b)^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] `Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]`

[Out] $-\left(\frac{(a+b)^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{d}\right) + \frac{b(3a^2+3ab+b^2) \operatorname{Sech}[c+dx]}{d} + \frac{b^2(3a+b) \operatorname{Sech}[c+dx]^3}{3d} + \frac{b^3 \operatorname{Sech}[c+dx]^5}{5d}$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 461

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])`

Rule 4133

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x^6(1-x^2)} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^6} + \frac{b^2(3a+b)}{x^4} + \frac{b(3a^2+3ab+b^2)}{x^2} - \frac{(a+b)^3}{-1+x^2}\right) dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d} + \frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d} \\
&= -\frac{(a+b)^3 \tanh^{-1}(\cosh(c+dx))}{d} + \frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 134, normalized size = 1.61

$$\frac{8\operatorname{sech}^5(c+dx) (a\cosh^2(c+dx)+b)^3 (-15b(3a^2+3ab+b^2)\cosh^4(c+dx)-5b^2(3a+b)\cosh^2(c+dx)+15d(a\cosh(2(c+dx))+a+2b))}{15d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*(-3*b^3 - 5*b^2*(3*a + b)*Cosh[c + d*x]^2 - 15*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]^4 + 15*(a + b)^3*Cosh[c + d*x]^5*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.44, size = 3443, normalized size = 41.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 135*a^2*b + 195*a*b^2 + 89*b^3 + 210*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 12*a*b^2 + 4*b^3 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) - 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^1 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^1)

$$\begin{aligned}
& 2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + \\
& c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 70* \\
& (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^ \\
& 3)*\cosh(d*x + c)^4 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 \\
& + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a \\
& *b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 7*(a^3 + 3* \\
& a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)* \\
& \cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c \\
& osh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 28* \\
& (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + \\
& 3*a*b^2 + b^3)*\cosh(d*x + c)^9 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d* \\
& x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 4*(a^3 + 3*a \\
& ^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos \\
& h(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 15*((a^ \\
& 3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 \\
& + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin \\
& h(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 5*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c) \\
& ^2)*\sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 \\
& + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 10*(21*(a^3 + 3 \\
& *a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 \\
&)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a \\
& *b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c) \\
& ^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3 \\
& *a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^ \\
& 3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3* \\
& a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2) \\
& *\sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^9 + 4* \\
& (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + b^3)*\cosh(d*x + c)^5 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^ \\
& 3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(\\
& d*x + c) + \sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + \\
& c)^8 + 28*(9*a^2*b + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^6 + 2*(135*a^2*b + 19 \\
& 5*a*b^2 + 89*b^3)*\cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b \\
& + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^10 + \\
& 10*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + d*\sinh(d*x + c)^10 + 5*d*\cosh(d*x + c) \\
& ^8 + 5*(9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 40*(3*d*\cosh(d*x + c)^3 \\
& + d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*d*\cosh(d*x + c)^6 + 10*(21*d*\cosh(d \\
& *x + c)^4 + 14*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 4*(63*d*\cosh(d*x + \\
& c)^5 + 70*d*\cosh(d*x + c)^3 + 15*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 10*d*co \\
& sh(d*x + c)^4 + 10*(21*d*\cosh(d*x + c)^6 + 35*d*\cosh(d*x + c)^4 + 15*d*\cosh \\
& (d*x + c)^2 + d)*\sinh(d*x + c)^4 + 40*(3*d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x +
\end{aligned}$$

$$c)^5 + 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 5*d*\cosh(d*x + c)^2 + 5*(9*d*\cosh(d*x + c)^8 + 28*d*\cosh(d*x + c)^6 + 30*d*\cosh(d*x + c)^4 + 12*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 10*(d*\cosh(d*x + c)^9 + 4*d*\cosh(d*x + c)^7 + 6*d*\cosh(d*x + c)^5 + 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.17, size = 228, normalized size = 2.75

$$15(a^3 + 3a^2b + 3ab^2 + b^3) \log(e^{dx+c} + e^{-dx-c} + 2) - 15(a^3 + 3a^2b + 3ab^2 + b^3) \log(e^{dx+c} + e^{-dx-c} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\log(e^{d*x + c} + e^{-d*x - c} + 2) - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\log(e^{d*x + c} + e^{-d*x - c} - 2) - 4*(45*a^2*b*(e^{d*x + c} + e^{-d*x - c})^4 + 45*a*b^2*(e^{d*x + c} + e^{-d*x - c})^4 + 15*b^3*(e^{d*x + c} + e^{-d*x - c})^4 + 60*a*b^2*(e^{d*x + c} + e^{-d*x - c})^2 + 20*b^3*(e^{d*x + c} + e^{-d*x - c})^2 + 48*b^3)/(e^{d*x + c} + e^{-d*x - c})^5)/d$$

maple [A] time = 0.24, size = 118, normalized size = 1.42

$$\frac{-2a^3 \operatorname{arctanh}(e^{dx+c}) + 3a^2b \left(\frac{1}{\cosh(dx+c)} - 2 \operatorname{arctanh}(e^{dx+c}) \right) + 3ab^2 \left(\frac{1}{3\cosh(dx+c)^3} + \frac{1}{\cosh(dx+c)} - 2 \operatorname{arctanh}(e^{dx+c}) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$1/d*(-2*a^3*\operatorname{arctanh}(\exp(d*x+c))+3*a^2*b*(1/\cosh(d*x+c)-2*\operatorname{arctanh}(\exp(d*x+c)))+3*a*b^2*(1/3/\cosh(d*x+c)^3+1/\cosh(d*x+c)-2*\operatorname{arctanh}(\exp(d*x+c)))+b^3*(1/5/\cosh(d*x+c)^5+1/3/\cosh(d*x+c)^3+1/\cosh(d*x+c)-2*\operatorname{arctanh}(\exp(d*x+c))))$$

maxima [B] time = 0.33, size = 358, normalized size = 4.31

$$-\frac{1}{15}b^3 \left(\frac{15 \log(e^{-dx-c} + 1)}{d} - \frac{15 \log(e^{-dx-c} - 1)}{d} - \frac{2(15e^{-dx-c} + 80e^{-3dx-3c} + 178e^{-5dx-5c} + 80e^{-7dx-7c} + 15e^{-9dx-9c})}{d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/15*b^3*(15*\log(e^{-d*x - c} + 1)/d - 15*\log(e^{-d*x - c} - 1)/d - 2*(15*e^{-d*x - c} + 80*e^{-3*d*x - 3*c} + 178*e^{-5*d*x - 5*c} + 80*e^{-7*d*x - 7*c} + 15*e^{-9*d*x - 9*c})/(d*(5*e^{-2*d*x - 2*c} + 10*e^{-4*d*x - 4*c} + 10*e^{-6*d*x - 6*c} + 5*e^{-8*d*x - 8*c} + e^{-10*d*x - 10*c} + 1))) - a*b^2*(3*\log(e^{-d*x - c} + 1)/d - 3*\log(e^{-d*x - c} - 1)/d - 2*(3*e^{-d*x - c} + 10*e^{-3*d*x - 3*c} + 3*e^{-5*d*x - 5*c})/(d*(3*e^{-2*d*x - 2*c} + 3*e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c} + 1))) - 3*a^2*b*(\log(e^{-d*x - c} + 1)/d - \log(e^{-d*x - c} - 1)/d - 2*e^{-d*x - c}/(d*(e^{-2*d*x - 2*c} + 1))) + a^3*\log(\tanh(1/2*d*x + 1/2*c))/d$$

mupad [B] time = 1.53, size = 434, normalized size = 5.23

$$\frac{2e^{c+dx} (3a^2b + 3ab^2 + b^3)}{d(e^{2c+2dx} + 1)} \frac{2 \operatorname{atan} \left(\frac{e^{dx} e^c (a^3 \sqrt{-d^2} + b^3 \sqrt{-d^2} + 3a^2b \sqrt{-d^2} + 3ab^2 \sqrt{-d^2})}{d \sqrt{a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6}} \right)}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*
atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*
b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*
a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (64*b^3*exp(c + d*
x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + ex
p(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 - 7*b^3))/(15*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d
*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (8*exp(c + d*x)*(3*a*b^2 +
b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x), x)
```


3.22 $\int \operatorname{csch}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=70

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d} - \frac{3b(a+b)^2\tanh(c+dx)}{d} - \frac{(a+b)^3\coth(c+dx)}{d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

[Out] $-(a+b)^3\coth(d*x+c)/d-3*b*(a+b)^2*\tanh(d*x+c)/d+b^2*(a+b)*\tanh(d*x+c)^3/d-1/5*b^3*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4132, 270}

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d} - \frac{3b(a+b)^2\tanh(c+dx)}{d} - \frac{(a+b)^3\coth(c+dx)}{d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3, x]

[Out] $-(((a+b)^3\coth[c+d*x])/d) - (3*b*(a+b)^2*\tanh[c+d*x])/d + (b^2*(a+b)*\tanh[c+d*x]^3)/d - (b^3*\tanh[c+d*x]^5)/(5*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 4132

Int[((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m+1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+b-bx^2)^3}{x^2} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(-3b(a+b)^2 + \frac{(a+b)^3}{x^2} + 3b^2(a+b)x^2 - b^3x^4 \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{(a+b)^3\coth(c+dx)}{d} - \frac{3b(a+b)^2\tanh(c+dx)}{d} + \frac{b^2(a+b)\tanh^3(c+dx)}{d} - \frac{b^3\tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [B] time = 2.79, size = 380, normalized size = 5.43

$$\frac{\operatorname{csch}(c)\operatorname{sech}(c)\coth(c+dx)\left(-25a^3\sinh(2(c+dx))-20a^3\sinh(4(c+dx))-5a^3\sinh(6(c+dx))-25a^3\sinh(8(c+dx))\right)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]

[Out]
$$\frac{-1/40*(\text{Coth}[c + d*x]*\text{Csch}[c]*\text{Sech}[c]*(a + b*\text{Sech}[c + d*x]^2)^3*(10*a*(5*a^2 + 12*a*b + 8*b^2)*\text{Sinh}[2*c] - 10*(5*a^3 + 18*a^2*b + 20*a*b^2 + 8*b^3)*\text{Sinh}[2*d*x] - 25*a^3*\text{Sinh}[2*(c + d*x)] + 50*a*b^2*\text{Sinh}[2*(c + d*x)] + 30*b^3*\text{Sinh}[2*(c + d*x)] - 20*a^3*\text{Sinh}[4*(c + d*x)] + 40*a*b^2*\text{Sinh}[4*(c + d*x)] + 24*b^3*\text{Sinh}[4*(c + d*x)] - 5*a^3*\text{Sinh}[6*(c + d*x)] + 10*a*b^2*\text{Sinh}[6*(c + d*x)] + 6*b^3*\text{Sinh}[6*(c + d*x)] - 25*a^3*\text{Sinh}[2*(c + 2*d*x)] - 120*a^2*b*\text{Sinh}[2*(c + 2*d*x)] - 160*a*b^2*\text{Sinh}[2*(c + 2*d*x)] - 64*b^3*\text{Sinh}[2*(c + 2*d*x)] + 25*a^3*\text{Sinh}[4*c + 2*d*x] + 30*a^2*b*\text{Sinh}[4*c + 2*d*x] + 5*a^3*\text{Sinh}[6*c + 4*d*x] - 5*a^3*\text{Sinh}[4*c + 6*d*x] - 30*a^2*b*\text{Sinh}[4*c + 6*d*x] - 40*a*b^2*\text{Sinh}[4*c + 6*d*x] - 16*b^3*\text{Sinh}[4*c + 6*d*x]))/(d*(a + 2*b + a*\text{Cosh}[2*(c + d*x)]))^3}$$

fricas [B] time = 0.47, size = 622, normalized size = 8.89

$$\frac{4\left((5a^3 + 15a^2b + 20ab^2 + 8b^3)\cosh(dx + c)^5 + 5(5a^3 + 15a^2b + 20ab^2 + 8b^3)\cosh(dx + c)\sinh(dx + c)^4\right)}{5\left(d\cosh(dx + c)^7 + 7d\cosh(dx + c)\sinh(dx + c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\frac{-4/5*((5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*\cosh(d*x + c)^5 + 5*(5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^4 - (15*a^2*b + 20*a*b^2 + 8*b^3)*\sinh(d*x + c)^5 + (25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*\cosh(d*x + c)^3 - (45*a^2*b + 80*a*b^2 + 32*b^3 + 10*(15*a^2*b + 20*a*b^2 + 8*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + (10*(5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*\cosh(d*x + c)^3 + 3*(25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 10*(5*a^3 + 15*a^2*b + 14*a*b^2 + 4*b^3)*\cosh(d*x + c) - (5*(15*a^2*b + 20*a*b^2 + 8*b^3)*\cosh(d*x + c)^4 + 30*a^2*b + 60*a*b^2 + 40*b^3 + 3*(45*a^2*b + 80*a*b^2 + 32*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + d*\sinh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + (21*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^5 + 5*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + d*\cosh(d*x + c)^3 + (35*d*\cosh(d*x + c)^4 + 50*d*\cosh(d*x + c)^2 + 9*d)*\sinh(d*x + c)^3 + 3*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^2 - 5*d*\cosh(d*x + c) + (7*d*\cosh(d*x + c)^6 + 25*d*\cosh(d*x + c)^4 + 27*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c))$$

giac [B] time = 0.18, size = 249, normalized size = 3.56

$$\frac{2\left(\frac{5(a^3 + 3a^2b + 3ab^2 + b^3)}{e^{2dx+2c}-1} - \frac{15a^2be^{8dx+8c} + 15ab^2e^{8dx+8c} + 5b^3e^{8dx+8c} + 60a^2be^{6dx+6c} + 90ab^2e^{6dx+6c} + 30b^3e^{6dx+6c} + 90a^2be^{4dx+4c} + 150ab^2e^{4dx+4c} + 50b^3e^{4dx+4c}}{e^{2dx+2c}+1}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\frac{-2/5*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^{(2*d*x + 2*c)} - 1) - (15*a^2*b*e^{(8*d*x + 8*c)} + 15*a*b^2*e^{(8*d*x + 8*c)} + 5*b^3*e^{(8*d*x + 8*c)} + 60*a^2*b*e^{(6*d*x + 6*c)} + 90*a*b^2*e^{(6*d*x + 6*c)} + 30*b^3*e^{(6*d*x + 6*c)} + 90*a^2*b*e^{(4*d*x + 4*c)} + 160*a*b^2*e^{(4*d*x + 4*c)} + 80*b^3*e^{(4*d*x + 4*c)} + 60*a^2*b*e^{(2*d*x + 2*c)} + 110*a*b^2*e^{(2*d*x + 2*c)} + 50*b^3*e^{(2*d*x + 2*c)} + 15*a^2*b + 25*a*b^2 + 11*b^3)/(e^{(2*d*x + 2*c)} + 1)^5)/d}$$

maple [B] time = 0.59, size = 148, normalized size = 2.11

$$\frac{-a^3 \coth(dx + c) + 3a^2b \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2 \tanh(dx + c) \right) + 3ab^2 \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3} - 4 \left(\frac{2}{3} + \frac{\text{sech}(dx+c)^2}{3} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)`

[Out] $1/d*(-a^3\coth(d*x+c)+3*a^2*b*(-1/\sinh(d*x+c)/\cosh(d*x+c)-2*\tanh(d*x+c))+3*a*b^2*(-1/\sinh(d*x+c)/\cosh(d*x+c)^3-4*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))+b^3*(-1/\sinh(d*x+c)/\cosh(d*x+c)^5-6*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.34, size = 358, normalized size = 5.11

$$-\frac{32}{5}b^3\left(\frac{4e^{(-2dx-2c)}}{d(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1)}+\frac{1}{d(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $-32/5*b^3*(4*e^{(-2*d*x-2*c)}/(d*(4*e^{(-2*d*x-2*c)}+5*e^{(-4*d*x-4*c)}-5*e^{(-8*d*x-8*c)}-4*e^{(-10*d*x-10*c)}-e^{(-12*d*x-12*c)}+1))+5*e^{(-4*d*x-4*c)}/(d*(4*e^{(-2*d*x-2*c)}+5*e^{(-4*d*x-4*c)}-5*e^{(-8*d*x-8*c)}-4*e^{(-10*d*x-10*c)}-e^{(-12*d*x-12*c)}+1))+1/(d*(4*e^{(-2*d*x-2*c)}+5*e^{(-4*d*x-4*c)}-5*e^{(-8*d*x-8*c)}-4*e^{(-10*d*x-10*c)}-e^{(-12*d*x-12*c)}+1))-16*a*b^2*(2*e^{(-2*d*x-2*c)}/(d*(2*e^{(-2*d*x-2*c)}-2*e^{(-6*d*x-6*c)}-e^{(-8*d*x-8*c)}+1))+1/(d*(2*e^{(-2*d*x-2*c)}-2*e^{(-6*d*x-6*c)}-e^{(-8*d*x-8*c)}+1)))+2*a^3/(d*(e^{(-2*d*x-2*c)}-1))+12*a^2*b/(d*(e^{(-4*d*x-4*c)}-1))$

mupad [B] time = 1.48, size = 644, normalized size = 9.20

$$\frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{2e^{2c+2dx}(3a^2b+3ab^2+b^3)}{5d} + \frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{2e^{6c+6dx}(3a^2b+3ab^2+b^3)}{5d} + \frac{6e^{2c+2dx}(3a^2b+7ab^2+5b^3)}{5d}$$

$$\frac{1}{2e^{2c+2dx} + e^{4c+4dx} + 1} + \frac{1}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2)^3/sinh(c + d*x)^2,x)`

[Out] $((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*\exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) + ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*\exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (6*\exp(2*c + 2*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*\exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) + ((2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (2*\exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (8*\exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (12*\exp(4*c + 4*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*\exp(6*c + 6*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) + ((2*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*\exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (4*\exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - (2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(\exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d*(\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**2, x)`

3.23 $\int \operatorname{csch}^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=144

$$\frac{b(6a^2 + 15ab + 7b^2) \operatorname{sech}^3(c + dx)}{6d} - \frac{b^2(5a + 7b) \operatorname{sech}^5(c + dx)}{10d} - \frac{(a + b)^2(a + 7b) \operatorname{sech}(c + dx)}{2d} + \frac{(a + b)^2(a + 7b)}{2d}$$

[Out] $1/2*(a+b)^2*(a+7*b)*\operatorname{arctanh}(\cosh(d*x+c))/d - 1/2*(a+b)^2*(a+7*b)*\operatorname{sech}(d*x+c)/d - 1/6*b*(6*a^2+15*a*b+7*b^2)*\operatorname{sech}(d*x+c)^3/d - 1/10*b^2*(5*a+7*b)*\operatorname{sech}(d*x+c)^5/d - 1/2*(a+b)*(b+a*\cosh(d*x+c)^2)^2*\operatorname{csch}(d*x+c)^2*\operatorname{sech}(d*x+c)^5/d$

Rubi [A] time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4133, 468, 570, 207}

$$\frac{b(6a^2 + 15ab + 7b^2) \operatorname{sech}^3(c + dx)}{6d} - \frac{b^2(5a + 7b) \operatorname{sech}^5(c + dx)}{10d} - \frac{(a + b)^2(a + 7b) \operatorname{sech}(c + dx)}{2d} + \frac{(a + b)^2(a + 7b)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^3*(a + b*\operatorname{Sech}[c + d*x]^2)^3, x]$

[Out] $((a + b)^2*(a + 7*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(2*d) - ((a + b)^2*(a + 7*b)*\operatorname{Sech}[c + d*x])/(2*d) - (b*(6*a^2 + 15*a*b + 7*b^2)*\operatorname{Sech}[c + d*x]^3)/(6*d) - (b^2*(5*a + 7*b)*\operatorname{Sech}[c + d*x]^5)/(10*d) - ((a + b)*(b + a*\operatorname{Cosh}[c + d*x]^2)^2*\operatorname{Csch}[c + d*x]^2*\operatorname{Sech}[c + d*x]^5)/(2*d)$

Rule 207

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[-a, 2]]/\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 468

$\operatorname{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}))^{(q_)}), x_Symbol] \rightarrow -\operatorname{Simp}[(c*b - a*d)*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}/(a*b*e*n*(p+1)), x] + \operatorname{Dist}[1/(a*b*n*(p+1)), \operatorname{Int}[(e*x)^m*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-2)}*\operatorname{Simp}[c*(c*b*n*(p+1) + (c*b - a*d)*(m+1)) + d*(c*b*n*(p+1) + (c*b - a*d)*(m+n*(q-1)+1)]*x^n, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 570

$\operatorname{Int}[(g_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}))^{(q_)}*((e_ + (f_)*(x_)^{(n_)})^{(r_)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 4133

$\operatorname{Int}[(a_ + (b_)*\operatorname{sec}[(e_ + (f_)*(x_))]^{(n_)})^{(p_)}*\sin[(e_ + (f_)*(x_))]^{(m_)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/ff, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(b + a*(ff*x)^n)^p]/(ff*x)^{(n*p)}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x^6(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
 &= -\frac{(a+b)(b+a\cosh^2(c+dx))^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \dots \\
 &= -\frac{(a+b)(b+a\cosh^2(c+dx))^2 \operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \dots \\
 &= -\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d} - \frac{b(6a^2+15ab+7b^2)\operatorname{sech}^3(c+dx)}{6d} + \dots \\
 &= \frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d} + \dots
 \end{aligned}$$

Mathematica [A] time = 3.89, size = 224, normalized size = 1.56

$$\operatorname{sech}^6(c+dx) (a\cosh^2(c+dx)+b)^3 \left(\frac{3}{4}(75a^3+195a^2b+165ab^2+77b^3)\sinh(4(c+dx))\operatorname{csch}^3(c+dx)+\dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]

[Out]
$$\begin{aligned}
 & -1/120*((b+a\cosh[c+d*x]^2)^3*\operatorname{Sech}[c+d*x]^6*((150*a^3+270*a^2*b-30*a*b^2-206*b^3+10*(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh[4*(c+d*x)] \\
 & +15*(a+b)^2*(a+7*b)*\cosh[6*(c+d*x)])*\operatorname{Coth}[c+d*x]*\operatorname{Csch}[c+d*x] \\
 & -480*(a+b)^2*(a+7*b)*\cosh[c+d*x]^6*(\operatorname{Log}[\cosh[(c+d*x)/2]]-\operatorname{Log}[\sinh[(c+d*x)/2]]) \\
 & + (3*(75*a^3+195*a^2*b+165*a*b^2+77*b^3)*\operatorname{Csch}[c+d*x]^3*\sinh[4*(c+d*x)]/4))/(d*(a+2*b+a*\cosh[2*(c+d*x)])^3)
 \end{aligned}$$

fricas [B] time = 0.60, size = 6717, normalized size = 46.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
 & -1/30*(30*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^{13}+390*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)*\sinh(d*x+c)^{12}+30*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\sinh(d*x+c)^{13}+20*(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh(d*x+c)^{11}+20*(9*a^3+45*a^2*b+75*a*b^2+35*b^3+117*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^2)*\sinh(d*x+c)^{11}+220*(39*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^3+(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh(d*x+c))*\sinh(d*x+c)^{10}+6*(75*a^3+195*a^2*b+165*a*b^2+77*b^3)*\cosh(d*x+c)^9+2*(10725*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^4+225*a^3+585*a^2*b+495*a*b^2+231*b^3+550*(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh(d*x+c)^2)*\sinh(d*x+c)^9+6*(6435*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^5+550*(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh(d*x+c)^3+9*(75*a^3+195*a^2*b+165*a*b^2+77*b^3)*\cosh(d*x+c))*\sinh(d*x+c)^8+8*(75*a^3+135*a^2*b-15*a*b^2-103*b^3)*\cosh(d*x+c)^7+8*(6435*(a^3+9*a^2*b+15*a*b^2+7*b^3)*\cosh(d*x+c)^6+825*(9*a^3+45*a^2*b+75*a*b^2+35*b^3)*\cosh(d*x+c)^4+75*a^3+135*a^2*b-15*a*b^2-103*b^3+27*(75*a^3+195*a^2*b+165*a*b^2+77*b^3)*\cosh(d*x+c)^2)*\sinh(d*x+c)^7+8*(6435*(a^3+9*a
 \end{aligned}$$

$$\begin{aligned}
& ^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^7 + 1155*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^5 + 63*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^3 + 7*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 6*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^5 + 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + 1540*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^6 + 126*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^4 + 75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3 + 28*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 + 3300*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^7 + 378*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^5 + 140*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^3 + 4*(2145*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^10 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^8 + 126*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^6 + 70*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)^4 + 45*a^3 + 225*a^2*b + 375*a*b^2 + 175*b^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(585*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^11 + 275*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^9 + 54*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^7 + 42*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)^5 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^3 + 15*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c) - 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))^14 + 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^13 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^12 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 - 350*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^11 + 165*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2
\end{aligned}$$

$$\begin{aligned}
& + 7*b^3)*\cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + \\
& c)^5 - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + (a^3 + 9*a^ \\
& 2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15 \\
& *a*b^2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2 + (91 \\
& *(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b + \\
& 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3) \\
& *\cosh(d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 - \\
& 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + \\
& 45*a*b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^13 \\
& + 18*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^11 + 5*(a^3 + 9*a^2*b \\
& + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 \\
&)*\cosh(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 + \\
& 2*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + \\
& 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d* \\
& x + c) + 1) + 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^14 + 14* \\
& (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^13 + (a^3 + \\
& 9*a^2*b + 15*a*b^2 + 7*b^3)*\sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a*b^2 \\
& + 7*b^3)*\cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91*(a^3 \\
& + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2))*\sinh(d*x + c)^12 + 4*(91*(a \\
& ^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b + 15*a* \\
& b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b^2 + \\
& 7*b^3)*\cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x \\
& + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a*b^2 \\
& + 7*b^3)*\cosh(d*x + c)^2))*\sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b + 15*a* \\
& b^2 + 7*b^3)*\cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(\\
& d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + (3003*(a^3 + \\
& 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 15*a*b \\
& ^2 + 7*b^3)*\cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 45*(a^ \\
& 3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2))*\sinh(d*x + c)^8 + 8*(429*(\\
& a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b + 15 \\
& *a*b^2 + 7*b^3)*\cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cos \\
& h(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 + (3003*(a^3 \\
& + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b + 15*a \\
& *b^2 + 7*b^3)*\cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh \\
& (d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*b + 1 \\
& 5*a*b^2 + 7*b^3)*\cosh(d*x + c)^2))*\sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a^2*b \\
& + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^ \\
& 3)*\cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 \\
& - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 - 15*(a^3 + 9*a^2 \\
& *b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + (a^3 + 9*a^2*b + 15 \\
& *a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)* \\
& \cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^8 \\
& + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 - 350*(a^3 + 9*a^2 \\
& *b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 - \\
& 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2))*\sinh(d*x + c)^4 + 4 \\
& *(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^11 + 165*(a^3 + 9*a^2 \\
& *b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b \\
& ^3)*\cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 \\
& - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + (a^3 + 9*a^2*b + \\
& 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15*a*b^ \\
& 2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2 + (91*(a^3 \\
& + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b + 15*a \\
& *b^2 + 7*b^3)*\cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh \\
& (d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^6 - 75*(\\
& a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + 45*a \\
& *b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^2))*\sinh(
\end{aligned}$$

$$d^2x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^{13} + 18*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^{11} + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^5 + 2*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(195*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\cosh(d*x + c)^{12} + 110*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^{10} + 27*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^8 + 28*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*\cosh(d*x + c)^6 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*\cosh(d*x + c)^4 + 15*a^3 + 135*a^2*b + 225*a*b^2 + 105*b^3 + 30*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^{14} + 14*d*\cosh(d*x + c)*\sinh(d*x + c)^{13} + d*\sinh(d*x + c)^{14} + 3*d*\cosh(d*x + c)^{12} + (91*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^{12} + 4*(91*d*\cosh(d*x + c)^3 + 9*d*\cosh(d*x + c))*\sinh(d*x + c)^{11} + d*\cosh(d*x + c)^{10} + (1001*d*\cosh(d*x + c)^4 + 198*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^{10} + 2*(1001*d*\cosh(d*x + c)^5 + 330*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^9 - 5*d*\cosh(d*x + c)^8 + (3003*d*\cosh(d*x + c)^6 + 1485*d*\cosh(d*x + c)^4 + 45*d*\cosh(d*x + c)^2 - 5*d)*\sinh(d*x + c)^8 + 8*(429*d*\cosh(d*x + c)^7 + 297*d*\cosh(d*x + c)^5 + 15*d*\cosh(d*x + c)^3 - 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 - 5*d*\cosh(d*x + c)^6 + (3003*d*\cosh(d*x + c)^8 + 2772*d*\cosh(d*x + c)^6 + 210*d*\cosh(d*x + c)^4 - 140*d*\cosh(d*x + c)^2 - 5*d)*\sinh(d*x + c)^6 + 2*(1001*d*\cosh(d*x + c)^9 + 1188*d*\cosh(d*x + c)^7 + 126*d*\cosh(d*x + c)^5 - 140*d*\cosh(d*x + c)^3 - 15*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + d*\cosh(d*x + c)^4 + (1001*d*\cosh(d*x + c)^{10} + 1485*d*\cosh(d*x + c)^8 + 210*d*\cosh(d*x + c)^6 - 350*d*\cosh(d*x + c)^4 - 75*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 4*(91*d*\cosh(d*x + c)^{11} + 165*d*\cosh(d*x + c)^9 + 30*d*\cosh(d*x + c)^7 - 70*d*\cosh(d*x + c)^5 - 25*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2 + (91*d*\cosh(d*x + c)^{12} + 198*d*\cosh(d*x + c)^{10} + 45*d*\cosh(d*x + c)^8 - 140*d*\cosh(d*x + c)^6 - 75*d*\cosh(d*x + c)^4 + 6*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^2 + 2*(7*d*\cosh(d*x + c)^{13} + 18*d*\cosh(d*x + c)^{11} + 5*d*\cosh(d*x + c)^9 - 20*d*\cosh(d*x + c)^7 - 15*d*\cosh(d*x + c)^5 + 2*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.20, size = 341, normalized size = 2.37

$$15(a^3 + 9a^2b + 15ab^2 + 7b^3) \log(e^{(dx+c)} + e^{(-dx-c)} + 2) - 15(a^3 + 9a^2b + 15ab^2 + 7b^3) \log(e^{(dx+c)} + e^{(-dx-c)} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{60}*(15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\log(e^{(d*x + c)} + e^{(-d*x - c)} + 2) - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*\log(e^{(d*x + c)} + e^{(-d*x - c)} - 2) - 60*(a^3*(e^{(d*x + c)} + e^{(-d*x - c)}) + 3*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)}) + 3*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)}) + b^3*(e^{(d*x + c)} + e^{(-d*x - c)})))/((e^{(d*x + c)} + e^{(-d*x - c)})^2 - 4) - 8*(45*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)})^4 + 90*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^4 + 45*b^3*(e^{(d*x + c)} + e^{(-d*x - c)})^4 + 60*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^2 + 40*b^3*(e^{(d*x + c)} + e^{(-d*x - c)})^2 + 48*b^3)/(e^{(d*x + c)} + e^{(-d*x - c)})^5)/d$

maple [A] time = 0.44, size = 192, normalized size = 1.33

$$a^3 \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{arctanh}(e^{dx+c}) \right) + 3a^2b \left(-\frac{1}{2 \sinh(dx+c)^2 \cosh(dx+c)} - \frac{3}{2 \cosh(dx+c)} + 3 \operatorname{arctanh}(e^{dx+c}) \right) + 3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c))^2)^3,x)

[Out] $\frac{1}{d} \left(a^3 \left(-\frac{1}{2} \operatorname{csch}(d*x+c) \operatorname{coth}(d*x+c) + \operatorname{arctanh}(\exp(d*x+c)) \right) + 3a^2 b \left(-\frac{1}{2} \frac{\sinh(d*x+c)^2}{\cosh(d*x+c)} - \frac{3}{2} \frac{1}{\cosh(d*x+c)} + 3 \operatorname{arctanh}(\exp(d*x+c)) \right) + 3a b^2 \left(-\frac{1}{2} \frac{1}{\sinh(d*x+c)^2 \cosh(d*x+c)^3} - \frac{5}{6} \frac{1}{\cosh(d*x+c)^3} - \frac{5}{2} \frac{1}{\cosh(d*x+c)} + 5 \operatorname{arctanh}(\exp(d*x+c)) \right) + b^3 \left(-\frac{1}{2} \frac{1}{\sinh(d*x+c)^2 \cosh(d*x+c)^5} - \frac{7}{10} \frac{1}{\cosh(d*x+c)^5} - \frac{7}{6} \frac{1}{\cosh(d*x+c)^3} - \frac{7}{2} \frac{1}{\cosh(d*x+c)} + 7 \operatorname{arctanh}(\exp(d*x+c)) \right) \right)$

maxima [B] time = 0.34, size = 556, normalized size = 3.86

$$\frac{1}{30} b^3 \left(\frac{105 \log(e^{(-dx-c)} + 1)}{d} - \frac{105 \log(e^{(-dx-c)} - 1)}{d} - \frac{2(105 e^{(-dx-c)} + 350 e^{(-3dx-3c)} + 231 e^{(-5dx-5c)} - 412 e^{(-7dx-7c)} + 231 e^{(-9dx-9c)} + 350 e^{(-11dx-11c)} + 105 e^{(-13dx-13c)})}{d(3 e^{(-2dx-2c)} + e^{(-4dx-4c)} - 5 e^{(-6dx-6c)} - 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 3 e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right) + \frac{1}{2} a b^2 \left(\frac{15 \log(e^{(-dx-c)} + 1)}{d} - \frac{15 \log(e^{(-dx-c)} - 1)}{d} - 2 \left(\frac{15 e^{(-dx-c)} + 20 e^{(-3dx-3c)} - 22 e^{(-5dx-5c)} + 20 e^{(-7dx-7c)} + 15 e^{(-9dx-9c)} \right)}{d(e^{(-2dx-2c)} - 2 e^{(-4dx-4c)} - 2 e^{(-6dx-6c)} + e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + \frac{3}{2} a^2 b \left(\frac{3 \log(e^{(-dx-c)} + 1)}{d} - \frac{3 \log(e^{(-dx-c)} - 1)}{d} + 2 \left(\frac{3 e^{(-dx-c)} - 2 e^{(-3dx-3c)} + 3 e^{(-5dx-5c)}}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)} - e^{(-6dx-6c)} - 1)} \right) + \frac{1}{2} a^3 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + 2 \left(\frac{e^{(-dx-c)} + e^{(-3dx-3c)}}{d(2 e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c))^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{30} b^3 \left(\frac{105 \log(e^{(-dx-c)} + 1)}{d} - \frac{105 \log(e^{(-dx-c)} - 1)}{d} - 2 \left(\frac{105 e^{(-dx-c)} + 350 e^{(-3dx-3c)} + 231 e^{(-5dx-5c)} - 412 e^{(-7dx-7c)} + 231 e^{(-9dx-9c)} + 350 e^{(-11dx-11c)} + 105 e^{(-13dx-13c)} \right)}{d(3 e^{(-2dx-2c)} + e^{(-4dx-4c)} - 5 e^{(-6dx-6c)} - 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 3 e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right) + \frac{1}{2} a b^2 \left(\frac{15 \log(e^{(-dx-c)} + 1)}{d} - \frac{15 \log(e^{(-dx-c)} - 1)}{d} - 2 \left(\frac{15 e^{(-dx-c)} + 20 e^{(-3dx-3c)} - 22 e^{(-5dx-5c)} + 20 e^{(-7dx-7c)} + 15 e^{(-9dx-9c)} \right)}{d(e^{(-2dx-2c)} - 2 e^{(-4dx-4c)} - 2 e^{(-6dx-6c)} + e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + \frac{3}{2} a^2 b \left(\frac{3 \log(e^{(-dx-c)} + 1)}{d} - \frac{3 \log(e^{(-dx-c)} - 1)}{d} + 2 \left(\frac{3 e^{(-dx-c)} - 2 e^{(-3dx-3c)} + 3 e^{(-5dx-5c)}}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)} - e^{(-6dx-6c)} - 1)} \right) + \frac{1}{2} a^3 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + 2 \left(\frac{e^{(-dx-c)} + e^{(-3dx-3c)}}{d(2 e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) \right)$

mupad [B] time = 1.64, size = 536, normalized size = 3.72

$$\frac{\operatorname{atan} \left(\frac{e^{dx} e^c (a^3 \sqrt{-d^2} + 7 b^3 \sqrt{-d^2} + 15 a b^2 \sqrt{-d^2} + 9 a^2 b \sqrt{-d^2})}{d \sqrt{a^6 + 18 a^5 b + 111 a^4 b^2 + 284 a^3 b^3 + 351 a^2 b^4 + 210 a b^5 + 49 b^6}} \right)}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x))^2)^3/sinh(c + d*x)^3,x)

[Out] $\frac{\operatorname{atan}((\exp(d*x) \exp(c) (a^3 (-d^2)^{(1/2)} + 7 b^3 (-d^2)^{(1/2)} + 15 a b^2 (-d^2)^{(1/2)} + 9 a^2 b (-d^2)^{(1/2)})) / (d(210 a b^5 + 18 a^5 b + a^6 + 49 b^6 + 351 a^2 b^4 + 284 a^3 b^3 + 111 a^4 b^2)^{(1/2)})) * (210 a b^5 + 18 a^5 b + a^6 + 49 b^6 + 351 a^2 b^4 + 284 a^3 b^3 + 111 a^4 b^2)^{(1/2)}}{(-d^2)^{(1/2)}} - \frac{(2 \exp(c + d*x) (3 a b^2 + 3 a^2 b + a^3 + b^3)) / (d(\exp(4*c + 4*d*x) - 2 \exp(2*c + 2*d*x) + 1)) - (8 \exp(c + d*x) (3 a b^2 + 2 b^3)) / (3 d (2 \exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (6 \exp(c + d*x) (2 a b^2 + a^2 b + b^3)) / (d(\exp(2*c + 2*d*x) + 1)) + (64 b^3 \exp(c + d*x)) / (5 d (4 \exp(2*c + 2*d*x) + 6 \exp(4*c + 4*d*x) + 4 \exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (8 \exp(c + d*x) (15 a b^2 - 2 b^3)) / (15 d (3 \exp(2*c + 2*d*x) + 3 \exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (32 b^3 \exp(c + d*x)) / (5 d (5 \exp(2*c + 2*d*x) + 10 \exp(4*c + 4*d*x) + 10 \exp(6*c + 6*d*x) + 5 \exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (\exp(c + d*x) (3 a b^2 + 3 a^2 b + a^3 + b^3)) / (d(\exp(2*c + 2*d*x) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

3.24 $\int \operatorname{csch}^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=104

$$\frac{b^2(3a + 4b) \tanh^3(c + dx)}{3d} + \frac{3b(a + b)(a + 2b) \tanh(c + dx)}{d} - \frac{(a + b)^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{(a + b)^2(a + 4b) \operatorname{coth}(c + dx)}{d}$$

[Out] $(a+b)^2*(a+4*b)*\operatorname{coth}(d*x+c)/d-1/3*(a+b)^3*\operatorname{coth}(d*x+c)^3/d+3*b*(a+b)*(a+2*b)*\tanh(d*x+c)/d-1/3*b^2*(3*a+4*b)*\tanh(d*x+c)^3/d+1/5*b^3*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4132, 448}

$$\frac{b^2(3a + 4b) \tanh^3(c + dx)}{3d} + \frac{3b(a + b)(a + 2b) \tanh(c + dx)}{d} - \frac{(a + b)^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{(a + b)^2(a + 4b) \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $((a + b)^2*(a + 4*b)*\operatorname{Coth}[c + d*x])/d - ((a + b)^3*\operatorname{Coth}[c + d*x]^3)/(3*d) + (3*b*(a + b)*(a + 2*b)*\operatorname{Tanh}[c + d*x])/d - (b^2*(3*a + 4*b)*\operatorname{Tanh}[c + d*x]^3)/(3*d) + (b^3*\operatorname{Tanh}[c + d*x]^5)/(5*d)$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(1-x^2)(a+b-bx^2)^3}{x^4} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(3b(a + b)(a + 2b) + \frac{(a+b)^3}{x^4} - \frac{(a+b)^2(a+4b)}{x^2} - b^2(3a + 4b)x \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{(a + b)^2(a + 4b) \operatorname{coth}(c + dx)}{d} - \frac{(a + b)^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{3b(a + b)(a + 2b) \tanh(c + dx)}{d} - \frac{b^2(3a + 4b) \tanh^3(c + dx)}{3d} + \frac{b^3 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 2.47, size = 213, normalized size = 2.05

$$\frac{8 \tanh(c) \operatorname{sech}^5(c + dx) \left(a \cosh^2(c + dx) + b \right)^3 \left(\operatorname{csch}(c) \sinh(dx) \cosh^4(c + dx) \left(5(a + b)^2(2a + 11b) \operatorname{coth}(c) \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(-8*(b + a*\cosh[c + d*x]^2)^3*\operatorname{sech}[c + d*x]^5*(-3*b^3*\cosh[c + d*x] + \cosh[c + d*x]^3*(-(b^2*(15*a + 14*b)) + 5*(a + b)^3*\operatorname{coth}[c]^2*\operatorname{coth}[c + d*x]^2) - 3*b^3*\operatorname{csch}[c]*\sinh[d*x] + \cosh[c + d*x]^4*(-(b*(45*a^2 + 120*a*b + 73*b^2)) + 5*(a + b)^2*(2*a + 11*b)*\operatorname{coth}[c]*\operatorname{coth}[c + d*x]))*\operatorname{csch}[c]*\sinh[d*x] - \cosh[c + d*x]^2*(b^2*(15*a + 14*b) + 5*(a + b)^3*\operatorname{coth}[c]*\operatorname{coth}[c + d*x]^3)*\operatorname{csch}[c]*\sinh[d*x])*\tanh[c])/(15*d*(a + 2*b + a*\cosh[2*(c + d*x)])^3)$

fricas [B] time = 0.59, size = 955, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $-8/15*((5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*\cosh(d*x + c)^6 + 12*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^5 + (5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*\sinh(d*x + c)^6 + 2*(15*a^3 - 60*a*b^2 - 32*b^3)*\cosh(d*x + c)^4 + (30*a^3 - 120*a*b^2 - 64*b^3 + 15*(5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*\cosh(d*x + c)^3 + 4*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 50*a^3 + 240*a^2*b + 360*a*b^2 + 192*b^3 + (75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3)*\cosh(d*x + c)^2 + (15*(5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*\cosh(d*x + c)^4 + 75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3 + 12*(15*a^3 - 60*a*b^2 - 32*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(3*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*\cosh(d*x + c)^5 + 8*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*\cosh(d*x + c)^3 + (25*a^3 + 75*a^2*b + 30*a*b^2 - 32*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^10 + 10*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + d*\sinh(d*x + c)^10 + 2*d*\cosh(d*x + c)^8 + (45*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^8 + 8*(15*d*\cosh(d*x + c)^3 + 2*d*\cosh(d*x + c))*\sinh(d*x + c)^7 - 3*d*\cosh(d*x + c)^6 + (210*d*\cosh(d*x + c)^4 + 56*d*\cosh(d*x + c)^2 - 3*d)*\sinh(d*x + c)^6 + 2*(126*d*\cosh(d*x + c)^5 + 56*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 - 8*d*\cosh(d*x + c)^4 + (210*d*\cosh(d*x + c)^6 + 140*d*\cosh(d*x + c)^4 - 45*d*\cosh(d*x + c)^2 - 8*d)*\sinh(d*x + c)^4 + 4*(30*d*\cosh(d*x + c)^7 + 28*d*\cosh(d*x + c)^5 - 5*d*\cosh(d*x + c)^3 - 4*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*d*\cosh(d*x + c)^2 + (45*d*\cosh(d*x + c)^8 + 56*d*\cosh(d*x + c)^6 - 45*d*\cosh(d*x + c)^4 - 48*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^2 + 2*(5*d*\cosh(d*x + c)^9 + 8*d*\cosh(d*x + c)^7 - 3*d*\cosh(d*x + c)^5 - 8*d*\cosh(d*x + c)^3 - 2*d*\cosh(d*x + c))*\sinh(d*x + c) + 6*d)$

giac [B] time = 0.19, size = 355, normalized size = 3.41

$$2 \left(\frac{5(9a^2be^{(4dx+4c)} + 18ab^2e^{(4dx+4c)} + 9b^3e^{(4dx+4c)} - 6a^3e^{(2dx+2c)} - 36a^2be^{(2dx+2c)} - 54ab^2e^{(2dx+2c)} - 24b^3e^{(2dx+2c)} + 2a^3 + 15a^2b + 24ab^2 + 11b^3)}{(e^{(2dx+2c)} - 1)^3} - \frac{45a^2be^{(8dx+8c)} + 90a^2be^{(8dx+8c)} + 180a^2be^{(6dx+6c)} + 450a^2be^{(6dx+6c)} + 240b^3e^{(6dx+6c)} + 270a^2be^{(4dx+4c)} + 750a^2be^{(4dx+4c)} + 490b^3e^{(4dx+4c)} + 180a^2be^{(2dx+2c)} + 510a^2be^{(2dx+2c)} + 320b^3e^{(2dx+2c)} + 45a^2b + 120a^2b^2 + 73b^3)}{(e^{(2dx+2c)} + 1)^5} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $2/15*(5*(9*a^2*b*e^{(4*d*x + 4*c)} + 18*a*b^2*e^{(4*d*x + 4*c)} + 9*b^3*e^{(4*d*x + 4*c)} - 6*a^3*e^{(2*d*x + 2*c)} - 36*a^2*b*e^{(2*d*x + 2*c)} - 54*a*b^2*e^{(2*d*x + 2*c)} - 24*b^3*e^{(2*d*x + 2*c)} + 2*a^3 + 15*a^2*b + 24*a*b^2 + 11*b^3)/(e^{(2*d*x + 2*c)} - 1)^3 - (45*a^2*b*e^{(8*d*x + 8*c)} + 90*a*b^2*e^{(8*d*x + 8*c)} + 45*b^3*e^{(8*d*x + 8*c)} + 180*a^2*b*e^{(6*d*x + 6*c)} + 450*a*b^2*e^{(6*d*x + 6*c)} + 240*b^3*e^{(6*d*x + 6*c)} + 270*a^2*b*e^{(4*d*x + 4*c)} + 750*a*b^2*e^{(4*d*x + 4*c)} + 490*b^3*e^{(4*d*x + 4*c)} + 180*a^2*b*e^{(2*d*x + 2*c)} + 510*a*b^2*e^{(2*d*x + 2*c)} + 320*b^3*e^{(2*d*x + 2*c)} + 45*a^2*b + 120*a^2*b^2 + 73*b^3)/(e^{(2*d*x + 2*c)} + 1)^5)/d$

maple [B] time = 0.66, size = 213, normalized size = 2.05

$$a^3 \left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3} \right) \coth(dx+c) + 3a^2b \left(-\frac{1}{3 \sinh(dx+c)^3 \cosh(dx+c)} + \frac{4}{3 \sinh(dx+c) \cosh(dx+c)} + \frac{8 \tanh(dx+c)}{3} \right) + 3a b^2 \left(-\frac{1}{3 \sinh(dx+c)^3 \cosh(dx+c)} + \frac{4}{3 \sinh(dx+c) \cosh(dx+c)} + \frac{8 \tanh(dx+c)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+3*a*b^2*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)^5+8/3/sinh(d*x+c)/cosh(d*x+c)^5+16*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [B] time = 0.34, size = 664, normalized size = 6.38

$$\frac{4}{3} a^3 \left(\frac{3 e^{(-2dx-2c)}}{d(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + \frac{256}{15} b^3 \left(\frac{1}{d(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 256/15*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 2*e^(-4*d*x - 4*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 6*e^(-6*d*x - 6*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1))) + 16*a^2*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 32*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))

mupad [B] time = 1.60, size = 745, normalized size = 7.16

$$\frac{6(a^2b + 2ab^2 + b^3)}{d(e^{2c+2dx} - 1)} - \frac{2(3a^2b + 9ab^2 + 5b^3)}{5d} + \frac{6e^{6c+6dx}(a^2b + 2ab^2 + b^3)}{5d} + \frac{6e^{4c+4dx}(3a^2b + 9ab^2 + 5b^3)}{5d} + \frac{2e^{2c+2dx}(9a^2b + 30ab^2 + 5b^3)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x))^2)^3/sinh(c + d*x)^4,x)

[Out] (6*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(30*a*b^2 + 9*a^2*b + 25*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((6*(2*a*b^2 + a^2*b + b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(30*a*b^2 + 9*a^2*b + 25*b^3))/(5*d))

```

25*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(30*a*b^2 + 9*a^2
*b + 25*b^3))/(15*d) + (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
(4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((2*(9*a*b^2 +
3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d
))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*(2*a*b^2 + a^2*b + b^3)
)/(5*d*(exp(2*c + 2*d*x) + 1)) - (8*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(3*d*(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**4, x)

$$3.25 \quad \int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=117

$$\frac{\sqrt{b}(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3 d} - \frac{(5a+4b) \sinh(c+dx) \cosh(c+dx)}{8a^2 d} + \frac{x(3a^2+12ab+8b^2)}{8a^3} + \frac{\sinh(c+dx)}{a^3}$$

[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^3/d

Rubi [A] time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{x(3a^2+12ab+8b^2)}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3 d} - \frac{(5a+4b) \sinh(c+dx) \cosh(c+dx)}{8a^2 d} + \frac{\sinh(c+dx)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*d) - ((5*a + 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +

$d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

Rule 4132

$\text{Int}[(a + b*x^n)^{(p+1)}*\text{sec}[(e + f*x)^m], x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff^{(m+1)}/f, \text{Subst}[\text{Int}[(x^m*\text{ExpandToSum}[a + b*(1 + ff^2*x^2)^{(n/2)}, x]^p)/(1 + ff^2*x^2)^{(m/2+1)}, x], x, \text{Tan}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n/2]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+3b)x^2}{(1-x^2)^2(a+b-x^2)} dx, x, \tanh(c+dx)\right)}{4ad} \\ &= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{-(a+b)}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{4ad} \\ &= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{(b(a+b)^2)\text{Su}}{4ad} \\ &= \frac{(3a^2+12ab+8b^2)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d} - \frac{(5a+4b)\cosh(c+dx)}{8a^2d} \end{aligned}$$

Mathematica [B] time = 2.48, size = 294, normalized size = 2.51

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx)) + a + 2b)\left(\sqrt{b}(3a^3 + 34a^2b + 64ab^2 + 32b^3)(\cosh(2c) - \sinh(2c))\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right) - (5a+4b)\cosh(c+dx)\right)}{8a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] $-1/64*((a + 2*b + a*\text{Cosh}[2*(c + d*x)])*\text{Sech}[c + d*x]^2*(\text{Sqrt}[b]*(3*a^3 + 34*a^2*b + 64*a*b^2 + 32*b^3)*\text{ArcTanh}[(\text{Sech}[d*x]*(\text{Cosh}[2*c] - \text{Sinh}[2*c])*(a + 2*b)*\text{Sinh}[d*x] - a*\text{Sinh}[2*c + d*x])]/(2*\text{Sqrt}[a + b]*\text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4])]*(\text{Cosh}[2*c] - \text{Sinh}[2*c]) - \text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4]*(a^2*(3*a + 2*b)*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/ \text{Sqrt}[a + b]] + \text{Sqrt}[b]*\text{Sqrt}[a + b]*(-2*a^2*c + 12*a^2*d*x + 48*a*b*d*x + 32*b^2*d*x - 8*a*(a + b)*\text{Sinh}[2*(c + d*x)] + a^2*\text{Sinh}[4*(c + d*x)])))/((a^3*\text{Sqrt}[b]*\text{Sqrt}[a + b]*d*(a + b*\text{Sech}[c + d*x]^2)*\text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4])$

fricas [B] time = 0.59, size = 1681, normalized size = 14.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 - 64*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]

giac [B] time = 2.88, size = 220, normalized size = 1.88

$$\frac{8(3a^2+12ab+8b^2)(dx+c)}{a^3} + \frac{ae^{4dx+4c}-8ae^{2dx+2c}-8be^{2dx+2c}}{a^2} - \frac{(18a^2e^{4dx+4c}+72abe^{4dx+4c}+48b^2e^{4dx+4c}-8a^2e^{2dx+2c}-8abe^{2dx+2c})}{a^3}$$

$64d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] 1/64*(8*(3*a^2 + 12*a*b + 8*b^2)*(d*x + c)/a^3 + (a*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/a^2 - (18*a^2*e^(4*d*x + 4*c) + 72*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^3 - 64*(a^2*b + 2*a*b^2 + b^3)*ar

$\text{ctan}(1/2*(a*e^{(2*d*x + 2*c)} + a + 2*b)/\text{sqrt}(-a*b - b^2))/(\text{sqrt}(-a*b - b^2)*a^3)/d$

maple [B] time = 0.41, size = 708, normalized size = 6.05

$$\frac{1}{4da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^4} + \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3} - \frac{1}{8da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} - \frac{b}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} - \frac{8da}{8da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)`

[Out] $\frac{1}{4}d/a/(\tanh(1/2*d*x+1/2*c)-1)^4 + \frac{1}{2}d/a/(\tanh(1/2*d*x+1/2*c)-1)^3 - \frac{1}{8}d/a/(\tanh(1/2*d*x+1/2*c)-1)^2 - \frac{1}{2}d/a^2/(\tanh(1/2*d*x+1/2*c)-1)^2 * b - \frac{3}{8}d/a/(\tanh(1/2*d*x+1/2*c)-1) - \frac{1}{2}d/a^2/(\tanh(1/2*d*x+1/2*c)-1) * b - \frac{3}{8}d/a * \ln(\tanh(1/2*d*x+1/2*c)-1) - \frac{3}{2}d/a^2 * \ln(\tanh(1/2*d*x+1/2*c)-1) * b - \frac{1}{d/a^3} * \ln(\tanh(1/2*d*x+1/2*c)-1) * b^2 - \frac{1}{4}d/a/(\tanh(1/2*d*x+1/2*c)+1)^4 + \frac{1}{2}d/a/(\tanh(1/2*d*x+1/2*c)+1)^3 + \frac{1}{8}d/a/(\tanh(1/2*d*x+1/2*c)+1)^2 + \frac{1}{2}d/a^2/(\tanh(1/2*d*x+1/2*c)+1)^2 * b - \frac{3}{8}d/a/(\tanh(1/2*d*x+1/2*c)+1) - \frac{1}{2}d/a^2/(\tanh(1/2*d*x+1/2*c)+1) * b + \frac{3}{8}d/a * \ln(\tanh(1/2*d*x+1/2*c)+1) + \frac{3}{2}d/a^2 * \ln(\tanh(1/2*d*x+1/2*c)+1) * b + \frac{1}{d/a^3} * \ln(\tanh(1/2*d*x+1/2*c)+1) * b^2 + \frac{1}{2}d/a * b^{(1/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 - 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) + \frac{1}{d/a^2} * b^{(3/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 - 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) + \frac{1}{2}d/a^3 * b^{(5/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 - 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) - \frac{1}{2}d/a * b^{(1/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) - \frac{1}{d/a^2} * b^{(3/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) - \frac{1}{2}d/a^3 * b^{(5/2)}/(a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)})$

maxima [B] time = 0.46, size = 526, normalized size = 4.50

$$\frac{3b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16\sqrt{(a+b)b}ad} + \frac{3(dx+c)}{8ad} - \frac{(8be^{(-2dx-2c)}-a)e^{(4dx+4c)}}{64a^2d} - \frac{e^{(2dx+2c)}}{8ad} + \frac{e^{(-2dx-2c)}}{8ad} + \frac{b \log(ae^{(4dx+4c)})}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $\frac{3}{16} * b * \log((a * e^{(-2 * d * x - 2 * c)} + a + 2 * b - 2 * \text{sqrt}((a + b) * b)) / (a * e^{(-2 * d * x - 2 * c)} + a + 2 * b + 2 * \text{sqrt}((a + b) * b))) / (\text{sqrt}((a + b) * b) * a * d) + \frac{3}{8} * (d * x + c) / (a * d) - \frac{1}{64} * (8 * b * e^{(-2 * d * x - 2 * c)} - a) * e^{(4 * d * x + 4 * c)} / (a^2 * d) - \frac{1}{8} * e^{(2 * d * x + 2 * c)} / (a * d) + \frac{1}{8} * e^{(-2 * d * x - 2 * c)} / (a * d) + \frac{1}{4} * b * \log(a * e^{(4 * d * x + 4 * c)} + 2 * (a + 2 * b) * e^{(2 * d * x + 2 * c)} + a) / (a^2 * d) - \frac{1}{4} * b * \log(2 * (a + 2 * b) * e^{(-2 * d * x - 2 * c)} + a * e^{(-4 * d * x - 4 * c)} + a) / (a^2 * d) - \frac{1}{8} * (a * b + 2 * b^2) * \log((a * e^{(2 * d * x + 2 * c)} + a + 2 * b - 2 * \text{sqrt}((a + b) * b)) / (a * e^{(2 * d * x + 2 * c)} + a + 2 * b + 2 * \text{sqrt}((a + b) * b))) / (\text{sqrt}((a + b) * b) * a^2 * d) + \frac{1}{8} * (a * b + 2 * b^2) * \log((a * e^{(-2 * d * x - 2 * c)} + a + 2 * b - 2 * \text{sqrt}((a + b) * b)) / (a * e^{(-2 * d * x - 2 * c)} + a + 2 * b + 2 * \text{sqrt}((a + b) * b))) / (\text{sqrt}((a + b) * b) * a^2 * d) + \frac{1}{2} * (a * b + 2 * b^2) * (d * x + c) / (a^3 * d) + \frac{1}{64} * (8 * b * e^{(-2 * d * x - 2 * c)} - a * e^{(-4 * d * x - 4 * c)}) / (a^2 * d) + \frac{1}{16} * (a^2 * b + 8 * a * b^2 + 8 * b^3) * \log((a * e^{(-2 * d * x - 2 * c)} + a + 2 * b - 2 * \text{sqrt}((a + b) * b)) / (a * e^{(-2 * d * x - 2 * c)} + a + 2 * b + 2 * \text{sqrt}((a + b) * b))) / (\text{sqrt}((a + b) * b) * a^3 * d)$

mupad [B] time = 2.43, size = 328, normalized size = 2.80

$$\frac{x(3a^2 + 12ab + 8b^2)}{8a^3} - \frac{e^{-4c-4dx}}{64ad} + \frac{e^{4c+4dx}}{64ad} + \frac{e^{-2c-2dx}(a+b)}{8a^2d} - \frac{e^{2c+2dx}(a+b)}{8a^2d} + \frac{\sqrt{b} \ln\left(\frac{4b(a+b)^3(2ab+a^2+a^2e^{2c+2dx})}{\dots}\right)}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2), x)`

[Out] $(x*(12*a*b + 3*a^2 + 8*b^2))/(8*a^3) - \exp(-4*c - 4*d*x)/(64*a*d) + \exp(4*c + 4*d*x)/(64*a*d) + (\exp(-2*c - 2*d*x)*(a + b))/(8*a^2*d) - (\exp(2*c + 2*d*x)*(a + b))/(8*a^2*d) + (b^{1/2}*\log((4*b*(a + b)^3*(2*a*b + a^2 + a^2*\exp(2*c + 2*d*x) + 8*b^2*\exp(2*c + 2*d*x) + 8*a*b*\exp(2*c + 2*d*x)))/a^8 - (8*b^{3/2}*(a + b)^{7/2}*(a + 2*a*\exp(2*c + 2*d*x) + 4*b*\exp(2*c + 2*d*x)))/a^8*(a + b)^{3/2})/(2*a^3*d) - (b^{1/2}*\log((8*b^{3/2}*(a + b)^{7/2}*(a + 2*a*\exp(2*c + 2*d*x) + 4*b*\exp(2*c + 2*d*x)))/a^8 + (4*b*(a + b)^3*(2*a*b + a^2 + a^2*\exp(2*c + 2*d*x) + 8*b^2*\exp(2*c + 2*d*x) + 8*a*b*\exp(2*c + 2*d*x)))/a^8)*(a + b)^{3/2})/(2*a^3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2), x)`

[Out] `Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)`

$$3.26 \quad \int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=71

$$\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}$$

[Out] $-(a+b)*\cosh(d*x+c)/a^2/d+1/3*\cosh(d*x+c)^3/a/d+(a+b)*\arctan(\cosh(d*x+c)*a^{(1/2)}/b^{(1/2)})*b^{(1/2)}/a^{(5/2)}/d$

Rubi [A] time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4133, 459, 321, 205}

$$-\frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} + \frac{\cosh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(5/2)*d) - ((a + b)*Cosh[c + d*x])/(a^2*d) + Cosh[c + d*x]^3/(3*a*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_))*sin[(e_.) + (f_.)*(x_)^(m_.)], x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)}{3ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2} dx, x, \cosh(c+dx)\right)}{ad} \\
&= -\frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \cosh(c+dx)\right)}{a^2d} \\
&= \frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [C] time = 2.23, size = 372, normalized size = 5.24

$$(a \cosh(2(c+dx)) + a + 2b) \left(2a^{3/2}\sqrt{b} \cosh(3(c+dx)) + 3(a^2 + 8ab + 8b^2) \tan^{-1} \left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right) (\sqrt{a-i\sqrt{a+b}} \sqrt{c+dx})}{\sqrt{a-i\sqrt{a+b}}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b] - 3*a^2*(ArcTan[(Sqrt[a] - I*Sqrt[a + b])*Tanh[(c + d*x)/2]]/Sqrt[b] + ArcTan[(Sqrt[a] + I*Sqrt[a + b])*Tanh[(c + d*x)/2]]/Sqrt[b])) - 6*Sqrt[a]*Sqrt[b]*(3*a + 4*b)*Cosh[c + d*x] + 2*a^(3/2)*Sqrt[b]*Cosh[3*(c + d*x)])/(48*a^(5/2)*Sqrt[b]*d*(b + a*Cosh[c + d*x]^2))

fricas [B] time = 0.49, size = 1246, normalized size = 17.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*(3*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 + 12*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 6*(a*c

```

osh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c)
*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d
*x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3), 1
/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c
)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)*s
inh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*sinh
(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*(3
*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 - 24*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(b/a)*arctan(1/2*(a*cosh(
d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4
*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a
)/b) + 24*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt
(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 6*(a*co
sh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))*
sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[-
65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[36,86]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[-59,-45]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[15,66]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[55,80]Undef/Unsigned Inf encountered in limitEvaluation ti
me: 1.19Limit: Max order reached or unable to make series expansion Error:
Bad Argument Value
```

maple [B] time = 0.34, size = 261, normalized size = 3.68

$$-\frac{1}{3da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3} - \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} + \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} + \frac{b}{da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} + \frac{1}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/3/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b+1/3/d/a/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d*b^2/a^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\left(3ae^{4c} + 4be^{4c}\right)e^{4dx} + 3\left(3ae^{2c} + 4be^{2c}\right)e^{2dx} - ae^{6dx+6c} - a\right)e^{(-3dx-3c)}}{24a^2d} + \frac{1}{8} \int \frac{16\left(\left(abe^{3c} + b^2e^{3c}\right)e^{3dx} - (a^2b + b^2a)e^{3dx} + a^3 + b^3\right)}{a^3e^{4dx+4c} + a^3 + b^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="maxima")
```

```
[Out] -1/24*(3*(3*a*e^(4*c) + 4*b*e^(4*c))*e^(4*d*x) + 3*(3*a*e^(2*c) + 4*b*e^(2*c))*e^(2*d*x) - a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integrate(16*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x)))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.84, size = 473, normalized size = 6.66

$$\frac{e^{-3c-3dx}}{24ad} \left(2 \operatorname{atan} \left(\frac{a^6 e^{dx} e^c \left(\frac{4(2a^4bd\sqrt{a^2b+2ab^2+b^3}+2a^2b^3d\sqrt{a^2b+2ab^2+b^3}+4a^3b^2d\sqrt{a^2b+2ab^2+b^3})}{a^{11}d^2(a+b)} + \frac{2(b^4\sqrt{a^5d^2}+3a^2b^2\sqrt{a^5d^2}+3ab^3\sqrt{a^5d^2})}{a^8d\sqrt{b(a+b)^2}\sqrt{a^5d^2}} \right)}{4a^2b+8ab^2+4b^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2), x)
```

```
[Out] exp(-3*c - 3*d*x)/(24*a*d) - ((2*atan((a^6*exp(d*x)*exp(c))*((4*(2*a^4*b*d*(2*a*b^2 + a^2*b + b^3)^(1/2) + 2*a^2*b^3*d*(2*a*b^2 + a^2*b + b^3)^(1/2) + 4*a^3*b^2*d*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(a^11*d^2*(a + b)) + (2*(b^4*(a^5*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*b*(a^5*d^2)^(1/2)))/(a^8*d*(b*(a + b)^2)^(1/2)*(a^5*d^2)^(1/2)))*(a^5*d^2)^(1/2))/(8*a*b^2 + 4*a^2*b + 4*b^3) + (2*exp(3*c)*exp(3*d*x)*(b^4*(a^5*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*b*(a^5*d^2)^(1/2)))/(a^2*d*(b*(a + b)^2)^(1/2)*(8*a*b^2 + 4*a^2*b + 4*b^3))) - 2*atan((exp(d*x)*exp(c)*(a + b)*(a^5*d^2)^(1/2))/(2*a^2*d*(b*(a + b)^2)^(1/2))))*(2*a*b^2 + a^2*b + b^3)^(1/2))/(2*(a^5*d^2)^(1/2)) + exp(3*c + 3*d*x)/(24*a*d) - (exp(c + d*x)*(3*a + 4*b))/(8*a^2*d) - (exp(-c - d*x)*(3*a + 4*b))/(8*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

$$3.27 \quad \int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=75

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{x(a+2b)}{2a^2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

[Out] $-1/2*(a+2*b)*x/a^2+1/2*\cosh(d*x+c)*\sinh(d*x+c)/a/d+\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}*(a+b)^{(1/2)}/a^2/d$

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 471, 522, 206, 208}

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{x(a+2b)}{2a^2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] $-((a + 2*b)*x)/(2*a^2) + (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[a + b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])])/(a^2*d) + (\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*a*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p},

x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{2ad} \\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2ad} \\ &= -\frac{(a+2b)x}{2a^2} + \frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} \end{aligned}$$

Mathematica [B] time = 1.00, size = 236, normalized size = 3.15

$$\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b) \left(\frac{(a^2+8ab+8b^2)(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)}{d\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}} - \frac{1}{a^2} \right)$$

$$16(a+b\operatorname{sech}^2(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(-ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)) + (-4*(a + 2*b)*x + ((a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])])*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*a*Cosh[2*d*x]*Sinh[2*c])/d + (2*a*Cosh[2*c]*Sinh[2*d*x])/d/a^2))/(16*(a + b*Sech[c + d*x]^2))

fricas [B] time = 0.48, size = 805, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [-1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(a*b + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)

*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), -1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*sqrt(-a*b - b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]

giac [B] time = 1.60, size = 132, normalized size = 1.76

$$\frac{\frac{4(dx+c)(a+2b)}{a^2} - \frac{e^{2dx+2c}}{a} - \frac{(2ae^{2dx+2c}+4be^{2dx+2c}-a)e^{-2dx-2c}}{a^2} - \frac{8(ab+b^2)\arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] -1/8*(4*(d*x + c)*(a + 2*b)/a^2 - e^(2*d*x + 2*c)/a - (2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^2 - 8*(a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2))/d

maple [B] time = 0.32, size = 383, normalized size = 5.11

$$\frac{1}{2da\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{1}{2da\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b}{da^2} - \frac{1}{2da\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)

[Out] 1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b-1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))

maxima [B] time = 0.42, size = 352, normalized size = 4.69

$$\frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}ad} - \frac{dx+c}{2ad} + \frac{e^{(2dx+2c)}}{8ad} - \frac{e^{(-2dx-2c)}}{8ad} - \frac{b \log\left(ae^{(4dx+4c)} + 2(a+2b)e^{(2dx+2c)} + a\right)}{4a^2d} + \frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] -1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) - 1/2*(d*x + c)/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log

$$g(ae^{4dx+4c} + 2(a+2b)e^{2dx+2c} + a)/(a^{2d}) + 1/4b \log(2(a+2b)e^{-2dx-2c} + ae^{-4dx-4c} + a)/(a^{2d}) + 1/8(a^2b + 2b^2) \log((ae^{2dx+2c} + a + 2b - 2\sqrt{(a+b)b})/(ae^{2dx+2c} + a + 2b + 2\sqrt{(a+b)b}))/(\sqrt{(a+b)b}a^{2d}) - 1/8(a^2b + 2b^2) \log((ae^{-2dx-2c} + a + 2b - 2\sqrt{(a+b)b})/(ae^{-2dx-2c} + a + 2b + 2\sqrt{(a+b)b}))/(\sqrt{(a+b)b}a^{2d})$$

mupad [B] time = 2.03, size = 276, normalized size = 3.68

$$\frac{e^{2c+2dx}}{8ad} - \frac{e^{-2c-2dx}}{8ad} - \frac{x(a+2b)}{2a^2} - \frac{\sqrt{b} \ln(2ab + a^2 + a^2 e^{2c+2dx} + 8b^2 e^{2c+2dx} - 2a\sqrt{b}\sqrt{a+b} - 8b^{3/2} e^{2c+2dx})}{2a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2), x)

[Out] $\frac{\exp(2c + 2dx)}{8ad} - \frac{\exp(-2c - 2dx)}{8ad} - \frac{x(a + 2b)}{2a^2} - \frac{(b^{1/2} \log(2ab + a^2 + a^2 \exp(2c + 2dx) + 8b^2 \exp(2c + 2dx) - 2ab^{1/2}(a + b)^{1/2} - 8b^{3/2} \exp(2c + 2dx) * (a + b)^{1/2} + 8ab \exp(2c + 2dx) - 4ab^{1/2} \exp(2c + 2dx) * (a + b)^{1/2})) * (a + b)^{1/2}}{2a^2 d} + \frac{(b^{1/2} \log(2ab + a^2 + a^2 \exp(2c + 2dx) + 8b^2 \exp(2c + 2dx) + 2ab^{1/2}(a + b)^{1/2} + 8b^{3/2} \exp(2c + 2dx) * (a + b)^{1/2} + 8ab \exp(2c + 2dx) + 4ab^{1/2} \exp(2c + 2dx) * (a + b)^{1/2})) * (a + b)^{1/2}}{2a^2 d}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2), x)

[Out] Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)

$$3.28 \quad \int \frac{\sinh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=47

$$\frac{\cosh(c+dx)}{ad} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

[Out] $\cosh(d*x+c)/a/d - \arctan(\cosh(d*x+c)*a^{(1/2)}/b^{(1/2)})*b^{(1/2)}/a^{(3/2)}/d$

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4133, 321, 205}

$$\frac{\cosh(c+dx)}{ad} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $-(\sqrt{b} \operatorname{ArcTan}[\sqrt{a} \operatorname{Cosh}[c + d*x]]/\sqrt{b})/(a^{(3/2)*d}) + \operatorname{Cosh}[c + d*x]/(a*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^(m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \cosh(c+dx)\right)}{ad} \\ &= -\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d} + \frac{\cosh(c+dx)}{ad} \end{aligned}$$

Mathematica [C] time = 1.02, size = 328, normalized size = 6.98

$$\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{a \left(\tan^{-1} \left(\frac{\sqrt{a} - i\sqrt{a+b} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}} \right) + \tan^{-1} \left(\frac{\sqrt{a} + i\sqrt{a+b} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}} \right) \right)}{\sqrt{b}} \right) - \frac{(a+4b) \left(\tan^{-1} \left(\frac{\sqrt{a} - i\sqrt{a+b} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}} \right) + \tan^{-1} \left(\frac{\sqrt{a} + i\sqrt{a+b} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}} \right) \right)}{\sqrt{b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $((-(((a + 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) * Sinh[c] * Tanh[(d*x)/2] + Cosh[c] * (Sqrt[a] - I*Sqrt[a + b] * Sqrt[(Cosh[c] - Sinh[c])^2]) * Tanh[(d*x)/2])))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b] * Sqrt[(Cosh[c] - Sinh[c])^2]) * Sinh[c] * Tanh[(d*x)/2] + Cosh[c] * (Sqrt[a] + I*Sqrt[a + b] * Sqrt[(Cosh[c] - Sinh[c])^2]) * Tanh[(d*x)/2])))/Sqrt[b]))/Sqrt[b]) + (a*(ArcTan[(Sqrt[a] - I*Sqrt[a + b] * Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a + b] * Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[b] + 4*Sqrt[a]*Cosh[c + d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2/(8*a^(3/2)*d*(a + b*Sech[c + d*x]^2))$

fricas [B] time = 0.51, size = 595, normalized size = 12.66

$$\sqrt{\frac{-b}{a}} (\cosh(dx + c) + \sinh(dx + c)) \log \left(\frac{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a-2b) \cosh(dx+c)^2 + 2(3a-b) \cosh(dx+c) \sinh(dx+c) + a \sinh(dx+c)^2}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a-2b) \cosh(dx+c)^2 + 2(3a-b) \cosh(dx+c) \sinh(dx+c) + a \sinh(dx+c)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] $[1/2*(\sqrt{-b/a}*(\cosh(d*x + c) + \sinh(d*x + c))*\log((a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a - 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c))*\sqrt{-b/a} + a)/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a) + \cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)/(a*d*\cosh(d*x + c) + a*d*\sinh(d*x + c)), 1/2*(2*\sqrt{b/a}*(\cosh(d*x + c) + \sinh(d*x + c))*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a + 4*b)*\sinh(d*x + c))*\sqrt{b/a}/b) - 2*\sqrt{b/a}*(\cosh(d*x + c) + \sinh(d*x + c))*\arctan(1/2*(a*\cosh(d*x + c) + a*\sinh(d*x + c))*\sqrt{b/a}/b) + \cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)/(a*d*\cosh(d*x + c) + a*d*\sinh(d*x + c))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
 lynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
 l with parameters. This might be wrong.The choice was done assuming [a,b]=[
 -65,-82]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
 Warning, need to choose a branch for the root of a polynomial with paramete
 rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[22,73]Undef/Unsigned Inf
 encountered in limitEvaluation time: 0.7Limit: Max order reached or unable
 to make series expansion Error: Bad Argument Value

maple [A] time = 0.08, size = 44, normalized size = 0.94

$$\frac{b \arctan\left(\frac{\operatorname{sech}(dx+c)b}{\sqrt{ab}}\right)}{da\sqrt{ab}} + \frac{1}{da \operatorname{sech}(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x)

[Out] 1/d/a*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1/2))+1/d/a/sech(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(e^{(2dx+2c)} + 1)e^{-dx-c}}{2ad} - \frac{1}{2} \int \frac{4(b e^{(3dx+3c)} - b e^{(dx+c)})}{a^2 e^{(4dx+4c)} + a^2 + 2(a^2 e^{(2c)} + 2 a b e^{(2c)}) e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
 + 3*c) - b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
 *e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 0.14, size = 42, normalized size = 0.89

$$\frac{\cosh(c + dx)}{ad} - \frac{b \operatorname{atan}\left(\frac{a \cosh(c+dx)}{\sqrt{ab}}\right)}{ad\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2),x)

[Out] cosh(c + d*x)/(a*d) - (b*atan((a*cosh(c + d*x))/(a*b)^(1/2)))/(a*d*(a*b)^(1
 /2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2),x)

[Out] Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2), x)

$$3.29 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=55

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a} d(a+b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

[Out] $-\operatorname{arctanh}(\cosh(d*x+c))/(a+b)/d+\operatorname{arctan}(\cosh(d*x+c)*a^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(a+b)/d/a^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4133, 481, 206, 205}

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a} d(a+b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $(\operatorname{Sqrt}[b]*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Cosh}[c + d*x])/ \operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[a]*(a + b)*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/((a + b)*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 481

Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m-n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m-n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*sin[(e_.) + (f_.)*(x_)^(n_)^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)} dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{(a+b)d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \cosh(c+dx)\right)}{(a+b)d}$$

$$= \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)d}$$

Mathematica [C] time = 0.93, size = 232, normalized size = 4.22

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right) \left(\sqrt{a-i\sqrt{a+b}} \sqrt{(\cosh(c)-\sinh(c))^2} + \cosh(c) \left(\sqrt{a-i\sqrt{a+b}} \sqrt{(\cosh(c)-\sinh(c))^2} \tanh\left(\frac{dx}{2}\right)\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b} \tan^{-1}\left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right) \left(\sqrt{a+i\sqrt{a+b}} \sqrt{(\cosh(c)-\sinh(c))^2} + \cosh(c) \left(\sqrt{a+i\sqrt{a+b}} \sqrt{(\cosh(c)-\sinh(c))^2} \tanh\left(\frac{dx}{2}\right)\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}}$$

$d(a+b)$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])/Sqrt[b]])/Sqrt[a] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]/((a + b)*d)

fricas [B] time = 0.51, size = 533, normalized size = 9.69

$$\left[\sqrt{-\frac{b}{a}} \log\left(\frac{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a-2b) \cosh(dx+c)^2 + 2(3a \cosh(dx+c)^2 + a-2b) \sinh(dx+c)^2 + 4(a \cosh(dx+c)^3 + a \sinh(dx+c)^3 + a \cosh(dx+c) + (3a \cosh(dx+c)^2 + a) \sinh(dx+c)) \sqrt{-b/a} + a}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a+2b) \cosh(dx+c)^2 + 2(3a \cosh(dx+c)^2 + a+2b) \sinh(dx+c)^2 + 4(a \cosh(dx+c)^3 + a \sinh(dx+c)^3 + a \cosh(dx+c) + (3a \cosh(dx+c)^2 + a) \sinh(dx+c)) \sqrt{b/a} + a}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d), -(sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b - sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-13,-93]Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.25, size = 67, normalized size = 1.22

$$\frac{b \arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2a - 2b}{4\sqrt{ab}}\right)}{d(a+b)\sqrt{ab}} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2),x)

[Out] 1/d*b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)} + 1\right)e^{(-c)}\right)}{ad + bd} + \frac{\log\left(\left(e^{(dx+c)} - 1\right)e^{(-c)}\right)}{ad + bd} + 2 \int \frac{be^{(3dx+3c)} - be^{(dx+c)}}{a^2 + ab + \left(a^2e^{(4c)} + abe^{(4c)}\right)e^{(4dx)} + 2\left(a^2e^{(2c)} + 3abe^{(2c)}\right)e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d + b*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d + b*d) + 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 2.18, size = 616, normalized size = 11.20

$$\frac{2 \operatorname{atan}\left(\frac{e^{dx} e^c \left(b^4 \sqrt{-a^2 d^2 - 2 a b d^2 - b^2 d^2} + 16 a^2 b^2 \sqrt{-a^2 d^2 - 2 a b d^2 - b^2 d^2} + 8 a b^3 \sqrt{-a^2 d^2 - 2 a b d^2 - b^2 d^2}\right)}{16 d a^3 b^2 + 24 d a^2 b^3 + 9 d a b^4 + d b^5}\right)}{\sqrt{-a^2 d^2 - 2 a b d^2 - b^2 d^2}} \sqrt{b} \left(2 \operatorname{atan}\left(\frac{e^{dx} e^c \left(\frac{\dots}{a^5}\right)}{\dots}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)

[Out] -(2*atan((exp(d*x)*exp(c))*(b^4*(-a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 16*a^2*b^2*(-a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 8*a*b^3*(-a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2)))/(b^5*d + 24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*d)))/(-a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan((exp(d*x)*exp(c))*((64*(2*b^(7/2)*d + 8*a^2*b^(3/2)*d + 10*a*b^(5/2)*d))/(a^5*(a + b)*

$$a*d^2*(a + b)^2)^{(1/2)}*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)) + (32*(b^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)} + 4*a*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)))/(a^5*b^{(1/2)}*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2))} + (32*\exp(3*c)*\exp(3*d*x)*(b^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)} + 4*a*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)))/(a^5*b^{(1/2)}*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2))}*(a^6*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)} + a^5*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2)))/(256*a*b + 64*b^2)) - 2*atan((\exp(d*x)*\exp(c)*(a*d^2*(a + b)^2)^{(1/2))/(2*b^{(1/2)}*d*(a + b))))/(2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^{(1/2))}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2), x)

[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2), x)

$$3.30 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=53

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}} - \frac{\operatorname{coth}(c+dx)}{d(a+b)}$$

[Out] $-\operatorname{coth}(d*x+c)/(a+b)/d+\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)/(a+b)^{(3/2)}/d}$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4132, 325, 208}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}} - \frac{\operatorname{coth}(c+dx)}{d(a+b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c+d*x]^2/(a+b*\operatorname{Sech}[c+d*x]^2), x]$

[Out] $(\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c+d*x])/(\operatorname{Sqrt}[a+b])])/((a+b)^{(3/2)*d}) - \operatorname{Coth}[c+d*x]/((a+b)*d)$

Rule 208

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 325

$\operatorname{Int}[(c_+)*(x_+)^{(m_+)}*((a_+ + (b_+)*(x_+)^n)^{(p_+)}), x_Symbol] \rightarrow \operatorname{Simp}[(c*x)^{(m+1)}*(a+b*x^n)^{(p+1)}/(a*c*(m+1)), x] - \operatorname{Dist}[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), \operatorname{Int}[(c*x)^{(m+n)}*(a+b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{LtQ}[m, -1] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 4132

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{sec}[(e_+ + (f_+)*(x_+)]^n)^{(p_+)}*\sin[(e_+ + (f_+)*(x_+)]^m), x_Symbol] \rightarrow \operatorname{With}\{\operatorname{ff} = \operatorname{FreeFactors}[\operatorname{Tan}[e+f*x], x]\}, \operatorname{Dist}[\operatorname{ff}^{(m+1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*\operatorname{ExpandToSum}[a+b*(1+\operatorname{ff}^2*x^2)^{(n/2)}, x]^p)/(1+\operatorname{ff}^2*x^2)^{(m/2+1)}, x], x, \operatorname{Tan}[e+f*x]/\operatorname{ff}], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \operatorname{IntegerQ}[m/2] \ \&\& \operatorname{IntegerQ}[n/2]$

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c+dx)}{(a+b)d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} \\ &= \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} - \frac{\operatorname{coth}(c+dx)}{(a+b)d} \end{aligned}$$

Mathematica [B] time = 0.75, size = 179, normalized size = 3.38

$$\frac{\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\sqrt{a + b} \operatorname{csch}(c) \sinh(dx) \sqrt{b(\cosh(c) - \sinh(c))^4} \operatorname{csch}(c + dx) + b(\cosh(c) - \sinh(c)) \right)}{2d(a + b)^{3/2} \sqrt{b(\cosh(c) - \sinh(c))^4} (a + b \operatorname{sech}^2(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b]*Csch[c]*Csch[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x])/(2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.56, size = 588, normalized size = 11.09

$$\left[\frac{(\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) \sqrt{\frac{b}{a+b}} \log \left(\frac{a^2 \cosh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c) + a^2 \sinh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c) + a^2 \sinh(dx+c)^4}{2((a + b) \cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) \sqrt{b/(a + b)}} \right)}{2((a + b) \cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) \sqrt{b/(a + b)}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 4)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d), ((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 2)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d)]

giac [A] time = 0.63, size = 75, normalized size = 1.42

$$\frac{b \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}(a+b)} - \frac{2}{(a+b)(e^{(2dx+2c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)) - 2/((a + b)*(e^(2*d*x + 2*c) - 1)))/d

maple [B] time = 0.30, size = 147, normalized size = 2.77

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d(a + b)} - \frac{1}{2d(a + b) \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)} - \frac{\sqrt{b} \ln\left(-\sqrt{a + b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{a + b}\right)}{2d(a + b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x)`

[Out]
$$-1/2/d/(a+b)*\tanh(1/2*d*x+1/2*c)-1/2/d/(a+b)/\tanh(1/2*d*x+1/2*c)-1/2/d*b^(1/2)/(a+b)^(3/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d*b^(1/2)/(a+b)^(3/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$$

maxima [B] time = 0.43, size = 100, normalized size = 1.89

$$-\frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2\sqrt{(a+b)b}(a+b)d} + \frac{2}{((a+b)e^{(-2dx-2c)} - a - b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out]
$$-1/2*b*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*(a + b)*d) + 2/(((a + b)*e^{(-2*d*x - 2*c)} - a - b)*d)$$

mupad [B] time = 2.47, size = 847, normalized size = 15.98

$$\frac{\sqrt{b} \operatorname{atan}\left(\frac{e^{2c} e^{2dx} \left(2\left(8b^{5/2} \sqrt{-a^3 d^2 - 3a^2 b d^2 - 3ab^2 d^2 - b^3 d^2} + 8ab^{3/2} \sqrt{-a^3 d^2 - 3a^2 b d^2 - 3ab^2 d^2 - b^3 d^2} + a^2 \sqrt{b}\right)}{a^5 d (a+b)^3 (a^2 + 2ab + b^2) \sqrt{-a^3 d^2 - 3a^2 b d^2 - 3ab^2 d^2 - b^3 d^2}}\right)}{2(e^{2c+2dx} - 1)(ad + bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)`

[Out]
$$-2/((\exp(2*c + 2*d*x) - 1)*(a*d + b*d)) - (b^(1/2)*\operatorname{atan}(((\exp(2*c)*\exp(2*d*x))*((2*(8*b^(5/2))*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 8*a*b^(3/2))*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^(1/2))*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2)))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(8*b^4*d + 16*a^2*b^2*d + 20*a*b^3*d + 4*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))) + (2*(2*a*b^(3/2))*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^(1/2))*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(4*a^2*b^2*d + 2*a*b^3*d + 2*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))*(a^5*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^4*b*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^3*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^3*b^2*(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*b))/(-a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

$$3.31 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2} + \frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

[Out] 1/2*(a-b)*arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 471, 522, 206, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2} + \frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] -((Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a + b)^2*d) + ((a - b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)^(n_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x

], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(b+ax^2)} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d} - \frac{\operatorname{Subst}\left(\int \frac{b-ax^2}{(1-x^2)(b+ax^2)} dx, x, \cosh(c + dx)\right)}{2(a + b)d} \\ &= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d} + \frac{(a - b)\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c + dx)\right)}{2(a + b)^2d} - \frac{(ab)\operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, \cosh(c + dx)\right)}{2(a + b)^2d} \\ &= -\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{(a + b)^2d} + \frac{(a - b)\tanh^{-1}(\cosh(c + dx))}{2(a + b)^2d} - \frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d} \end{aligned}$$

Mathematica [C] time = 2.05, size = 338, normalized size = 3.89

$$\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left((a + b)\operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) + (a + b)\operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) + 8\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] $-1/16*((a + 2*b + a*\operatorname{Cosh}[2*(c + d*x)])*(8*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*\operatorname{ArcTan}[\frac{(\operatorname{Sqrt}[a] - I*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2])*\operatorname{Sinh}[c]*\operatorname{Tanh}[(d*x)/2] + \operatorname{Cosh}[c]*(\operatorname{Sqrt}[a] - I*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2])*\operatorname{Tanh}[(d*x)/2])}{\operatorname{Sqrt}[b]} + 8*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*\operatorname{ArcTan}[\frac{(\operatorname{Sqrt}[a] + I*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2])*\operatorname{Sinh}[c]*\operatorname{Tanh}[(d*x)/2] + \operatorname{Cosh}[c]*(\operatorname{Sqrt}[a] + I*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2])*\operatorname{Tanh}[(d*x)/2])}{\operatorname{Sqrt}[b]}] + (a + b)*\operatorname{Csch}[(c + d*x)/2]^2 - 4*a*\operatorname{Log}[\operatorname{Cosh}[(c + d*x)/2]] + 4*b*\operatorname{Log}[\operatorname{Cosh}[(c + d*x)/2]] + 4*a*\operatorname{Log}[\operatorname{Sinh}[(c + d*x)/2]] - 4*b*\operatorname{Log}[\operatorname{Sinh}[(c + d*x)/2]] + (a + b)*\operatorname{Sech}[(c + d*x)/2]^2)*\operatorname{Sech}[c + d*x]^2)/((a + b)^2*d*(a + b*\operatorname{Sech}[c + d*x]^2))$

fricas [B] time = 0.46, size = 1881, normalized size = 21.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] $[-1/2*(2*(a + b)*\cosh(d*x + c)^3 + 6*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a + b)*\sinh(d*x + c)^3 - (\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*\operatorname{sqrt}(-a*b)*\operatorname{log}((a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a - 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c) + \cosh(d*x + c))*\operatorname{sqrt}(-a*b) + a)/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4) + \operatorname{csch}(d*x + c)^3]/d$

$$\begin{aligned}
& 4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + \\
& a) + 2*(a + b)*\cosh(d*x + c) - ((a - b)*\cosh(d*x + c)^4 + 4*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - b)*\sinh(d*x + c)^4 - 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a - b)*\cosh(d*x + c)^2 - a + b)*\sinh(d*x + c)^2 + 4*((a - b)*\cosh(d*x + c)^3 - (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a - b)*\cosh(d*x + c)^4 + 4*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - b)*\sinh(d*x + c)^4 - 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a - b)*\cosh(d*x + c)^2 - a + b)*\sinh(d*x + c)^2 + 4*((a - b)*\cosh(d*x + c)^3 - (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c))/ \\
& ((a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*\sinh(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*\sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(a + b)*\cosh(d*x + c)^3 + 6*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a + b)*\sinh(d*x + c)^3 + 2*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/b) - 2*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*\sqrt{a*b}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a + 4*b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)) + 2*(a + b)*\cosh(d*x + c) - ((a - b)*\cosh(d*x + c)^4 + 4*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - b)*\sinh(d*x + c)^4 - 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a - b)*\cosh(d*x + c)^2 - a + b)*\sinh(d*x + c)^2 + 4*((a - b)*\cosh(d*x + c)^3 - (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a - b)*\cosh(d*x + c)^4 + 4*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - b)*\sinh(d*x + c)^4 - 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a - b)*\cosh(d*x + c)^2 - a + b)*\sinh(d*x + c)^2 + 4*((a - b)*\cosh(d*x + c)^3 - (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c))/ \\
& ((a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*\sinh(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*\sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2*a*b + b^2)*d*\cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-65,-82]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56] Undef/Unsigned Inf encountered in limitEvaluation time: 0.45Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.32, size = 134, normalized size = 1.54

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d(a+b)} - \frac{ab \arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2a - 2b}{4\sqrt{ab}}\right)}{d(a+b)^2\sqrt{ab}} - \frac{1}{8d(a+b)\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a}{2d(a+b)^2} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2), x)

[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d*a*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*a+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a-b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\left(a^2d+2abd+b^2d\right)} - \frac{(a-b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\left(a^2d+2abd+b^2d\right)} - \frac{e^{(3dx+3c)}+e^{(dx+c)}}{ad+bd+\left(ade^{(4c)}+bde^{(4c)}\right)e^{(4dx)}-2\left(ade^{(2c)}+bde^{(2c)}\right)e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*(a-b)*log((e^(d*x+c)+1)*e^(-c))/(a^2*d+2*a*b*d+b^2*d) - 1/2*(a-b)*log((e^(d*x+c)-1)*e^(-c))/(a^2*d+2*a*b*d+b^2*d) - (e^(3*d*x+3*c)+e^(d*x+c))/(a*d+b*d+(a*d*e^(4*c)+b*d*e^(4*c))*e^(4*d*x) - 2*(a*d*e^(2*c)+b*d*e^(2*c))*e^(2*d*x) - 8*integrate(1/4*(a*b*e^(3*d*x+3*c)-a*b*e^(d*x+c))/(a^3+2*a^2*b+a*b^2+(a^3*e^(4*c)+2*a^2*b*e^(4*c)+a*b^2*e^(4*c))*e^(4*d*x)+2*(a^3*e^(2*c)+4*a^2*b*e^(2*c)+5*a*b^2*e^(2*c)+2*b^3*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 3.07, size = 1586, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c+d*x)^3*(a+b/cosh(c+d*x)^2)), x)

[Out] ((a*b)^(1/2)*(2*atan(((exp(d*x)*exp(c))*((64*(2*b^5*d*(a*b)^(1/2)+2*a*b^4*d*(a*b)^(1/2)+2*a^4*b*d*(a*b)^(1/2)+2*a^3*b^2*d*(a*b)^(1/2)))/(a^4*(a+b)^3*(d^2*(a+b)^4)^(1/2)*(2*a*b+a^2+b^2)*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))+(32*(a*b^3*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+a^3*b*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)-a^2*b^2*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))/(a^3*d*(a*b)^(1/2)*(a+b)^5*(2*a*b+a^2+b^2)*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))+(32*exp(3*c)*exp(3*d*x)*(a*b^3*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+a^3*b*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)-a^2*b^2*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))/(a^3*d*(a*b)^(1/2)*(a+b)^5*(2*a*b+a^2+b^2)*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))+(a^8*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+5*a^7*b*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+a^3*b^5*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+5*a^4*b^4*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+10*a^5*b^3*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+10*a^6*b^2*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+10*a^7*b*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)+10*a^8*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2)))/(a^3*d*(a*b)^(1/2)*(a+b)^5*(2*a*b+a^2+b^2)*(a^4*d^2+b^4*d^2+4*a*b^3*d^2+4*a^3*b*d^2+6*a^2*b^2*d^2)^(1/2))

```

*b^2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))
)/(64*a^2*b - 64*a*b^2 + 64*b^3)) - 2*atan((a*exp(d*x)*exp(c)*(d^2*(a + b)^
4)^(1/2))/(2*d*(a*b)^(1/2)*(a + b)^2)))/(2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^
2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) - exp(c + d*x)/((exp(2*c + 2*d*x) -
1)*(a*d + b*d)) - (atan((exp(d*x)*exp(c)*(b^7*(- a^4*d^2 - b^4*d^2 - 4*a*b
^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 3*a*b^6*(- a^4*d^2 - b^4*d^2
- 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 5*a^2*b^5*(- a^4*d^2 -
b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 5*a^3*b^4*(-
a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 3*a^
4*b^3*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/
2) - a^5*b^2*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d
^2)^(1/2))))/(b^8*d*(a^2 - 2*a*b + b^2)^(1/2) + 2*a^3*b^5*d*(a^2 - 2*a*b + b
^2)^(1/2) + a^6*b^2*d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2)
)/(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) -
(2*exp(c + d*x))/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2), x)

[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2), x)

$$3.32 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=75

$$-\frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}} - \frac{\operatorname{coth}^3(c+dx)}{3d(a+b)} + \frac{a \operatorname{coth}(c+dx)}{d(a+b)^2}$$

[Out] $a*\operatorname{coth}(d*x+c)/(a+b)^2/d-1/3*\operatorname{coth}(d*x+c)^3/(a+b)/d-a*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}/(a+b)^{(5/2)}/d$

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 453, 325, 208}

$$-\frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}} - \frac{\operatorname{coth}^3(c+dx)}{3d(a+b)} + \frac{a \operatorname{coth}(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] $-((a*\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])])/((a + b)^{(5/2)*d})) + (a*\operatorname{Coth}[c + d*x])/((a + b)^2*d) - \operatorname{Coth}[c + d*x]^3/(3*(a + b)*d)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m+1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{coth}^3(c+dx)}{3(a+b)d} - \frac{a \operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{(a+b)d} \\
&= \frac{a \operatorname{coth}(c+dx)}{(a+b)^2 d} - \frac{\operatorname{coth}^3(c+dx)}{3(a+b)d} - \frac{(ab) \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} \\
&= -\frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2} d} + \frac{a \operatorname{coth}(c+dx)}{(a+b)^2 d} - \frac{\operatorname{coth}^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [B] time = 2.08, size = 216, normalized size = 2.88

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(\frac{1}{4} \sqrt{a+b} \operatorname{csch}(c) \sqrt{b(\cosh(c) - \sinh(c))^4} \operatorname{csch}^3(c+dx) ((b-2a) \sin(2(c+dx)) + 2b) + 6d(a+b)^{5/2} \sqrt{b(c+dx)}\right)}{6d(a+b)^{5/2} \sqrt{b(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a*b*ArcTanh[(Sech[d*x]*Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a + b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(6*a*Sinh[d*x] - 3*b*Sinh[2*c + d*x] + (-2*a + b)*Sinh[2*c + 3*d*x]))/4)/(6*(a + b)^(5/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.52, size = 1753, normalized size = 23.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [-1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh(d*x + c)^4 + 24*a*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 - 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - a)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 48*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 8*a + 4*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*d*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*d*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*d*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*d*sqrt(b/(a + b)))]

2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d*x + c)^3 + 6*b*sinh(d*x + c)^4 + 12*a*cosh(d*x + c)^2 + 12*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - a)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 24*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 4*a + 2*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [A] time = 0.67, size = 123, normalized size = 1.64

$$\frac{3ab \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^2+2ab+b^2)\sqrt{-ab-b^2}} + \frac{2(3be^{(4dx+4c)+6ae^{(2dx+2c)-2a+b}})}{(a^2+2ab+b^2)(e^{(2dx+2c)-1})^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] -1/3*(3*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2)))/(a^2 + 2*a*b + b^2)*sqrt(-a*b - b^2) + 2*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 2*a + b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^3))/d

maple [B] time = 0.36, size = 258, normalized size = 3.44

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{24d(a+b)^2} - \frac{\left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b}{24d(a+b)^2} + \frac{3a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{8d(a+b)^2} - \frac{\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) b}{8d(a+b)^2} + \frac{a\sqrt{b} \ln \left(-\sqrt{a+b} \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \right)}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x)

[Out] -1/24/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3*b+3/8/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)-1/8/d/(a+b)^2*tanh(1/2*d*x+1/2*c)*b+1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b)/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*a-1/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*b

maxima [B] time = 0.46, size = 195, normalized size = 2.60

$$\frac{ab \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2(a^2+2ab+b^2)\sqrt{(a+b)bd}} - \frac{2(6ae^{(-2dx-2c)}+3be^{(-4dx-4c)}-2a+b)}{3(a^2+2ab+b^2-3(a^2+2ab+b^2)e^{(-2dx-2c)}+3(a^2+2ab+b^2)e^{(-4dx-4c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) - 2/3*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)

mupad [B] time = 2.17, size = 248, normalized size = 3.31

$$\frac{a\sqrt{b} \ln\left(\frac{4be^{2c+2dx}}{(a+b)^2} - \frac{2\sqrt{b}(a+ae^{2c+2dx}+2be^{2c+2dx})}{(a+b)^{5/2}}\right)}{2d(a+b)^{5/2}} - \frac{8}{3(ad+bd)(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1)} - \frac{1}{e^{2c+2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)

[Out] (a*b^(1/2)*log((4*b*exp(2*c + 2*d*x))/(a + b)^2 - (2*b^(1/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2)) - 8/(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*b)/((exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d)) - 4/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (a*b^(1/2)*log((4*b*exp(2*c + 2*d*x))/(a + b)^2 + (2*b^(1/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2),x)

[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2), x)

$$3.33 \quad \int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=194

$$\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{3b(3a+4b)\tanh(c+dx)}{8a^3d(a-b\tanh^2(c+dx)+b)} - \frac{(5a+6b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b\tanh^2(c+dx)+b)}$$

[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^4/d-1/8*(5*a+6*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)-3/8*b*(3*a+4*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{3x(a^2+8ab+8b^2)}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{3b(3a+4b)\tanh(c+dx)}{8a^3d(a-b\tanh^2(c+dx)+b)} - \frac{(5a+6b)\sinh(c+dx)}{8a^2d(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*sqrt[b]*sqrt[a + b]*(a + 2*b)*ArcTanh[(sqrt[b]*Tanh[c + d*x])/sqrt[a + b]])/(2*a^4*d) - ((5*a + 6*b)*Cosh[c + d*x]*sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^3*sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)) - (3*b*(3*a + 4*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,

c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_)*sin[(e_.) + (f_.)*(x_)^(m_)], x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f*ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+5b)x^2}{(1-x^2)^2(a+b-x^2)^2} dx, x, \tanh(c + dx)\right)}{4ad}$$

$$= -\frac{(5a + 6b) \cosh(c + dx) \sinh(c + dx)}{8a^2d(a + b - b \tanh^2(c + dx))} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{3bx^2}{(1-x^2)^2(a+b-x^2)^2} dx, x, \tanh(c + dx)\right)}{8a^3d}$$

$$= -\frac{(5a + 6b) \cosh(c + dx) \sinh(c + dx)}{8a^2d(a + b - b \tanh^2(c + dx))} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))} - \frac{3b \operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-x^2)^2} dx, x, \tanh(c + dx)\right)}{8a^3d}$$

$$= \frac{3(a^2 + 8ab + 8b^2)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{(5a + 6b)}{8a^2d(a + b - b \tanh^2(c + dx))}$$

Mathematica [C] time = 14.25, size = 1330, normalized size = 6.86

$$\frac{(\cosh(2c + 2dx)a + a + 2b)^2 \left(16x + \frac{(a^3 - 6ba^2 - 24b^2a - 16b^3) \tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c) - \sinh(2c))((a+2b)\sinh(dx) - a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c) - \sinh(c))^4}}\right)}{b(a+b)^{3/2}d\sqrt{b(\cosh(c) - \sinh(c))^4}} \right)}{256a^2(b \operatorname{sech}^2(c + dx) + a)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out]
$$-1/256*((a + 2*b + a*\cosh[2*c + 2*d*x])^2*\operatorname{sech}[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*\operatorname{ArcTanh}[(\operatorname{sech}[d*x]*(\cosh[2*c] - \sinh[2*c]))*((a + 2*b)*\sinh[d*x] - a*\sinh[2*c + d*x]))/(2*\sqrt{a + b}*\sqrt{b*(\cosh[c] - \sinh[c])^4}))*(\cosh[2*c] - \sinh[2*c]))/(b*(a + b)^{(3/2)}*d*\sqrt{b*(\cosh[c] - \sinh[c])^4}) + ((a^2 + 8*a*b + 8*b^2)*\operatorname{sech}[2*c]*((a + 2*b)*\sinh[2*c] - a*\sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*\cosh[2*(c + d*x)])))/((a^2*(a + b*\operatorname{sech}[c + d*x]^2)^2) + (3*(a + 2*b + a*\cosh[2*c + 2*d*x])^2*\operatorname{sech}[c + d*x]^4*((a + 2*b)*\operatorname{ArcTanh}[(\sqrt{b}*\tanh[c + d*x])/(\sqrt{a + b})])/(8*b^{(3/2)}*(a + b)^{(3/2)}*d) - (a*\sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*\cosh[2*(c + d*x)])))/((128*(a + b*\operatorname{sech}[c + d*x]^2)^2) + ((a + 2*b + a*\cosh[2*c + 2*d*x])^2*\operatorname{sech}[c + d*x]^4*((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 - 1920*a*b^4 - 768*b^5)*(((-1/8*I)*\operatorname{ArcTan}[\operatorname{sech}[d*x]*(((-1/2*I)*\cosh[2*c]))/(\sqrt{a + b}*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]}) + ((I/2)*\sinh[2*c])/(\sqrt{a + b}*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]})))*(-a*\sinh[d*x]) - 2*b*\sinh[d*x] + a*\sinh[2*c + d*x])*\cosh[2*c])/(a^4*b*\sqrt{a + b}*d*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]}) + ((I/8)*\operatorname{ArcTan}[\operatorname{sech}[d*x]*(((-1/2*I)*\cosh[2*c]))/(\sqrt{a + b}*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]})]) + ((I/2)*\sinh[2*c])/(\sqrt{a + b}*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]})*(-a*\sinh[d*x]) - 2*b*\sinh[d*x] + a*\sinh[2*c + d*x]))*\sinh[2*c])/(a^4*b*\sqrt{a + b}*d*\sqrt{b*\cosh[4*c] - b*\sinh[4*c]})))/((a + b) + (\operatorname{sech}[2*c]*(160*a^4*b*d*x*\cosh[2*c] + 1248*a^3*b^2*d*x*\cosh[2*c] + 3392*a^2*b^3*d*x*\cosh[2*c] + 3840*a*b^4*d*x*\cosh[2*c] + 1536*b^5*d*x*\cosh[2*c] + 80*a^4*b*d*x*\cosh[2*d*x] + 464*a^3*b^2*d*x*\cosh[2*d*x] + 768*a^2*b^3*d*x*\cosh[2*d*x] + 384*a*b^4*d*x*\cosh[2*d*x] + 80*a^4*b*d*x*\cosh[4*c + 2*d*x] + 464*a^3*b^2*d*x*\cosh[4*c + 2*d*x] + 768*a^2*b^3*d*x*\cosh[4*c + 2*d*x] + 384*a*b^4*d*x*\cosh[4*c + 2*d*x] + a^5*\sinh[2*c] + 34*a^4*b*\sinh[2*c] + 224*a^3*b^2*\sinh[2*c] + 576*a^2*b^3*\sinh[2*c] + 640*a*b^4*\sinh[2*c] + 256*b^5*\sinh[2*c] - a^5*\sinh[2*d*x] - 62*a^4*b*\sinh[2*d*x] - 318*a^3*b^2*\sinh[2*d*x] - 512*a^2*b^3*\sinh[2*d*x] - 256*a*b^4*\sinh[2*d*x] - 30*a^4*b*\sinh[4*c + 2*d*x] - 158*a^3*b^2*\sinh[4*c + 2*d*x] - 256*a^2*b^3*\sinh[4*c + 2*d*x] - 128*a*b^4*\sinh[4*c + 2*d*x] - 12*a^4*b*\sinh[2*c + 4*d*x] - 36*a^3*b^2*\sinh[2*c + 4*d*x] - 24*a^2*b^3*\sinh[2*c + 4*d*x] - 12*a^4*b*\sinh[6*c + 4*d*x] - 36*a^3*b^2*\sinh[6*c + 4*d*x] - 24*a^2*b^3*\sinh[6*c + 4*d*x] + 2*a^4*b*\sinh[4*c + 6*d*x] + 2*a^3*b^2*\sinh[4*c + 6*d*x] + 2*a^4*b*\sinh[8*c + 6*d*x] + 2*a^3*b^2*\sinh[8*c + 6*d*x]))/(8*a^4*b*(a + b)*d*(a + 2*b + a*\cosh[2*c + 2*d*x])))/((128*(a + b*\operatorname{sech}[c + d*x]^2)^2)$$

fricas [B] time = 0.56, size = 5169, normalized size = 26.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$[1/64*(a^3*\cosh(d*x + c)^{12} + 12*a^3*\cosh(d*x + c)*\sinh(d*x + c)^{11} + a^3*\sinh(d*x + c)^{12} - 6*(a^3 + 2*a^2*b)*\cosh(d*x + c)^{10} + 6*(11*a^3*\cosh(d*x + c)^2 - a^3 - 2*a^2*b)*\sinh(d*x + c)^{10} + 20*(11*a^3*\cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^8 + (495*a^3*\cosh(d*x + c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x - 270*(a^3 + 2*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*a^3*\cosh(d*x + c)^5 - 90*(a^3 + 2*a^2*b)*\cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 16*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*\cosh(d*x + c)^6 + 4*(231*a^3*\cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*\cosh(d*x + c)^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(99*a^3*\cosh(d*x + c)^7 - 189*(a^3 + 2*a^2*b)*\cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2$$

$$\begin{aligned}
& *b + 8*a*b^2)*d*x)*\cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 \\
& + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + (15 \\
& *a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + \\
& c)^4 + (495*a^3*\cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*\cosh(d*x + c)^6 - 7 \\
& 0*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x \\
& + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x \\
& + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3) \\
&)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(55*a^3*\cosh(d*x + c)^9 - 180*(\\
& a^3 + 2*a^2*b)*\cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 \\
& + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 + 8*b^3 \\
& + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*\cosh(d*x + c)^3 + (15*a^3 + \\
& 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*\cosh(d*x + c)^2 + 2*(33*a^3*\cosh(d*x \\
& + c)^10 - 135*(a^3 + 2*a^2*b)*\cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*b + 6 \\
& 4*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^6 + 120*(4*a^2*b \\
& + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*\cosh(d*x + \\
& c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a \\
& ^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 48*((a^2 + 2*a*b)*c \\
& osh(d*x + c)^8 + 8*(a^2 + 2*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 + 2*a \\
& *b)*\sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x + c)^6 + 2*(14*(a^2 \\
& + 2*a*b)*\cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*\sinh(d*x + c)^6 + 4*(14*(a^ \\
& 2 + 2*a*b)*\cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d* \\
& x + c)^5 + (a^2 + 2*a*b)*\cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*\cosh(d*x + c)^ \\
& 4 + 30*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^4 \\
& + 4*(14*(a^2 + 2*a*b)*\cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x \\
& + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(14*(a^2 + 2*a*b) \\
& *\cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x + c)^4 + 3*(a^2 + 2*a* \\
& b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*\cosh(d*x + c)^7 + \\
& 3*(a^2 + 4*a*b + 4*b^2)*\cosh(d*x + c)^5 + (a^2 + 2*a*b)*\cosh(d*x + c)^3)*\si \\
& nh(d*x + c))*\sqrt{a*b + b^2}*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c) \\
& *\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + \\
& 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b \\
& ^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + \\
& 4*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 \\
& + a + 2*b)*\sqrt{a*b + b^2}))/((a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x \\
& + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x \\
& + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(\\
& d*x + c))*\sinh(d*x + c) + a)) + 4*(3*a^3*\cosh(d*x + c)^11 - 15*(a^3 + 2*a^2 \\
& *b)*\cosh(d*x + c)^9 - 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + \\
& 8*a*b^2)*d*x)*\cosh(d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + \\
& 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*\cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b + \\
& 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^3 + 3*(a^3 + 2* \\
& a^2*b)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*d*\cosh(d*x + c)^8 + 8*a^5*d*\cosh(\\
& d*x + c)*\sinh(d*x + c)^7 + a^5*d*\sinh(d*x + c)^8 + a^5*d*\cosh(d*x + c)^4 + \\
& 2*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^6 + 2*(14*a^5*d*\cosh(d*x + c)^2 + (a^5 + \\
& 2*a^4*b)*d)*\sinh(d*x + c)^6 + 4*(14*a^5*d*\cosh(d*x + c)^3 + 3*(a^5 + 2*a^4* \\
& b)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + (70*a^5*d*\cosh(d*x + c)^4 + a^5*d + 3 \\
& 0*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(14*a^5*d*\cosh(d*x \\
& + c)^5 + a^5*d*\cosh(d*x + c) + 10*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^3)*\sinh(\\
& d*x + c)^3 + 2*(14*a^5*d*\cosh(d*x + c)^6 + 3*a^5*d*\cosh(d*x + c)^2 + 15*(a^ \\
& 5 + 2*a^4*b)*d*\cosh(d*x + c)^4)*\sinh(d*x + c)^2 + 4*(2*a^5*d*\cosh(d*x + c)^ \\
& 7 + a^5*d*\cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^5)*\sinh(d*x + \\
& c)), 1/64*(a^3*\cosh(d*x + c)^12 + 12*a^3*\cosh(d*x + c)*\sinh(d*x + c)^11 + \\
& a^3*\sinh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*\cosh(d*x + c)^10 + 6*(11*a^3*\cosh(\\
& d*x + c)^2 - a^3 - 2*a^2*b)*\sinh(d*x + c)^10 + 20*(11*a^3*\cosh(d*x + c)^3 - \\
& 3*(a^3 + 2*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64 \\
& *a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*\cosh(d*x + c)^8 + (495*a^3*\cosh(\\
& d*x + c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d* \\
& x - 270*(a^3 + 2*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*a^3*\cosh(d
\end{aligned}$$

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*x + c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*
b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16
*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*
cosh(d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x
+ c)^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16
*b^3)*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^
3 + 2*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 +
8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 +
3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
+ (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^
6 - 70*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*co
sh(d*x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^
2)*d*x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 -
180*(a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24
*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 +
8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a
^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*co
sh(d*x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*b
+ 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 96*((a^2 + 2*a
*b)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(1
4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x
+ c)^4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
c)^4 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2
*a*b)*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 +
2*a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)
^7 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^
3)*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*
b + b^2)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2*b)*cosh(d*x + c)^9
- 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2
+ 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 + 2*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x
+ c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 30*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x + c)^5 + a^5*d*cosh
(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(14*a
^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^5 + 2*a^4*b)*d*cosh(
d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^7 + a^5*d*cosh(d*x +
c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5)*sinh(d*x + c))]

```

giac [A] time = 3.73, size = 323, normalized size = 1.66

$$\frac{24(a^2+8ab+8b^2)(dx+c)}{a^4} - \frac{(18a^2e^{(4dx+4c)}+144abc^{(4dx+4c)}+144b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}-16abc^{(2dx+2c)}+a^2)e^{(-4dx-4c)}}{a^4} - \frac{96(a^2b+3ab^2+2b^3)a}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (24 \cdot (a^2 + 8 \cdot a \cdot b + 8 \cdot b^2) \cdot (d \cdot x + c) / a^4 - (18 \cdot a^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 14 \cdot 4 \cdot a \cdot b \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 144 \cdot b^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} - 8 \cdot a^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 16 \cdot a \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a^2) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} / a^4 - 96 \cdot (a^2 \cdot b + 3 \cdot a \cdot b^2 + 2 \cdot b^3) \cdot \arctan(1/2 \cdot (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b) / \sqrt{-a \cdot b - b^2}) / (\sqrt{-a \cdot b - b^2}) \cdot a^4 + (a^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} - 8 \cdot a^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 16 \cdot a \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) / a^4 + 64 \cdot (a^2 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 3 \cdot a \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot b^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a^2 \cdot b + a \cdot b^2) / ((a \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 2 \cdot a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 4 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a) \cdot a^4) / d$

maple [B] time = 0.40, size = 1034, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-\frac{9}{4} \cdot \frac{d}{a^3 \cdot b^{(3/2)}} \cdot (a+b)^{(1/2)} \cdot \ln((a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{(1/2)}) - \frac{1}{d} \cdot \frac{1}{a^3} \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^2 \cdot b^{-1} / a^3 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot b^{-3} / d \cdot a^3 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot b^{-3} / d \cdot a^4 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot b^2 + 1/d \cdot a^3 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^2 \cdot b^{-1} / d \cdot a^3 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot b^3 / d \cdot a^3 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot b^3 / d \cdot a^4 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) \cdot b^2 + 1/4 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^4 + 1/2 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^3 - 1/8 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^2 - 3/8 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) - 3/8 \cdot d \cdot a^2 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) - 1/4 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^4 + 1/2 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^3 + 1/8 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^2 - 3/8 \cdot d \cdot a^2 \cdot (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) + 3/8 \cdot d \cdot a^2 \cdot \ln(\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1) - 3/2 \cdot d \cdot a^4 \cdot b^{(5/2)} \cdot (a+b)^{(1/2)} \cdot \ln((a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{(1/2)}) + 9/4 \cdot d \cdot a^3 \cdot b^{(3/2)} \cdot (a+b)^{(1/2)} \cdot \ln(-(a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - (a+b)^{(1/2)}) - 1/d \cdot a^2 \cdot b / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c))^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 1/d \cdot a^3 \cdot b^2 / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c))^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - 1/d \cdot a^2 \cdot b / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c))^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1/d \cdot a^3 \cdot b^2 / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c))^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + 3/4 \cdot d \cdot a^2 \cdot b^{(1/2)} \cdot (a+b)^{(1/2)} \cdot \ln(-(a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - (a+b)^{(1/2)}) - 3/4 \cdot d \cdot a^2 \cdot b^{(1/2)} \cdot (a+b)^{(1/2)} \cdot \ln((a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{(1/2)}) + 3/2 \cdot d \cdot a^4 \cdot b^{(5/2)} \cdot (a+b)^{(1/2)} \cdot \ln(-(a+b)^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{(1/2)} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - (a+b)^{(1/2)})$

maxima [B] time = 0.51, size = 1299, normalized size = 6.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-\frac{1}{64} \cdot (3 \cdot a^3 \cdot b + 42 \cdot a^2 \cdot b^2 + 88 \cdot a \cdot b^3 + 48 \cdot b^4) \cdot \log((a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a + b) \cdot b}) / (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a + b) \cdot b})) / ((a^5 + a^4 \cdot b) \cdot \sqrt{(a + b) \cdot b} \cdot d) - \frac{1}{16} \cdot (3 \cdot a^2 \cdot b + 12 \cdot a \cdot b^2 + 8 \cdot b^3) \cdot \log((a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a + b) \cdot b}) / (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a + b) \cdot b})) / ((a^4 + a^3 \cdot b) \cdot \sqrt{(a + b) \cdot b} \cdot d) + \frac{1}{64} \cdot (3 \cdot a^3 \cdot b + 42 \cdot a^2 \cdot b^2 + 88 \cdot a \cdot b^3 + 48 \cdot b^4) \cdot \log((a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a + b) \cdot b}) / (a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a + b) \cdot b})) / ((a^5$

$$\begin{aligned}
& + a^4 b \sqrt{(a+b)b} d + \frac{1}{16} (3a^2 b + 12ab^2 + 8b^3) \log\left(\frac{a e^{(-2dx - 2c)} + a + 2b - 2\sqrt{(a+b)b}}{a e^{(-2dx - 2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) \\
& + \frac{3}{32} (3ab + 2b^2) \log\left(\frac{a e^{(-2dx - 2c)} + a + 2b - 2\sqrt{(a+b)b}}{a e^{(-2dx - 2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) \\
& + \frac{1}{16} (a^3 b + 8a^2 b^2 + 8ab^3 + (a^3 b + 18a^2 b^2 + 48ab^3 + 32b^4) e^{(2dx + 2c)}) \\
& + \frac{1}{16} (a^3 b + 8a^2 b^2 + 8ab^3 + (a^3 b + 18a^2 b^2 + 48ab^3 + 32b^4) e^{(-2dx - 2c)}) \\
& + \frac{1}{4} (a^2 b + 2ab^2 + (a^2 b + 8ab^2 + 8b^3) e^{(2dx + 2c)}) \\
& + \frac{1}{4} (a^2 b + 2ab^2 + (a^2 b + 8ab^2 + 8b^3) e^{(-2dx - 2c)}) \\
& + \frac{3}{8} (ab + (ab + 2b^2) e^{(-2dx - 2c)}) \\
& + \frac{1}{8} e^{(2dx + 2c)} / (a^2 d) + \frac{1}{8} e^{(-2dx - 2c)} / (a^2 d) + \frac{1}{2} b \log(a e^{(4dx + 4c)} + 2(a + 2b) e^{(2dx + 2c)} + a) / (a^3 d) \\
& - \frac{1}{2} b \log(2(a + 2b) e^{(-2dx - 2c)} + a e^{(-4dx - 4c)} + a) / (a^3 d) + \frac{1}{64} (a e^{(4dx + 4c)} - 16b e^{(2dx + 2c)}) / (a^3 d) \\
& + \frac{1}{64} (16b e^{(-2dx - 2c)} - a e^{(-4dx - 4c)}) / (a^3 d) + \frac{1}{4} (ab + 3b^2) \log(a e^{(4dx + 4c)} + 2(a + 2b) e^{(2dx + 2c)} + a) / (a^4 d) \\
& - \frac{1}{4} (ab + 3b^2) \log(2(a + 2b) e^{(-2dx - 2c)} + a e^{(-4dx - 4c)} + a) / (a^4 d)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \sinh(c + dx)^4}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.34 \quad \int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=114

$$\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d(a\cosh^2(c+dx)+b)} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d}$$

[Out] $-(a+2*b)*\cosh(d*x+c)/a^3/d+1/3*\cosh(d*x+c)^3/a^2/d-1/2*b*(a+b)*\cosh(d*x+c)/a^3/d/(b+a*\cosh(d*x+c)^2)+1/2*(3*a+5*b)*\arctan(\cosh(d*x+c)*a^{(1/2)}/b^{(1/2)})*b^{(1/2)}/a^{(7/2)}/d$

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4133, 455, 1153, 205}

$$-\frac{b(a+b)\cosh(c+dx)}{2a^3d(a\cosh^2(c+dx)+b)} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} + \frac{\cosh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] $(\text{Sqrt}[b]*(3*a + 5*b)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cosh}[c + d*x])/\text{Sqrt}[b]])/(2*a^{(7/2)}*d) - ((a + 2*b)*\text{Cosh}[c + d*x])/(a^3*d) + \text{Cosh}[c + d*x]^3/(3*a^2*d) - (b*(a + b)*\text{Cosh}[c + d*x])/(2*a^3*d*(b + a*\text{Cosh}[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{b(a+b)\cosh(c+dx)}{2a^3d(b+a\cosh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2} dx, x, \cosh(c+dx)\right)}{2a^3d} \\
&= -\frac{b(a+b)\cosh(c+dx)}{2a^3d(b+a\cosh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right) dx, x, \cosh(c+dx)\right)}{2a^3d} \\
&= -\frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d(b+a\cosh^2(c+dx))} + \frac{(b(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right))}{2a^{7/2}d} \\
&= \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d(b+a\cosh^2(c+dx))}
\end{aligned}$$

Mathematica [C] time = 4.82, size = 861, normalized size = 7.55

$$\frac{(\cosh(2(c+dx))a+a+2b)^2\operatorname{sech}^4(c+dx)}{b^{3/2}} \left(\frac{9 \tan^{-1}\left(\frac{\sqrt{a-i\sqrt{a+b}}\sqrt{(\cosh(c)-\sinh(c))^2}}{\sqrt{b}}\right) \sinh(c) \tanh\left(\frac{dx}{2}\right) + \cosh(c) \left(\sqrt{a-i\sqrt{a+b}}\sqrt{(\cosh(c)-\sinh(c))^2}\right)}{\sqrt{b}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((9*a^3*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b]))/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b] + 960*b^(3/2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b] + (9*a^3*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b]))/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b] + 960*b^(3/2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])]/Sqrt[b] - (9*a^3*ArcTan[(Sqrt[a] - I*Sqrt[a + b])*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] + I*Sqrt[a + b])*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - 96*Sqrt[a]*(3*a + 8*b)*Cosh[c]*Cosh[d*x] + 32*a^(3/2)*Cosh[3*c]*Cosh[3*d*x] - (384*a^(3/2)*b*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - (384*Sqrt[a]*b^2*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - 288*a^(3/2)*Sinh[c]*Sinh[d*x] - 768*Sqrt[a]*b*Sinh[c]*Sinh[d*x] + 32*a^(3/2)*Sinh[3*c]*Sinh[3*d*x]))/(1536*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.49, size = 3804, normalized size = 33.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
[Out] [1/24*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*si
nh(d*x + c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2
- 7*a^2 - 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 2
0*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d
*x + c)^6 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^
2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 -
14*(7*a^2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*
(105*a^2*cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2
+ 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x +
c)^4 + 8*(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(
13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cos
h(d*x + c)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b +
60*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7
*a^2 - 20*a*b)*sinh(d*x + c)^2 + 6*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*
a^2 + 5*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^
7 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b
)*cosh(d*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^4 + (3*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4
+ 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x +
c)^3 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*
cosh(d*x + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*
a^2 + 5*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4
+ 3*(3*a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh
(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cos
h(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*co
sh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(
a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^
2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + a
^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)*cosh(d*x + c)^7 - 6*(13*
a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 + 66*a*b + 60*b^2)*cosh(
d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x
+ c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 + a^4*d*sinh(d*x + c)^7 + a
^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^5 + (21*a^4*d*cosh
(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5 + 5*(7*a^4*d*cosh(d*x +
c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^4*d*cosh(
d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c) + 20*(a^4 + 2*a^3*b)*d*
cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d*x + c)^6 + 3*a^4*d*cosh(
d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x +
c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2 - 7*a^2
- 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^6
+ 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^2 - 13*a^
2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 - 14*(7*a^
2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c))*s
```

```

inh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*(105*a^2*
cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2 + 66*a*b
+ 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^4 + 8*
(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(13*a^2 +
66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cosh(d*x + c)
)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b + 60*b^2)*c
osh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7*a^2 - 20
*a*b)*sinh(d*x + c)^2 - 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*
a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)
^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d
*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3
*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 +
(21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x
+ c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*
a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*
a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d
*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*
b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)
/b) + 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*a*b)*cosh(d*x + c)
*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^2 + 6*a^2 + 22*a*
b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d*x + c)^3 + 2*(3*a
^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 5*a*b)*cosh
(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 + (21*(3*a^2 + 5*a*b)
)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
+ 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*a*b)*cosh(d*x + c)
^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d
*x + c))*sqrt(b/a)/b) + a^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)
*cosh(d*x + c)^7 - 6*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2
+ 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(
d*x + c))/(a^4*d*cosh(d*x + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 +
a^4*d*sinh(d*x + c)^7 + a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*
x + c)^5 + (21*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5
+ 5*(7*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x
+ c)^4 + (35*a^4*d*cosh(d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x
+ c)^2)*sinh(d*x + c)^3 + (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c)
) + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d
*x + c)^6 + 3*a^4*d*cosh(d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)
*sinh(d*x + c))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
```

rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-37,-59]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-67,22]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[80,-46]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-72,77]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[43,41]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[37,80]Undef/Unsigned Inf encountered in limitEvaluation time: 1.43Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.37, size = 561, normalized size = 4.92

$$\frac{1}{3d a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{1}{2d a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{1}{2d a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{2b}{d a^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$\begin{aligned} & -1/3/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)^2+1/ \\ & 2/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)+2/d/a^3/(\tanh(1/2*d*x+1/2*c)-1)*b+1/3/d/a^2 \\ & /(\tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a^2/(\tanh(1/2*d*x+1/2*c)+1) \\ & -2/d/a^3/(\tanh(1/2*d*x+1/2*c)+1)*b-1/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^2+1/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^2-1/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d/a^2*b/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/2/d/a^3*b^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 e^{(10 dx + 10 c)} + a^2 - (7 a^2 e^{(8 c)} + 20 a b e^{(8 c)}) e^{(8 dx)} - 2 (13 a^2 e^{(6 c)} + 66 a b e^{(6 c)} + 60 b^2 e^{(6 c)}) e^{(6 dx)} - 2 (13 a^2 e^{(4 c)} + 66 a b e^{(4 c)} + 60 b^2 e^{(4 c)}) e^{(4 dx)} - 2 (7 a^2 e^{(2 c)} + 20 a b e^{(2 c)}) e^{(2 dx)}}{24 (a^4 d e^{(7 dx + 7 c)} + a^4 d e^{(3 dx + 3 c)} + 2 (a^4 d e^{(5 c)} + 2 a^3 b d e^{(5 c)}) e^{(5 dx)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/24*(a^2*e^{(10*d*x + 10*c)} + a^2 - (7*a^2*e^{(8*c)} + 20*a*b*e^{(8*c)})*e^{(8*d*x)} \\ & - 2*(13*a^2*e^{(6*c)} + 66*a*b*e^{(6*c)} + 60*b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(13*a^2*e^{(4*c)} + 66*a*b*e^{(4*c)} + 60*b^2*e^{(4*c)})*e^{(4*d*x)} - (7*a^2*e^{(2*c)} \\ & + 20*a*b*e^{(2*c)})*e^{(2*d*x)})/(a^4*d*e^{(7*d*x + 7*c)} + a^4*d*e^{(3*d*x + 3*c)} \\ & + 2*(a^4*d*e^{(5*c)} + 2*a^3*b*d*e^{(5*c)})*e^{(5*d*x)}) + 1/8*integrate(8*((3*a*b*e^{(3*c)} + 5*b^2*e^{(3*c)})*e^{(3*d*x)} - (3*a*b*e^c + 5*b^2*e^c)*e^{(d*x)})/(a^4*e^{(4*d*x + 4*c)} + a^4 + 2*(a^4*e^{(2*c)} + 2*a^3*b*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \sinh(c + dx)^3}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.35 \quad \int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=131

$$\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} - \frac{x(a+4b)}{2a^3} + \frac{b\tanh(c+dx)}{a^2d(a-b\tanh^2(c+dx)+b)} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad(a-b\tanh^2(c+dx)+b)}$$

[Out] $-1/2*(a+4*b)*x/a^3+1/2*(3*a+4*b)*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}/a^3/d/(a+b)^{(1/2)}+1/2*\cosh(d*x+c)*\sinh(d*x+c)/a/d/(a+b-b*\tanh(d*x+c)^2)+b*\tanh(d*x+c)/a^2/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} + \frac{b\tanh(c+dx)}{a^2d(a-b\tanh^2(c+dx)+b)} - \frac{x(a+4b)}{2a^3} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2, x]

[Out] $-((a+4*b)*x)/(2*a^3) + (\operatorname{Sqrt}[b]*(3*a+4*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c+d*x])/(\operatorname{Sqrt}[a+b])])/(2*a^3*\operatorname{Sqrt}[a+b]*d) + (\operatorname{Cosh}[c+d*x]*\operatorname{Sinh}[c+d*x])/(2*a*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2)) + (b*\operatorname{Tanh}[c+d*x])/(a^2*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)]^(p_))*sin[(e_) + (f_)*(x_)^(m_)], x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a+b+3bx^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{b \tanh(c + dx)}{a^2d(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{-2(a+b)}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{b \tanh(c + dx)}{a^2d(a + b - b \tanh^2(c + dx))} - \frac{(a + 4b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= -\frac{(a + 4b)x}{2a^3} + \frac{\sqrt{b}(3a + 4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a + b}d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))}$$

Mathematica [B] time = 11.33, size = 791, normalized size = 6.04

$$\frac{\operatorname{sech}^4(c + dx)(a \cosh(2c + 2dx) + a + 2b)^2 \left(\frac{(a^2+8ab+8b^2)\operatorname{sech}(2c)((a+2b) \sinh(2c)-a \sinh(2dx))}{bd(a+b)(a \cosh(2(c+dx))+a+2b)} + \frac{(a^3-6a^2b-24ab^2-16b^3)(\cosh(2c)-\sinh(2c))}{(a+b)^2} \right)}{128a^2(a + b \operatorname{sech}^2(c + dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])))/(128*a^2*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-64*(a +
```

$$2*b)*x + ((-a^4 + 16*a^3*b + 144*a^2*b^2 + 256*a*b^3 + 128*b^4)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*sqrt[a + b]*sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*sqrt[b*(Cosh[c] - Sinh[c])^4]) + (16*a*Cosh[2*d*x]*Sinh[2*c])/d + (16*a*Cosh[2*c]*Sinh[2*d*x])/d - ((a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])))/(256*a^3*(a + b*Sech[c + d*x]^2)^2) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a*ArcTanh[(sqrt[b]*Tanh[c + d*x])/sqrt[a + b]])/(a + b)^(3/2)) + (sqrt[b]*(a + 2*b)*Sinh[2*(c + d*x)])/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])))))/(256*b^(3/2)*d*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-1/8*((a + 2*b)*ArcTanh[(sqrt[b]*Tanh[c + d*x])/sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) + (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])))))/(16*(a + b*Sech[c + d*x]^2)^2)$$

fricas [B] time = 0.50, size = 2925, normalized size = 22.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(14*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 2*((3*a^2 + 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 + 4*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c) + (a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - a^2 + 4*(2*a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^3 - (2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x + c)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d*sinh(d*x + c)^6 + a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4 + (15*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*(5*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4*d*cosh(d*x + c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*

```
(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)), 1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(14*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 4*((3*a^2 + 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 + 4*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b)))/b - a^2 + 4*(2*a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^3 - (2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x + c)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d*sinh(d*x + c)^6 + a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4 + (15*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*(5*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4*d*cosh(d*x + c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

giac [A] time = 2.05, size = 234, normalized size = 1.79

$$\frac{\frac{12(dx+c)(a+4b)}{a^3} - \frac{3e^{2dx+2c}}{a^2} - \frac{12(3ab+4b^2)\arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^3} - \frac{2a^2e^{6dx+6c}+8abe^{6dx+6c}+a^2e^{4dx+4c}-16b^2e^{4dx+4c}-4a^2e^{2dx+2c}}{(ae^{6dx+6c}+2ae^{4dx+4c}+4be^{4dx+4c}+ae^{2dx+2c})}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] -1/24*(12*(d*x + c)*(a + 4*b)/a^3 - 3*e^(2*d*x + 2*c)/a^2 - 12*(3*a*b + 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/sqrt(-a*b - b^2)*a^3) - (2*a^2*e^(6*d*x + 6*c) + 8*a*b*e^(6*d*x + 6*c) + a^2*e^(4*d*x + 4*c) - 16*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x + 2*c) - 3*a^2)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))*a^3)/d
```

maple [B] time = 0.37, size = 543, normalized size = 4.15

$$\frac{1}{2d a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{1}{2d a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2d a^2} + \frac{2 \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) b}{d a^3} - \frac{b}{2d a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)

[Out] $\frac{1}{2} \frac{d}{a^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2} + \frac{1}{2} \frac{d}{a^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)} + \frac{1}{2} \frac{d}{a^2} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1) + \frac{2}{d} \frac{1}{a^3} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1) * b - \frac{1}{2} \frac{d}{a^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2} + \frac{1}{2} \frac{d}{a^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)} - \frac{1}{2} \frac{d}{a^2} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1) - \frac{2}{d} \frac{1}{a^3} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1) * b + \frac{1}{d} \frac{1}{a^2} * b / (\tanh(\frac{1}{2}d*x+\frac{1}{2}c)^4 * a + b * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^4 + 2 * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 * a - 2 * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 * b + a + b) * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^3 + \frac{1}{d} \frac{1}{a^2} * b / (\tanh(\frac{1}{2}d*x+\frac{1}{2}c)^4 * a + b * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^4 + 2 * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 * a - 2 * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 * b + a + b) * \tanh(\frac{1}{2}d*x+\frac{1}{2}c) - \frac{3}{4} \frac{d}{a^2} * b^{(1/2)} / (a+b)^{(1/2)} * \ln(-(a+b)^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 + 2 * b^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c) - (a+b)^{(1/2)}) + \frac{3}{4} \frac{d}{a^2} * b^{(1/2)} / (a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 + 2 * b^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c) + (a+b)^{(1/2)}) - \frac{1}{d} \frac{1}{a^3} * b^{(3/2)} / (a+b)^{(1/2)} * \ln(-(a+b)^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 + 2 * b^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c) - (a+b)^{(1/2)}) + \frac{1}{d} \frac{1}{a^3} * b^{(3/2)} / (a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 + 2 * b^{(1/2)} * \tanh(\frac{1}{2}d*x+\frac{1}{2}c) + (a+b)^{(1/2)})$

maxima [B] time = 0.46, size = 696, normalized size = 5.31

$$\frac{(3a^2b + 12ab^2 + 8b^3) \log\left(\frac{ae^{(2dx+2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(2dx+2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^4 + a^3b)\sqrt{(a+b)bd}} - \frac{(3a^2b + 12ab^2 + 8b^3) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^4 + a^3b)\sqrt{(a+b)bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{16} * (3 * a^2 * b + 12 * a * b^2 + 8 * b^3) * \log((a * e^{(2 * d * x + 2 * c)} + a + 2 * b - 2 * \sqrt{(a + b) * b}) / (a * e^{(2 * d * x + 2 * c)} + a + 2 * b + 2 * \sqrt{(a + b) * b})) / ((a^4 + a^3 * b) * \sqrt{(a + b) * b} * d) - \frac{1}{16} * (3 * a^2 * b + 12 * a * b^2 + 8 * b^3) * \log((a * e^{(-2 * d * x - 2 * c)} + a + 2 * b - 2 * \sqrt{(a + b) * b}) / (a * e^{(-2 * d * x - 2 * c)} + a + 2 * b + 2 * \sqrt{(a + b) * b})) / ((a^4 + a^3 * b) * \sqrt{(a + b) * b} * d) - \frac{1}{8} * (3 * a * b + 2 * b^2) * \log((a * e^{(-2 * d * x - 2 * c)} + a + 2 * b - 2 * \sqrt{(a + b) * b}) / (a * e^{(-2 * d * x - 2 * c)} + a + 2 * b + 2 * \sqrt{(a + b) * b})) / ((a^3 + a^2 * b) * \sqrt{(a + b) * b} * d) - \frac{1}{4} * (a^2 * b + 2 * a * b^2 + (a^2 * b + 8 * a * b^2 + 8 * b^3) * e^{(2 * d * x + 2 * c)}) / ((a^5 + a^4 * b + (a^5 + a^4 * b) * e^{(4 * d * x + 4 * c)} + 2 * (a^5 + 3 * a^4 * b + 2 * a^3 * b^2) * e^{(2 * d * x + 2 * c)}) * d) + \frac{1}{4} * (a^2 * b + 2 * a * b^2 + (a^2 * b + 8 * a * b^2 + 8 * b^3) * e^{(-2 * d * x - 2 * c)}) / ((a^5 + a^4 * b + 2 * (a^5 + 3 * a^4 * b + 2 * a^3 * b^2) * e^{(-2 * d * x - 2 * c)} + (a^5 + a^4 * b) * e^{(-4 * d * x - 4 * c)}) * d) + \frac{1}{2} * (a * b + (a * b + 2 * b^2) * e^{(-2 * d * x - 2 * c)}) / ((a^4 + a^3 * b + 2 * (a^4 + 3 * a^3 * b + 2 * a^2 * b^2) * e^{(-2 * d * x - 2 * c)} + (a^4 + a^3 * b) * e^{(-4 * d * x - 4 * c)}) * d) - \frac{1}{2} * (d * x + c) / (a^2 * d) + \frac{1}{8} * e^{(2 * d * x + 2 * c)} / (a^2 * d) - \frac{1}{8} * e^{(-2 * d * x - 2 * c)} / (a^2 * d) - \frac{1}{2} * b * \log(a * e^{(4 * d * x + 4 * c)} + 2 * (a + 2 * b) * e^{(2 * d * x + 2 * c)} + a) / (a^3 * d) + \frac{1}{2} * b * \log(2 * (a + 2 * b) * e^{(-2 * d * x - 2 * c)} + a * e^{(-4 * d * x - 4 * c)} + a) / (a^3 * d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \sinh(c + dx)^2}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

$$3.36 \quad \int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=84

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3 \cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad(a \cosh^2(c+dx) + b)}$$

[Out] $3/2*\cosh(d*x+c)/a^{2/d}-1/2*\cosh(d*x+c)^3/a/d/(b+a*\cosh(d*x+c)^2)-3/2*\arctan(\cosh(d*x+c)*a^{(1/2)/b^{(1/2)}}*b^{(1/2)/a^{(5/2)/d}}$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4133, 288, 321, 205}

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3 \cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad(a \cosh^2(c+dx) + b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]

[Out] $(-3*\sqrt{b}*ArcTan[(\sqrt{a}*\cosh[c + d*x])/(\sqrt{b})])/(2*a^{(5/2)*d}) + (3*\cosh[c + d*x])/(2*a^{2*d}) - \cosh[c + d*x]^3/(2*a*d*(b + a*\cosh[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\cosh^3(c + dx)}{2ad(b + a \cosh^2(c + dx))} + \frac{3 \operatorname{Subst}\left(\int \frac{x^2}{b+ax^2} dx, x, \cosh(c + dx)\right)}{2ad} \\
&= \frac{3 \cosh(c + dx)}{2a^2d} - \frac{\cosh^3(c + dx)}{2ad(b + a \cosh^2(c + dx))} - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \cosh(c + dx)\right)}{2a^2d} \\
&= -\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3 \cosh(c + dx)}{2a^2d} - \frac{\cosh^3(c + dx)}{2ad(b + a \cosh^2(c + dx))}
\end{aligned}$$

Mathematica [C] time = 2.76, size = 479, normalized size = 5.70

$$\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b)^2 \left(\frac{32b \cosh(c+dx)}{a^2(a \cosh(2(c+dx))+a+2b)} + \frac{32 \cosh(c) \cosh(dx)}{a^2} + \frac{2 \left(-(a^2+24b^2) \tan^{-1}\left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right)}{\sqrt{a+2b}}\right) \right)}{a^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((32*Cosh[c]*Cosh[d*x])/
a^2 + (32*b*Cosh[c + d*x])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*(-((a
^2 + 24*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]))*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b])) - a^2*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 24*b^2*Ar
cTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
(d*x)/2]))/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2]
)/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]
] + 16*Sqrt[a]*b^(3/2)*Sinh[c]*Sinh[d*x])/(a^(5/2)*b^(3/2)))/(128*d*(a +
b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.54, size = 1780, normalized size = 21.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
[Out] [1/4*(2*a*cosh(d*x + c)^6 + 12*a*cosh(d*x + c)*sinh(d*x + c)^5 + 2*a*sinh(d
*x + c)^6 + 6*(a + 2*b)*cosh(d*x + c)^4 + 6*(5*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^4 + 8*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + 6*(a + 2*b)*cosh(d*x + c)^2 + 6*(5*a*cosh(d*x + c)^4 + 6*(a +
2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5
*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x
+ c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d
```

```

*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) +
(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt
(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b
)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqr
t(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sin
h(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(
d*x + c) + a)) + 12*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a +
2*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
+ 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a*cosh(d*x + c)^6
+ 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 + 3*(a + 2*b)*cosh(
d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^4 + 4*(5*a*cos
h(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + 2*b)*cos
h(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c
)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(
d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a +
2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*
cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b)
- 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x +
c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh
(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)
^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x +
c))*sqrt(b/a)/b) + 6*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
+ 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[67,31]Warning, need
to choose a branch for the root of a polynomial with parameters. This migh
```

t be wrong. The choice was done assuming [a,b]=[-88,66] Undef/Unsigned Inf encountered in limitEvaluation time: 0.78 Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.17, size = 74, normalized size = 0.88

$$\frac{b \operatorname{sech}(dx+c)}{2d a^2 (a+b \operatorname{sech}(dx+c))^2} + \frac{3b \arctan\left(\frac{\operatorname{sech}(dx+c)b}{\sqrt{ab}}\right)}{2d a^2 \sqrt{ab}} + \frac{1}{d a^2 \operatorname{sech}(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/2/d/a^2*b*sech(d*x+c)/(a+b*sech(d*x+c)^2)+3/2/d/a^2*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1/2))+1/d/a^2/sech(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3(ae^{4c} + 2be^{4c})e^{4dx} + 3(ae^{2c} + 2be^{2c})e^{2dx} + ae^{6dx+6c} + a}{2(a^3de^{5dx+5c} + a^3de^{dx+c} + 2(a^3de^{3c} + 2a^2bde^{3c})e^{3dx})} - \frac{1}{2} \int \frac{6(be^{3dx+3c} - be^{dx+c})}{a^3e^{4dx+4c} + a^3 + 2(a^3e^{2c} + 2a^2be^{2c})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(3*(a*e^(4*c) + 2*b*e^(4*c))*e^(4*d*x) + 3*(a*e^(2*c) + 2*b*e^(2*c))*e^(2*d*x) + a*e^(6*d*x + 6*c) + a)/(a^3*d*e^(5*d*x + 5*c) + a^3*d*e^(d*x + c) + 2*(a^3*d*e^(3*c) + 2*a^2*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(6*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 1.65, size = 71, normalized size = 0.85

$$\frac{b \cosh(c+dx)}{2(d a^3 \cosh(c+dx)^2 + b d a^2)} + \frac{\cosh(c+dx)}{a^2 d} - \frac{3\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{2 a^{5/2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c+d*x)/(a+b/cosh(c+d*x)^2)^2,x)

[Out] (b*cosh(c+d*x))/(2*(a^3*d*cosh(c+d*x)^2 + a^2*b*d)) + cosh(c+d*x)/(a^2*d) - (3*b^(1/2)*atan((a^(1/2)*cosh(c+d*x))/b^(1/2)))/(2*a^(5/2)*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.37 \quad \int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=99

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2} - \frac{b\cosh(c+dx)}{2ad(a+b)(a\cosh^2(c+dx)+b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

[Out] $-\operatorname{arctanh}(\cosh(dx+c))/(a+b)^{2/d-1/2}b*\cosh(dx+c)/a/(a+b)/d/(b+a*\cosh(dx+c)^2)+1/2*(3*a+b)*\operatorname{arctan}(\cosh(dx+c)*a^{(1/2)}/b^{(1/2)})*b^{(1/2)}/a^{(3/2)}/(a+b)^{2/d}$

Rubi [A] time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4133, 470, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2} - \frac{b\cosh(c+dx)}{2ad(a+b)(a\cosh^2(c+dx)+b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] `Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2), x]`

[Out] `(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(3/2)*(a + b)^2*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^2*d) - (b*Cosh[c + d*x])/(2*a*(a + b)*d*(b + a*Cosh[c + d*x]^2))`

Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 470

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]`

Rule 522

`Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]`

Rule 4133

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^2)^2} dx, x, \operatorname{cosh}(c + dx)\right)}{d}$$

$$= -\frac{b \operatorname{cosh}(c + dx)}{2a(a + b)d(b + a \operatorname{cosh}^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)(b+ax^2)} dx, x, \operatorname{cosh}(c + dx)\right)}{2a(a + b)d}$$

$$= -\frac{b \operatorname{cosh}(c + dx)}{2a(a + b)d(b + a \operatorname{cosh}^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \operatorname{cosh}(c + dx)\right)}{(a + b)^2d} + \frac{b(3a + b)}{2a(a + b)d(b + a \operatorname{cosh}^2(c + dx))}$$

$$= \frac{\sqrt{b}(3a + b) \tan^{-1}\left(\frac{\sqrt{a} \operatorname{cosh}(c + dx)}{\sqrt{b}}\right)}{2a^{3/2}(a + b)^2d} - \frac{\tanh^{-1}(\operatorname{cosh}(c + dx))}{(a + b)^2d} - \frac{b \operatorname{cosh}(c + dx)}{2a(a + b)d(b + a \operatorname{cosh}^2(c + dx))}$$

Mathematica [C] time = 1.25, size = 377, normalized size = 3.81

$$\operatorname{sech}^3(c + dx)(a \operatorname{cosh}(2(c + dx)) + a + 2b) \left(\frac{\sqrt{b}(3a + b) \operatorname{sech}(c + dx)(a \operatorname{cosh}(2(c + dx)) + a + 2b) \tan^{-1}\left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right) \left(\sqrt{a - i\sqrt{a + b}} \sqrt{\operatorname{cosh}(c) - \sinh(c)}\right)}{a^{3/2}}\right)}{a^{3/2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(-2*b*(a + b))/a + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) - 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Log[Cosh[(c + d*x)/2]*Sech[c + d*x] + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Log[Sinh[(c + d*x)/2]*Sech[c + d*x]])/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.58, size = 2376, normalized size = 24.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```



```
[Out] [-1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 4*(a*b + b^2)*cosh(d*x + c) + 4*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a*b + b^2)*cosh(d*x + c)^3 + 6*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2)*sinh(d*x + c)^3 + ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) - ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(a*b + b^2)*cosh(d*x + c) + 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
```

c))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[89,-63]Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.34, size = 431, normalized size = 4.35

$$\frac{b \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d(a+b)^2 \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right)^2} + d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$-1/d*b/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^2+1/d*b^2/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/a*\tanh(1/2*d*x+1/2*c)^2-1/d*b/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d*b^2/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/a+3/2/d*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c))^2+2*a-2*b)/(a*b)^(1/2))+1/2/d*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c))^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^2*ln(\tanh(1/2*d*x+1/2*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{be^{(3dx+3c)} + be^{(dx+c)}}{a^3d + a^2bd + (a^3de^{(4c)} + a^2bde^{(4c)})e^{(4dx)} + 2(a^3de^{(2c)} + 3a^2bde^{(2c)} + 2ab^2de^{(2c)})e^{(2dx)}} \frac{\log((e^{(dx+c)} + 1)e^{(-c)})}{a^2d + 2abd + b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-(b*e^{(3*d*x + 3*c)} + b*e^{(d*x + c)})/(a^3*d + a^2*b*d + (a^3*d*e^{(4*c)} + a^2*b*d*e^{(4*c)})*e^{(4*d*x)} + 2*(a^3*d*e^{(2*c)} + 3*a^2*b*d*e^{(2*c)} + 2*a*b^2*d*e^{(2*c)})*e^{(2*d*x)}) - \log((e^{(d*x + c)} + 1)*e^{(-c)})/(a^2*d + 2*a*b*d + b^2*d) + \log((e^{(d*x + c)} - 1)*e^{(-c)})/(a^2*d + 2*a*b*d + b^2*d) + 2*\integrate(1/2*((3*a*b*e^{(3*c)} + b^2*e^{(3*c)})*e^{(3*d*x)} - (3*a*b*e^c + b^2*e^c)*e^{(d*x)})/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^4*e^{(2*c)} + 4*a^3*b*e^{(2*c)} + 5*a^2*b^2*e^{(2*c)} + 2*a*b^3*e^{(2*c)})*e^{(2*d*x)}), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4}{\sinh(c + dx) (a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)`

[Out] `int(cosh(c + d*x)^4/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**2, x)`

[Out] `Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)`

$$3.38 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=92

$$\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}} - \frac{3 \coth(c+dx)}{2d(a+b)^2} + \frac{\coth(c+dx)}{2d(a+b)(a-b \tanh^2(c+dx)+b)}$$

[Out] $-3/2*\coth(d*x+c)/(a+b)^2/d+3/2*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}/(a+b)^{(5/2)}/d+1/2*\coth(d*x+c)/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 290, 325, 208}

$$\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}} - \frac{3 \coth(c+dx)}{2d(a+b)^2} + \frac{\coth(c+dx)}{2d(a+b)(a-b \tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] $(3*\sqrt{b}*\operatorname{ArcTanh}[(\sqrt{b}*\operatorname{Tanh}[c + d*x])/(\sqrt{a + b})])/(2*(a + b)^{(5/2)*d}) - (3*\operatorname{Coth}[c + d*x])/(2*(a + b)^2*d) + \operatorname{Coth}[c + d*x]/(2*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{coth}(c+dx)}{2(a+b)d(a+b-b\tanh^2(c+dx))} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\
&= -\frac{3\operatorname{coth}(c+dx)}{2(a+b)^2d} + \frac{\operatorname{coth}(c+dx)}{2(a+b)d(a+b-b\tanh^2(c+dx))} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\
&= \frac{3\sqrt{b}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b}\operatorname{tanh}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{3\operatorname{coth}(c+dx)}{2(a+b)^2d} + \frac{\operatorname{coth}(c+dx)}{2(a+b)d(a+b-b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [B] time = 2.62, size = 220, normalized size = 2.39

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\operatorname{csch}(c)\sinh(dx)\operatorname{csch}(c+dx)(a\cosh(2(c+dx))+a+2b)+\frac{3b\operatorname{coth}(c+dx)}{2(a+b)d}\right)}{8d(a+b)^2(a+b-b\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((3*b*ArcTanh[(Sech[d*x]*Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + b*Sech[2*c]*Sinh[2*d*x] - (b*(a + 2*b)*Tanh[2*c])/a)/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.62, size = 2407, normalized size = 26.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + a*b + 2*b^2)*sinh(d*x + c)^4 + 8*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 4*a*b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 4*a*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*

```
(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2
+ a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 -
4*a*b + 16*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 +
6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*c
osh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 6
*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b + 9*a^2*b
^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
+ c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*
cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/2*(2*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + a*
b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(2*a^2 + a*b + 2*b^2)*sinh(d*x
+ c)^4 + 4*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 4*(3*(2*a^2 + a*b + 2*b
^2)*cosh(d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*
x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2
+ 4*a*b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x
+ c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 +
4*a*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cos
h(d*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
+ 4*a^2 - 2*a*b + 8*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b
- b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
+ c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^
4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d
*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 +
6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*
b^2 + 4*a*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4
+ 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*
a*b^3)*d*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x
+ c))*sinh(d*x + c)]]
```

giac [B] time = 0.73, size = 239, normalized size = 2.60

$$\frac{3b \arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^2+2ab+b^2)\sqrt{-ab-b^2}} - \frac{2(2a^2e^{4dx+4c}+abe^{4dx+4c}+2b^2e^{4dx+4c}+4a^2e^{2dx+2c}+8abe^{2dx+2c}-2b^2e^{2dx+2c}+2a^2-ab)}{(a^3+2a^2b+ab^2)(ae^{6dx+6c}+ae^{4dx+4c}+4be^{4dx+4c}-ae^{2dx+2c}-4be^{2dx+2c}-a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + 2*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(2*a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x + 4*c) + 2*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*c) - 2*b^2*e^(2*d*x + 2*c) + 2*a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(6*

$d*x + 6*c) + a*e^{(4*d*x + 4*c)} + 4*b*e^{(4*d*x + 4*c)} - a*e^{(2*d*x + 2*c)} - 4*b*e^{(2*d*x + 2*c)} - a)))/d$

maple [B] time = 0.39, size = 313, normalized size = 3.40

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d(a^2 + 2ab + b^2)} + \frac{b\left(\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d(a+b)^2\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2), x)

[Out] $-1/2/d/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)+1/d*b/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^3+1/d*b/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)-3/4/d*b^(1/2)/(a+b)^(5/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/4/d*b^(1/2)/(a+b)^(5/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^2/\tanh(1/2*d*x+1/2*c)$

maxima [B] time = 0.46, size = 262, normalized size = 2.85

$$\frac{3b \log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4(a^2 + 2ab + b^2)\sqrt{(a+b)bd}} - \frac{2a^2 - ab + 2(2a^2 + 4ab - b^2)e^{(-2dx-2c)} + (a^4 + 2a^3b + a^2b^2 + (a^4 + 6a^3b + 9a^2b^2 + 4ab^3)e^{(-2dx-2c)} - (a^4 + 6a^3b + 9a^2b^2 + 4ab^3)e^{(-4dx-4c)} - (a^4 + 2a^3b + a^2b^2)e^{(-6dx-6c)})d}{(a^4 + 2a^3b + a^2b^2 + (a^4 + 6a^3b + 9a^2b^2 + 4ab^3)e^{(-2dx-2c)} - (a^4 + 6a^3b + 9a^2b^2 + 4ab^3)e^{(-4dx-4c)} - (a^4 + 2a^3b + a^2b^2)e^{(-6dx-6c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $-3/4*b*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b}))/((a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^2 + 2*a*b + b^2)*\sqrt{(a + b)*b}*d) - (2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^{(-2*d*x - 2*c)} + (2*a^2 + a*b + 2*b^2)*e^{(-4*d*x - 4*c)})/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^{(-2*d*x - 2*c)} - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^{(-4*d*x - 4*c)} - (a^4 + 2*a^3*b + a^2*b^2)*e^{(-6*d*x - 6*c)})*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4}{\sinh(c + dx)^2 (a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)

$$3.39 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=147

$$\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2(a\cosh^2(c+dx)+b)} - \frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}d(a+b)^3} + \frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3} - \frac{\coth(c+dx)}{2d(a+b)}$$

[Out] 1/2*(a-3*b)*arctanh(cosh(d*x+c))/d/(a+b)^3-1/2*(a-b)*cosh(d*x+c)/(a+b)^2/d/(b+a*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/d/(b+a*cosh(d*x+c)^2)-1/2*(3*a-b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/d/a^(1/2)

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4133, 470, 527, 522, 206, 205}

$$\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2(a\cosh^2(c+dx)+b)} - \frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}d(a+b)^3} + \frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3} - \frac{\coth(c+dx)}{2d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] -((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*Sqrt[a]*(a + b)^3*d) + ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^3*d) - ((a - b)*Cosh[c + d*x])/(2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d(b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{2(a + b)d}$$

$$= -\frac{(a - b)\cosh(c + dx)}{2(a + b)^2d(b + a \cosh^2(c + dx))} - \frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d(b + a \cosh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{(-a+2b)x^2}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{2(a + b)d}$$

$$= -\frac{(a - b)\cosh(c + dx)}{2(a + b)^2d(b + a \cosh^2(c + dx))} - \frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2(a + b)d(b + a \cosh^2(c + dx))} + \frac{(a - 3b)\cosh(c + dx)}{2(a + b)d(b + a \cosh^2(c + dx))}$$

$$= -\frac{(3a - b)\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c + dx)}{\sqrt{b}}\right)}{2\sqrt{a}(a + b)^3d} + \frac{(a - 3b)\tanh^{-1}(\cosh(c + dx))}{2(a + b)^3d} - \frac{(a - 3b)\cosh(c + dx)}{2(a + b)d(b + a \cosh^2(c + dx))}$$

Mathematica [C] time = 2.23, size = 462, normalized size = 3.14

$$\operatorname{sech}^3(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(-(a + b)\operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) \operatorname{sech}(c + dx)(a \cosh(2(c + dx)) + a + 2b) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(8*b*(a + b) + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b])*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x])/Sqrt[a] + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[
```

$$\begin{aligned} & (\text{Cosh}[c] - \text{Sinh}[c])^2) * \text{Sinh}[c] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] + I * \text{Sqrt}[a \\ & + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2 * \text{Tanh}[(d*x)/2]]) / \text{Sqrt}[b] * (a + 2*b + a * \text{Cos} \\ & \text{h}[2*(c + d*x)]) * \text{Sech}[c + d*x] / \text{Sqrt}[a] - (a + b) * (a + 2*b + a * \text{Cosh}[2*(c + d \\ & *x)]) * \text{Csch}[(c + d*x)/2]^2 * \text{Sech}[c + d*x] + 4*(a - 3*b) * (a + 2*b + a * \text{Cosh}[2*(\\ & c + d*x)]) * \text{Log}[\text{Cosh}[(c + d*x)/2]] * \text{Sech}[c + d*x] - 4*(a - 3*b) * (a + 2*b + a * \\ & \text{Cosh}[2*(c + d*x)]) * \text{Log}[\text{Sinh}[(c + d*x)/2]] * \text{Sech}[c + d*x] - (a + b) * (a + 2*b \\ & + a * \text{Cosh}[2*(c + d*x)]) * \text{Sech}[(c + d*x)/2]^2 * \text{Sech}[c + d*x]) / (32*(a + b)^3*d \\ & (a + b * \text{Sech}[c + d*x]^2)^2) \end{aligned}$$

fricas [B] time = 0.73, size = 6878, normalized size = 46.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(4*(a^2 - b^2)*\cosh(d*x + c)^7 + 28*(a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x \\ & + c)^6 + 4*(a^2 - b^2)*\sinh(d*x + c)^7 + 4*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d \\ & *x + c)^5 + 4*(21*(a^2 - b^2)*\cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*\sinh \\ & (d*x + c)^5 + 20*(7*(a^2 - b^2)*\cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 5*b^2)*\c \\ & \text{osh}(d*x + c))*\sinh(d*x + c)^4 + 4*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^3 + \\ & 4*(35*(a^2 - b^2)*\cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + \\ & c)^2 + 3*a^2 + 8*a*b + 5*b^2)*\sinh(d*x + c)^3 + 4*(21*(a^2 - b^2)*\cosh(d*x \\ & + c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b + 5* \\ & b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((3*a^2 - a*b)*\cosh(d*x + c)^8 + 8*(3 \\ & *a^2 - a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 - a*b)*\sinh(d*x + c)^8 + \\ & 4*(3*a*b - b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*\cosh(d*x + c)^2 + 3*a \\ & *b - b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*\cosh(d*x + c)^3 + 3*(3*a*b - \\ & b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x \\ & + c)^4 + 2*(35*(3*a^2 - a*b)*\cosh(d*x + c)^4 + 30*(3*a*b - b^2)*\cosh(d*x + \\ & c)^2 - 3*a^2 - 11*a*b + 4*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 - a*b)*\cosh(d \\ & x + c)^5 + 10*(3*a*b - b^2)*\cosh(d*x + c)^3 - (3*a^2 + 11*a*b - 4*b^2)*\cosh \\ & (d*x + c))*\sinh(d*x + c)^3 + 4*(3*a*b - b^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 \\ & - a*b)*\cosh(d*x + c)^6 + 15*(3*a*b - b^2)*\cosh(d*x + c)^4 - 3*(3*a^2 + 11*a \\ & *b - 4*b^2)*\cosh(d*x + c)^2 + 3*a*b - b^2)*\sinh(d*x + c)^2 + 3*a^2 - a*b + \\ & 8*((3*a^2 - a*b)*\cosh(d*x + c)^7 + 3*(3*a*b - b^2)*\cosh(d*x + c)^5 - (3*a^2 \\ & + 11*a*b - 4*b^2)*\cosh(d*x + c)^3 + (3*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x \\ & + c))*\sqrt{-b/a} * \log((a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 \\ & + a*\sinh(d*x + c)^4 + 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 \\ & + a - 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a - 2*b)*\cosh(d*x + c \\ &))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 \\ & + a*\sinh(d*x + c)^3 + a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a)*\sinh(d*x \\ & + c))*\sqrt{-b/a} + a) / (a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c) \\ & ^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c) \\ & ^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + \\ & c))*\sinh(d*x + c) + a) + 4*(a^2 - b^2)*\cosh(d*x + c) - 2*((a^2 - 3*a*b)*\c \\ & \text{osh}(d*x + c)^8 + 8*(a^2 - 3*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 - 3*a \\ & *b)*\sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)* \\ & \cosh(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*\cosh(d \\ & x + c)^3 + 3*(a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - \\ & 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*\cosh(d*x + c)^4 + 30*(a*b - 3 \\ & *b^2)*\cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 - 3 \\ & *a*b)*\cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12* \\ & b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^2 + 4*(\\ & 7*(a^2 - 3*a*b)*\cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*\cosh(d*x + c)^4 - 3*(a^2 \\ & + a*b - 12*b^2)*\cosh(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^2 + a^2 - 3*a \\ & *b + 8*((a^2 - 3*a*b)*\cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*\cosh(d*x + c)^5 - (\\ & a^2 + a*b - 12*b^2)*\cosh(d*x + c)^3 + (a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x \\ & + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 2*((a^2 - 3*a*b)*\cosh(d*x + \\ & c)^8 + 8*(a^2 - 3*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 - 3*a*b)*\sinh(\end{aligned}$$

$$\begin{aligned}
& d*x + c)^8 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*\cosh(d*x \\
& + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*\cosh(d*x + c)^3 \\
& + 3*(a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*\cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*\cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*\cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^2 + 4*(7*(a^2 - 3*a*b)*\cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*\cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((a^2 - 3*a*b)*\cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*\cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c)^3 + (a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 4*(7*(a^2 - b^2)*\cosh(d*x + c)^6 + 5*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\sinh(d*x + c)^8 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d)*\sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^3 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^4 + 30*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d)*\sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^5 + 10*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^3 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^6 + 15*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d)*\sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(a^2 - b^2)*\cosh(d*x + c)^7 + 14*(a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(a^2 - b^2)*\sinh(d*x + c)^7 + 2*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^5 + 2*(21*(a^2 - b^2)*\cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*\sinh(d*x + c)^5 + 10*(7*(a^2 - b^2)*\cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^3 + 2*(35*(a^2 - b^2)*\cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*\sinh(d*x + c)^3 + 2*(21*(a^2 - b^2)*\cosh(d*x + c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((3*a^2 - a*b)*\cosh(d*x + c)^8 + 8*(3*a^2 - a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 - a*b)*\sinh(d*x + c)^8 + 4*(3*a*b - b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*\cosh(d*x + c)^2 + 3*a*b - b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*\cosh(d*x + c)^3 + 3*(3*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*\cosh(d*x + c)^4 + 30*(3*a*b - b^2)*\cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 - a*b)*\cosh(d*x + c)^5 + 10*(3*a*b - b^2)*\cosh(d*x + c)^3 - (3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a*b - b^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 - a*b)*\cosh(d*x + c)^6 + 15*(3*a*b - b^2)*\cosh(d*x + c)^4 - 3*(3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c)^2 + 3*a*b - b^2)*\sinh(d*x + c)^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*\cosh(d*x + c)^7 + 3*(3*a*b - b^2)*\cosh(d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c)^3 + (3*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a + 4*b)*\sinh(d*x + c))*\sqrt{b/a}/b) + ((3*a^2 - a*b)*\cosh(d*x + c)^8 + 8*(3*a^2 - a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 -
\end{aligned}$$

$$\begin{aligned}
& a*b)*\sinh(d*x + c)^8 + 4*(3*a*b - b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b) \\
&)*\cosh(d*x + c)^2 + 3*a*b - b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*\cosh(\\
& d*x + c)^3 + 3*(3*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(3*a^2 + 11 \\
& *a*b - 4*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*\cosh(d*x + c)^4 + 30*(3 \\
& *a*b - b^2)*\cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*\sinh(d*x + c)^4 + 8*(\\
& 7*(3*a^2 - a*b)*\cosh(d*x + c)^5 + 10*(3*a*b - b^2)*\cosh(d*x + c)^3 - (3*a^2 \\
& + 11*a*b - 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a*b - b^2)*\cosh(d* \\
& x + c)^2 + 4*(7*(3*a^2 - a*b)*\cosh(d*x + c)^6 + 15*(3*a*b - b^2)*\cosh(d*x + \\
& c)^4 - 3*(3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c)^2 + 3*a*b - b^2)*\sinh(d*x \\
& + c)^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*\cosh(d*x + c)^7 + 3*(3*a*b - b^2)*c \\
& osh(d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*\cosh(d*x + c)^3 + (3*a*b - b^2)*c \\
& osh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*(a*\cosh(d*x + c) + a*\sinh \\
& (d*x + c))*\sqrt{b/a}/b) + 2*(a^2 - b^2)*\cosh(d*x + c) - ((a^2 - 3*a*b)*\cosh \\
& (d*x + c)^8 + 8*(a^2 - 3*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 - 3*a*b) \\
& *\sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cos \\
& h(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*\cosh(d*x + \\
& c)^3 + 3*(a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - 12* \\
& b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*\cosh(d*x + c)^4 + 30*(a*b - 3*b^ \\
& 2)*\cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a* \\
& b)*\cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2 \\
&)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^2 + 4*(7*(\\
& a^2 - 3*a*b)*\cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*\cosh(d*x + c)^4 - 3*(a^2 + \\
& a*b - 12*b^2)*\cosh(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^2 + a^2 - 3*a*b \\
& + 8*((a^2 - 3*a*b)*\cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*\cosh(d*x + c)^5 - (a^2 \\
& + a*b - 12*b^2)*\cosh(d*x + c)^3 + (a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a^2 - 3*a*b)*\cosh(d*x + c)^8 \\
& + 8*(a^2 - 3*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 - 3*a*b)*\sinh(d*x + \\
& c)^8 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*\cosh(d*x + c)^ \\
& 2 + a*b - 3*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*\cosh(d*x + c)^3 + 3*(\\
& a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*\cosh(d \\
& *x + c)^4 + 2*(35*(a^2 - 3*a*b)*\cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*\cosh(d*x \\
& + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*\cosh(d*x \\
& + c)^5 + 10*(a*b - 3*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*\cosh(d*x + c)^2 + 4*(7*(a^2 - 3*a*b) \\
&)*\cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*\cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^ \\
& 2)*\cosh(d*x + c)^2 + a*b - 3*b^2)*\sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((a^2 - \\
& 3*a*b)*\cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*\cosh(d*x + c)^5 - (a^2 + a*b - 12 \\
& *b^2)*\cosh(d*x + c)^3 + (a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(cos \\
& h(d*x + c) + \sinh(d*x + c) - 1) + 2*(7*(a^2 - b^2)*\cosh(d*x + c)^6 + 5*(3*a \\
& ^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + \\
& c)^2 + a^2 - b^2)*\sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cos \\
& h(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)*\sinh(d \\
& *x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\sinh(d*x + c)^8 + 4*(a^3*b \\
& + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b + 3* \\
& a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d) \\
& *\sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh \\
& (d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^3 + \\
& 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2* \\
& (35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^4 + 30*(a^3*b + 3*a \\
& ^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 1 \\
& 3*a*b^3 + 4*b^4)*d)*\sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4) \\
& *d*\cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + \\
& c)^5 + 10*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^3 - (a^4 + 7* \\
& a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4 \\
& *(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^6 + 15*(a^3*b + 3*a \\
& ^2*b^2 + 3*a*b^3 + b^4)*d*\cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*b^2 + \\
& 13*a*b^3 + 4*b^4)*d*\cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)* \\
& d)*\sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 + 3*a^ \\
& 3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
\end{aligned}$$

+ b^4)*d*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)
*d*cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.7Limit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.42, size = 496, normalized size = 3.37

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d(a^2 + 2ab + b^2)} + \frac{ba\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d(a+b)^3\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)^2-1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a+1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-3/2/d*b/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))*a+1/2/d*b^2/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*a+3/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a-3b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\left(a^3d+3a^2bd+3ab^2d+b^3d\right)} - \frac{(a-3b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\left(a^3d+3a^2bd+3ab^2d+b^3d\right)} - \frac{a^3d+2a^2bd+ab^2d+\left(a^3de^{(8c)}+2a^2bde^{(8c)}\right)}{2\left(a^3d+3a^2bd+3ab^2d+b^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(a-3*b)*log((e^(d*x+c)+1)*e^(-c))/(a^3*d+3*a^2*b*d+3*a*b^2*d+b^3*d)-1/2*(a-3*b)*log((e^(d*x+c)-1)*e^(-c))/(a^3*d+3*a^2*b*d+3*a*b^2*d+b^3*d)-((a*e^(7*c)-b*e^(7*c))*e^(7*d*x)+(3*a*e^(5*c)+5*b*e^(5*c))*e^(5*d*x)+(3*a*e^(3*c)+5*b*e^(3*c))*e^(3*d*x)+(a*e^c-b

$$e^c * e^{dx}) / (a^3 * d + 2 * a^2 * b * d + a * b^2 * d + (a^3 * d * e^{8c} + 2 * a^2 * b * d * e^{8c} + a * b^2 * d * e^{8c})) * e^{8dx} + 4 * (a^2 * b * d * e^{6c} + 2 * a * b^2 * d * e^{6c} + b^3 * d * e^{6c}) * e^{6dx} - 2 * (a^3 * d * e^{4c} + 6 * a^2 * b * d * e^{4c} + 9 * a * b^2 * d * e^{4c} + 4 * b^3 * d * e^{4c}) * e^{4dx} + 4 * (a^2 * b * d * e^{2c} + 2 * a * b^2 * d * e^{2c} + b^3 * d * e^{2c}) * e^{2dx} - 8 * \text{integrate}(1/8 * ((3 * a * b * e^{3c} - b^2 * e^{3c})) * e^{3dx} - (3 * a * b * e^c - b^2 * e^c) * e^{dx}) / (a^4 + 3 * a^3 * b + 3 * a^2 * b^2 + a * b^3 + (a^4 * e^{4c} + 3 * a^3 * b * e^{4c} + 3 * a^2 * b^2 * e^{4c} + a * b^3 * e^{4c})) * e^{4dx} + 2 * (a^4 * e^{2c} + 5 * a^3 * b * e^{2c} + 9 * a^2 * b^2 * e^{2c} + 7 * a * b^3 * e^{2c} + 2 * b^4 * e^{2c}) * e^{2dx}), x$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4}{\sinh(c + dx)^3 (a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)

$$3.40 \quad \int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=123

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}} - \frac{ab\tanh(c+dx)}{2d(a+b)^3(a-b\tanh^2(c+dx)+b)} - \frac{\coth^3(c+dx)}{3d(a+b)^2} + \frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

[Out] (a-b)*coth(d*x+c)/d/(a+b)^3-1/3*coth(d*x+c)^3/(a+b)^2/d-1/2*(3*a-2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d-1/2*a*b*tanh(d*x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 456, 1261, 208}

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}} - \frac{ab\tanh(c+dx)}{2d(a+b)^3(a-b\tanh^2(c+dx)+b)} - \frac{\coth^3(c+dx)}{3d(a+b)^2} + \frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] -((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(7/2)*d) + ((a - b)*Coth[c + d*x])/((a + b)^3*d) - Coth[c + d*x]^3/(3*(a + b)^2*d) - (a*b*Tanh[c + d*x])/((2*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{ab \tanh(c+dx)}{2(a+b)^3 d (a+b-b \tanh^2(c+dx))} + \frac{b \operatorname{Subst}\left(\int \frac{\frac{2}{b(a+b)} - \frac{2ax^2}{b(a+b)^2} - \frac{ax^4}{(a+b)^3}}{x^4(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{2d} \\
&= -\frac{ab \tanh(c+dx)}{2(a+b)^3 d (a+b-b \tanh^2(c+dx))} + \frac{b \operatorname{Subst}\left(\int \left(\frac{2}{b(a+b)^2 x^4} - \frac{2(a-b)}{b(a+b)^3 x^2} + \frac{-3a}{(a+b)^3(a+b-bx^2)}\right) dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{(a-b) \operatorname{coth}(c+dx)}{(a+b)^3 d} - \frac{\operatorname{coth}^3(c+dx)}{3(a+b)^2 d} - \frac{ab \tanh(c+dx)}{2(a+b)^3 d (a+b-b \tanh^2(c+dx))} - \frac{((3a-2b)\sqrt{b} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right))}{2(a+b)^{7/2} d} + \frac{(a-b) \operatorname{coth}(c+dx)}{(a+b)^3 d} - \frac{\operatorname{coth}^3(c+dx)}{3(a+b)^2 d} - \frac{2(a+b) \operatorname{coth}(c) \operatorname{csch}^2(c+dx) (a \cosh(2(c+dx)) + a + 2b)}{2(a+b)^3 d}
\end{aligned}$$

Mathematica [B] time = 6.08, size = 295, normalized size = 2.40

$$\operatorname{sech}^4(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(-3ab \operatorname{sech}(2c) \sinh(2dx) - 2(a+b) \operatorname{coth}(c) \operatorname{csch}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-2*(a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]^2 - (3*(3*a - 2*b)*b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - 4*(a - 2*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + 2*(a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x] - 3*a*b*Sech[2*c]*Sinh[2*d*x] + 3*b*(a + 2*b)*Tanh[2*c]))/(24*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.53, size = 6143, normalized size = 49.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/12*(12*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 96*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 24*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(10*a^2 +

$$\begin{aligned}
& 22*a*b - 33*b^2)*\cosh(d*x + c)^4 + 8*(105*(3*a*b - 2*b^2)*\cosh(d*x + c)^4 + \\
& 45*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*\sinh(d*x + c)^4 + 32*(21*(3*a*b - 2*b^2)*\cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*b \\
& + 11*b^2)*\cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 8*(2*a^2 - 9*a*b + 19*b^2)*\cosh(d*x + c)^2 + 8*(42*(3*a*b - 2* \\
& b^2)*\cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^4 + 6*(10* \\
& a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*\sinh(d*x + \\
& c)^2 + 3*((3*a^2 - 2*a*b)*\cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*\sinh(d*x + c)^10 - (3*a^2 - 14*a*b + \\
& 8*b^2)*\cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*\cosh(d*x + c)^2 - 3*a^2 + 14*a \\
& *b - 8*b^2)*\sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*\cosh(d*x + c)^3 - (3*a^2 \\
& - 14*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b - 12 \\
& *b^2)*\cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*\cosh(d*x + c)^4 - 14*(3*a^2 \\
& - 14*a*b + 8*b^2)*\cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*\sinh(d*x + c)^6 \\
& + 4*(63*(3*a^2 - 2*a*b)*\cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + \\
& 2*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*\cosh(d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^4 - 15*(3*a^2 + 16*a \\
& *b - 12*b^2)*\cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*\sinh(d*x + c)^4 + 8 \\
& *(15*(3*a^2 - 2*a*b)*\cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x \\
& + c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^3 + (3*a^2 + 16*a*b - 12 \\
& *b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + \\
& c)^2 + (45*(3*a^2 - 2*a*b)*\cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^4 + 12*(3*a^2 + \\
& 16*a*b - 12*b^2)*\cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*\sinh(d*x + c)^2 \\
& - 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*\cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*b \\
& + 8*b^2)*\cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^5 + 4* \\
& (3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c))*\sqrt{b/(a + b)}*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c))*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a) - 16*a^2 + 44*a*b + 16*(6*(3*a*b - 2*b^2)*\cosh(d*x + c)^7 + 9*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c)^3 + (2*a^2 - 9*a*b + 19*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^2 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*\sinh(d*x + c)^8 - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^6 + 8*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*\sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^6 - 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^4 - 15*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^2 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*\sinh(d*x + c)^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b +
\end{aligned}$$

$$\begin{aligned}
& 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^8 - 28*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^6 - 30*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^4 + 12*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^2 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*\sinh(d*x + c)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 2*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^9 - 4*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^7 - 6*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^5 + 4*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^3 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/6*(6*(3*a*b - 2*b^2)*\cosh(d*x + c)^8 + 48*(3*a*b - 2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + 6*(3*a*b - 2*b^2)*\sinh(d*x + c)^8 + 12*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^6 + 12*(14*(3*a*b - 2*b^2)*\cosh(d*x + c)^2 + 2*a^2 + 3*a*b + 11*b^2)*\sinh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*\cosh(d*x + c)^3 + 3*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c)^4 + 4*(105*(3*a*b - 2*b^2)*\cosh(d*x + c)^4 + 45*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*\sinh(d*x + c)^4 + 16*(21*(3*a*b - 2*b^2)*\cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(2*a^2 - 9*a*b + 19*b^2)*\cosh(d*x + c)^2 + 4*(42*(3*a*b - 2*b^2)*\cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^4 + 6*(10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*\sinh(d*x + c)^2 + 3*((3*a^2 - 2*a*b)*\cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*\sinh(d*x + c)^10 - (3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*\cosh(d*x + c)^2 - 3*a^2 + 14*a*b - 8*b^2)*\sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*\cosh(d*x + c)^3 - (3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*\cosh(d*x + c)^4 - 14*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*\sinh(d*x + c)^6 + 4*(63*(3*a^2 - 2*a*b)*\cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*\cosh(d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^4 - 15*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*\sinh(d*x + c)^4 + 8*(15*(3*a^2 - 2*a*b)*\cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^3 + (3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^2 + (45*(3*a^2 - 2*a*b)*\cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^4 + 12*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*\sinh(d*x + c)^2 - 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*\cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^5 + 4*(3*a^2 + 16*a*b - 12*b^2)*\cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a + b)}*\arctan(1/2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b)*\sqrt{-b/(a + b)})/b) - 8*a^2 + 22*a*b + 8*(6*(3*a*b - 2*b^2)*\cosh(d*x + c)^7 + 9*(2*a^2 + 3*a*b + 11*b^2)*\cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*\cosh(d*x + c)^3 + (2*a^2 - 9*a*b + 19*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^2 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*\sinh(d*x + c)^8 - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*\cosh(d*x + c)^6 + 8*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*\cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3*b
\end{aligned}$$

$$\begin{aligned}
& + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^4 - 14(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^2 - (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \sinh(dx + c)^6 + 2(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^4 + 4(63(a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^5 - 14(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^3 - 3(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2(105(a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^6 - 35(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^4 - 15(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^2 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d) * \sinh(dx + c)^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^2 + 8(15(a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^7 - 7(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^5 - 5(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^3 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45(a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^8 - 28(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^6 - 30(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^4 + 12(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^2 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d) * \sinh(dx + c)^2 - (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d + 2(5(a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * \cosh(dx + c)^9 - 4(a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)^7 - 6(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^5 + 4(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * \cosh(dx + c)^3 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * \cosh(dx + c)) * \sinh(dx + c)))]
\end{aligned}$$

giac [B] time = 0.77, size = 253, normalized size = 2.06

$$\frac{3(3ab-2b^2) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{-ab-b^2}} - \frac{6(abe^{(2dx+2c)+2b^2e^{(2dx+2c)+ab}})}{(a^3+3a^2b+3ab^2+b^3)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}})}} + \frac{8(3be^{(4dx+4c)+3ae^{(2dx+2c)-3be^{(2dx+2c)+a}})}}{(a^3+3a^2b+3ab^2+b^3)(e^{(2dx+2c)+a})}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^4/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/6*(3*(3*a*b - 2*b^2)*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + a + 2*b)/\sqrt{-a*b - b^2}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{-a*b - b^2}) - 6*(a*b*e^{(2*d*x + 2*c)} + 2*b^2*e^{(2*d*x + 2*c)} + a*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)) + 8*(3*b*e^{(4*d*x + 4*c)} + 3*a*e^{(2*d*x + 2*c)} - 3*b*e^{(2*d*x + 2*c)} - a + 2*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^{(2*d*x + 2*c)} - 1)^3))/d
\end{aligned}$$

maple [B] time = 0.50, size = 577, normalized size = 4.69

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{24d(a+b)(a^2+2ab+b^2)} - \frac{\left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b}{24d(a+b)(a^2+2ab+b^2)} + \frac{3a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{8d(a+b)(a^2+2ab+b^2)} - \frac{5 \tanh \left(\frac{dx}{2} + \frac{c}{2} \right) b}{8d(a+b)(a^2+2ab+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(dx+c)^4/(a+b*sech(dx+c)^2)^2,x)

[Out]
$$\begin{aligned}
& -1/24/d/(a+b)/(a^2+2*a*b+b^2)*a*\tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3*b+3/8/d/(a+b)/(a^2+2*a*b+b^2)*a*\tanh(1/2*d*x+1/2*c)-5/8/d/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)*b-1/d*b/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*a*\tanh(1/2*d*x+1/2*c)^3-1/d*b/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*a*\tanh(1/2*d*x+1/2*c)+3/4/d*b^(1/2)/(a+b)^(7/2)*a*\ln(-
\end{aligned}$$

$$(a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) - (a+b)^{1/2} - 3/4 d b^{1/2} / (a+b)^{7/2} * a \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 1/2 d b^{3/2} / (a+b)^{7/2} * \ln(-(a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) - (a+b)^{1/2}) + 1/2 d b^{3/2} / (a+b)^{7/2} * \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 1/24 d / (a+b)^2 / \tanh(1/2 dx + 1/2 c)^3 + 3/8 d / (a+b)^3 / \tanh(1/2 dx + 1/2 c) * a - 5/8 d / (a+b)^3 / \tanh(1/2 dx + 1/2 c) * b$$

maxima [B] time = 0.52, size = 430, normalized size = 3.50

$$\frac{(3ab - 2b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4(a^3 + 3a^2b + 3ab^2 + b^3)\sqrt{(a+b)bd}} + \frac{4a^2 - 3(a^4 + 3a^3b + 3a^2b^2 + ab^3 - (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4)e^{(-2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1/4*(3*a*b - 2*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{(a + b)*b}*d) + 1/3*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b + 19*b^2)*e^{(-2*d*x - 2*c)} - 2*(10*a^2 + 22*a*b - 33*b^2)*e^{(-4*d*x - 4*c)} - 6*(2*a^2 + 3*a*b + 11*b^2)*e^{(-6*d*x - 6*c)} - 3*(3*a*b - 2*b^2)*e^{(-8*d*x - 8*c)})}{(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^{(-2*d*x - 2*c)} - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^{(-4*d*x - 4*c)} + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^{(-6*d*x - 6*c)} + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^{(-8*d*x - 8*c)} - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^{(-10*d*x - 10*c)})*d}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4}{\sinh(c + dx)^4 (a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)

$$3.41 \quad \int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=242

$$\frac{3b(a+2b)\tanh(c+dx)}{2a^4d(a-b\tanh^2(c+dx)+b)} - \frac{b(7a+12b)\tanh(c+dx)}{8a^3d(a-b\tanh^2(c+dx)+b)^2} - \frac{(5a+8b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b\tanh^2(c+dx)+b)^2} - \frac{3\sqrt{b}}{8a}$$

[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^5/d/(a+b)^(1/2)-1/8*(5*a+8*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(7*a+12*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+c)^2)^2-3/2*b*(a+2*b)*tanh(d*x+c)/a^4/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.40, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{3\sqrt{b}(5a^2+20ab+16b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5d\sqrt{a+b}} + \frac{3x(a^2+12ab+16b^2)}{8a^5} - \frac{3b(a+2b)\tanh(c+dx)}{2a^4d(a-b\tanh^2(c+dx)+b)} - \frac{3\sqrt{b}}{8a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3, x]

[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*sqrt[b]*(5*a^2 + 20*a*b + 16*b^2)*ArcTanh[(sqrt[b]*Tanh[c + d*x])/sqrt[a + b]])/(8*a^5*sqrt[a + b]*d) - ((5*a + 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/((8*a^2*d*(a + b - b*Tanh[c + d*x]^2)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 12*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2)^2) - (3*b*(a + 2*b)*Tanh[c + d*x])/(2*a^4*d*(a + b - b*Tanh[c + d*x]^2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 4132

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)]^(p_))*sin[(e_) + (f_)*(x_)^(n_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f*ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+7b)x^2}{(1-x^2)^2(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= -\frac{(5a + 8b) \cosh(c + dx) \sinh(c + dx)}{8a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+7b)x^2}{(1-x^2)^2(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= -\frac{(5a + 8b) \cosh(c + dx) \sinh(c + dx)}{8a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{b(7a^2 + 12ab + 16b^2)}{8a^3d(a + b - b \tanh^2(c + dx))} \\ &= -\frac{(5a + 8b) \cosh(c + dx) \sinh(c + dx)}{8a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{b(7a^2 + 12ab + 16b^2)}{8a^3d(a + b - b \tanh^2(c + dx))} \\ &= \frac{3(a^2 + 12ab + 16b^2)x}{8a^5} - \frac{3\sqrt{b}(5a^2 + 20ab + 16b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}d} - \frac{b(7a^2 + 12ab + 16b^2)}{8a^3d(a + b - b \tanh^2(c + dx))} \end{aligned}$$

Mathematica [B] time = 27.32, size = 3080, normalized size = 12.73

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(3*(a + 2*b + a*\cosh[2*c + 2*d*x])^3*\operatorname{sech}[c + d*x]^6*((3*a^2 + 8*a*b + 8*b^2)*\operatorname{ArcTanh}[(\sqrt{b}*\tanh[c + d*x])/\sqrt{a + b}])/(a + b)^{(5/2)} - (a*\sqrt{b}*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*\cosh[2*(c + d*x)])*\sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*\cosh[2*(c + d*x)])^2))/((16384*b^{(5/2)}*d*(a + b*\operatorname{sech}[c + d*x]^2)^3) + ((a + 2*b + a*\cosh[2*c + 2*d*x])^3*\operatorname{sech}[c + d*x]^6*((-3*a*(a + 2*b)*\operatorname{ArcTanh}[(\sqrt{b}*\tanh[c + d*x])/\sqrt{a + b}])/(a + b)^{(5/2)} + (\sqrt{b}*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*\cosh[2*(c + d*x)])*\sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*\cosh[2*(c + d*x)])^2)))/((16384*b^{(5/2)}*d*(a + b*\operatorname{sech}[c + d*x]^2)^3) - (3*(a + 2*b + a*\cosh[2*c + 2*d*x])^3*\operatorname{sech}[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*\operatorname{ArcTanh}[(\operatorname{sech}[d*x]*(\cosh[2*c] - \sinh[2*c]))*(a + 2*b)*\sinh[d*x] - a*\sinh[2*c + d*x]])/(2*\sqrt{a + b}*\sqrt{b*(\cosh[c] - \sinh[c])^4}))*(\cosh[2*c] - \sinh[2*c]))/(\sqrt{a + b}*\sqrt{b*(\cosh[c] - \sinh[c])^4}) + (\operatorname{sech}[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*\cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*\cosh[2*d*x] + 128*a^4*b^2*d*x*\cosh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*\cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*\cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*\cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*\cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*\cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*\cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*\cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*\cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*\cosh[6*c + 4*d*x] - 9*a^6*\sinh[2*c] + 12*a^5*b*\sinh[2*c] + 684*a^4*b^2*\sinh[2*c] + 2880*a^3*b^3*\sinh[2*c] + 5280*a^2*b^4*\sinh[2*c] + 4608*a*b^5*\sinh[2*c] + 1536*b^6*\sinh[2*c] + 9*a^6*\sinh[2*d*x] - 14*a^5*b*\sinh[2*d*x] - 608*a^4*b^2*\sinh[2*d*x] - 2112*a^3*b^3*\sinh[2*d*x] - 2560*a^2*b^4*\sinh[2*d*x] - 1024*a*b^5*\sinh[2*d*x] + 3*a^6*\sinh[2*(c + 2*d*x)] - 12*a^5*b*\sinh[2*(c + 2*d*x)] - 204*a^4*b^2*\sinh[2*(c + 2*d*x)] - 384*a^3*b^3*\sinh[2*(c + 2*d*x)] - 192*a^2*b^4*\sinh[2*(c + 2*d*x)] - 3*a^6*\sinh[4*c + 2*d*x] + 10*a^5*b*\sinh[4*c + 2*d*x] + 304*a^4*b^2*\sinh[4*c + 2*d*x] + 1056*a^3*b^3*\sinh[4*c + 2*d*x] + 1280*a^2*b^4*\sinh[4*c + 2*d*x] + 512*a*b^5*\sinh[4*c + 2*d*x]))/(a + 2*b + a*\cosh[2*(c + d*x)])^2))/((65536*a^3*b^2*(a + b)^2*d*(a + b*\operatorname{sech}[c + d*x]^2)^3) + ((a + 2*b + a*\cosh[2*c + 2*d*x])^3*\operatorname{sech}[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*\operatorname{ArcTanh}[(\operatorname{sech}[d*x]*(\cosh[2*c] - \sinh[2*c]))*(a + 2*b)*\sinh[d*x] - a*\sinh[2*c + d*x]])/(2*\sqrt{a + b}*\sqrt{b*(\cosh[c] - \sinh[c])^4}))*(\cosh[2*c] - \sinh[2*c]))/(\sqrt{a + b}*\sqrt{b*(\cosh[c] - \sinh[c])^4}) + (\operatorname{sech}[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3)*d*x*\cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*\cosh[2*d*x] - 768*a^5*b^2*d*x*\cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*\cosh[2*(c + 2*d*x)] - 3840*a^3*b^4*d*x*\cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*\cosh[2*(c + 2*d*x)] - 3072*a^5*b^2*d*x*\cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*\cosh[4*c + 2*d*x] - 39936*a^3*b^4*d*x*\cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*\cosh[4*c + 2*d*x] - 12288*a*b^6*d*x*\cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*\cosh[6*c + 4*d*x] - 3072*a^4*b^3*d*x*\cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*\cosh[6*c + 4*d*x] - 1536*a^2*b^5*d*x*\cosh[6*c + 4*d*x] + 9*a^7*\sinh[2*c] - 54*a^6*b*\sinh[2*c] - 2392*a^5*b^2*\sinh[2*c] - 13968*a^4*b^3*\sinh[2*c] - 36480*a^3*b^4*\sinh[2*c] - 50432*a^2*b^5*\sinh[2*c] - 35840*a*b^6*\sinh[2*c] - 10240*b^7*\sinh[2*c] - 9*a^7*\sinh[2*d*x] + 56*a^6*b*\sinh[2*d*x] + 2552*a^5*b^2*\sinh[2*d*x] + 13184*a^4*b^3*\sinh[2*d*x] + 27072*a^3*b^4*\sinh[2*d*x] + 24576*a^2*b^5*\sinh[2*d*x] + 8192*a*b^6*\sinh[2*d*x] - 3*a^7*\sinh[2*(c + 2*d*x)] + 26*a^6*b*\sinh[2*(c + 2*d*x)] + 992*a^5*b^2*\sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*\sinh[2*(c + 2*d*x)] + 4480*a^3*b^4*\sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*\sinh[2*(c + 2*d*x)] + 3*a^7*\sinh[4*c + 2*d*x] - 24*a^6*b*\sinh[4*c + 2*d*x] - 600*a^5*b^2*\sinh[4*c + 2*d*x] - 3200*a^4*b^3*\sinh[4*c + 2*d*x] - 6720*a^3*b^4*\sinh[4*c + 2*d*x] - 6144*a^2*b^5*\sinh[4*c + 2*d*x] - 2048*a*b^6*\sinh[4*c + 2*d*x] + 256*a^5*b^2*\sinh[6*c + 4*d*x] + 1024*a^4*b^3*\sinh[6*c + 4*d*x] + 1280*a^3*b^4*\sinh[6*c + 4*d*x] + 512*a^2*b^5*\sinh[6*c + 4*d*x] + 64*a^5*b^2*\sinh[4*c + 6*d*x] + 128*a^4*b^3*\sinh[4*c + 6*d*x] + 64*a^3*b^4*\sinh[4*c + 6*d*x] + 64*a^5*b^2*\sinh[8*c + 6*d*x] + 128*a^4*b^3*\sinh[8*c + 6*d*x] + 64*a^3*b^4*\sinh[8*c + 6*d*x]))/(a + 2*b + a*\cosh[2*(c + d*x)])^2))/((32768*a^4*b^2*(a + b)^2$

$$\begin{aligned} & *d*(a + b*\text{Sech}[c + d*x]^2)^3 - ((a + 2*b + a*\text{Cosh}[2*c + 2*d*x])^3*\text{Sech}[c + \\ & d*x]^6*((6*a^2*\text{ArcTanh}[(\text{Sech}[d*x]*(\text{Cosh}[2*c] - \text{Sinh}[2*c])*((a + 2*b)*\text{Sinh}[\\ & d*x] - a*\text{Sinh}[2*c + d*x]))/(2*\text{Sqrt}[a + b]*\text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4]))*(\\ & \text{Cosh}[2*c] - \text{Sinh}[2*c]))/(\text{Sqrt}[a + b]*\text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4]) + (a*\text{Se} \\ & \text{ch}[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*\text{Sinh}[2*d*x] \\ & + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*\text{Sinh}[2*(c + 2*d*x)] + (3*a^4 - 6 \\ & 4*a^2*b^2 - 128*a*b^3 - 64*b^4)*\text{Sinh}[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64 \\ & *a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*\text{Tanh}[2*c])/(a^2*(a + 2*b + a* \\ & \text{Cosh}[2*(c + d*x)]^2)))/(8192*b^2*(a + b)^2*d*(a + b*\text{Sech}[c + d*x]^2)^3 + \\ & ((a + 2*b + a*\text{Cosh}[2*c + 2*d*x])^3*\text{Sech}[c + d*x]^6*(768*(7*a^2 + 32*a*b + 3 \\ & 2*b^2)*x - (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600*a^4*b^3 + 22400*a^3*b^4 \\ & + 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*\text{ArcTanh}[(\text{Sech}[d*x]*(\text{Cosh}[2*c] - \text{S} \\ & \text{inh}[2*c])*((a + 2*b)*\text{Sinh}[d*x] - a*\text{Sinh}[2*c + d*x]))/(2*\text{Sqrt}[a + b]*\text{Sqrt}[b* \\ & (\text{Cosh}[c] - \text{Sinh}[c])^4]))*(\text{Cosh}[2*c] - \text{Sinh}[2*c]))/(b^2*(a + b)^(5/2)*d*\text{Sqrt} \\ & [b*(\text{Cosh}[c] - \text{Sinh}[c])^4]) - (4*(a^5 + 50*a^4*b + 400*a^3*b^2 + 1120*a^2*b^ \\ & 3 + 1280*a*b^4 + 512*b^5)*\text{Sech}[2*c]*((a + 2*b)*\text{Sinh}[2*c] - a*\text{Sinh}[2*d*x]))/ \\ & (b*(a + b)*d*(a + 2*b + a*\text{Cosh}[2*(c + d*x)]^2) + (768*a*(a + 2*b)*(\text{Cosh}[2* \\ & (c + d*x)] - \text{Sinh}[2*(c + d*x)]))/d - (768*a*(a + 2*b)*(\text{Cosh}[2*(c + d*x)] + \\ & \text{Sinh}[2*(c + d*x)]))/d + (128*a^2*\text{Sinh}[4*(c + d*x)]/d + (a*(3*a^6 - 44*a^5* \\ & b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4 - 21504*a*b^5 - 7168*b^6)* \\ & \text{Sech}[2*c]*\text{Sinh}[2*d*x] + (-3*a^7 + 42*a^6*b + 2192*a^5*b^2 + 16480*a^4*b^3 + \\ & 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 16384*b^7)*\text{Tanh}[2*c])/(b^2*(\\ & a + b)^2*d*(a + 2*b + a*\text{Cosh}[2*(c + d*x)]^2)))/(32768*a^5*(a + b*\text{Sech}[c + d* \\ & x]^2)^3) \end{aligned}$$

fricas [B] time = 0.64, size = 12353, normalized size = 51.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*sinh(d*x + c)^15 + a^4*s
inh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14 + 4*(30*a^4*cosh(d*x +
c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*cosh(d*x + c)^3 - (a^4
+ 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a^4 + 72*a^3*b + 88*a^2
b^2 - 12(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^12 + 2*(910*a^4*
cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*
a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(5
46*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 - 3*(13*a^4 +
72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)
) *sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
+ 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2002*a^4*cos
h(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 - 9*a^4 - 24*a^3*b + 16
*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x - 33*
(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x + c)^7 - 1001*(a^4 +
4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 1
2*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b
^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3
*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^
8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 - 49
5*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*c
osh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3 + 896*b^4 + 24*(3*a^4
+ 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x - 90*(9*a^4 + 24*a^3*b
- 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(d*x + c)^9 - 858*(a^4
+ 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 +
12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*a^4 + 24*a^3*b - 16*a^

$$\begin{aligned}
& 2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^6 + 4*(2002*a^4*\cosh(d*x + c)^10 - 3003*(a^4 + 4*a^3*b)*\cosh(d*x + c)^8 - 462*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^6 - 210*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(546*a^4*\cosh(d*x + c)^11 - 1001*(a^4 + 4*a^3*b)*\cosh(d*x + c)^9 - 198*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^7 - 126*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^4 + 2*(910*a^4*\cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3*b)*\cosh(d*x + c)^10 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^6 + 560*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^4 + 13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 + 8*(70*a^4*\cosh(d*x + c)^13 - 182*(a^4 + 4*a^3*b)*\cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^9 - 60*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^5 + 10*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b)*\cosh(d*x + c)^2 + 4*(30*a^4*\cosh(d*x + c)^14 - 91*(a^4 + 4*a^3*b)*\cosh(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^8 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^6 + 15*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 12*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^12 + 12*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^11 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*\sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 20*(11*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^3 + 2*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^8 + (495*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b^2 + 576*a*b^3 + 256*b^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^5 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^3 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 4
\end{aligned}$$

$$\begin{aligned}
&*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^6 + 4*(231*(5*a^4 \\
&+ 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^6 + 210*(5*a^4 + 30*a^3*b + 56*a^2* \\
&b^2 + 32*a*b^3)*\cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3 \\
&+ 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c) \\
&^2)*\sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^7 \\
&+ 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^5 + 14*(15* \\
&a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^3 + 3*(5 \\
&*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + (\\
&5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^4 + (495*(5*a^4 + 20*a^3*b + 1 \\
&6*a^2*b^2)*\cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3) \\
&*\cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128* \\
&b^4)*\cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2 + 60*(5*a^4 + 30*a^3*b \\
&+ 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(55*(5*a^4 + \\
&20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^9 + 120*(5*a^4 + 30*a^3*b + 56*a^2*b^ \\
&2 + 32*a*b^3)*\cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288* \\
&a*b^3 + 128*b^4)*\cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a \\
&*b^3)*\cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c))*\sinh \\
&(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^10 + 90*(\\
&5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^8 + 28*(15*a^4 + 10 \\
&0*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^6 + 30*(5*a^4 + \\
&30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^4 + 3*(5*a^4 + 20*a^3*b + 1 \\
&6*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(3*(5*a^4 + 20*a^3*b + 16*a \\
&^2*b^2)*\cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\co \\
&sh(d*x + c)^9 + 4*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)* \\
&\cosh(d*x + c)^7 + 6*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c \\
&)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^3)*\sinh(d*x + c))*\sqrt{ \\
&b/(a + b))*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
&a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x \\
&+ c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d \\
&>*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*((a^2 + a*b)*\cos \\
&h(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh \\
&(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4* \\
&a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x \\
&+ c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x \\
&+ c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 8*(2*a^4*\cosh(d*x + \\
&c)^15 - 7*(a^4 + 4*a^3*b)*\cosh(d*x + c)^13 - 3*(13*a^4 + 72*a^3*b + 88*a^2 \\
&*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^11 - 5*(9*a^4 + \\
&24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b \\
&^3)*d*x)*\cosh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 \\
&+ 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + \\
&c)^7 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b \\
&+ 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^5 + (13*a^4 + 144*a^3*b + 200* \\
&a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^3 + (a^4 + 4* \\
&a^3*b)*\cosh(d*x + c))*\sinh(d*x + c))/(a^7*d*\cosh(d*x + c)^12 + 12*a^7*d*\cos \\
&h(d*x + c)*\sinh(d*x + c)^11 + a^7*d*\sinh(d*x + c)^12 + a^7*d*\cosh(d*x + c)^ \\
&4 + 4*(a^7 + 2*a^6*b)*d*\cosh(d*x + c)^10 + 2*(33*a^7*d*\cosh(d*x + c)^2 + 2* \\
&(a^7 + 2*a^6*b)*d)*\sinh(d*x + c)^10 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*\cos \\
&h(d*x + c)^8 + 20*(11*a^7*d*\cosh(d*x + c)^3 + 2*(a^7 + 2*a^6*b)*d*\cosh(d*x \\
&+ c))*\sinh(d*x + c)^9 + (495*a^7*d*\cosh(d*x + c)^4 + 180*(a^7 + 2*a^6*b)*d* \\
&\cosh(d*x + c)^2 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d)*\sinh(d*x + c)^8 + 4*(a \\
&^7 + 2*a^6*b)*d*\cosh(d*x + c)^6 + 8*(99*a^7*d*\cosh(d*x + c)^5 + 60*(a^7 + 2 \\
&*a^6*b)*d*\cosh(d*x + c)^3 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^7 + 4*(231*a^7*d*\cosh(d*x + c)^6 + 210*(a^7 + 2*a^6*b)*d*\co \\
&sh(d*x + c)^4 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*\cosh(d*x + c)^2 + (a^7 + \\
&2*a^6*b)*d)*\sinh(d*x + c)^6 + 8*(99*a^7*d*\cosh(d*x + c)^7 + 126*(a^7 + 2*a \\
&^6*b)*d*\cosh(d*x + c)^5 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*\cosh(d*x + c)^ \\
&3 + 3*(a^7 + 2*a^6*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + (495*a^7*d*\cosh(d \\
&x + c)^8 + a^7*d + 840*(a^7 + 2*a^6*b)*d*\cosh(d*x + c)^6 + 140*(3*a^7 + 8*a \\
&^6*b + 8*a^5*b^2)*d*\cosh(d*x + c)^4 + 60*(a^7 + 2*a^6*b)*d*\cosh(d*x + c)^2)
\end{aligned}$$

$$\begin{aligned}
& * \sinh(dx + c)^4 + 4*(55*a^7*d*cosh(dx + c)^9 + a^7*d*cosh(dx + c) + 120* \\
& (a^7 + 2*a^6*b)*d*cosh(dx + c)^7 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh \\
& (dx + c)^5 + 20*(a^7 + 2*a^6*b)*d*cosh(dx + c)^3)*\sinh(dx + c)^3 + 2*(33 \\
& *a^7*d*cosh(dx + c)^10 + 3*a^7*d*cosh(dx + c)^2 + 90*(a^7 + 2*a^6*b)*d*co \\
& sh(dx + c)^8 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(dx + c)^6 + 30*(a^ \\
& 7 + 2*a^6*b)*d*cosh(dx + c)^4)*\sinh(dx + c)^2 + 4*(3*a^7*d*cosh(dx + c)^ \\
& 11 + a^7*d*cosh(dx + c)^3 + 10*(a^7 + 2*a^6*b)*d*cosh(dx + c)^9 + 4*(3*a^ \\
& 7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(dx + c)^7 + 6*(a^7 + 2*a^6*b)*d*cosh(dx + \\
& c)^5)*\sinh(dx + c)), 1/64*(a^4*cosh(dx + c)^16 + 16*a^4*cosh(dx + c)*si \\
& nh(dx + c)^15 + a^4*\sinh(dx + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(dx + c)^14 \\
& + 4*(30*a^4*cosh(dx + c)^2 - a^4 - 4*a^3*b)*\sinh(dx + c)^14 + 56*(10*a^4* \\
& cosh(dx + c)^3 - (a^4 + 4*a^3*b)*cosh(dx + c))*\sinh(dx + c)^13 - 2*(13*a \\
& ^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(dx \\
& + c)^12 + 2*(910*a^4*cosh(dx + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12 \\
& *(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(dx + c)^2)*s \\
& inh(dx + c)^12 + 8*(546*a^4*cosh(dx + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(dx \\
& + c)^3 - 3*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b \\
& ^2)*d*x)*cosh(dx + c))*\sinh(dx + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2 \\
& - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c \\
&)^10 + 4*(2002*a^4*cosh(dx + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(dx + c)^4 - \\
& 9*a^4 - 24*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 \\
& + 32*a*b^3)*d*x - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b \\
& + 16*a^2*b^2)*d*x)*cosh(dx + c)^2)*\sinh(dx + c)^10 + 8*(1430*a^4*cosh(dx \\
& + c)^7 - 1001*(a^4 + 4*a^3*b)*cosh(dx + c)^5 - 55*(13*a^4 + 72*a^3*b + 88 \\
& *a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(dx + c)^3 - 5*(9*a^4 \\
& + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32* \\
& a*b^3)*d*x)*cosh(dx + c))*\sinh(dx + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 1 \\
& 84*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^ \\
& 4)*d*x)*cosh(dx + c)^8 + 2*(6435*a^4*cosh(dx + c)^8 - 6006*(a^4 + 4*a^3*b \\
&)*cosh(dx + c)^6 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3* \\
& b + 16*a^2*b^2)*d*x)*cosh(dx + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3 \\
& + 896*b^4 + 24*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x \\
& - 90*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^ \\
& 2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c)^2)*\sinh(dx + c)^8 + 16*(715*a^4*cosh(\\
& dx + c)^9 - 858*(a^4 + 4*a^3*b)*cosh(dx + c)^7 - 99*(13*a^4 + 72*a^3*b + \\
& 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(dx + c)^5 - 30*(9* \\
& a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + \\
& 32*a*b^3)*d*x)*cosh(dx + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 11 \\
& 2*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(\\
& dx + c))*\sinh(dx + c)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 \\
& + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c)^6 + 4*(200 \\
& 2*a^4*cosh(dx + c)^10 - 3003*(a^4 + 4*a^3*b)*cosh(dx + c)^8 - 462*(13*a^4 \\
& + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(dx + \\
& c)^6 - 210*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b \\
& + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2 \\
& *b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(2 \\
& 7*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2 \\
& *b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(dx + c)^2)*\sinh(dx + c)^6 + 8*(546* \\
& a^4*cosh(dx + c)^11 - 1001*(a^4 + 4*a^3*b)*cosh(dx + c)^9 - 198*(13*a^4 + \\
& 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(dx + c \\
&)^7 - 126*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + \\
& 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 + \\
& 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128* \\
& b^4)*d*x)*cosh(dx + c)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 \\
& + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(dx + c))*\sinh(dx \\
& + c)^5 + 2*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2* \\
& b^2)*d*x)*cosh(dx + c)^4 + 2*(910*a^4*cosh(dx + c)^12 - 2002*(a^4 + 4*a^3 \\
& *b)*cosh(dx + c)^10 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a \\
& ^3*b + 16*a^2*b^2)*d*x)*cosh(dx + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + \\
& c)^6 + 560*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^ \\
& 3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^4 + 13*a^4 + 14 \\
& 4*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 + \\
& 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32* \\
& a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 + 8*(70*a^4*\cosh(d*x + c \\
&)^13 - 182*(a^4 + 4*a^3*b)*\cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^ \\
& 2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^9 - 60*(9*a^4 + \\
& 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a* \\
& b^3)*d*x)*\cosh(d*x + c)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b \\
& ^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x \\
& + c)^5 + 10*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^ \\
& 3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 2 \\
& 00*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 + 4*(a^4 + 4*a^3*b)*\cosh(d*x + c)^2 + 4*(30*a^4*\cosh(d*x + c)^14 - 9 \\
& 1*(a^4 + 4*a^3*b)*\cosh(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 1 \\
& 2*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3* \\
& b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x \\
&)*\cosh(d*x + c)^8 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(\\
& 3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*\cosh(d*x + c)^6 \\
& + 15*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40 \\
& *a^2*b^2 + 32*a*b^3)*d*x)*\cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144 \\
& *a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^2 - 24*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^12 + \\
& 12*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^11 + (5*a^4 \\
& + 20*a^3*b + 16*a^2*b^2)*\sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b \\
& ^2 + 32*a*b^3)*\cosh(d*x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a \\
& *b^3 + 33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 \\
& + 20*(11*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^3 + 2*(5*a^4 + 30*a \\
& ^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 2*(15*a^4 + \\
& 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^8 + (495*(5*a^ \\
& 4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b \\
& ^2 + 576*a*b^3 + 256*b^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*c \\
& osh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos \\
& h(d*x + c)^5 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^ \\
& 3 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c \\
&))*\sinh(d*x + c)^7 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x \\
& + c)^6 + 4*(231*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^6 + 210*(5*a^ \\
& 4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b + \\
& 56*a^2*b^2 + 32*a*b^3 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + \\
& 128*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^ \\
& 2*b^2)*\cosh(d*x + c)^7 + 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cos \\
& h(d*x + c)^5 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)* \\
& cosh(d*x + c)^3 + 3*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c \\
&))*\sinh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^4 + (495 \\
& *(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b + \\
& 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2* \\
& b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2 \\
& + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 4*(55*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^9 + 120*(5*a^4 \\
& + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3* \\
& b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3 \\
& *b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^ \\
& 2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)* \\
& osh(d*x + c)^10 + 90*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + \\
& c)^8 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*\cosh(d*x \\
& + c)^6 + 30*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*\cosh(d*x + c)^4 + 3 \\
& *(5*a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(3*(5 \\
& *a^4 + 20*a^3*b + 16*a^2*b^2)*\cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 + 32 a^3 b^3) \cosh(dx + c)^9 + 4(15 a^4 + 100 a^3 b + 248 a^2 b^2 + \\
& 288 a b^3 + 128 b^4) \cosh(dx + c)^7 + 6(5 a^4 + 30 a^3 b + 56 a^2 b^2 + \\
& 32 a b^3) \cosh(dx + c)^5 + (5 a^4 + 20 a^3 b + 16 a^2 b^2) \cosh(dx + c)^3 \\
&) \sinh(dx + c) \sqrt{-b/(a + b)} \arctan(1/2(a \cosh(dx + c)^2 + 2 a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2 b) \sqrt{-b/(a + b)})/b) + \\
& 8(2 a^4 \cosh(dx + c)^{15} - 7(a^4 + 4 a^3 b) \cosh(dx + c)^{13} - 3(13 a^4 + \\
& 72 a^3 b + 88 a^2 b^2 - 12(a^4 + 12 a^3 b + 16 a^2 b^2) dx) \cosh(dx + c)^{11} - 5(9 a^4 + 24 a^3 b - 16 a^2 b^2 - 32 a b^3 - 24(a^4 + 14 a^3 b + \\
& 40 a^2 b^2 + 32 a b^3) dx) \cosh(dx + c)^9 + 16(27 a^3 b + 114 a^2 b^2 + 184 a b^3 + 112 b^4 + 3(3 a^4 + 44 a^3 b + 152 a^2 b^2 + 224 a b^3 + 128 b^4) dx) \cosh(dx + c)^7 + 3(9 a^4 + 168 a^3 b + 496 a^2 b^2 + 416 a b^3 + 24(a^4 + 14 a^3 b + 40 a^2 b^2 + 32 a b^3) dx) \cosh(dx + c)^5 + (13 a^4 + 144 a^3 b + 200 a^2 b^2 + 12(a^4 + 12 a^3 b + 16 a^2 b^2) dx) \cosh(dx + c)^3 + (a^4 + 4 a^3 b) \cosh(dx + c) \sinh(dx + c) / (a^7 d \cosh(dx + c)^{12} + 12 a^7 d \cosh(dx + c) \sinh(dx + c)^{11} + a^7 d \sinh(dx + c)^{12} + a^7 d \cosh(dx + c)^4 + 4(a^7 + 2 a^6 b) d \cosh(dx + c)^{10} + 2(33 a^7 d \cosh(dx + c)^2 + 2(a^7 + 2 a^6 b) d) \sinh(dx + c)^{10} + 2(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^8 + 20(11 a^7 d \cosh(dx + c)^3 + 2(a^7 + 2 a^6 b) d \cosh(dx + c) \sinh(dx + c)^9 + (495 a^7 d \cosh(dx + c)^4 + 180(a^7 + 2 a^6 b) d \cosh(dx + c)^2 + 2(3 a^7 + 8 a^6 b + 8 a^5 b^2) d) \sinh(dx + c)^8 + 4(a^7 + 2 a^6 b) d \cosh(dx + c)^6 + 8(99 a^7 d \cosh(dx + c)^5 + 60(a^7 + 2 a^6 b) d \cosh(dx + c)^3 + 2(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c) \sinh(dx + c)^7 + 4(231 a^7 d \cosh(dx + c)^6 + 210(a^7 + 2 a^6 b) d \cosh(dx + c)^4 + 14(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^2 + (a^7 + 2 a^6 b) d) \sinh(dx + c)^6 + 8(99 a^7 d \cosh(dx + c)^7 + 126(a^7 + 2 a^6 b) d \cosh(dx + c)^5 + 14(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^3 + 3(a^7 + 2 a^6 b) d \cosh(dx + c) \sinh(dx + c)^5 + (495 a^7 d \cosh(dx + c)^8 + a^7 d + 840(a^7 + 2 a^6 b) d \cosh(dx + c)^6 + 140(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^4 + 60(a^7 + 2 a^6 b) d \cosh(dx + c)^2) \sinh(dx + c)^4 + 4(55 a^7 d \cosh(dx + c)^9 + a^7 d \cosh(dx + c) + 120(a^7 + 2 a^6 b) d \cosh(dx + c)^7 + 28(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^5 + 20(a^7 + 2 a^6 b) d \cosh(dx + c)^3) \sinh(dx + c)^3 + 2(33 a^7 d \cosh(dx + c)^{10} + 3 a^7 d \cosh(dx + c)^2 + 90(a^7 + 2 a^6 b) d \cosh(dx + c)^8 + 28(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^6 + 30(a^7 + 2 a^6 b) d \cosh(dx + c)^4) \sinh(dx + c)^2 + 4(3 a^7 d \cosh(dx + c)^{11} + a^7 d \cosh(dx + c)^3 + 10(a^7 + 2 a^6 b) d \cosh(dx + c)^9 + 4(3 a^7 + 8 a^6 b + 8 a^5 b^2) d \cosh(dx + c)^7 + 6(a^7 + 2 a^6 b) d \cosh(dx + c)^5) \sinh(dx + c))].
\end{aligned}$$

giac [B] time = 6.13, size = 518, normalized size = 2.14

$$\frac{24(a^2 + 12ab + 16b^2)(dx+c)}{a^5} - \frac{24(5a^2b + 20ab^2 + 16b^3) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2} a^5} + \frac{a^3 e^{(4dx+4c)} - 8a^3 e^{(2dx+2c)} - 24a^2 b e^{(2dx+2c)}}{a^6} - \frac{6a^4 e^{(12dx+12c)}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^4/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/64*(24*(a^2 + 12*a*b + 16*b^2)*(dx + c)/a^5 - 24*(5*a^2*b + 20*a*b^2 + 16*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^5) + (a^3*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c))/a^6 - (6*a^4*e^(12*d*x + 12*c) + 72*a^3*b*e^(12*d*x + 12*c) + 96*a^2*b^2*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) + 168*a^3*b*e^(10*d*x + 10*c) + 384*a^2*b^2*e^(10*d*x + 10*c) + 256*a*b^3*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) - 64*a^3*b*e^(8*d*x + 8*c) - 192*a^2*b^2*e^(8*d*x + 8*c) - 256*a*b^3*e^(8*d*x + 8*c) - 256*b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*x + 6*c) - 360*a^3*b*e^(6*d*x + 6*c) - 1024*a^2*b^2*e^(6*d*x + 6*c) - 896*a*b^3*e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) - 216*a^3*b*e^(4*d*x + 4*c) - 304*a^2*b^2*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2*c) - 16*a^3*b*e^(2*d*x +

$$2*c) + a^4)/((a*e^{(6*d*x + 6*c)} + 2*a*e^{(4*d*x + 4*c)} + 4*b*e^{(4*d*x + 4*c)} + a*e^{(2*d*x + 2*c)})^2*a^5))/d$$

maple [B] time = 0.43, size = 1668, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)`

[Out]
$$\begin{aligned} & 3/8/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)+1)-3/8/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/8 \\ & /d/a^3/(\tanh(1/2*d*x+1/2*c)+1)^2-3/8/d/a^3/(\tanh(1/2*d*x+1/2*c)+1)+1/4/d/a^3/ \\ & (\tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a^3/(\tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a^3/ \\ & (\tanh(1/2*d*x+1/2*c)-1)^2-3/8/d/a^3/(\tanh(1/2*d*x+1/2*c)-1)-1/4/d/a^3/(\tanh(\\ & 1/2*d*x+1/2*c)+1)^4+1/2/d/a^3/(\tanh(1/2*d*x+1/2*c)+1)^3-3/2/d/a^4/(\tanh(1/2 \\ & *d*x+1/2*c)-1)*b-9/2/d/a^4*\ln(\tanh(1/2*d*x+1/2*c)-1)*b-6/d/a^5*\ln(\tanh(1/2* \\ & d*x+1/2*c)-1)*b^2+3/2/d/a^4/(\tanh(1/2*d*x+1/2*c)+1)^2*b-3/2/d/a^4/(\tanh(1/2 \\ & *d*x+1/2*c)+1)*b+9/2/d/a^4*\ln(\tanh(1/2*d*x+1/2*c)+1)*b+6/d/a^5*\ln(\tanh(1/2* \\ & d*x+1/2*c)+1)*b^2-15/16/d/a^3*b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d \\ & *x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-35/4/d/a^3*b^2/(\tanh \\ & (1/2*d*x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh \\ & (1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^5+3/d/a^4*b^3/(\tanh(1/2*d*x \\ & +1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d* \\ & x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^3-21/4/d/a^3*b^2/(\tanh(1/2*d*x+1/2* \\ & c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2 \\ & *c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)-35/4/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+ \\ & b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b \\ & +a+b)^2*\tanh(1/2*d*x+1/2*c)^3-3/2/d/a^4/(\tanh(1/2*d*x+1/2*c)-1)^2*b+15/4/d/ \\ & a^4*b^{(3/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh \\ & (1/2*d*x+1/2*c)+(a+b)^{(1/2)})-27/4/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh(1 \\ & /2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2* \\ & \tanh(1/2*d*x+1/2*c)^5+15/16/d/a^3*b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1 \\ & /2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-9/4/d/a^2*b/(\tanh \\ & (1/2*d*x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh \\ & (1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^7+3/d/a^4*b^3/(\tanh(1/2*d* \\ & x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d \\ & *x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^5-27/4/d/a^2*b/(\tanh(1/2*d*x+1/2*c \\ &)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2* \\ & c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^3-3/d/a^4*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b* \\ & \tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & +b)^2*\tanh(1/2*d*x+1/2*c)^7-21/4/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh(\\ & 1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2 \\ & *\tanh(1/2*d*x+1/2*c)^7-15/4/d/a^4*b^{(3/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1 \\ & /2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-3/d/a^5*b^{(5/2)}/ \\ & (a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2 \\ & *c)+(a+b)^{(1/2)})-9/4/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c) \\ & ^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+ \\ & 1/2*c)-3/d/a^4*b^3/(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(\\ & 1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)+3/d \\ & /a^5*b^{(5/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh \\ & (1/2*d*x+1/2*c)+(a+b)^{(1/2)}) \end{aligned}$$

maxima [B] time = 0.60, size = 2468, normalized size = 10.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")`

```
[Out] -3/256*(5*a^4*b + 100*a^3*b^2 + 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e
^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) - 3/64*
(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b
- 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((
a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) + 3/256*(5*a^4*b + 100*a^3*b^2
+ 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*
sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7
+ 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) + 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a
*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a^4*b^2)*s
qrt((a + b)*b)*d) + 3/128*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x -
2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(9*a^5*b
+ 110*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (9*a^5*b + 228*a^4*b^2 + 920*a^
3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(6*d*x + 6*c) + (27*a^5*b + 594*a^4*b^2
+ 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(4*d*x + 4*c) + (
27*a^5*b + 476*a^4*b^2 + 1720*a^3*b^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(2*d*x
+ 2*c))/((a^9 + 2*a^8*b + a^7*b^2 + (a^9 + 2*a^8*b + a^7*b^2)*e^(8*d*x + 8*
c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(6*d*x + 6*c) + 2*(3*a^9 +
14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b^4)*e^(4*d*x + 4*c) + 4*(a^9 +
4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(2*d*x + 2*c))*d) - 1/64*(9*a^5*b + 110
*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (27*a^5*b + 476*a^4*b^2 + 1720*a^3*b
^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(-2*d*x - 2*c) + (27*a^5*b + 594*a^4*b^2 +
2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(-4*d*x - 4*c) + (9
*a^5*b + 228*a^4*b^2 + 920*a^3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(-6*d*x -
6*c))/((a^9 + 2*a^8*b + a^7*b^2 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)
*e^(-2*d*x - 2*c) + 2*(3*a^9 + 14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b
^4)*e^(-4*d*x - 4*c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(-6*d*x
- 6*c) + (a^9 + 2*a^8*b + a^7*b^2)*e^(-8*d*x - 8*c))*d) + 1/16*(9*a^4*b + 3
2*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^4)*e
^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 + 320*b^
5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*b^4)*e^(
2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b^2)*e^(8*d*
x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x + 6*c) + 2*(3
*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x + 4*c) + 4*
(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b
+ 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*
b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 +
320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^
4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^
5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*
b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*c))*d) - 3/
32*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*
c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3
*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4
*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a
^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a
^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*
e^(-8*d*x - 8*c))*d) + 3/8*(d*x + c)/(a^3*d) - 1/8*e^(2*d*x + 2*c)/(a^3*d)
+ 1/8*e^(-2*d*x - 2*c)/(a^3*d) + 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*
e^(2*d*x + 2*c) + a)/(a^4*d) - 3/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e
^(-4*d*x - 4*c) + a)/(a^4*d) + 1/64*(a*e^(4*d*x + 4*c) - 24*b*e^(2*d*x + 2*
c))/(a^4*d) + 1/64*(24*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^4*d) + 3
/8*(a*b + 4*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(
a^5*d) - 3/8*(a*b + 4*b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x -
4*c) + a)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)^6 \sinh(c + dx)^4}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.42 \quad \int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=154

$$\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} + \frac{b^2(a+b)\cosh(c+dx)}{4a^4d(a\cosh^2(c+dx)+b)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d(a\cosh^2(c+dx)+b)} - \frac{(a+3b)\cosh(c+dx)}{a^4d}$$

[Out] $-(a+3*b)*\cosh(d*x+c)/a^{4/d+1/3}*\cosh(d*x+c)^3/a^{3/d+1/4}*b^2*(a+b)*\cosh(d*x+c)/a^{4/d}/(b+a*\cosh(d*x+c)^2)^2-1/8*b*(9*a+13*b)*\cosh(d*x+c)/a^{4/d}/(b+a*\cosh(d*x+c)^2)+5/8*(3*a+7*b)*\arctan(\cosh(d*x+c)*a^{1/2}/b^{1/2})*b^{1/2}/a^{9/2}/d$

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4133, 455, 1814, 1153, 205}

$$\frac{b^2(a+b)\cosh(c+dx)}{4a^4d(a\cosh^2(c+dx)+b)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d(a\cosh^2(c+dx)+b)} - \frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3, x]

[Out] $(5*\text{Sqrt}[b]*(3*a + 7*b)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cosh}[c + d*x])/\text{Sqrt}[b]])/(8*a^{9/2}*d) - ((a + 3*b)*\text{Cosh}[c + d*x])/(a^4*d) + \text{Cosh}[c + d*x]^3/(3*a^3*d) + (b^2*(a + b)*\text{Cosh}[c + d*x])/(4*a^4*d*(b + a*\text{Cosh}[c + d*x]^2)^2) - (b*(9*a + 13*b)*\text{Cosh}[c + d*x])/(8*a^4*d*(b + a*\text{Cosh}[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int

`[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]`

Rule 4133

`Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*sin[(e_) + (f_)*(x_)]
^(m_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f,
Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x
, x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= - \frac{\operatorname{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\
 &= \frac{b^2(a + b) \cosh(c + dx)}{4a^4d (b + a \cosh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{4a^4d} \\
 &= \frac{b^2(a + b) \cosh(c + dx)}{4a^4d (b + a \cosh^2(c + dx))^2} - \frac{b(9a + 13b) \cosh(c + dx)}{8a^4d (b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{-b^2(7a+11b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)} dx, x, \cosh(c + dx)\right)}{4a^4d} \\
 &= \frac{b^2(a + b) \cosh(c + dx)}{4a^4d (b + a \cosh^2(c + dx))^2} - \frac{b(9a + 13b) \cosh(c + dx)}{8a^4d (b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int (8b(a + 3b) + 4a^2x^2 - 4a^2x^4 + 4a^3x^6) dx, x, \cosh(c + dx)\right)}{4a^4d} \\
 &= -\frac{(a + 3b) \cosh(c + dx)}{a^4d} + \frac{\cosh^3(c + dx)}{3a^3d} + \frac{b^2(a + b) \cosh(c + dx)}{4a^4d (b + a \cosh^2(c + dx))^2} - \frac{b(9a + 13b) \cosh(c + dx)}{8a^4d (b + a \cosh^2(c + dx))} \\
 &= \frac{5\sqrt{b}(3a + 7b) \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} - \frac{(a + 3b) \cosh(c + dx)}{a^4d} + \frac{\cosh^3(c + dx)}{3a^3d} + \frac{b^2(a + b) \cosh(c + dx)}{4a^4d (b + a \cosh^2(c + dx))^2} - \frac{b(9a + 13b) \cosh(c + dx)}{8a^4d (b + a \cosh^2(c + dx))}
 \end{aligned}$$

Mathematica [C] time = 10.38, size = 1217, normalized size = 7.90

result too large to display

Warning: Unable to verify antiderivative.

[In] `Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3, x]`

[Out] `((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((24*(3*a - 4*b)*(ArcTan
(((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2)])*Sinh[c]*Tanh[(d*x)/
2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Tanh[(d*x)
) / 2])) / Sqrt[b]] + ArcTan[(((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])
^2)])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c]
- Sinh[c])^2])*Tanh[(d*x)/2])) / Sqrt[b]])) / (a^(3/2)*b^(5/2)) - (54*(ArcTan[(
Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2]) / Sqrt[b]] + ArcTan[(Sqrt[a] + I*S
qrt[a + b]*Tanh[(c + d*x)/2]) / Sqrt[b]]) / (Sqrt[a]*b^(5/2)) - (36*Cosh[c + d
x](3*a + 10*b + 3*a*Cosh[2*(c + d*x)])) / (b^2*(a + 2*b + a*Cosh[2*(c + d*x
)])^2) + (48*Cosh[c + d*x]*(3*a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*(c
+ d*x)])) / (a*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (3*(3*a^4 - 40*a^3*b
+ 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[(((Sqrt[a] - I*Sqrt[a + b]*Sq`

$$\begin{aligned} & \text{rt}[(\text{Cosh}[c] - \text{Sinh}[c])^2)] * \text{Sinh}[c] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] - I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b] + 3 * (3*a^4 - 40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4) * \text{ArcTan}[(\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b]] + (2 * \text{Sqrt}[a] * \text{Sqrt}[b] * \text{Cosh}[c + d*x] * (9*a^5 - 90*a^4*b - 10144*a^3*b^2 - 48672*a^2*b^3 - 85120*a*b^4 - 53760*b^5 + a * (9*a^4 - 120*a^3*b - 12432*a^2*b^2 - 47936*a*b^3 - 44800*b^4) * \text{Cosh}[2*(c + d*x)] - 128*a^2*b^2 * (15*a + 28*b) * \text{Cosh}[4*(c + d*x)] + 128*a^3*b^2 * \text{Cosh}[6*(c + d*x)])) / (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2) / (a^(9/2) * b^(5/2)) + (9 * ((-3 * (a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3) * \text{ArcTan}[(\text{Sqrt}[a] - I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] - I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b]])) / b^(5/2) - (3 * (a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3) * \text{ArcTan}[(\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b]])) / b^(5/2) + 512 * \text{Sqrt}[a] * \text{Cosh}[c] * \text{Cosh}[d*x] - (8 * \text{Sqrt}[a] * (a^3 + 24*a^2*b + 80*a*b^2 + 64*b^3) * \text{Cosh}[c + d*x]) / (b * (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2) - (2 * \text{Sqrt}[a] * (3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3) * \text{Cosh}[c + d*x]) / (b^2 * (a + 2*b + a * \text{Cosh}[2*(c + d*x)])) + 512 * \text{Sqrt}[a] * \text{Sinh}[c] * \text{Sinh}[d*x]) / a^(7/2)) / (49152 * d * (a + b * \text{Sech}[c + d*x]^2)^3) \end{aligned}$$

fricas [B] time = 0.56, size = 8667, normalized size = 56.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/48*(2*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + 2*a^3*sinh(d*x + c)^14 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + 2*(91*a^3*cosh(d*x + c)^2 - 5*a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 8*(91*a^3*cosh(d*x + c)^3 - 3*(5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^10 + 2*(1001*a^3*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(1001*a^3*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^3 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + 2*(3003*a^3*cosh(d*x + c)^6 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^5 - 15*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + 2*(3003*a^3*cosh(d*x + c)^8 - 924*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(1001*a^3*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^7 - 126*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 + 2*(1001*a^3*cosh(d*x + c)^10 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^6 - 350*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(91*a^3*cosh(d*x + c)^11 - 55*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^7 - 70*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 25*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a^3 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + 2*(91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 +

$$\begin{aligned}
& 28a^2b) \cosh(dx + c)^{10} - 45(39a^3 + 290a^2b + 350ab^2) \cosh(dx + c)^8 - 140(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c)^6 - 75(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c)^4 - 5a^3 - 28a^2b - 6(39a^3 + 290a^2b + 350ab^2) \cosh(dx + c)^2 \sinh(dx + c)^2 \\
& + 15((3a^3 + 7a^2b) \cosh(dx + c)^{11} + 11(3a^3 + 7a^2b) \cosh(dx + c) \sinh(dx + c)^{10} + (3a^3 + 7a^2b) \sinh(dx + c)^{11} + 4(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^9 + (12a^3 + 52a^2b + 56ab^2 + 55(3a^3 + 7a^2b) \cosh(dx + c)^2) \sinh(dx + c)^9 + 3(55(3a^3 + 7a^2b) \cosh(dx + c)^3 + 12(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)) \sinh(dx + c)^8 + 2(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^7 + 2(165(3a^3 + 7a^2b) \cosh(dx + c)^4 + 9a^3 + 45a^2b + 80ab^2 + 56b^3 + 72(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^2) \sinh(dx + c)^7 + 14(33(3a^3 + 7a^2b) \cosh(dx + c)^5 + 24(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^3 + (9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)) \sinh(dx + c)^6 + 4(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^5 + 2(231(3a^3 + 7a^2b) \cosh(dx + c)^6 + 252(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^4 + 6a^3 + 26a^2b + 28ab^2 + 21(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^2) \sinh(dx + c)^5 + 2(165(3a^3 + 7a^2b) \cosh(dx + c)^7 + 252(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^5 + 35(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^3 + 10(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)) \sinh(dx + c)^4 + (3a^3 + 7a^2b) \cosh(dx + c)^3 + (165(3a^3 + 7a^2b) \cosh(dx + c)^8 + 336(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^6 + 70(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^4 + 3a^3 + 7a^2b + 40(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^2) \sinh(dx + c)^3 + (55(3a^3 + 7a^2b) \cosh(dx + c)^9 + 144(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^7 + 42(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^5 + 40(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^3 + 3(3a^3 + 7a^2b) \cosh(dx + c)) \sinh(dx + c)^2 + (11(3a^3 + 7a^2b) \cosh(dx + c)^{10} + 36(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^8 + 14(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^6 + 20(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^4 + 3(3a^3 + 7a^2b) \cosh(dx + c)^2) \sinh(dx + c)) \sqrt{-b/a} \log((a \cosh(dx + c)^4 + 4a \cosh(dx + c) \sinh(dx + c)^3 + a \sinh(dx + c)^4 + 2(a - 2b) \cosh(dx + c)^2 + 2(3a \cosh(dx + c)^2 + a - 2b) \sinh(dx + c)^2 + 4(a \cosh(dx + c)^3 + (a - 2b) \cosh(dx + c)) \sinh(dx + c) + 4(a \cosh(dx + c)^3 + 3a \cosh(dx + c) \sinh(dx + c)^2 + a \sinh(dx + c)^3 + a \cosh(dx + c) + (3a \cosh(dx + c)^2 + a) \sinh(dx + c)) \sqrt{-b/a} + a) / (a \cosh(dx + c)^4 + 4a \cosh(dx + c) \sinh(dx + c)^3 + a \sinh(dx + c)^4 + 2(a + 2b) \cosh(dx + c)^2 + 2(3a \cosh(dx + c)^2 + a + 2b) \sinh(dx + c)^2 + 4(a \cosh(dx + c)^3 + (a + 2b) \cosh(dx + c)) \sinh(dx + c) + a)) + 4(7a^3 \cosh(dx + c)^{13} - 6(5a^3 + 28a^2b) \cosh(dx + c)^{11} - 5(39a^3 + 290a^2b + 350ab^2) \cosh(dx + c)^9 - 20(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c)^7 - 15(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c)^5 - 2(39a^3 + 290a^2b + 350ab^2) \cosh(dx + c)^3 - (5a^3 + 28a^2b) \cosh(dx + c)) \sinh(dx + c)) / (a^6 d \cosh(dx + c)^{11} + 11a^6 d \cosh(dx + c) \sinh(dx + c)^{10} + a^6 d \sinh(dx + c)^{11} + 4(a^6 + 2a^5b) d \cosh(dx + c)^9 + a^6 d \cosh(dx + c)^3 + (55a^6 d \cosh(dx + c)^2 + 4(a^6 + 2a^5b) d) \sinh(dx + c)^9 + 2(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^7 + 3(55a^6 d \cosh(dx + c)^3 + 12(a^6 + 2a^5b) d \cosh(dx + c)) \sinh(dx + c)^8 + 2(165a^6 d \cosh(dx + c)^4 + 72(a^6 + 2a^5b) d \cosh(dx + c)^2 + (3a^6 + 8a^5b + 8a^4b^2) d) \sinh(dx + c)^7 + 4(a^6 + 2a^5b) d \cosh(dx + c)^5 + 14(33a^6 d \cosh(dx + c)^5 + 24(a^6 + 2a^5b) d \cosh(dx + c)^3 + (3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)) \sinh(dx + c)^6 + 2(231a^6 d \cosh(dx + c)^6 + 252(a^6 + 2a^5b) d \cosh(dx + c)^4 + 21(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^2 + 2(a^6 + 2a^5b) d) \sinh(dx + c)^5 + 2(165a^6 d \cosh(dx + c)^7 + 252(a^6 + 2a^5b) d \cosh(dx + c)^5 + 35(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^3 + 10(a^6 + 2a^5b) d \cosh(dx + c)) \sinh(dx + c)^4 + (165a^6 d \cosh(dx + c)^8 + 336(a^6 + 2a^5b) d \cosh(dx + c)^6 + a^6 d + 70(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^4 + 40(a^6 + 2a^5b)
\end{aligned}$$

$$\begin{aligned}
& b) * d * \cosh(dx + c)^2 * \sinh(dx + c)^3 + (55 * a^6 * d * \cosh(dx + c)^9 + 144 * (a^6 + 2 * a^5 * b) * d * \cosh(dx + c)^7 + 3 * a^6 * d * \cosh(dx + c) + 42 * (3 * a^6 + 8 * a^5 * b + 8 * a^4 * b^2) * d * \cosh(dx + c)^5 + 40 * (a^6 + 2 * a^5 * b) * d * \cosh(dx + c)^3) * \sinh(dx + c)^2 + (11 * a^6 * d * \cosh(dx + c)^10 + 36 * (a^6 + 2 * a^5 * b) * d * \cosh(dx + c)^8 + 3 * a^6 * d * \cosh(dx + c)^2 + 14 * (3 * a^6 + 8 * a^5 * b + 8 * a^4 * b^2) * d * \cosh(dx + c)^6 + 20 * (a^6 + 2 * a^5 * b) * d * \cosh(dx + c)^4) * \sinh(dx + c)), 1/24 * (a^3 * \cosh(dx + c)^14 + 14 * a^3 * \cosh(dx + c) * \sinh(dx + c)^13 + a^3 * \sinh(dx + c)^14 - (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^12 + (91 * a^3 * \cosh(dx + c)^2 - 5 * a^3 - 28 * a^2 * b) * \sinh(dx + c)^12 + 4 * (91 * a^3 * \cosh(dx + c)^3 - 3 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)) * \sinh(dx + c)^11 - (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^10 + (1001 * a^3 * \cosh(dx + c)^4 - 39 * a^3 - 290 * a^2 * b - 350 * a * b^2 - 66 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^10 + 2 * (1001 * a^3 * \cosh(dx + c)^5 - 110 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^3 - 5 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^9 - 5 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^8 + (3003 * a^3 * \cosh(dx + c)^6 - 495 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^4 - 85 * a^3 - 730 * a^2 * b - 1410 * a * b^2 - 840 * b^3 - 45 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 8 * (429 * a^3 * \cosh(dx + c)^7 - 99 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^5 - 15 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^3 - 5 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)) * \sinh(dx + c)^7 - 5 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^6 + (3003 * a^3 * \cosh(dx + c)^8 - 924 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^6 - 210 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^4 - 85 * a^3 - 730 * a^2 * b - 1410 * a * b^2 - 840 * b^3 - 140 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 2 * (1001 * a^3 * \cosh(dx + c)^9 - 396 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^7 - 126 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^5 - 140 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^3 - 15 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)) * \sinh(dx + c)^5 - (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^4 + (1001 * a^3 * \cosh(dx + c)^10 - 495 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^8 - 210 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^6 - 350 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^4 - 39 * a^3 - 290 * a^2 * b - 350 * a * b^2 - 75 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 4 * (91 * a^3 * \cosh(dx + c)^11 - 55 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^9 - 30 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^7 - 70 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^5 - 25 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^3 - (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + a^3 - (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^2 + (91 * a^3 * \cosh(dx + c)^12 - 66 * (5 * a^3 + 28 * a^2 * b) * \cosh(dx + c)^10 - 45 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^8 - 140 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^6 - 75 * (17 * a^3 + 146 * a^2 * b + 282 * a * b^2 + 168 * b^3) * \cosh(dx + c)^4 - 5 * a^3 - 28 * a^2 * b - 6 * (39 * a^3 + 290 * a^2 * b + 350 * a * b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^2 - 15 * ((3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^11 + 11 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c) * \sinh(dx + c)^10 + (3 * a^3 + 7 * a^2 * b) * \sinh(dx + c)^11 + 4 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^9 + (12 * a^3 + 52 * a^2 * b + 56 * a * b^2 + 55 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^9 + 3 * (55 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^3 + 12 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^8 + 2 * (9 * a^3 + 45 * a^2 * b + 80 * a * b^2 + 56 * b^3) * \cosh(dx + c)^7 + 2 * (165 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^4 + 9 * a^3 + 45 * a^2 * b + 80 * a * b^2 + 56 * b^3 + 72 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^7 + 14 * (33 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^5 + 24 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^3 + (9 * a^3 + 45 * a^2 * b + 80 * a * b^2 + 56 * b^3) * \cosh(dx + c)) * \sinh(dx + c)^6 + 4 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^5 + 2 * (231 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^6 + 252 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^4 + 6 * a^3 + 26 * a^2 * b + 28 * a * b^2 + 21 * (9 * a^3 + 45 * a^2 * b + 80 * a * b^2 + 56 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^5 + 2 * (165 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^7 + 252 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)^5 + 35 * (9 * a^3 + 45 * a^2 * b + 80 * a * b^2 + 56 * b^3) * \cosh(dx + c)^3 + 10 * (3 * a^3 + 13 * a^2 * b + 14 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^4 + (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^3 + (165 * (3 * a^3 + 7 * a^2 * b) * \cosh(dx + c)^8 + 336 * (3 * a
\end{aligned}$$

$$\begin{aligned}
&^3 + 13a^2b + 14ab^2) \cosh(dx + c)^6 + 70(9a^3 + 45a^2b + 80ab^2 \\
&+ 56b^3) \cosh(dx + c)^4 + 3a^3 + 7a^2b + 40(3a^3 + 13a^2b + 14ab^2) \\
&b^2) \cosh(dx + c)^2) \sinh(dx + c)^3 + (55(3a^3 + 7a^2b) \cosh(dx + c) \\
&^9 + 144(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^7 + 42(9a^3 + 45a^2 \\
&*b + 80ab^2 + 56b^3) \cosh(dx + c)^5 + 40(3a^3 + 13a^2b + 14ab^2) * \\
&\cosh(dx + c)^3 + 3(3a^3 + 7a^2b) \cosh(dx + c)) \sinh(dx + c)^2 + (11 * \\
&(3a^3 + 7a^2b) \cosh(dx + c)^10 + 36(3a^3 + 13a^2b + 14ab^2) \cosh(\\
&dx + c)^8 + 14(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^6 + 20 \\
&*(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^4 + 3(3a^3 + 7a^2b) \cosh(d \\
&*x + c)^2) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2*(a \cosh(dx + c)^3 + 3a \cos \\
&h(dx + c) \sinh(dx + c)^2 + a \sinh(dx + c)^3 + (a + 4b) \cosh(dx + c) + \\
&(3a \cosh(dx + c)^2 + a + 4b) \sinh(dx + c)) \sqrt{b/a}/b) + 15*((3a^3 + \\
&7a^2b) \cosh(dx + c)^11 + 11(3a^3 + 7a^2b) \cosh(dx + c) \sinh(dx + c \\
&)^10 + (3a^3 + 7a^2b) \sinh(dx + c)^11 + 4(3a^3 + 13a^2b + 14ab^2) \\
&)* \cosh(dx + c)^9 + (12a^3 + 52a^2b + 56ab^2 + 55(3a^3 + 7a^2b) \cos \\
&h(dx + c)^2) \sinh(dx + c)^9 + 3(55(3a^3 + 7a^2b) \cosh(dx + c)^3 + 1 \\
&2*(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)) \sinh(dx + c)^8 + 2(9a^3 + \\
&45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^7 + 2(165(3a^3 + 7a^2b) *c \\
&osh(dx + c)^4 + 9a^3 + 45a^2b + 80ab^2 + 56b^3 + 72(3a^3 + 13a^2* \\
&b + 14ab^2) \cosh(dx + c)^2) \sinh(dx + c)^7 + 14(33(3a^3 + 7a^2b) *c \\
&osh(dx + c)^5 + 24(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^3 + (9a^3 \\
&+ 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)) \sinh(dx + c)^6 + 4(3a^3 + \\
&13a^2b + 14ab^2) \cosh(dx + c)^5 + 2(231(3a^3 + 7a^2b) \cosh(dx + \\
&c)^6 + 252(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^4 + 6a^3 + 26a^2* \\
&b + 28ab^2 + 21(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^2) *s \\
&inh(dx + c)^5 + 2(165(3a^3 + 7a^2b) \cosh(dx + c)^7 + 252(3a^3 + 13 \\
&a^2b + 14ab^2) \cosh(dx + c)^5 + 35(9a^3 + 45a^2b + 80ab^2 + 56b \\
&^3) \cosh(dx + c)^3 + 10(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)) \sinh(\\
&dx + c)^4 + (3a^3 + 7a^2b) \cosh(dx + c)^3 + (165(3a^3 + 7a^2b) \cos \\
&h(dx + c)^8 + 336(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^6 + 70(9a^ \\
&3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^4 + 3a^3 + 7a^2b + 40(3 \\
&a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^2) \sinh(dx + c)^3 + (55(3a^3 + \\
&7a^2b) \cosh(dx + c)^9 + 144(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c) \\
&^7 + 42(9a^3 + 45a^2b + 80ab^2 + 56b^3) \cosh(dx + c)^5 + 40(3a^3 \\
&+ 13a^2b + 14ab^2) \cosh(dx + c)^3 + 3(3a^3 + 7a^2b) \cosh(dx + c)) \\
&*\sinh(dx + c)^2 + (11(3a^3 + 7a^2b) \cosh(dx + c)^10 + 36(3a^3 + 13* \\
&a^2b + 14ab^2) \cosh(dx + c)^8 + 14(9a^3 + 45a^2b + 80ab^2 + 56b^ \\
&3) \cosh(dx + c)^6 + 20(3a^3 + 13a^2b + 14ab^2) \cosh(dx + c)^4 + 3*(\\
&3a^3 + 7a^2b) \cosh(dx + c)^2) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2*(a \cos \\
&h(dx + c) + a \sinh(dx + c)) \sqrt{b/a}/b) + 2(7a^3 \cosh(dx + c)^13 - 6 \\
&*(5a^3 + 28a^2b) \cosh(dx + c)^11 - 5(39a^3 + 290a^2b + 350ab^2) *c \\
&osh(dx + c)^9 - 20(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c \\
&)^7 - 15(17a^3 + 146a^2b + 282ab^2 + 168b^3) \cosh(dx + c)^5 - 2(39 \\
&a^3 + 290a^2b + 350ab^2) \cosh(dx + c)^3 - (5a^3 + 28a^2b) \cosh(dx \\
&+ c)) \sinh(dx + c)) / (a^6 d \cosh(dx + c)^11 + 11a^6 d \cosh(dx + c) \sinh \\
&(dx + c)^10 + a^6 d \sinh(dx + c)^11 + 4(a^6 + 2a^5b) d \cosh(dx + c)^9 \\
&+ a^6 d \cosh(dx + c)^3 + (55a^6 d \cosh(dx + c)^2 + 4(a^6 + 2a^5b) d) \\
&*\sinh(dx + c)^9 + 2(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^7 + 3(5 \\
&5a^6 d \cosh(dx + c)^3 + 12(a^6 + 2a^5b) d \cosh(dx + c)) \sinh(dx + c) \\
&^8 + 2(165a^6 d \cosh(dx + c)^4 + 72(a^6 + 2a^5b) d \cosh(dx + c)^2 + \\
&(3a^6 + 8a^5b + 8a^4b^2) d) \sinh(dx + c)^7 + 4(a^6 + 2a^5b) d \cosh \\
&(dx + c)^5 + 14(33a^6 d \cosh(dx + c)^5 + 24(a^6 + 2a^5b) d \cosh(dx \\
&+ c)^3 + (3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)) \sinh(dx + c)^6 + 2 \\
&*(231a^6 d \cosh(dx + c)^6 + 252(a^6 + 2a^5b) d \cosh(dx + c)^4 + 21(3 \\
&a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^2 + 2(a^6 + 2a^5b) d) \sinh(d \\
&*x + c)^5 + 2(165a^6 d \cosh(dx + c)^7 + 252(a^6 + 2a^5b) d \cosh(dx + \\
&c)^5 + 35(3a^6 + 8a^5b + 8a^4b^2) d \cosh(dx + c)^3 + 10(a^6 + 2a^ \\
&5b) d \cosh(dx + c)) \sinh(dx + c)^4 + (165a^6 d \cosh(dx + c)^8 + 336(a \\
&^6 + 2a^5b) d \cosh(dx + c)^6 + a^6 d + 70(3a^6 + 8a^5b + 8a^4b^2) *
\end{aligned}$$

$$d \cdot \cosh(dx + c)^4 + 40 \cdot (a^6 + 2 \cdot a^5 \cdot b) \cdot d \cdot \cosh(dx + c)^2 \cdot \sinh(dx + c)^3 + (55 \cdot a^6 \cdot d \cdot \cosh(dx + c)^9 + 144 \cdot (a^6 + 2 \cdot a^5 \cdot b) \cdot d \cdot \cosh(dx + c)^7 + 3 \cdot a^6 \cdot d \cdot \cosh(dx + c) + 42 \cdot (3 \cdot a^6 + 8 \cdot a^5 \cdot b + 8 \cdot a^4 \cdot b^2) \cdot d \cdot \cosh(dx + c)^5 + 40 \cdot (a^6 + 2 \cdot a^5 \cdot b) \cdot d \cdot \cosh(dx + c)^3) \cdot \sinh(dx + c)^2 + (11 \cdot a^6 \cdot d \cdot \cosh(dx + c)^{10} + 36 \cdot (a^6 + 2 \cdot a^5 \cdot b) \cdot d \cdot \cosh(dx + c)^8 + 3 \cdot a^6 \cdot d \cdot \cosh(dx + c)^2 + 14 \cdot (3 \cdot a^6 + 8 \cdot a^5 \cdot b + 8 \cdot a^4 \cdot b^2) \cdot d \cdot \cosh(dx + c)^6 + 20 \cdot (a^6 + 2 \cdot a^5 \cdot b) \cdot d \cdot \cosh(dx + c)^4) \cdot \sinh(dx + c)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^3/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-43,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-5,48]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-33,-84]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[90,-39]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[70,15]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-11,29]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[84,29]Unassigned Inf encountered in limitEvaluation time: 3.48Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.38, size = 1296, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(dx+c)^3/(a+b*sech(dx+c)^2)^3,x)

[Out]
$$-1/3/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) - 1)^3 - 1/2/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) - 1)^2 + 1/2/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) - 1) + 3/d/a^4/(\tanh(1/2 \cdot dx + 1/2 \cdot c) - 1) \cdot b + 1/3/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) + 1)^3 - 1/2/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) + 1)^2 - 1/2/d/a^3/(\tanh(1/2 \cdot dx + 1/2 \cdot c) + 1) - 3/d/a^4/(\tanh(1/2 \cdot dx + 1/2 \cdot c) + 1) \cdot b - 9/4/d/a^2 \cdot b/(\tanh(1/2 \cdot dx + 1/2 \cdot c))^4 \cdot a + b \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^2 \cdot b + a \cdot b)^2 \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^6 + 1/2/d/a^3 \cdot b^2/(\tanh(1/2 \cdot dx + 1/2 \cdot c))^3$$

$$\begin{aligned}
& +1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6+11/4/d/a^4*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6-27/4/d/a*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^4-15/4/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^4-5/4/d/a^3*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^4-33/4/d/a^4*b^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^4-27/4/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^2-5/2/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^2+33/4/d/a^4*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^2-9/4/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-5/d/a^3*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-11/4/d/a^4*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+15/8/d/a^3*b/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+35/8/d/a^4*b^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*\tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^6 \sinh(c+dx)^3}{(a \cosh(c+dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c+d*x)^3/(a+b/cosh(c+d*x)^2)^3,x)

[Out] int((cosh(c+d*x)^6*sinh(c+d*x)^3)/(b+a*cosh(c+d*x)^2)^3,x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.43 \quad \int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=187

$$-\frac{x(a+6b)}{2a^4} + \frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)(a-b\tanh^2(c+dx)+b)} + \frac{3b\tanh(c+dx)}{4a^2d(a-b\tanh^2(c+dx)+b)^2} + \frac{\sqrt{b}(15a^2+40ab+24b^2)}{8a^4d(a+b)}$$

[Out] $-1/2*(a+6*b)*x/a^4+1/8*(15*a^2+40*a*b+24*b^2)*\operatorname{arctanh}(b^{1/2}*\tanh(d*x+c)/(a+b)^{1/2})*b^{1/2}/a^4/(a+b)^{3/2}/d+1/2*\cosh(d*x+c)*\sinh(d*x+c)/a/d/(a+b-b*\tanh(d*x+c)^2)+3/4*b*\tanh(d*x+c)/a^2/d/(a+b-b*\tanh(d*x+c)^2)+1/8*b*(1+12*b)*\tanh(d*x+c)/a^3/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}} + \frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)(a-b\tanh^2(c+dx)+b)} + \frac{3b\tanh(c+dx)}{4a^2d(a-b\tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-((a+6*b)*x)/(2*a^4) + (\operatorname{Sqrt}[b]*(15*a^2+40*a*b+24*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c+d*x])/\operatorname{Sqrt}[a+b]])/(8*a^4*(a+b)^{3/2}*d) + (\operatorname{Cosh}[c+d*x]*\operatorname{Sinh}[c+d*x])/(2*a*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2)^2) + (3*b*\operatorname{Tanh}[c+d*x])/(4*a^2*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2)^2) + (b*(11*a+12*b)*\operatorname{Tanh}[c+d*x])/(8*a^3*(a+b)*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{3b \tanh(c + dx)}{4a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-2(a+b-bx^2)}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{3b \tanh(c + dx)}{4a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{b(11a + 12b)}{8a^3(a + b)d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{3b \tanh(c + dx)}{4a^2d(a + b - b \tanh^2(c + dx))^2} + \frac{b(11a + 12b)}{8a^3(a + b)d}$$

$$= -\frac{(a + 6b)x}{2a^4} + \frac{\sqrt{b} (15a^2 + 40ab + 24b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a + b)^{3/2}d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2}$$

Mathematica [B] time = 17.84, size = 2544, normalized size = 13.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
[Out] (-5*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)]*Sinh[2*(c + d*x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)]^2)))/(8192*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)]*Sinh[2*(c + d*x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)]^2)))/(8192*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3)
```

$$\begin{aligned}
& 2) + (\text{Sqrt}[b] * (3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2) * \text{Cosh}[2*(c + d*x)]) * \text{Sinh}[2*(c + d*x)]) / ((a + b)^2 * (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2) / (2048*b^{(5/2)} * d * (a + b * \text{Sech}[c + d*x]^2)^3 + ((a + 2*b + a * \text{Cosh}[2*c + 2*d*x])^3 * \text{Sech}[c + d*x]^6 * ((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5) * \text{ArcTanh}[(\text{Sech}[d*x] * (\text{Cosh}[2*c] - \text{Sinh}[2*c]) * ((a + 2*b) * \text{Sinh}[d*x] - a * \text{Sinh}[2*c + d*x])) / (2 * \text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4])]) * (\text{Cosh}[2*c] - \text{Sinh}[2*c])) / (\text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4]) + (\text{Sech}[2*c] * (256*b^2 * (a + b)^2 * (3*a^2 + 8*a*b + 8*b^2) * d * x * \text{Cosh}[2*c] + 512*a*b^2 * (a + b)^2 * (a + 2*b) * d * x * \text{Cosh}[2*d*x] + 128*a^4*b^2 * d * x * \text{Cosh}[2*(c + 2*d*x)] + 256*a^3*b^3 * d * x * \text{Cosh}[2*(c + 2*d*x)] + 128*a^2*b^4 * d * x * \text{Cosh}[2*(c + 2*d*x)] + 512*a^4*b^2 * d * x * \text{Cosh}[4*c + 2*d*x] + 2048*a^3*b^3 * d * x * \text{Cosh}[4*c + 2*d*x] + 2560*a^2*b^4 * d * x * \text{Cosh}[4*c + 2*d*x] + 1024*a*b^5 * d * x * \text{Cosh}[4*c + 2*d*x] + 128*a^4*b^2 * d * x * \text{Cosh}[6*c + 4*d*x] + 256*a^3*b^3 * d * x * \text{Cosh}[6*c + 4*d*x] + 128*a^2*b^4 * d * x * \text{Cosh}[6*c + 4*d*x] - 9*a^6 * \text{Sinh}[2*c] + 12*a^5*b * \text{Sinh}[2*c] + 684*a^4*b^2 * \text{Sinh}[2*c] + 2880*a^3*b^3 * \text{Sinh}[2*c] + 5280*a^2*b^4 * \text{Sinh}[2*c] + 4608*a*b^5 * \text{Sinh}[2*c] + 1536*b^6 * \text{Sinh}[2*c] + 9*a^6 * \text{Sinh}[2*d*x] - 14*a^5*b * \text{Sinh}[2*d*x] - 608*a^4*b^2 * \text{Sinh}[2*d*x] - 2112*a^3*b^3 * \text{Sinh}[2*d*x] - 2560*a^2*b^4 * \text{Sinh}[2*d*x] - 1024*a*b^5 * \text{Sinh}[2*d*x] + 3*a^6 * \text{Sinh}[2*(c + 2*d*x)] - 12*a^5*b * \text{Sinh}[2*(c + 2*d*x)] - 204*a^4*b^2 * \text{Sinh}[2*(c + 2*d*x)] - 384*a^3*b^3 * \text{Sinh}[2*(c + 2*d*x)] - 192*a^2*b^4 * \text{Sinh}[2*(c + 2*d*x)] - 3*a^6 * \text{Sinh}[4*c + 2*d*x] + 10*a^5*b * \text{Sinh}[4*c + 2*d*x] + 304*a^4*b^2 * \text{Sinh}[4*c + 2*d*x] + 1056*a^3*b^3 * \text{Sinh}[4*c + 2*d*x] + 1280*a^2*b^4 * \text{Sinh}[4*c + 2*d*x] + 512*a*b^5 * \text{Sinh}[4*c + 2*d*x])) / (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2) / (4096*a^3*b^2 * (a + b)^2 * d * (a + b * \text{Sech}[c + d*x]^2)^3 + ((a + 2*b + a * \text{Cosh}[2*c + 2*d*x])^3 * \text{Sech}[c + d*x]^6 * ((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6) * \text{ArcTanh}[(\text{Sech}[d*x] * (\text{Cosh}[2*c] - \text{Sinh}[2*c]) * ((a + 2*b) * \text{Sinh}[d*x] - a * \text{Sinh}[2*c + d*x])) / (2 * \text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4])]) * (\text{Cosh}[2*c] - \text{Sinh}[2*c])) / (\text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4]) + (\text{Sech}[2*c] * (-1536*b^2 * (a + b)^2 * (3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3) * d * x * \text{Cosh}[2*c] - 3072*a*b^2 * (a^2 + 3*a*b + 2*b^2)^2 * d * x * \text{Cosh}[2*d*x] - 768*a^5*b^2 * d * x * \text{Cosh}[2*(c + 2*d*x)] - 3072*a^4*b^3 * d * x * \text{Cosh}[2*(c + 2*d*x)] - 3840*a^3*b^4 * d * x * \text{Cosh}[2*(c + 2*d*x)] - 1536*a^2*b^5 * d * x * \text{Cosh}[2*(c + 2*d*x)] - 3072*a^5*b^2 * d * x * \text{Cosh}[4*c + 2*d*x] - 18432*a^4*b^3 * d * x * \text{Cosh}[4*c + 2*d*x] - 39936*a^3*b^4 * d * x * \text{Cosh}[4*c + 2*d*x] - 36864*a^2*b^5 * d * x * \text{Cosh}[4*c + 2*d*x] - 12288*a*b^6 * d * x * \text{Cosh}[4*c + 2*d*x] - 768*a^5*b^2 * d * x * \text{Cosh}[6*c + 4*d*x] - 3072*a^4*b^3 * d * x * \text{Cosh}[6*c + 4*d*x] - 3840*a^3*b^4 * d * x * \text{Cosh}[6*c + 4*d*x] - 1536*a^2*b^5 * d * x * \text{Cosh}[6*c + 4*d*x] + 9*a^7 * \text{Sinh}[2*c] - 54*a^6*b * \text{Sinh}[2*c] - 2392*a^5*b^2 * \text{Sinh}[2*c] - 13968*a^4*b^3 * \text{Sinh}[2*c] - 36480*a^3*b^4 * \text{Sinh}[2*c] - 50432*a^2*b^5 * \text{Sinh}[2*c] - 35840*a*b^6 * \text{Sinh}[2*c] - 10240*b^7 * \text{Sinh}[2*c] - 9*a^7 * \text{Sinh}[2*d*x] + 56*a^6*b * \text{Sinh}[2*d*x] + 2552*a^5*b^2 * \text{Sinh}[2*d*x] + 13184*a^4*b^3 * \text{Sinh}[2*d*x] + 27072*a^3*b^4 * \text{Sinh}[2*d*x] + 24576*a^2*b^5 * \text{Sinh}[2*d*x] + 8192*a*b^6 * \text{Sinh}[2*d*x] - 3*a^7 * \text{Sinh}[2*(c + 2*d*x)] + 26*a^6*b * \text{Sinh}[2*(c + 2*d*x)] + 992*a^5*b^2 * \text{Sinh}[2*(c + 2*d*x)] + 3648*a^4*b^3 * \text{Sinh}[2*(c + 2*d*x)] + 4480*a^3*b^4 * \text{Sinh}[2*(c + 2*d*x)] + 1792*a^2*b^5 * \text{Sinh}[2*(c + 2*d*x)] + 3*a^7 * \text{Sinh}[4*c + 2*d*x] - 24*a^6*b * \text{Sinh}[4*c + 2*d*x] - 600*a^5*b^2 * \text{Sinh}[4*c + 2*d*x] - 3200*a^4*b^3 * \text{Sinh}[4*c + 2*d*x] - 6720*a^3*b^4 * \text{Sinh}[4*c + 2*d*x] - 6144*a^2*b^5 * \text{Sinh}[4*c + 2*d*x] - 2048*a*b^6 * \text{Sinh}[4*c + 2*d*x] + 256*a^5*b^2 * \text{Sinh}[6*c + 4*d*x] + 1024*a^4*b^3 * \text{Sinh}[6*c + 4*d*x] + 1280*a^3*b^4 * \text{Sinh}[6*c + 4*d*x] + 512*a^2*b^5 * \text{Sinh}[6*c + 4*d*x] + 64*a^5*b^2 * \text{Sinh}[4*c + 6*d*x] + 128*a^4*b^3 * \text{Sinh}[4*c + 6*d*x] + 64*a^3*b^4 * \text{Sinh}[4*c + 6*d*x] + 64*a^5*b^2 * \text{Sinh}[8*c + 6*d*x] + 128*a^4*b^3 * \text{Sinh}[8*c + 6*d*x] + 64*a^3*b^4 * \text{Sinh}[8*c + 6*d*x])) / (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2) / (16384*a^4*b^2 * (a + b)^2 * d * (a + b * \text{Sech}[c + d*x]^2)^3 + ((a + 2*b + a * \text{Cosh}[2*c + 2*d*x])^3 * \text{Sech}[c + d*x]^6 * ((6*a^2 * \text{ArcTanh}[(\text{Sech}[d*x] * (\text{Cosh}[2*c] - \text{Sinh}[2*c]) * ((a + 2*b) * \text{Sinh}[d*x] - a * \text{Sinh}[2*c + d*x])) / (2 * \text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4])]) * (\text{Cosh}[2*c] - \text{Sinh}[2*c])) / (\text{Sqrt}[a + b] * \text{Sqrt}[b * (\text{Cosh}[c] - \text{Sinh}[c])^4]) + (a * \text{Sech}[2*c] * ((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4) * \text{Sinh}[2*d*x] + a * (-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3) * \text{Sinh}[2*(c + 2*d*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4) * \text{Sinh}[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3
\end{aligned}$$

$*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*\text{Tanh}[2*c]/(a^2*(a + 2*b + a*\text{Cosh}[2*(c + d*x)])^2)))/(4096*b^2*(a + b)^2*d*(a + b*\text{Sech}[c + d*x]^2)^3)$

fricas [B] time = 0.60, size = 9730, normalized size = 52.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $[1/16*(2*(a^4 + a^3*b)*\cosh(d*x + c)^{12} + 24*(a^4 + a^3*b)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + 2*(a^4 + a^3*b)*\sinh(d*x + c)^{12} + 8*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^{10} + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x + 33*(a^4 + a^3*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 40*(11*(a^4 + a^3*b)*\cosh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 2*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^8 + 2*(495*(a^4 + a^3*b)*\cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(99*(a^4 + a^3*b)*\cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 4*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*\cosh(d*x + c)^6 + 4*(462*(a^4 + a^3*b)*\cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 - 152*a*b^3 - 80*b^4 - 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(198*(a^4 + a^3*b)*\cosh(d*x + c)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^3 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^4 + 2*(495*(a^4 + a^3*b)*\cosh(d*x + c)^8 + 840*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^6 + 70*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 2*a^4 - 2*a^3*b + 8*(55*(a^4 + a^3*b)*\cosh(d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^5 - 10*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*\cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^2 + 4*(33*(a^4 + a^3*b)*\cosh(d*x + c)^{10} + 90*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*\cosh(d*x + c)^8 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*\cosh(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*\cosh(d*x + c)^{10} + 10*(15*a^4 + 40*a^3*b + 24*a^2*b^2$

$$\begin{aligned}
&) * \cosh(dx + c) * \sinh(dx + c)^9 + (15a^4 + 40a^3b + 24a^2b^2) * \sinh(dx + c)^{10} + 4 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^8 + \\
& (60a^4 + 280a^3b + 416a^2b^2 + 192ab^3 + 45 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 8 * (15 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^3 + 4 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)) * \sinh(dx + c)^7 + 2 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)^6 + 2 * (105 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^4 + 45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4 + 56 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 4 * (63 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^5 + 56 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^3 + 3 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 + 4 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^4 + 2 * (105 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^6 + 140 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^4 + 30a^4 + 140a^3b + 208a^2b^2 + 96ab^3 + 15 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 8 * (15 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^7 + 28 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^5 + 5 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)^3 + 2 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^2 + (45 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^8 + 112 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^6 + 30 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)^4 + 15a^4 + 40a^3b + 24a^2b^2 + 24 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 2 * (5 * (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)^9 + 16 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^7 + 6 * (45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) * \cosh(dx + c)^5 + 8 * (15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) * \cosh(dx + c)^3 + (15a^4 + 40a^3b + 24a^2b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{b / (a + b)} * \log((a^2 * \cosh(dx + c)^4 + 4a^2 * \cosh(dx + c) * \sinh(dx + c)^3 + a^2 * \sinh(dx + c)^4 + 2 * (a^2 + 2ab) * \cosh(dx + c)^2 + 2 * (3a^2 * \cosh(dx + c)^2 + a^2 + 2ab) * \sinh(dx + c)^2 + a^2 + 8ab + 8b^2 + 4 * (a^2 * \cosh(dx + c)^3 + (a^2 + 2ab) * \cosh(dx + c)) * \sinh(dx + c) - 4 * ((a^2 + ab) * \cosh(dx + c)^2 + 2 * (a^2 + ab) * \cosh(dx + c) * \sinh(dx + c) + (a^2 + ab) * \sinh(dx + c)^2 + a^2 + 3ab + 2b^2) * \sqrt{b / (a + b)}) / (a * \cosh(dx + c)^4 + 4a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 + 2 * (a + 2b) * \cosh(dx + c)^2 + 2 * (3a * \cosh(dx + c)^2 + a + 2b) * \sinh(dx + c)^2 + 4 * (a * \cosh(dx + c)^3 + (a + 2b) * \cosh(dx + c)) * \sinh(dx + c) + a) + 8 * (3 * (a^4 + a^3b) * \cosh(dx + c)^{11} + 10 * (a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2) * dx) * \cosh(dx + c)^9 + 2 * (5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16 * (a^4 + 9a^3b + 20a^2b^2 + 12ab^3) * dx) * \cosh(dx + c)^7 - 3 * (27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4 * (3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4) * dx) * \cosh(dx + c)^5 - (5a^4 + 75a^3b + 192a^2b^2 + 128ab^3 + 16 * (a^4 + 9a^3b + 20a^2b^2 + 12ab^3) * dx) * \cosh(dx + c)^3 - (2a^4 + 15a^3b + 14a^2b^2 + 2 * (a^4 + 7a^3b + 6a^2b^2) * dx) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^7 + a^6b) * d * \cosh(dx + c)^{10} + 10 * (a^7 + a^6b) * d * \cosh(dx + c) * \sinh(dx + c)^9 + (a^7 + a^6b) * d * \sinh(dx + c)^{10} + 4 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)^8 + (45 * (a^7 + a^6b) * d * \cosh(dx + c)^2 + 4 * (a^7 + 3a^6b + 2a^5b^2) * d) * \sinh(dx + c)^8 + 2 * (3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) * d * \cosh(dx + c)^6 + 8 * (15 * (a^7 + a^6b) * d * \cosh(dx + c)^3 + 4 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 2 * (105 * (a^7 + a^6b) * d * \cosh(dx + c)^4 + 56 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)^2 + (3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) * d) * \sinh(dx + c)^6 + 4 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)^4 + 4 * (63 * (a^7 + a^6b) * d * \cosh(dx + c)^5 + 56 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)^3 + 3 * (3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (105 * (a^7 + a^6b) * d * \cosh(dx + c)^6 + 140 * (a^7 + 3a^6b + 2a^5b^2) * d * \cosh(dx + c)^4 + 15 * (3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) * d * \cosh(dx + c)^2 + 2 * (a^7 + 3a^6b + 2a
\end{aligned}$$

$$\begin{aligned}
& ^5b^2)d*\sinh(dx + c)^4 + (a^7 + a^6b)*d*\cosh(dx + c)^2 + 8*(15*(a^7 + \\
& a^6b)*d*\cosh(dx + c)^7 + 28*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c)^5 + 5*(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3)*d*\cosh(dx + c)^3 + 2*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c))*\sinh(dx + c)^3 + (45*(a^7 + a^6b)*d*\cosh(dx + c)^8 + 112*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c)^6 + 30*(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3)*d*\cosh(dx + c)^4 + 24*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c)^2 + (a^7 + a^6b)d*\sinh(dx + c)^2 + 2*(5*(a^7 + a^6b)*d*\cosh(dx + c)^9 + 16*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c)^7 + 6*(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3)*d*\cosh(dx + c)^5 + 8*(a^7 + 3a^6b + 2a^5b^2)*d*\cosh(dx + c)^3 + (a^7 + a^6b)d*\cosh(dx + c))*\sinh(dx + c)), 1/8*((a^4 + a^3b)*\cosh(dx + c)^12 + 12*(a^4 + a^3b)*\cosh(dx + c)*\sinh(dx + c)^11 + (a^4 + a^3b)*\sinh(dx + c)^12 + 4*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^10 + 2*(2a^4 + 6a^3b + 4a^2b^2 - 2*(a^4 + 7a^3b + 6a^2b^2)*dx + 33*(a^4 + a^3b)*\cosh(dx + c)^2)*\sinh(dx + c)^10 + 20*(11*(a^4 + a^3b)*\cosh(dx + c)^3 + 2*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c))*\sinh(dx + c)^9 + (5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^8 + (495*(a^4 + a^3b)*\cosh(dx + c)^4 + 5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx + 180*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^8 + 8*(99*(a^4 + a^3b)*\cosh(dx + c)^5 + 60*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^3 + (5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^7 - 2*(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c)^6 + 2*(462*(a^4 + a^3b)*\cosh(dx + c)^6 + 420*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^4 - 27a^3b - 102a^2b^2 - 152ab^3 - 80b^4 - 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^6 + 4*(198*(a^4 + a^3b)*\cosh(dx + c)^7 + 252*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^5 + 14*(5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^3 - 3*(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c))*\sinh(dx + c)^5 - (5a^4 + 75a^3b + 192a^2b^2 + 128ab^3 + 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^4 + (495*(a^4 + a^3b)*\cosh(dx + c)^8 + 840*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^6 + 70*(5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^4 - 5a^4 - 75a^3b - 192a^2b^2 - 128ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx - 30*(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^4 - a^4 - a^3b + 4*(55*(a^4 + a^3b)*\cosh(dx + c)^9 + 120*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^7 + 14*(5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^5 - 10*(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c)^3 - (5a^4 + 75a^3b + 192a^2b^2 + 128ab^3 + 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^3 - 2*(2a^4 + 15a^3b + 14a^2b^2 + 2*(a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^2 + 2*(33*(a^4 + a^3b)*\cosh(dx + c)^10 + 90*(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2)*dx)*\cosh(dx + c)^8 + 14*(5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^6 - 15*(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4*(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4)*dx)*\cosh(dx + c)^4 - 2a^4 - 15a^3b - 14a^2b^2 - 2*(a^4 + 7a^3b + 6a^2b^2)*dx - 3*(5a^4 + 75a^3b + 192a^2b^2 + 128ab^3 + 16*(a^4 + 9a^3b + 20a^2b^2 + 12ab^3)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& ((15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^{10} + 10(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c) \sinh(dx + c)^9 + (15a^4 + 40a^3b + 24a^2b^2) \sinh(dx + c)^{10} + 4(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^8 + (60a^4 + 280a^3b + 416a^2b^2 + 192ab^3 + 45(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^8 + 8(15(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^3 + 4(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)) \sinh(dx + c)^7 + 2(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)^6 + 2(105(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^4 + 45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4 + 56(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^2) \sinh(dx + c)^6 + 4(63(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^5 + 56(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^3 + 3(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 4(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^4 + 2(105(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^6 + 140(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^4 + 30a^4 + 140a^3b + 208a^2b^2 + 96ab^3 + 15(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(15(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^7 + 28(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^5 + 5(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)^3 + 2(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)) \sinh(dx + c)^3 + (15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^2 + (45(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^8 + 112(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^6 + 30(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)^4 + 15a^4 + 40a^3b + 24a^2b^2 + 24(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(5(15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)^9 + 16(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^7 + 6(45a^4 + 240a^3b + 512a^2b^2 + 512ab^3 + 192b^4) \cosh(dx + c)^5 + 8(15a^4 + 70a^3b + 104a^2b^2 + 48ab^3) \cosh(dx + c)^3 + (15a^4 + 40a^3b + 24a^2b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{-b/(a + b)} \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-b/(a + b)})/b) + 4(3(a^4 + a^3b) \cosh(dx + c)^{11} + 10(a^4 + 3a^3b + 2a^2b^2 - (a^4 + 7a^3b + 6a^2b^2) dx) \cosh(dx + c)^9 + 2(5a^4 + 3a^3b - 32a^2b^2 - 32ab^3 - 16(a^4 + 9a^3b + 20a^2b^2 + 12ab^3) dx) \cosh(dx + c)^7 - 3(27a^3b + 102a^2b^2 + 152ab^3 + 80b^4 + 4(3a^4 + 29a^3b + 82a^2b^2 + 104ab^3 + 48b^4) dx) \cosh(dx + c)^5 - (5a^4 + 75a^3b + 192a^2b^2 + 128ab^3 + 16(a^4 + 9a^3b + 20a^2b^2 + 12ab^3) dx) \cosh(dx + c)^3 - (2a^4 + 15a^3b + 14a^2b^2 + 2(a^4 + 7a^3b + 6a^2b^2) dx) \cosh(dx + c)) \sinh(dx + c)) / ((a^7 + a^6b) d \cosh(dx + c)^{10} + 10(a^7 + a^6b) d \cosh(dx + c) \sinh(dx + c)^9 + (a^7 + a^6b) d \sinh(dx + c)^{10} + 4(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^8 + (45(a^7 + a^6b) d \cosh(dx + c)^2 + 4(a^7 + 3a^6b + 2a^5b^2) d) \sinh(dx + c)^8 + 2(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d \cosh(dx + c)^6 + 8(15(a^7 + a^6b) d \cosh(dx + c)^3 + 4(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)) \sinh(dx + c)^7 + 2(105(a^7 + a^6b) d \cosh(dx + c)^4 + 56(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^2 + (3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d) \sinh(dx + c)^6 + 4(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^4 + 4(63(a^7 + a^6b) d \cosh(dx + c)^5 + 56(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^3 + 3(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(105(a^7 + a^6b) d \cosh(dx + c)^6 + 140(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^4 + 15(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d \cosh(dx + c)^2 + 2(a^7 + 3a^6b + 2a^5b^2) d) \sinh(dx + c)^4 + (a^7 + a^6b) d \cosh(dx + c)^2 + 8(15(a^7 + a^6b) d \cosh(dx + c)^7 + 28(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^5 + 5(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d \cosh(dx + c)^3 + 2(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)) \sinh(dx + c)^3 + (45(a^7 + a^6b) d \cosh(dx + c)^8 + 112(a^7 + 3a^6b + 2a^5b^2) d \cosh(dx + c)^6 + 30(3a^7 + 11a^6b + 16a^5b^2 + 8a^4b^3) d \cosh(dx + c)^4
\end{aligned}$$

+ 24*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + a^6*b)*d)*sinh(d*x + c)^2 + 2*(5*(a^7 + a^6*b)*d*cosh(d*x + c)^9 + 16*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^5 + 8*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + (a^7 + a^6*b)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [B] time = 3.70, size = 370, normalized size = 1.98

$$\frac{(15 a^2 b+40 a b^2+24 b^3) \arctan\left(\frac{a e^{(2 d x+2 c)+a+2 b}}{2 \sqrt{-a b-b^2}}\right)}{\left(a^5+a^4 b\right) \sqrt{-a b-b^2}} - \frac{2\left(9 a^3 b e^{(6 d x+6 c)}+32 a^2 b^2 e^{(6 d x+6 c)}+24 a b^3 e^{(6 d x+6 c)}+27 a^3 b e^{(4 d x+4 c)}+102 a^2 b^2 e^{(4 d x+4 c)}+152 a b^3 e^{(4 d x+4 c)}+27 a^3 b e^{(2 d x+2 c)}+80 a^2 b^2 e^{(2 d x+2 c)}+56 a a b^3 e^{(2 d x+2 c)}+9 a^3 b+10 a^2 b^2\right)}{\left(a^5+a^4 b\right)\left(a e^{(4 d x+4 c)}+2 a e^{(2 d x+2 c)}+a\right)^2} - \frac{4\left(d x+c\right)\left(a+6 b\right)}{a^4} + \frac{e^{(2 d x+2 c)}}{a^3} + \frac{2 a e^{(2 d x+2 c)}+12 b e^{(2 d x+2 c)}-a}{a^4} e^{(-2 d x-2 c)}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^5 + a^4*b)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 32*a^2*b^2*e^(6*d*x + 6*c) + 24*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 102*a^2*b^2*e^(4*d*x + 4*c) + 152*a*b^3*e^(4*d*x + 4*c) + 80*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 80*a^2*b^2*e^(2*d*x + 2*c) + 56*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 10*a^2*b^2)/((a^5 + a^4*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)^2) - 4*(d*x + c)*(a + 6*b)/a^4 + e^(2*d*x + 2*c)/a^3 + (2*a*e^(2*d*x + 2*c) + 12*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^4)/d

maple [B] time = 0.39, size = 1329, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+2/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+23/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+23/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+2/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-15/16/d*b^(1/2)/a^2/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-5/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-3/2/d*b^(5/2)/a^4/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))

$$2*c)^2+2*b^{(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^{(1/2))+15/16/d*b^{(1/2)/a^2/(a+b)^{(3/2)*\ln((a+b)^{(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2))+5/2/d*b^{(3/2)/a^3/(a+b)^{(3/2)*\ln((a+b)^{(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2))+3/2/d*b^{(5/2)/a^4/(a+b)^{(3/2)*\ln((a+b)^{(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2))}}$$

maxima [B] time = 0.53, size = 1373, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\frac{3}{64} \cdot (5a^3b + 30a^2b^2 + 40ab^3 + 16b^4) \cdot \log\left(\frac{a e^{(2dx+2c)} + a + 2b - 2\sqrt{(a+b)b}}{a e^{(2dx+2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) + \frac{3}{64} \cdot (5a^3b + 30a^2b^2 + 40ab^3 + 16b^4) \cdot \log\left(\frac{a e^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{a e^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) - \frac{1}{32} \cdot (15a^2b + 20ab^2 + 8b^3) \cdot \log\left(\frac{a e^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{a e^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) - \frac{1}{16} \cdot (9a^4b + 32a^3b^2 + 20a^2b^3 + 3(3a^4b + 34a^3b^2 + 64a^2b^3 + 32ab^4) e^{(6dx+6c)} + (27a^4b + 264a^3b^2 + 740a^2b^3 + 832ab^4 + 320b^5) e^{(4dx+4c)} + (27a^4b + 194a^3b^2 + 336a^2b^3 + 160ab^4) e^{(2dx+2c)}) / ((a^8 + 2a^7b + a^6b^2 + (a^8 + 2a^7b + a^6b^2) e^{(8dx+8c)} + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) e^{(6dx+6c)} + 2(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) e^{(4dx+4c)} + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) e^{(2dx+2c)}) * d) + \frac{1}{16} \cdot (9a^4b + 32a^3b^2 + 20a^2b^3 + (27a^4b + 194a^3b^2 + 336a^2b^3 + 160ab^4) e^{(-2dx-2c)} + (27a^4b + 264a^3b^2 + 740a^2b^3 + 832ab^4 + 320b^5) e^{(-4dx-4c)} + 3(3a^4b + 34a^3b^2 + 64a^2b^3 + 32ab^4) e^{(-6dx-6c)}) / ((a^8 + 2a^7b + a^6b^2 + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) e^{(-2dx-2c)} + 2(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) e^{(-4dx-4c)} + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) e^{(-6dx-6c)} + (a^8 + 2a^7b + a^6b^2) e^{(-8dx-8c)}) * d) + \frac{1}{8} \cdot (9a^3b + 6a^2b^2 + (27a^3b + 68a^2b^2 + 32ab^3) e^{(-2dx-2c)} + 3(9a^3b + 30a^2b^2 + 40ab^3 + 16b^4) e^{(-4dx-4c)} + (9a^3b + 28a^2b^2 + 16ab^3) e^{(-6dx-6c)}) / ((a^7 + 2a^6b + a^5b^2 + 4(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) e^{(-2dx-2c)} + 2(3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) e^{(-4dx-4c)} + 4(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) e^{(-6dx-6c)} + (a^7 + 2a^6b + a^5b^2) e^{(-8dx-8c)}) * d) - \frac{1}{2} \cdot (dx + c) / (a^3d) + \frac{1}{8} \cdot e^{(2dx+2c)} / (a^3d) - \frac{1}{8} \cdot e^{(-2dx-2c)} / (a^3d) - \frac{3}{4} \cdot b \cdot \log(a e^{(4dx+4c)} + 2(a + 2b) e^{(2dx+2c)} + a) / (a^4d) + \frac{3}{4} \cdot b \cdot \log(2(a + 2b) e^{(-2dx-2c)} + a e^{(-4dx-4c)} + a) / (a^4d)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^6 \sinh(c+dx)^2}{(a \cosh(c+dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

$$3.44 \quad \int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=116

$$\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15 \cosh(c+dx)}{8a^3d} - \frac{5 \cosh^3(c+dx)}{8a^2d(a \cosh^2(c+dx) + b)} - \frac{\cosh^5(c+dx)}{4ad(a \cosh^2(c+dx) + b)^2}$$

[Out] 15/8*cosh(d*x+c)/a^3/d-1/4*cosh(d*x+c)^5/a/d/(b+a*cosh(d*x+c)^2)^2-5/8*cosh(d*x+c)^3/a^2/d/(b+a*cosh(d*x+c)^2)-15/8*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(7/2)/d

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4133, 288, 321, 205}

$$\frac{5 \cosh^3(c+dx)}{8a^2d(a \cosh^2(c+dx) + b)} - \frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15 \cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad(a \cosh^2(c+dx) + b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-15*sqrt[b]*ArcTan[(sqrt[a]*Cosh[c + d*x])/sqrt[b]])/(8*a^(7/2)*d) + (15*Cosh[c + d*x])/(8*a^3*d) - Cosh[c + d*x]^5/(4*a*d*(b + a*Cosh[c + d*x]^2)^2) - (5*Cosh[c + d*x]^3)/(8*a^2*d*(b + a*Cosh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4133

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[((1 - ff^2*x^2)^((m-1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^6}{(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\cosh^5(c+dx)}{4ad(b+a\cosh^2(c+dx))^2} + \frac{5\operatorname{Subst}\left(\int \frac{x^4}{(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{4ad} \\
&= -\frac{\cosh^5(c+dx)}{4ad(b+a\cosh^2(c+dx))^2} - \frac{5\cosh^3(c+dx)}{8a^2d(b+a\cosh^2(c+dx))} + \frac{15\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2} dx, x, \cosh(c+dx)\right)}{8a^2d} \\
&= \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad(b+a\cosh^2(c+dx))^2} - \frac{5\cosh^3(c+dx)}{8a^2d(b+a\cosh^2(c+dx))} - \frac{15\cosh(c+dx)}{8a^3d} \\
&= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad(b+a\cosh^2(c+dx))^2} - \frac{5\cosh^3(c+dx)}{8a^2d(b+a\cosh^2(c+dx))}
\end{aligned}$$

Mathematica [C] time = 9.60, size = 453, normalized size = 3.91

$$\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3 \left(\frac{512\sinh(c)\sinh(dx)}{a^3} + \frac{512\cosh(c)\cosh(dx)}{a^3} - \frac{15\left((a^3+64b^3)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt{a}\sqrt{b}}\right)\right)}{a^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((-15*((a^3 + 64*b^3)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Tanh[(d*x)/2]))/Sqrt[b]] + (a^3 + 64*b^3)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Tanh[(d*x)/2]))/Sqrt[b]] - a^3*(ArcTan[(Sqrt[a] - I*Sqrt[a + b])*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a + b])*Tanh[(c + d*x)/2])/Sqrt[b]])))/(a^(7/2)*b^(5/2)) + (512*Cosh[c]*Cosh[d*x])/a^3 + (8*Cosh[c + d*x]*(16*b^3*(9*a + 14*b) + 3*(a^4 + 48*a*b^3)*Cosh[2*(c + d*x)]))/(a^3*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (512*Sinh[c]*Sinh[d*x])/a^3 - (6*a*Csch[c + d*x]*Sinh[4*(c + d*x)]/(b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.51, size = 4829, normalized size = 41.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(8*a^2*cosh(d*x + c)^10 + 80*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 8*a^2*sinh(d*x + c)^10 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^8 + 20*(18*a^2*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^8 + 160*(6*a^2*cosh(d*x + c)^3 + (

$$\begin{aligned}
& 2*a^2 + 5*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*(4*a^2 + 15*a*b + 12*b^2) \\
&)*\cosh(d*x + c)^6 + 20*(84*a^2*\cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*\cosh(d* \\
& x + c)^2 + 4*a^2 + 15*a*b + 12*b^2))*\sinh(d*x + c)^6 + 8*(252*a^2*\cosh(d*x + \\
& c)^5 + 140*(2*a^2 + 5*a*b)*\cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)* \\
& \cosh(d*x + c))*\sinh(d*x + c)^5 + 20*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c) \\
& ^4 + 20*(84*a^2*\cosh(d*x + c)^6 + 70*(2*a^2 + 5*a*b)*\cosh(d*x + c)^4 + 15*(\\
& 4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2))*\sinh(d* \\
& x + c)^4 + 80*(12*a^2*\cosh(d*x + c)^7 + 14*(2*a^2 + 5*a*b)*\cosh(d*x + c)^5 \\
& + 5*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*\c \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + 20*(2*a^2 + 5*a*b)*\cosh(d*x + c)^2 + 20*(18 \\
& *a^2*\cosh(d*x + c)^8 + 28*(2*a^2 + 5*a*b)*\cosh(d*x + c)^6 + 15*(4*a^2 + 15* \\
& a*b + 12*b^2)*\cosh(d*x + c)^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c)^2 \\
& + 2*a^2 + 5*a*b)*\sinh(d*x + c)^2 + 15*(a^2*\cosh(d*x + c)^9 + 9*a^2*\cosh(d* \\
& x + c))*\sinh(d*x + c)^8 + a^2*\sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*\cosh(d*x + c) \\
&)^7 + 4*(9*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^7 + 28*(3*a^2*c \\
& \cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(3*a^2 + \\
& 8*a*b + 8*b^2)*\cosh(d*x + c)^5 + 2*(63*a^2*\cosh(d*x + c)^4 + 42*(a^2 + 2*a* \\
& b)*\cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2))*\sinh(d*x + c)^5 + 2*(63*a^2*\cos \\
& h(d*x + c)^5 + 70*(a^2 + 2*a*b)*\cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2) \\
&)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*\cosh(d*x + c)^3 + 4*(21*a \\
& ^2*\cosh(d*x + c)^6 + 35*(a^2 + 2*a*b)*\cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b + \\
& 8*b^2)*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^3 + a^2*\cosh(d*x + c) + \\
& 4*(9*a^2*\cosh(d*x + c)^7 + 21*(a^2 + 2*a*b)*\cosh(d*x + c)^5 + 5*(3*a^2 + 8 \\
& *a*b + 8*b^2)*\cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) \\
&)^2 + (9*a^2*\cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*\cosh(d*x + c)^6 + 10*(3*a^2 \\
& + 8*a*b + 8*b^2)*\cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + a^2) \\
&)*\sinh(d*x + c))*\sqrt{-b/a}*\log((a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(\\
& d*x + c)^2 + a - 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a - 2*b)*\co \\
& sh(d*x + c))*\sinh(d*x + c) - 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c))*\sinh(\\
& d*x + c)^2 + a*\sinh(d*x + c)^3 + a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a \\
&)*\sinh(d*x + c))*\sqrt{-b/a} + a)/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c))*\sin \\
& h(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cos \\
& h(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)* \\
& \cosh(d*x + c))*\sinh(d*x + c) + a)) + 8*a^2 + 40*(2*a^2*\cosh(d*x + c)^9 + 4* \\
& (2*a^2 + 5*a*b)*\cosh(d*x + c)^7 + 3*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c) \\
& ^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*\cosh(d*x \\
& + c))*\sinh(d*x + c))/(a^5*d*\cosh(d*x + c)^9 + 9*a^5*d*\cosh(d*x + c))*\sinh(d \\
& *x + c)^8 + a^5*d*\sinh(d*x + c)^9 + 4*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^7 + 4 \\
& *(9*a^5*d*\cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*\sinh(d*x + c)^7 + a^5*d*\cosh \\
& (d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c)^5 + 28*(3*a^5*d \\
& *\cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(63 \\
& *a^5*d*\cosh(d*x + c)^4 + 42*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^2 + (3*a^5 + 8* \\
& a^4*b + 8*a^3*b^2)*d)*\sinh(d*x + c)^5 + 4*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^3 \\
& + 2*(63*a^5*d*\cosh(d*x + c)^5 + 70*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^3 + 5*(\\
& 3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*a^5*d \\
& *\cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4* \\
& b + 8*a^3*b^2)*d*\cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*\sinh(d*x + c)^3 + 4*(\\
& 9*a^5*d*\cosh(d*x + c)^7 + 21*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^5 + 5*(3*a^5 + \\
& 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^2 + (9*a^5*d*\cosh(d*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*\cosh(d* \\
& x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c)^4 + 12* \\
& (a^5 + 2*a^4*b)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)), 1/8*(4*a^2*\cosh(d*x + c) \\
& ^10 + 40*a^2*\cosh(d*x + c))*\sinh(d*x + c)^9 + 4*a^2*\sinh(d*x + c)^10 + 10*(2 \\
& *a^2 + 5*a*b)*\cosh(d*x + c)^8 + 10*(18*a^2*\cosh(d*x + c)^2 + 2*a^2 + 5*a*b) \\
&)*\sinh(d*x + c)^8 + 80*(6*a^2*\cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^7 + 10*(4*a^2 + 15*a*b + 12*b^2)*\cosh(d*x + c)^6 + 10*(84* \\
& a^2*\cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*\cosh(d*x + c)^2 + 4*a^2 + 15*a*b + \\
& 12*b^2))*\sinh(d*x + c)^6 + 4*(252*a^2*\cosh(d*x + c)^5 + 140*(2*a^2 + 5*a*b)
\end{aligned}$$

$$\begin{aligned}
& * \cosh(dx + c)^3 + 15*(4a^2 + 15ab + 12b^2)*\cosh(dx + c)) * \sinh(dx + c) \\
&)^5 + 10*(4a^2 + 15ab + 12b^2)*\cosh(dx + c)^4 + 10*(84a^2*\cosh(dx + \\
& c)^6 + 70*(2a^2 + 5ab)*\cosh(dx + c)^4 + 15*(4a^2 + 15ab + 12b^2)*\co \\
& sh(dx + c)^2 + 4a^2 + 15ab + 12b^2)*\sinh(dx + c)^4 + 40*(12a^2*\cosh(\\
& dx + c)^7 + 14*(2a^2 + 5ab)*\cosh(dx + c)^5 + 5*(4a^2 + 15ab + 12b^ \\
& 2)*\cosh(dx + c)^3 + (4a^2 + 15ab + 12b^2)*\cosh(dx + c))*\sinh(dx + c) \\
& ^3 + 10*(2a^2 + 5ab)*\cosh(dx + c)^2 + 10*(18a^2*\cosh(dx + c)^8 + 28*(\\
& 2a^2 + 5ab)*\cosh(dx + c)^6 + 15*(4a^2 + 15ab + 12b^2)*\cosh(dx + c) \\
& ^4 + 6*(4a^2 + 15ab + 12b^2)*\cosh(dx + c)^2 + 2a^2 + 5ab)*\sinh(dx \\
& + c)^2 + 15*(a^2*\cosh(dx + c)^9 + 9a^2*\cosh(dx + c)*\sinh(dx + c)^8 + a^ \\
& 2*\sinh(dx + c)^9 + 4*(a^2 + 2ab)*\cosh(dx + c)^7 + 4*(9a^2*\cosh(dx + c) \\
&)^2 + a^2 + 2ab)*\sinh(dx + c)^7 + 28*(3a^2*\cosh(dx + c)^3 + (a^2 + 2a \\
& *b)*\cosh(dx + c))*\sinh(dx + c)^6 + 2*(3a^2 + 8ab + 8b^2)*\cosh(dx + c) \\
&)^5 + 2*(63a^2*\cosh(dx + c)^4 + 42*(a^2 + 2ab)*\cosh(dx + c)^2 + 3a^2 \\
& + 8ab + 8b^2)*\sinh(dx + c)^5 + 2*(63a^2*\cosh(dx + c)^5 + 70*(a^2 + 2 \\
& ab)*\cosh(dx + c)^3 + 5*(3a^2 + 8ab + 8b^2)*\cosh(dx + c))*\sinh(dx + \\
& c)^4 + 4*(a^2 + 2ab)*\cosh(dx + c)^3 + 4*(21a^2*\cosh(dx + c)^6 + 35*(a^ \\
& 2 + 2ab)*\cosh(dx + c)^4 + 5*(3a^2 + 8ab + 8b^2)*\cosh(dx + c)^2 + a^ \\
& 2 + 2ab)*\sinh(dx + c)^3 + a^2*\cosh(dx + c) + 4*(9a^2*\cosh(dx + c)^7 + \\
& 21*(a^2 + 2ab)*\cosh(dx + c)^5 + 5*(3a^2 + 8ab + 8b^2)*\cosh(dx + c) \\
& ^3 + 3*(a^2 + 2ab)*\cosh(dx + c))*\sinh(dx + c)^2 + (9a^2*\cosh(dx + c)^ \\
& 8 + 28*(a^2 + 2ab)*\cosh(dx + c)^6 + 10*(3a^2 + 8ab + 8b^2)*\cosh(dx \\
& + c)^4 + 12*(a^2 + 2ab)*\cosh(dx + c)^2 + a^2)*\sinh(dx + c))*\sqrt{b/a}*a \\
& rctan(1/2*(a*\cosh(dx + c)^3 + 3a*\cosh(dx + c)*\sinh(dx + c)^2 + a*\sinh(d \\
& *x + c)^3 + (a + 4b)*\cosh(dx + c) + (3a*\cosh(dx + c)^2 + a + 4b)*\sinh(\\
& dx + c))*\sqrt{b/a}/b) - 15*(a^2*\cosh(dx + c)^9 + 9a^2*\cosh(dx + c)*\sinh \\
& (dx + c)^8 + a^2*\sinh(dx + c)^9 + 4*(a^2 + 2ab)*\cosh(dx + c)^7 + 4*(9 \\
& a^2*\cosh(dx + c)^2 + a^2 + 2ab)*\sinh(dx + c)^7 + 28*(3a^2*\cosh(dx + c) \\
&)^3 + (a^2 + 2ab)*\cosh(dx + c))*\sinh(dx + c)^6 + 2*(3a^2 + 8ab + 8b \\
& ^2)*\cosh(dx + c)^5 + 2*(63a^2*\cosh(dx + c)^4 + 42*(a^2 + 2ab)*\cosh(dx \\
& + c)^2 + 3a^2 + 8ab + 8b^2)*\sinh(dx + c)^5 + 2*(63a^2*\cosh(dx + c)^ \\
& 5 + 70*(a^2 + 2ab)*\cosh(dx + c)^3 + 5*(3a^2 + 8ab + 8b^2)*\cosh(dx + \\
& c))*\sinh(dx + c)^4 + 4*(a^2 + 2ab)*\cosh(dx + c)^3 + 4*(21a^2*\cosh(dx \\
& + c)^6 + 35*(a^2 + 2ab)*\cosh(dx + c)^4 + 5*(3a^2 + 8ab + 8b^2)*\cosh \\
& (dx + c)^2 + a^2 + 2ab)*\sinh(dx + c)^3 + a^2*\cosh(dx + c) + 4*(9a^2*c \\
& osh(dx + c)^7 + 21*(a^2 + 2ab)*\cosh(dx + c)^5 + 5*(3a^2 + 8ab + 8b^ \\
& 2)*\cosh(dx + c)^3 + 3*(a^2 + 2ab)*\cosh(dx + c))*\sinh(dx + c)^2 + (9a^ \\
& 2*\cosh(dx + c)^8 + 28*(a^2 + 2ab)*\cosh(dx + c)^6 + 10*(3a^2 + 8ab + \\
& 8b^2)*\cosh(dx + c)^4 + 12*(a^2 + 2ab)*\cosh(dx + c)^2 + a^2)*\sinh(dx + \\
& c))*\sqrt{b/a}*\arctan(1/2*(a*\cosh(dx + c) + a*\sinh(dx + c))*\sqrt{b/a}/b) \\
& + 4a^2 + 20*(2a^2*\cosh(dx + c)^9 + 4*(2a^2 + 5ab)*\cosh(dx + c)^7 + 3 \\
& *(4a^2 + 15ab + 12b^2)*\cosh(dx + c)^5 + 2*(4a^2 + 15ab + 12b^2)*\co \\
& sh(dx + c)^3 + (2a^2 + 5ab)*\cosh(dx + c))*\sinh(dx + c))/(a^5*d*\cosh(d \\
& *x + c)^9 + 9a^5*d*\cosh(dx + c)*\sinh(dx + c)^8 + a^5*d*\sinh(dx + c)^9 + \\
& 4*(a^5 + 2a^4*b)*d*\cosh(dx + c)^7 + 4*(9a^5*d*\cosh(dx + c)^2 + (a^5 + \\
& 2a^4*b)*d)*\sinh(dx + c)^7 + a^5*d*\cosh(dx + c) + 2*(3a^5 + 8a^4*b + 8 \\
& a^3*b^2)*d*\cosh(dx + c)^5 + 28*(3a^5*d*\cosh(dx + c)^3 + (a^5 + 2a^4*b)* \\
& d*\cosh(dx + c))*\sinh(dx + c)^6 + 2*(63a^5*d*\cosh(dx + c)^4 + 42*(a^5 + \\
& 2a^4*b)*d*\cosh(dx + c)^2 + (3a^5 + 8a^4*b + 8a^3*b^2)*d)*\sinh(dx + c) \\
& ^5 + 4*(a^5 + 2a^4*b)*d*\cosh(dx + c)^3 + 2*(63a^5*d*\cosh(dx + c)^5 + 70 \\
& *(a^5 + 2a^4*b)*d*\cosh(dx + c)^3 + 5*(3a^5 + 8a^4*b + 8a^3*b^2)*d*\cosh \\
& (dx + c))*\sinh(dx + c)^4 + 4*(21a^5*d*\cosh(dx + c)^6 + 35*(a^5 + 2a^4 \\
& *b)*d*\cosh(dx + c)^4 + 5*(3a^5 + 8a^4*b + 8a^3*b^2)*d*\cosh(dx + c)^2 + \\
& (a^5 + 2a^4*b)*d)*\sinh(dx + c)^3 + 4*(9a^5*d*\cosh(dx + c)^7 + 21*(a^5 + \\
& 2a^4*b)*d*\cosh(dx + c)^5 + 5*(3a^5 + 8a^4*b + 8a^3*b^2)*d*\cosh(dx + \\
& c)^3 + 3*(a^5 + 2a^4*b)*d*\cosh(dx + c))*\sinh(dx + c)^2 + (9a^5*d*\cosh(d \\
& *x + c)^8 + 28*(a^5 + 2a^4*b)*d*\cosh(dx + c)^6 + a^5*d + 10*(3a^5 + 8a^ \\
& 4*b + 8a^3*b^2)*d*\cosh(dx + c)^4 + 12*(a^5 + 2a^4*b)*d*\cosh(dx + c)^2)* \\
& \sinh(dx + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
 olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
 al with parameters. This might be wrong.The choice was done assuming [a,b]=
 [-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
 ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
 o choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
 he choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
 ce was done assuming [a,b]=[-82,81]Precision problem choosing root in commo
 n_EXT, current precision 14Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.The choice was done assu
 ming [a,b]=[-60,-34]Undef/Unsigned Inf encountered in limitEvaluation time:
 2.06Limit: Max order reached or unable to make series expansion Error: Bad
 Argument Value

maple [A] time = 0.17, size = 107, normalized size = 0.92

$$\frac{7b^2 \operatorname{sech}(dx+c)^3}{8d a^3 (a+b \operatorname{sech}(dx+c)^2)^2} + \frac{9b \operatorname{sech}(dx+c)}{8d a^2 (a+b \operatorname{sech}(dx+c)^2)^2} + \frac{15b \arctan\left(\frac{\operatorname{sech}(dx+c)b}{\sqrt{ab}}\right)}{8d a^3 \sqrt{ab}} + \frac{1}{d a^3 \operatorname{sech}(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)

[Out] 7/8/d/a^3*b^2/(a+b*sech(d*x+c)^2)^2*sech(d*x+c)^3+9/8/d/a^2*b/(a+b*sech(d*x
 +c)^2)^2*sech(d*x+c)+15/8/d/a^3*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1
 /2))+1/d/a^3/sech(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 a^2 e^{(10 dx+10 c)} + 2 a^2 + 5 \left(2 a^2 e^{(8 c)} + 5 a b e^{(8 c)} \right) e^{(8 dx)} + 5 \left(4 a^2 e^{(6 c)} + 15 a b e^{(6 c)} + 12 b^2 e^{(6 c)} \right) e^{(6 dx)} + 5 \left(4 a^2 e^{(4 c)} + 15 a b e^{(4 c)} + 12 b^2 e^{(4 c)} \right) e^{(4 dx)} + 5 \left(4 a^2 e^{(2 c)} + 15 a b e^{(2 c)} + 12 b^2 e^{(2 c)} \right) e^{(2 dx)}}{4 \left(a^5 d e^{(9 dx+9 c)} + a^5 d e^{(dx+c)} + 4 \left(a^5 d e^{(7 c)} + 2 a^4 b d e^{(7 c)} \right) e^{(7 dx)} + 2 \left(3 a^5 d e^{(5 c)} + 8 a^4 b d e^{(5 c)} + 8 a^3 b^2 d e^{(5 c)} \right) e^{(5 dx)} + 2 \left(3 a^5 d e^{(3 c)} + 8 a^4 b d e^{(3 c)} + 8 a^3 b^2 d e^{(3 c)} \right) e^{(3 dx)} + 2 \left(3 a^5 d e^{(c)} + 8 a^4 b d e^{(c)} + 8 a^3 b^2 d e^{(c)} \right) e^{(c)}} e^{(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*(2*a^2*e^(10*d*x + 10*c) + 2*a^2 + 5*(2*a^2*e^(8*c) + 5*a*b*e^(8*c))*e^(
 8*d*x) + 5*(4*a^2*e^(6*c) + 15*a*b*e^(6*c) + 12*b^2*e^(6*c))*e^(6*d*x) + 5
 *(4*a^2*e^(4*c) + 15*a*b*e^(4*c) + 12*b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(
 2*c) + 5*a*b*e^(2*c))*e^(2*d*x))/(a^5*d*e^(9*d*x + 9*c) + a^5*d*e^(d*x + c)
 + 4*(a^5*d*e^(7*c) + 2*a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*d*e^(5*c) + 8

```
*a^4*b*d*e^(5*c) + 8*a^3*b^2*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 2*a^
4*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*
x + c))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2
*d*x)), x)
```

mupad [B] time = 1.61, size = 103, normalized size = 0.89

$$\frac{\frac{7b^2 \cosh(c+dx)}{8} + \frac{9ab \cosh(c+dx)^3}{8}}{da^5 \cosh(c+dx)^4 + 2da^4b \cosh(c+dx)^2 + da^3b^2} + \frac{\cosh(c+dx)}{a^3d} - \frac{15\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)
```

```
[Out] ((7*b^2*cosh(c + d*x))/8 + (9*a*b*cosh(c + d*x)^3)/8)/(a^5*d*cosh(c + d*x)^
4 + a^3*b^2*d + 2*a^4*b*d*cosh(c + d*x)^2) + cosh(c + d*x)/(a^3*d) - (15*b^
(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(8*a^(7/2)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**3, x)
```

```
[Out] Timed out
```


$$3.45 \quad \int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=154

$$-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2(a\cosh^2(c+dx)+b)} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3} - \frac{b\cosh^3(c+dx)}{4ad(a+b)(a\cosh^2(c+dx)+b)}$$

[Out] $-\operatorname{arctanh}(\cosh(d*x+c))/d/(a+b)^3-1/4*b*\cosh(d*x+c)^3/a/(a+b)/d/(b+a*\cosh(d*x+c)^2)^2-1/8*b*(7*a+3*b)*\cosh(d*x+c)/a^2/(a+b)^2/d/(b+a*\cosh(d*x+c)^2)+1/8*(15*a^2+10*a*b+3*b^2)*\operatorname{arctan}(\cosh(d*x+c)*a^{1/2}/b^{1/2})*b^{1/2}/a^{5/2}/(a+b)^3/d$

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4133, 470, 578, 522, 206, 205}

$$\frac{\sqrt{b}(15a^2+10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2(a\cosh^2(c+dx)+b)} - \frac{b\cosh^3(c+dx)}{4ad(a+b)(a\cosh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(\operatorname{Sqrt}[b]*(15*a^2+10*a*b+3*b^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Cosh}[c+d*x])/(\operatorname{Sqrt}[b])])/(8*a^{5/2}*(a+b)^3*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]/((a+b)^3*d) - (b*\operatorname{Cosh}[c+d*x]^3)/(4*a*(a+b)*d*(b+a*\operatorname{Cosh}[c+d*x]^2)^2) - (b*(7*a+3*b)*\operatorname{Cosh}[c+d*x])/(8*a^2*(a+b)^2*d*(b+a*\operatorname{Cosh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> -Simp[(a*e^(2*n-1)*(e*x)^(m-2*n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(b*n*(b*c-a*d)*(p+1)), x] + Dist[e^(2*n)/(b*n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-2*n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[a*c*(m-2*n+1)+(a*d*(m-n+n*q+1)+b*c*n*(p+1)]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m-n+1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)]^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{b \cosh^3(c + dx)}{4a(a + b)d (b + a \cosh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{4a(a + b)d}$$

$$= -\frac{b \cosh^3(c + dx)}{4a(a + b)d (b + a \cosh^2(c + dx))^2} - \frac{b(7a + 3b) \cosh(c + dx)}{8a^2(a + b)^2d (b + a \cosh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{4a(a + b)d}$$

$$= -\frac{b \cosh^3(c + dx)}{4a(a + b)d (b + a \cosh^2(c + dx))^2} - \frac{b(7a + 3b) \cosh(c + dx)}{8a^2(a + b)^2d (b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{4a(a + b)d}$$

$$= \frac{\sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a + b)^3d} - \frac{\tanh^{-1}(\cosh(c + dx))}{(a + b)^3d} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c + dx)\right)}{4a(a + b)d}$$

Mathematica [C] time = 2.50, size = 440, normalized size = 2.86

$$\operatorname{sech}^5(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(9a+5b)(a+b)(a \cosh(2(c+dx))+a+2b)}{a^2} + \frac{\sqrt{b} (15a^2+10ab+3b^2) \operatorname{sech}(c+dx)}{4a(a + b)d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*b^2*(a + b)^2)/a^2 - (2*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sin
```

$$\begin{aligned} & h[c]^2) * \text{Sinh}[c] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] - I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b] * (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2 * \text{Sech}[c + d*x] / a^{5/2} + (\text{Sqrt}[b] * (15*a^2 + 10*a*b + 3*b^2) * \text{ArcTan}[(\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2] + \text{Cosh}[c] * (\text{Sqrt}[a] + I * \text{Sqrt}[a + b] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[(d*x)/2])) / \text{Sqrt}[b] * (a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2 * \text{Sech}[c + d*x] / a^{5/2} - 8*(a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2 * \text{Log}[\text{Cosh}[(c + d*x)/2]] * \text{Sech}[c + d*x] + 8*(a + 2*b + a * \text{Cosh}[2*(c + d*x)])^2 * \text{Log}[\text{Sinh}[(c + d*x)/2]] * \text{Sech}[c + d*x])) / (64*(a + b)^3 * d * (a + b * \text{Sech}[c + d*x]^2)^3) \end{aligned}$$

fricas [B] time = 0.64, size = 8742, normalized size = 56.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^7 + 28*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\sinh(d*x + c)^7 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^5 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 20*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + 4*(35*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(21*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^5 + 10*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*\sinh(d*x + c)^8 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^3 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^4 + 45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^5 + 10*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^7 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c) * \text{sqrt}(-b/a) * \text{log}((a * \cosh(d*x + c))^4 + 4*a * \cosh(d*x + c) * \sinh(d*x + c)^3 + a * \sinh(d*x + c)^4 + 2*(a - 2*b) * \cosh(d*x + c)^2 + 2*(3*a * \cosh(d*x + c)^2 + a - 2*b) * \sinh(d*x + c)^2 + 4*(a * \cosh(d*x + c)^3 + (a - 2*b) * \cosh(d*x + c)) * \sinh(d*x + c) + 4*(a * \cosh(d*x + c)^3 + 3*a * \cosh(d*x + c) * \sinh(d*x + c)^2 + a * \sinh(d*x + c)^3 + a * \cosh(d*x + c) + (3*a * \cosh(d*x + c)^2 + a) * \sinh(d*x + c)) * \text{sqrt}(-b/a) + a) / (a * \cosh(d*x + c))^4 + 4*a * \cosh(d*x + c) * \sinh(d*x + c)^3 + a * \sinh(d*x + c)^4 + 2*(a + 2*b) * \cosh(d*x + c)^2 + 2*(3*a * \cosh(d*x + c)^2 + a + 2*b) * \sinh(d*x + c)^2 + 4*(a * \cosh(d*x + c)^3 + (a + 2*b) * \cosh(d*x + c)) * \sinh(d*x + c) + a) + 4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3) * \cosh(d*x + c) + 16*(a^4 * \cosh(d*x + c))^8 + 8*a^4 * \cosh(d*x + c) \end{aligned}$$

$$\begin{aligned}
&^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + 2*(35*(9*a^3*b + 14*a^2*b^2 + 5*a \\
&*b^3)*\cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27* \\
&a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + \\
&2*(21*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c)^5 + 10*(27*a^3*b + 70* \\
&a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 5 \\
&5*a*b^3 + 12*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((15*a^4 + 10*a^3*b + 3* \\
&a^2*b^2)*\cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)* \\
&\sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*\sinh(d*x + c)^8 + 4*(15*a \\
&^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3* \\
&b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^ \\
&2)*\sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^3 + \\
&3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^ \\
&5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c) \\
&^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^4 + 45*a^4 + 150*a \\
&^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^ \\
&2 + 6*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b \\
&^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^5 + 10*(15*a^4 + 40 \\
&a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209* \\
&a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(15*a^4 + \\
&40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b \\
&+ 3*a^2*b^2)*\cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 \\
&)*\cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 + \\
&150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c \\
&)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^7 + 3*(15*a^4 + 40*a \\
&^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^ \\
&2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b \\
&^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*(a*\cosh(d* \\
&x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (a + 4*b \\
&)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + a + 4*b)*\sinh(d*x + c))*\sqrt{b/a}/ \\
&b) - ((15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3* \\
&b + 3*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b \\
&^2)*\sinh(d*x + c)^8 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x \\
&+ c)^6 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3* \\
&b + 3*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + \\
&3*a^2*b^2)*\cosh(d*x + c)^3 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)* \\
&\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104* \\
&a*b^3 + 24*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cos \\
&h(d*x + c)^4 + 45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(\\
&15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 \\
&+ 15*a^4 + 10*a^3*b + 3*a^2*b^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh \\
&(d*x + c)^5 + 10*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^3 \\
&+ (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c))*s \\
&\sinh(d*x + c)^3 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c) \\
&^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^6 + 15*(15*a^4 + 40 \\
&a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2 \\
&*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)* \\
&\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*\cosh(\\
&d*x + c)^7 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^5 + \\
&(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*\cosh(d*x + c)^3 + \\
&(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sq \\
&\sqrt{b/a}*\arctan(1/2*(a*\cosh(d*x + c) + a*\sinh(d*x + c))*\sqrt{b/a}/b) + 2*(9* \\
&a^3*b + 14*a^2*b^2 + 5*a*b^3)*\cosh(d*x + c) + 8*(a^4*\cosh(d*x + c)^8 + 8*a^ \\
&4*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^4*\sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*\c \\
&osh(d*x + c)^6 + 4*(7*a^4*\cosh(d*x + c)^2 + a^4 + 2*a^3*b)*\sinh(d*x + c)^6 \\
&+ 8*(7*a^4*\cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*\cosh(d*x + c))*\sinh(d*x + c) \\
&^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^4 + 2*(35*a^4*\cosh(d*x + \\
&c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*\cosh(d*x + c)^2)*s \\
&\sinh(d*x + c)^4 + a^4 + 8*(7*a^4*\cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*\cosh(d \\
&*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 +
\end{aligned}$$

$$\begin{aligned}
& 4*(a^4 + 2*a^3*b)*\cosh(d*x + c)^2 + 4*(7*a^4*\cosh(d*x + c)^6 + 15*(a^4 + 2* \\
& a^3*b)*\cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(a^4*\cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b) \\
& *\cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^3 + (a^4 + 2 \\
& *a^3*b)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1 \\
&) - 8*(a^4*\cosh(d*x + c)^8 + 8*a^4*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^4*\sinh \\
& (d*x + c)^8 + 4*(a^4 + 2*a^3*b)*\cosh(d*x + c)^6 + 4*(7*a^4*\cosh(d*x + c)^2 \\
& + a^4 + 2*a^3*b)*\sinh(d*x + c)^6 + 8*(7*a^4*\cosh(d*x + c)^3 + 3*(a^4 + 2*a^ \\
& 3*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(\\
& d*x + c)^4 + 2*(35*a^4*\cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(\\
& a^4 + 2*a^3*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 8*(7*a^4*\cosh(d*x + \\
& c)^5 + 10*(a^4 + 2*a^3*b)*\cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)* \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*\cosh(d*x + c)^2 + 4*(7*a \\
& ^4*\cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*\cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3 \\
& *(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(a^4*\co \\
& sh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*\cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^ \\
& 2*b^2)*\cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\\
& \cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)* \\
& \cosh(d*x + c)^6 + 5*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*\cosh(d*x + \\
& c)^4 + 9*a^3*b + 14*a^2*b^2 + 5*a*b^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 \\
& + 12*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^ \\
& 4*b^3)*d*\cosh(d*x + c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d \\
& *x + c)*\sinh(d*x + c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\sinh(d*x \\
& + c)^8 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + \\
& c)^6 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^2 + (a^7 \\
& + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(3*a \\
& ^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*\cosh(d* \\
& x + c)^4 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^3 + 3 \\
& *(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + c))*\sinh(\\
& d*x + c)^5 + 2*(35*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^4 \\
& + 30*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + c)^2 \\
& + (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)* \\
& \sinh(d*x + c)^4 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*c \\
& osh(d*x + c)^2 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c) \\
& ^5 + 10*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + c) \\
& ^3 + (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)* \\
& d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^ \\
& 3)*d*\cosh(d*x + c)^6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^ \\
& 4)*d*\cosh(d*x + c)^4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a \\
& ^3*b^4 + 8*a^2*b^5)*d*\cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4* \\
& b^3 + 2*a^3*b^4)*d)*\sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) \\
& *d + 8*((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^7 + 3*(a^7 + \\
& 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + c)^5 + (3*a^7 + 1 \\
& 7*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*\cosh(d*x + c) \\
& ^3 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*\cosh(d*x + c))*s \\
& inh(d*x + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi

al with parameters. This might be wrong. The choice was done assuming [a,b]= [-43,-99] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]= [-28,94] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]= [-7,46] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]= [-35,-99] Undef/Unsigned Inf encountered in limitEvaluation time: 1.19 Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.36, size = 1476, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$-9/4/d*b/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*\tanh(1/2*d*x+1/2*c)^6+1/4/d*b^2/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6+13/4/d*b^3/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^6+3/4/d*b^4/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*\tanh(1/2*d*x+1/2*c)^6-27/4/d*b/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^4*a+9/4/d*b^2/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^4-21/4/d*b^3/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^4-9/4/d*b^4/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*\tanh(1/2*d*x+1/2*c)^4-27/4/d*b/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a-21/4/d*b^2/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-15/4/d*b^3/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a-3/4/d*b^4/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2+15/8/d*b/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/4/d*b^2/(a+b)^3/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+3/8/d*b^3/(a+b)^3/a^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(9a^2be^{7c} + 5ab^2e^{7c})e^{7dx} + (27a^2be^{5c} + 4(a^6d + 2a^5bd + a^4b^2d + (a^6de^{8c} + 2a^5bde^{8c} + a^4b^2de^{8c}))e^{8dx} + 4(a^6de^{6c} + 4a^5bde^{6c} + 5a^4b^2de^{6c})e^{6dx}}{4(a^6d + 2a^5bd + a^4b^2d + (a^6de^{8c} + 2a^5bde^{8c} + a^4b^2de^{8c}))e^{8dx} + 4(a^6de^{6c} + 4a^5bde^{6c} + 5a^4b^2de^{6c})e^{6dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/4*((9*a^2*b*e^{(7*c)} + 5*a*b^2*e^{(7*c)})e^{(7*d*x)} + (27*a^2*b*e^{(5*c)} + 4*3*a*b^2*e^{(5*c)} + 12*b^3*e^{(5*c)})e^{(5*d*x)} + (27*a^2*b*e^{(3*c)} + 43*a*b^2*e^{(3*c)} + 12*b^3*e^{(3*c)})e^{(3*d*x)} + (9*a^2*b*e^c + 5*a*b^2*e^c)e^{(d*x)}) / (a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^{(8*c)} + 2*a^5*b*d*e^{(8*c)} + a^4*b^2*d*e^{(8*c)})e^{(8*d*x)} + 4*(a^6*d*e^{(6*c)} + 4*a^5*b*d*e^{(6*c)} + 5*a^4*b^2*d*e^{(6*c)} + 2*a^3*b^3*d*e^{(6*c)})e^{(6*d*x)} + 2*(3*a^6*d*e^{(4*c)} + 14*a^5*b*d*e^{(4*c)} + 27*a^4*b^2*d*e^{(4*c)} + 24*a^3*b^3*d*e^{(4*c)} + 8*a^2*b^4*d*e^{(4*c)})e^{(4*d*x)} + 4*(a^6*d*e^{(2*c)} + 4*a^5*b*d*e^{(2*c)} + 5*a^4*b^2*d*e^{(2*c)} + 2*a^3*b^3*d*e^{(2*c)})e^{(2*d*x)}) - \log((e^{(d*x+c)} + 1)e^{(-c)}) / (a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + \log((e^{(d*x+c)} - 1)e^{(-c)}) / (a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + 2*\integrate(1/8*((15*a^2*b*e^{(3*c)} + 10*a*b^2*e^{(3*c)} + 3*b^3*e^{(3*c)})e^{(3*d*x)} - (15*a^2*b*e^c + 10*a*b^2*e^c + 3*b^3*e^c)e^{(d*x)}) / (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^{(4*c)} + 3*a^5*b*e^{(4*c)} + 3*a^4*b^2*e^{(4*c)} + a^3*b^3*e^{(4*c)})e^{(4*d*x)} + 2*(a^6*e^{(2*c)} + 5*a^5*b*e^{(2*c)} + 9*a^4*b^2*e^{(2*c)} + 7*a^3*b^3*e^{(2*c)} + 2*a^2*b^4*e^{(2*c)})e^{(2*d*x)}), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^6}{\sinh(c+dx) (a \cosh(c+dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c+d*x)*(a+b/cosh(c+d*x)^2)^3),x)

[Out] int(cosh(c+d*x)^6/(sinh(c+d*x)*(b+a*cosh(c+d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b \operatorname{sech}^2(c+dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(csch(c+d*x)/(a+b*sech(c+d*x)**2)**3, x)

$$3.46 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=126

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}} - \frac{15 \coth(c+dx)}{8d(a+b)^3} + \frac{5 \coth(c+dx)}{8d(a+b)^2(a-b \tanh^2(c+dx)+b)} + \frac{\coth(c+dx)}{4d(a+b)(a-b \tanh^2(c+dx))}$$

[Out] $-15/8*\coth(d*x+c)/d/(a+b)^3+15/8*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}/(a+b)^{(7/2)}/d+1/4*\coth(d*x+c)/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)^2+5/8*\coth(d*x+c)/(a+b)^2/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4132, 290, 325, 208}

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}} - \frac{15 \coth(c+dx)}{8d(a+b)^3} + \frac{5 \coth(c+dx)}{8d(a+b)^2(a-b \tanh^2(c+dx)+b)} + \frac{\coth(c+dx)}{4d(a+b)(a-b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3, x]

[Out] $(15*\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])]/(8*(a + b)^{(7/2)}*d) - (15*\operatorname{Coth}[c + d*x])/(8*(a + b)^3*d) + \operatorname{Coth}[c + d*x]/(4*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^2) + (5*\operatorname{Coth}[c + d*x])/(8*(a + b)^2*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c*n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4132

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m+1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^3} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{coth}(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{5 \operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\operatorname{coth}(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{5 \operatorname{coth}(c+dx)}{8(a+b)^2d(a+b-b\tanh^2(c+dx))} + \frac{15}{8(a+b)^2d(a+b-b\tanh^2(c+dx))}$$

$$= -\frac{15 \operatorname{coth}(c+dx)}{8(a+b)^3d} + \frac{\operatorname{coth}(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{5 \operatorname{coth}(c+dx)}{8(a+b)^2d(a+b-b\tanh^2(c+dx))}$$

$$= \frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} - \frac{15 \operatorname{coth}(c+dx)}{8(a+b)^3d} + \frac{\operatorname{coth}(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))}$$

Mathematica [C] time = 6.92, size = 981, normalized size = 7.79

$$\frac{(\cosh(2c+2dx)a+a+2b)^3 \left(\frac{15ib \tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i \sinh(2c)}{2\sqrt{a+b}\sqrt{b \cosh(4c)-b \sinh(4c)}} - \frac{i \cosh(2c)}{2\sqrt{a+b}\sqrt{b \cosh(4c)-b \sinh(4c)}}\right)\right)}{64\sqrt{a+b}d\sqrt{b \cosh(4c)-b \sinh(4c)}} \right) (-a \sinh(dx) - 2b \sinh(dx) + a \sinh(dx))}{(a+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((( (-15*I)/64)*b*ArcTan[Sech[d*x]*(( (-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) )]*(- (a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]))*Cosh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + (((15*I)/64)*b*ArcTan[Sech[d*x]*(( (-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) )]*(- (a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]))*Sinh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[c + d*x]^6*(-32*a^4*Sinh[d*x] - 64*a^3*b*Sinh[d*x] + 22*a^2*b^2*Sinh[d*x] + 80*a*b^3*Sinh[d*x] + 16*b^4*Sinh[d*x] + 32*a^4*Sinh[3*d*x] + 46*a^3*b*Sinh[3*d*x] - 54*a^2*b^2*Sinh[3*d*x] - 8*a*b^3*Sinh[3*d*x] - 48*a^4*Sinh[2*c - d*x] - 128*a^3*b*Sinh[2*c - d*x] - 106*a^2*b^2*Sinh[2*c - d*x] + 80*a*b^3*Sinh[2*c - d*x] + 16*b^4*Sinh[2*c - d*x] + 48*a^4*Sinh[2*c + d*x] + 146*a^3*b*Sinh[2*c + d*x] + 182*a^2*b^2*Sinh[2*c + d*x] + 80*a*b^3*Sinh[2*c + d*x] + 16*b^4*Sinh[2*c + d*x] - 32*a^4*Sinh[4*c + d*x] - 82*a^3*b*Sinh[4*c + d*x] - 54*a^2*b^2*Sinh[4*c + d*x] - 80*a*b^3*Sinh[4*c + d*x] - 16*b^4*Sinh[4*c + d*x] - 8*a^4*Sinh[2*c + 3*d*x] + 18*a^3*b*Sinh[2*c + 3*d*x] + 54*a^2*b^2*Sinh[2*c + 3*d*x] + 8*a*b^3*Sinh[2*c + 3*d*x] + 32*a^4*Sinh[4*c + 3*d*x] + 73*a^3*b*Sinh[4*c + 3*d*x] + 24*a^2*b^2*Sinh[4*c + 3*d*x] + 8*a*b^3*Sinh[4*c + 3*d*x] - 8*a^4*Sinh[6*c + 3*d*x] - 9*a^3*b*Sinh[6*c + 3*d*x] - 24*a^2*b^2*Sinh[6*c + 3*d*x] - 8*a*b^3*Sinh[6*c + 3*d*x] + 8*a^4*Sinh[2*c + 5*d*x] - 9*a^3*b*Sinh[2*c + 5*d*x] + 16*b^4*Sinh[2*c + 5*d*x]))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3)
```

$d*x] - 2*a^2*b^2*\text{Sinh}[2*c + 5*d*x] + 9*a^3*b*\text{Sinh}[4*c + 5*d*x] + 2*a^2*b^2*\text{Sinh}[4*c + 5*d*x] + 8*a^4*\text{Sinh}[6*c + 5*d*x]) / (512*a^2*(a + b)^3*d*(a + b*\text{Sech}[c + d*x]^2)^3)$

fricas [B] time = 0.55, size = 7275, normalized size = 57.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^8 + 32*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + 4*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\sinh(d*x + c)^8 + 8*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c)^6 + 8*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4 + 14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3) * \cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 16*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^3 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 8*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*\cosh(d*x + c)^4 + 8*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^4 + 24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c)^2) * \sinh(d*x + c)^4 + 32*a^4 - 36*a^3*b - 8*a^2*b^2 + 32*(7*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 8*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^2 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^6 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c)^4 + 16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 15*(a^4*\cosh(d*x + c)^10 + 10*a^4*\cosh(d*x + c)*\sinh(d*x + c)^9 + a^4*\sinh(d*x + c)^10 + (3*a^4 + 8*a^3*b)*\cosh(d*x + c)^8 + (45*a^4*\cosh(d*x + c)^2 + 3*a^4 + 8*a^3*b)*\sinh(d*x + c)^8 + 8*(15*a^4*\cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^6 + 2*(105*a^4*\cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 + 14*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*a^4*\cosh(d*x + c)^5 + 14*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b^2)^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^4 + 2*(105*a^4*\cosh(d*x + c)^6 + 35*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^4 - a^4 - 4*a^3*b - 8*a^2*b^2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 + 8*(15*a^4*\cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^5 + 5*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^3 - (a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*\cosh(d*x + c)^2 + (45*a^4*\cosh(d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^6 + 30*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*a^4*\cosh(d*x + c)^9 + 4*(3*a^4 + 8*a^3*b)*\cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^5 - 4*(a^4 + 4*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/(a + b)}*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 16*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\cosh(d*x + c)^7 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\cosh(d*x + c)^5 + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*co$$

$$\begin{aligned}
& \text{sh}(d*x + c)^3 + (16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)^{10} + 1 \\
& 0*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^9 + (\\
& a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{sinh}(d*x + c)^{10} + (3*a^7 + 17*a^6*b \\
& + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^8 + (45*(a^7 + 3*a^6*b \\
& + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b \\
& ^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*\text{sinh}(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5 \\
& *b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*\text{cosh}(d*x + c)^6 + 8*(15*(a^7 \\
& + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33 \\
& *a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^7 + 2*(10 \\
& 5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)^4 + 14*(3*a^7 + 17* \\
& a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^2 + (a^7 + 7*a \\
& ^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*\text{sinh}(d*x + c)^6 \\
& - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*c \\
& \text{osh}(d*x + c)^4 + 4*(63*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c \\
&)^5 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d* \\
& x + c)^3 + 3*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2* \\
& b^5)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 + \\
& a^4*b^3)*d*\text{cosh}(d*x + c)^6 + 35*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^ \\
& 3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^4 + 15*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4* \\
& b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*\text{cosh}(d*x + c)^2 - (a^7 + 7*a^6*b + 23*a^5*b \\
& ^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*\text{sinh}(d*x + c)^4 - (3*a^7 + 17* \\
& a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^2 + 8*(15*(a^7 \\
& + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)^7 + 7*(3*a^7 + 17*a^6*b + \\
& 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c)^5 + 5*(a^7 + 7*a^6*b \\
& + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*\text{cosh}(d*x + c)^3 - (a^ \\
& 7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*\text{cosh}(d*x \\
& + c))*\text{sinh}(d*x + c)^3 + (45*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d \\
& x + c)^8 + 28*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*c \\
& \text{osh}(d*x + c)^6 + 30*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + \\
& 8*a^2*b^5)*d*\text{cosh}(d*x + c)^4 - 12*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 \\
& + 28*a^3*b^4 + 8*a^2*b^5)*d*\text{cosh}(d*x + c)^2 - (3*a^7 + 17*a^6*b + 33*a^5*b^ \\
& 2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*\text{sinh}(d*x + c)^2 - (a^7 + 3*a^6*b + 3*a^5*b^2 \\
& + a^4*b^3)*d + 2*(5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*\text{cosh}(d*x + c)^ \\
& 9 + 4*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + \\
& c)^7 + 6*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5 \\
&)*d*\text{cosh}(d*x + c)^5 - 4*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b \\
& ^4 + 8*a^2*b^5)*d*\text{cosh}(d*x + c)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4 \\
& *b^3 + 8*a^3*b^4)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)), -1/8*(2*(8*a^4 + 9*a^3*b \\
& + 24*a^2*b^2 + 8*a*b^3)*\text{cosh}(d*x + c)^8 + 16*(8*a^4 + 9*a^3*b + 24*a^2*b^2 \\
& + 8*a*b^3)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + 2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 \\
& + 8*a*b^3)*\text{sinh}(d*x + c)^8 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 \\
& + 8*b^4)*\text{cosh}(d*x + c)^6 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8 \\
& *b^4 + 14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\text{cosh}(d*x + c)^2)*\text{sinh}(d* \\
& x + c)^6 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\text{cosh}(d*x + c)^3 + \\
& 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\text{cosh}(d*x + c))*\text{sinh}(\\
& d*x + c)^5 + 4*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*\text{cosh}(d*x \\
& + c)^4 + 4*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\text{cosh}(d*x + c)^4 + \\
& 24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4 + 15*(16*a^4 + 41*a^3*b + \\
& 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^4 + 16*a^4 - \\
& 18*a^3*b - 4*a^2*b^2 + 16*(7*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\text{cosh} \\
& (d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\text{cosh}(d* \\
& x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*\text{cosh}(d*x + c \\
&))*\text{sinh}(d*x + c)^3 + 4*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*\text{cosh}(d*x \\
& + c)^2 + 4*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*\text{cosh}(d*x + c)^6 + 1 \\
& 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*\text{cosh}(d*x + c)^4 + 16* \\
& a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - \\
& 40*a*b^3 - 8*b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 - 15*(a^4*\text{cosh}(d*x + c) \\
& ^{10} + 10*a^4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^9 + a^4*\text{sinh}(d*x + c)^{10} + (3*a^4
\end{aligned}$$

$$\begin{aligned}
& + 8a^3b) \cosh(dx + c)^8 + (45a^4 \cosh(dx + c)^2 + 3a^4 + 8a^3b) \sinh(dx + c)^8 + 8(15a^4 \cosh(dx + c)^3 + (3a^4 + 8a^3b) \cosh(dx + c)) \\
& * \sinh(dx + c)^7 + 2(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)^6 + 2(105a^4 \cosh(dx + c)^4 + a^4 + 4a^3b + 8a^2b^2 + 14(3a^4 + 8a^3b) \cosh(dx + c)^2) \\
& * \sinh(dx + c)^6 + 4(63a^4 \cosh(dx + c)^5 + 14(3a^4 + 8a^3b) \cosh(dx + c)^3 + 3(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)) * \sinh(dx + c)^5 - 2(a^4 + 4a^3b + 8a^2b^2) \\
& * \cosh(dx + c)^4 + 2(105a^4 \cosh(dx + c)^6 + 35(3a^4 + 8a^3b) \cosh(dx + c)^4 - a^4 - 4a^3b - 8a^2b^2 + 15(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)^2) * \sinh(dx + c)^4 - a^4 + \\
& 8(15a^4 \cosh(dx + c)^7 + 7(3a^4 + 8a^3b) \cosh(dx + c)^5 + 5(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)^3 - (a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)) * \sinh(dx + c)^3 - (3a^4 + 8a^3b) \\
& * \cosh(dx + c)^2 + (45a^4 \cosh(dx + c)^8 + 28(3a^4 + 8a^3b) \cosh(dx + c)^6 + 30(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)^4 - 3a^4 - 8a^3b - 12(a^4 + 4a^3b + 8a^2b^2) \\
& * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 2(5a^4 \cosh(dx + c)^9 + 4(3a^4 + 8a^3b) \cosh(dx + c)^7 + 6(a^4 + 4a^3b + 8a^2b^2) \cosh(dx + c)^5 - 4(a^4 + 4a^3b + 8a^2b^2) \\
& * \cosh(dx + c)^3 - (3a^4 + 8a^3b) \cosh(dx + c)) * \sinh(dx + c) * \sqrt{-b/(a + b)} * \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \\
& * \sqrt{-b/(a + b)})/b + 8(2(8a^4 + 9a^3b + 24a^2b^2 + 8a^2b^3) \cosh(dx + c)^7 + 3(16a^4 + 41a^3b + 27a^2b^2 + 40a^2b^3 + 8b^4) \cosh(dx + c)^5 + 2(24a^4 + 64a^3b + 53a^2b^2 - 40a^2b^3 - 8b^4) \\
& * \cosh(dx + c)^3 + (16a^4 + 23a^3b - 27a^2b^2 - 4a^2b^3) \cosh(dx + c)) * \sinh(dx + c) / ((a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^10 + 10(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) \\
& * d * \cosh(dx + c) * \sinh(dx + c)^9 + (a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \sinh(dx + c)^10 + (3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^8 + (45(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^2 + (3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d) * \sinh(dx + c)^8 + 2(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^6 + 8(15(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^3 + (3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 2(105(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^4 + 14(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^2 + (a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d) * \sinh(dx + c)^6 - 2(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^4 + 4(63(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^5 + 14(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^3 + 3(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2(105(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^6 + 35(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^4 + 15(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^2 - (a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d) * \sinh(dx + c)^4 - (3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^2 + 8(15(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^7 + 7(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^5 + 5(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^3 - (a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^8 + 28(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^6 + 30(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^4 - 12(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^2 - (3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d) * \sinh(dx + c)^2 - (a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d + 2(5(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) * d * \cosh(dx + c)^9 + 4(3a^7 + 17a^6b + 33a^5b^2 + 27a^4b^3 + 8a^3b^4) * d * \cosh(dx + c)^7 + 6(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)^5 - 4(a^7 + 7a^6b + 23a^5b^2 + 37a^4b^3 + 28a^3b^4 + 8a^2b^5) * d * \cosh(dx + c)
\end{aligned}$$

)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [B] time = 1.76, size = 347, normalized size = 2.75

$$\frac{15 b \arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{-ab-b^2}} - \frac{2(9a^3be^{6dx+6c}+24a^2b^2e^{6dx+6c}+8ab^3e^{6dx+6c}+27a^3be^{4dx+4c}+78a^2b^2e^{4dx+4c}+88ab^3e^{4dx+4c}+16b^4e^{4dx+4c})}{(a^5+3a^4b+3a^3b^2+a^2b^3)(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c})}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 24*a^2*b^2*e^(6*d*x + 6*c) + 8*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 78*a^2*b^2*e^(4*d*x + 4*c) + 88*a*b^3*e^(4*d*x + 4*c) + 16*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 56*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 2*a^2*b^2)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) - 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1))/d

maple [B] time = 0.41, size = 816, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)

[Out] -1/2/d/(a^3+3a^2b+3ab^2+b^3)*tanh(1/2*d*x+1/2*c)+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-15/16/d*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+15/16/d*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^3/tanh(1/2*d*x+1/2*c)

maxima [B] time = 0.53, size = 533, normalized size = 4.23

$$\frac{15 b \log\left(\frac{ae^{-2dx-2c}+a+2b-2\sqrt{(a+b)b}}{ae^{-2dx-2c}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^3+3a^2b+3ab^2+b^3)\sqrt{(a+b)b}d} - \frac{8a^4-9a^3b-2a^2b^2+2a^2b^3-9a^2b^2+2a^2b^3}{4(a^7+3a^6b+3a^5b^2+a^4b^3+(3a^7+17a^6b+33a^5b^2+27a^4b^3+8a^3b^4)d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

```
[Out] -15/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) - 1/4*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*e^(-8*d*x - 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-6*d*x - 6*c) - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6}{\sinh(c + dx)^2 (a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3), x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**3, x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

$$3.47 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=213

$$\frac{(4a^2 - 9ab - b^2) \cosh(c + dx)}{8ad(a + b)^3 (a \cosh^2(c + dx) + b)} - \frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a + b)^4} + \frac{b(2a - b) \cosh(c + dx)}{4ad(a + b)^2 (a \cosh^2(c + dx) + b)}$$

[Out] 1/2*(a-5*b)*arctanh(cosh(d*x+c))/d/(a+b)^4+1/4*(2*a-b)*b*cosh(d*x+c)/a/(a+b)^2/d/(b+a*cosh(d*x+c)^2)^2-1/8*(4*a^2-9*a*b-b^2)*cosh(d*x+c)/a/(a+b)^3/d/(b+a*cosh(d*x+c)^2)-1/2*cosh(d*x+c)*coth(d*x+c)^2/(a+b)/d/(b+a*cosh(d*x+c)^2)^2-1/8*(15*a^2-10*a*b-b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^4/d

Rubi [A] time = 0.34, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$\frac{(4a^2 - 9ab - b^2) \cosh(c + dx)}{8ad(a + b)^3 (a \cosh^2(c + dx) + b)} - \frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a + b)^4} + \frac{b(2a - b) \cosh(c + dx)}{4ad(a + b)^2 (a \cosh^2(c + dx) + b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] -(Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(3/2)*(a + b)^4*d) + ((a - 5*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^4*d) + ((2*a - b)*b*Cosh[c + d*x])/(4*a*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2) - ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(8*a*(a + b)^3*d*(b + a*Cosh[c + d*x]^2)^2) - (Cosh[c + d*x]*Coth[c + d*x]^2)/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 578

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(g^n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 4133

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)]^(p_))*sin[(e_) + (f_)*(x_)^(n_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/ff, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{csch}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\
 &= -\frac{\cosh(c + dx) \operatorname{coth}^2(c + dx)}{2(a + b)d (b + a \cosh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{2(a + b)d} \\
 &= \frac{(2a - b)b \cosh(c + dx)}{4a(a + b)^2d (b + a \cosh^2(c + dx))^2} - \frac{\cosh(c + dx) \operatorname{coth}^2(c + dx)}{2(a + b)d (b + a \cosh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{2(a + b)d} \\
 &= \frac{(2a - b)b \cosh(c + dx)}{4a(a + b)^2d (b + a \cosh^2(c + dx))^2} - \frac{(4a^2 - 9ab - b^2) \cosh(c + dx)}{8a(a + b)^3d (b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{2(a + b)d} \\
 &= \frac{(2a - b)b \cosh(c + dx)}{4a(a + b)^2d (b + a \cosh^2(c + dx))^2} - \frac{(4a^2 - 9ab - b^2) \cosh(c + dx)}{8a(a + b)^3d (b + a \cosh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c + dx)\right)}{2(a + b)d} \\
 &= -\frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1}\left(\frac{\sqrt{a} \cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a + b)^4d} + \frac{(a - 5b) \tanh^{-1}(\cosh(c + dx))}{2(a + b)^4d}
 \end{aligned}$$

Mathematica [C] time = 4.12, size = 524, normalized size = 2.46

$$\operatorname{sech}^5(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{\sqrt{b(-15a^2 + 10ab + b^2)} \operatorname{sech}(c + dx)(a \cosh(2(c + dx)) + a + 2b)^2 \tan^{-1} \left(\frac{\sinh(c) \tanh\left(\frac{dx}{2}\right) (\sqrt{a - i\sqrt{a+b}})}{\sqrt{a - i\sqrt{a+b}}} \right)}{a^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((-8*b^2*(a + b)^2)/a + (2*b*(a + b)*(9*a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b])*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Sech[c + d*x])/a^(3/2) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b])*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b])*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Sech[c + d*x])/a^(3/2) - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] - 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x] - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]^2*Sech[(c + d*x)/2]^2*Sech[c + d*x]))/(64*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-43,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a

branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming $[a,b]=[-63,-70]$ Undefined/Unsigned Inf encountered in limitEvaluation time: 1.54Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.45, size = 1555, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\operatorname{csch}(dx+c))^3 / (a+b \operatorname{sech}(dx+c))^2 dx, x$

[Out] $\frac{1}{8} \frac{d \tanh(1/2 dx + 1/2 c)^2}{(a^3 + 3 a^2 b + 3 a b^2 + b^3)} + \frac{9}{4} \frac{d b}{(a+b)^4} \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a^2 \tanh(1/2 dx + 1/2 c)^6 - 5/4 d b^2 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a \tanh(1/2 dx + 1/2 c)^6 - 13/4 d b^3 / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^6 + 1/4 d b^4 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / a \tanh(1/2 dx + 1/2 c)^6 + 27/4 d b / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a^2 \tanh(1/2 dx + 1/2 c)^4 - 21/4 d b^2 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a \tanh(1/2 dx + 1/2 c)^4 + 27/4 d b / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a^2 \tanh(1/2 dx + 1/2 c)^2 + 1/4 d b^2 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a \tanh(1/2 dx + 1/2 c)^2 - 23/4 d b^3 / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^2 + 3/4 d b^4 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / a \tanh(1/2 dx + 1/2 c)^2 + 9/4 d b / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a^2 + 17/4 d b^2 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a + 7/4 d b^3 / (a+b)^4} + \frac{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a - 1/4 d b^4 / (a+b)^4}{(\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / a - 15/8 d b / (a+b)^4} + \frac{a}{(a*b)^{(1/2)} \arctan(1/4 * (2*(a+b) * \tanh(1/2 dx + 1/2 c)^2 + 2*a - 2*b) / (a*b)^{(1/2)})} + 5/4 d b^2 / (a+b)^4} + \frac{1/8 d b^3 / (a+b)^4}{a / (a*b)^{(1/2)} \arctan(1/4 * (2*(a+b) * \tanh(1/2 dx + 1/2 c)^2 + 2*a - 2*b) / (a*b)^{(1/2)})} + 1/8 d / (a+b)^3} + \frac{1/2 d / (a+b)^4 \ln(\tanh(1/2 dx + 1/2 c))}{a + 5/2 d / (a+b)^4 \ln(\tanh(1/2 dx + 1/2 c))} * b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\operatorname{csch}(dx+c))^3 / (a+b \operatorname{sech}(dx+c))^2 dx, x, \text{algorithm}="maxima"$

[Out] $\frac{1}{2} (a - 5b) \log((e^{(dx+c)} + 1) e^{-c}) / (a^4 d + 4 a^3 b d + 6 a^2 b^2 d + 4 a b^3 d + b^4 d) - \frac{1}{2} (a - 5b) \log((e^{(dx+c)} - 1) e^{-c}) / (a^4 d + 4 a^3 b d + 6 a^2 b^2 d + 4 a b^3 d + b^4 d) - \frac{1}{4} ((4 a^3 e^{(11c)} - 9$

$$\begin{aligned}
& a^2 b e^{(11c)} - a b^2 e^{(11c)} e^{(11dx)} + (20 a^3 e^{(9c)} + 23 a^2 b e^{(9c)} - 29 a b^2 e^{(9c)} + 4 b^3 e^{(9c)}) e^{(9dx)} + 2(20 a^3 e^{(7c)} + 57 a^2 b e^{(7c)} + 47 a b^2 e^{(7c)} - 2 b^3 e^{(7c)}) e^{(7dx)} + 2(20 a^3 e^{(5c)} + 57 a^2 b e^{(5c)} + 47 a b^2 e^{(5c)} - 2 b^3 e^{(5c)}) e^{(5dx)} + (20 a^3 e^{(3c)} + 23 a^2 b e^{(3c)} - 29 a b^2 e^{(3c)} + 4 b^3 e^{(3c)}) e^{(3dx)} + (4 a^3 e^c - 9 a^2 b e^c - a b^2 e^c) e^{(dx)} \\
& / (a^6 d + 3 a^5 b d + 3 a^4 b^2 d + a^3 b^3 d + (a^6 d e^{(12c)} + 3 a^5 b d e^{(12c)} + 3 a^4 b^2 d e^{(12c)} + a^3 b^3 d e^{(12c)}) e^{(12dx)} + 2(a^6 d e^{(10c)} + 7 a^5 b d e^{(10c)} + 15 a^4 b^2 d e^{(10c)} + 13 a^3 b^3 d e^{(10c)} + 4 a^2 b^4 d e^{(10c)}) e^{(10dx)} - (a^6 d e^{(8c)} + 3 a^5 b d e^{(8c)} - 13 a^4 b^2 d e^{(8c)} - 47 a^3 b^3 d e^{(8c)} - 48 a^2 b^4 d e^{(8c)} - 16 a b^5 d e^{(8c)}) e^{(8dx)} - 4(a^6 d e^{(6c)} + 7 a^5 b d e^{(6c)} + 23 a^4 b^2 d e^{(6c)} + 37 a^3 b^3 d e^{(6c)} + 28 a^2 b^4 d e^{(6c)} + 8 a b^5 d e^{(6c)}) e^{(6dx)} - (a^6 d e^{(4c)} + 3 a^5 b d e^{(4c)} - 13 a^4 b^2 d e^{(4c)} - 47 a^3 b^3 d e^{(4c)} - 48 a^2 b^4 d e^{(4c)} - 16 a b^5 d e^{(4c)}) e^{(4dx)} + 2(a^6 d e^{(2c)} + 7 a^5 b d e^{(2c)} + 15 a^4 b^2 d e^{(2c)} + 13 a^3 b^3 d e^{(2c)} + 4 a^2 b^4 d e^{(2c)}) e^{(2dx)} - 8 \int \frac{1}{32} ((15 a^2 b e^{(3c)} - 10 a b^2 e^{(3c)} - b^3 e^{(3c)}) e^{(3dx)} - (15 a^2 b e^c - 10 a b^2 e^c - b^3 e^c) e^{(dx)}) / (a^6 + 4 a^5 b + 6 a^4 b^2 + 4 a^3 b^3 + a^2 b^4 + (a^6 e^{(4c)} + 4 a^5 b e^{(4c)} + 6 a^4 b^2 e^{(4c)} + 4 a^3 b^3 e^{(4c)} + a^2 b^4 e^{(4c)}) e^{(4dx)} + 2(a^6 e^{(2c)} + 6 a^5 b e^{(2c)} + 14 a^4 b^2 e^{(2c)} + 16 a^3 b^3 e^{(2c)} + 9 a^2 b^4 e^{(2c)} + 2 a b^5 e^{(2c)}) e^{(2dx)}), x
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)^6}{\sinh(c + dx)^3 (a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**3, x)

[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)

$$3.48 \quad \int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=165

$$-\frac{5\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}} - \frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4(a-b\tanh^2(c+dx)+b)} - \frac{ab\tanh(c+dx)}{4d(a+b)^3(a-b\tanh^2(c+dx))}$$

[Out] (a-2*b)*coth(d*x+c)/d/(a+b)^4-1/3*coth(d*x+c)^3/d/(a+b)^3-5/8*(3*a-4*b)*arc tanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/d-1/4*a*b*tanh(d*x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(7*a-4*b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.27, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4132, 456, 1259, 1261, 208}

$$-\frac{5\sqrt{b}(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}} - \frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4(a-b\tanh^2(c+dx)+b)} - \frac{ab\tanh(c+dx)}{4d(a+b)^3(a-b\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(8*(a + b)^(9/2)*d) + ((a - 2*b)*Coth[c + d*x])/((a + b)^4*d) - Coth[c + d*x]^3/(3*(a + b)^3*d) - (a*b*Tanh[c + d*x])/(4*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Tanh[c + d*x])/(8*(a + b)^4*d*(a + b - b*Tanh[c + d*x]^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2))]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(m/2 - 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.))^p_.)*sin[(e_.) + (f_.)*(x_)^(m_)]^p_., x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2)], x]^p]/(1 + f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{ab \tanh(c + dx)}{4(a + b)^3 d (a + b - b \tanh^2(c + dx))^2} + \frac{b \operatorname{Subst}\left(\int \frac{\frac{4}{b(a+b)} - \frac{4ax^2}{b(a+b)^2} - \frac{3ax^4}{(a+b)^3}}{x^4(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{4d}$$

$$= -\frac{ab \tanh(c + dx)}{4(a + b)^3 d (a + b - b \tanh^2(c + dx))^2} - \frac{(7a - 4b)b \tanh(c + dx)}{8(a + b)^4 d (a + b - b \tanh^2(c + dx))} + \dots$$

$$= -\frac{ab \tanh(c + dx)}{4(a + b)^3 d (a + b - b \tanh^2(c + dx))^2} - \frac{(7a - 4b)b \tanh(c + dx)}{8(a + b)^4 d (a + b - b \tanh^2(c + dx))} + \dots$$

$$= \frac{(a - 2b) \operatorname{coth}(c + dx)}{(a + b)^4 d} - \frac{\operatorname{coth}^3(c + dx)}{3(a + b)^3 d} - \frac{ab \tanh(c + dx)}{4(a + b)^3 d (a + b - b \tanh^2(c + dx))^2} - \dots$$

$$= -\frac{5(3a - 4b)\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{8(a + b)^{9/2} d} + \frac{(a - 2b) \operatorname{coth}(c + dx)}{(a + b)^4 d} - \frac{\operatorname{coth}^3(c + dx)}{3(a + b)^3 d} - \dots$$

Mathematica [B] time = 5.53, size = 985, normalized size = 5.97

$$\frac{(\cosh(2(c + dx))a + a + 2b)\operatorname{sech}^6(c + dx)}{\operatorname{csch}(c)\operatorname{sech}(2c)(224 \sinh(2c-dx)a^4 - 224 \sinh(2c+dx)a^4 + 176 \sinh(4c+dx)a^4 + 48 \sinh(2c+3dx)a^4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
[Out] -1/6144*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((480*(3*a - 4*b)*b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Co
```

```

sh[2*(c + d*x)]^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4]) + (Csch[c]*Csch[c + d*x]^3*Sech[2*c]*(4*(44*a^4 + 122*a^3*b +
63*a^2*b^2 + 126*a*b^3 + 36*b^4)*Sinh[d*x] + (-96*a^4 - 71*a^3*b + 344*a^2*
b^2 - 1208*a*b^3 + 48*b^4)*Sinh[3*d*x] + 224*a^4*Sinh[2*c - d*x] + 576*a^3*
b*Sinh[2*c - d*x] + 124*a^2*b^2*Sinh[2*c - d*x] - 2184*a*b^3*Sinh[2*c - d*x
] + 144*b^4*Sinh[2*c - d*x] - 224*a^4*Sinh[2*c + d*x] - 657*a^3*b*Sinh[2*c
+ d*x] - 538*a^2*b^2*Sinh[2*c + d*x] + 984*a*b^3*Sinh[2*c + d*x] + 144*b^4*
Sinh[2*c + d*x] + 176*a^4*Sinh[4*c + d*x] + 569*a^3*b*Sinh[4*c + d*x] + 666
*a^2*b^2*Sinh[4*c + d*x] + 1704*a*b^3*Sinh[4*c + d*x] - 144*b^4*Sinh[4*c +
d*x] + 48*a^4*Sinh[2*c + 3*d*x] + 111*a^3*b*Sinh[2*c + 3*d*x] + 360*a^2*b^2
*Sinh[2*c + 3*d*x] + 312*a*b^3*Sinh[2*c + 3*d*x] - 48*b^4*Sinh[2*c + 3*d*x]
- 96*a^4*Sinh[4*c + 3*d*x] - 152*a^3*b*Sinh[4*c + 3*d*x] + 146*a^2*b^2*Sin
h[4*c + 3*d*x] - 728*a*b^3*Sinh[4*c + 3*d*x] - 48*b^4*Sinh[4*c + 3*d*x] + 4
8*a^4*Sinh[6*c + 3*d*x] + 192*a^3*b*Sinh[6*c + 3*d*x] + 558*a^2*b^2*Sinh[6*
c + 3*d*x] - 168*a*b^3*Sinh[6*c + 3*d*x] + 48*b^4*Sinh[6*c + 3*d*x] + 16*a^
4*Sinh[2*c + 5*d*x] - 598*a^2*b^2*Sinh[2*c + 5*d*x] + 48*a*b^3*Sinh[2*c + 5
*d*x] + 72*a^3*b*Sinh[4*c + 5*d*x] + 150*a^2*b^2*Sinh[4*c + 5*d*x] - 48*a*b
^3*Sinh[4*c + 5*d*x] + 16*a^4*Sinh[6*c + 5*d*x] + 27*a^3*b*Sinh[6*c + 5*d*x
] - 388*a^2*b^2*Sinh[6*c + 5*d*x] + 45*a^3*b*Sinh[8*c + 5*d*x] - 60*a^2*b^2
*Sinh[8*c + 5*d*x] + 16*a^4*Sinh[4*c + 7*d*x] - 83*a^3*b*Sinh[4*c + 7*d*x]
+ 6*a^2*b^2*Sinh[4*c + 7*d*x] + 27*a^3*b*Sinh[6*c + 7*d*x] - 6*a^2*b^2*Sin
h[6*c + 7*d*x] + 16*a^4*Sinh[8*c + 7*d*x] - 56*a^3*b*Sinh[8*c + 7*d*x]))/a))
/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 1.66, size = 406, normalized size = 2.46

$$\frac{15(3ab-4b^2)\arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{-ab-b^2}} - \frac{6(9a^3be^{6dx+6c}+20a^2b^2e^{6dx+6c}+27a^3be^{4dx+4c}+66a^2b^2e^{4dx+4c}+56ab^3e^{4dx+4c}-16b^4e^{4dx+4c})}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)(ae^{4dx+4c}+2ae^{2dx+2c}+a)}$$

24d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*
b - b^2)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(-a*b - b^2)) -
6*(9*a^3*b*e^(6*d*x + 6*c) + 20*a^2*b^2*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x
+ 4*c) + 66*a^2*b^2*e^(4*d*x + 4*c) + 56*a*b^3*e^(4*d*x + 4*c) - 16*b^4*e^(
4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 44*a^2*b^2*e^(2*d*x + 2*c) - 16*
a*b^3*e^(2*d*x + 2*c) + 9*a^3*b - 2*a^2*b^2)/((a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x
+ 2*c) + a)^2) + 16*(9*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*
d*x + 2*c) - 2*a + 7*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*
d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.48, size = 1443, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$\begin{aligned} & -1/24/d/(a^3+3a^2b+3ab^2+b^3)/(a+b)*a*\tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3 \\ & +3a^2b+3ab^2+b^3)/(a+b)*\tanh(1/2*d*x+1/2*c)^3*b+3/8/d/(a^3+3a^2b+3a \\ & b^2+b^3)/(a+b)*a*\tanh(1/2*d*x+1/2*c)-9/8/d/(a^3+3a^2b+3ab^2+b^3)/(a+b)* \\ & \tanh(1/2*d*x+1/2*c)*b-9/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d \\ & *x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh \\ & (1/2*d*x+1/2*c)^7*a^2-5/4/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2 \\ & *d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh \\ & (1/2*d*x+1/2*c)^7*a+1/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d \\ & *x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh \\ & (1/2*d*x+1/2*c)^7-27/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+ \\ & 1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/ \\ & 2*d*x+1/2*c)^5*a^2+13/4/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d \\ & *x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh \\ & (1/2*d*x+1/2*c)^5*a-1/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x \\ & +1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/ \\ & 2*d*x+1/2*c)^5-27/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/ \\ & 2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2* \\ & d*x+1/2*c)^3*a^2+13/4/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x \\ & +1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/ \\ & 2*d*x+1/2*c)^3*a-1/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1 \\ & /2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2 \\ & *d*x+1/2*c)^3-9/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c \\ &)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x \\ & +1/2*c)*a^2-5/4/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c \\ &)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x \\ & +1/2*c)*a+1/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+ \\ & 2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2 \\ & *c)+15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b \\ & ^{(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln((a \\ & +b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))- \\ & 5/4/d*b^(3/2)/(a+b)^(9/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*t \\ & anh(1/2*d*x+1/2*c)-(a+b)^(1/2))+5/4/d*b^(3/2)/(a+b)^(9/2)*ln((a+b)^(1/2)*tanh \\ & (1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b) \\ & ^3/\tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^4/\tanh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^4/ \\ & \tanh(1/2*d*x+1/2*c)*b \end{aligned}$$

maxima [B] time = 0.60, size = 782, normalized size = 4.74

$$\frac{5(3ab - 4b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)\sqrt{(a+b)bd} + 12(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4 + (a^7 + 12a^6b + 38a^5b^2 + 4a^4b^3 + 6a^3b^4 + 4a^2b^3 + b^4)\sqrt{(a+b)b})} + \frac{1}{12(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4 + (a^7 + 12a^6b + 38a^5b^2 + 4a^4b^3 + 6a^3b^4 + 4a^2b^3 + b^4)\sqrt{(a+b)b})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 5/16*(3*a*b - 4*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b}) \\ & / (a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^4 + 4*a^3*b + 6*a^2 \\ & *b^2 + 4*a*b^3 + b^4)*\sqrt{(a + b)*b}*d) + 1/12*(16*a^4 - 83*a^3*b + 6*a^2 \\ & *b^2 + 2*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*e^{(-2*d*x - 2*c)} - (96*a^4 + 71*a \\ & ^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*e^{(-4*d*x - 4*c)} - 4*(56*a^4 + 14 \\ & 4*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*e^{(-6*d*x - 6*c)} - (176*a^4 + 56 \\ & 9*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*e^{(-8*d*x - 8*c)} - 6*(8*a^4 + \\ & 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*e^{(-10*d*x - 10*c)} - 15*(3*a^3*b \\ & - 4*a^2*b^2)*e^{(-12*d*x - 12*c)})/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + \\ & a^3*b^4 + (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b \\ & ^5)*e^{(-2*d*x - 2*c)} - (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3 - 61*a^3* \\ & b^4 - 56*a^2*b^5 - 16*a*b^6)*e^{(-4*d*x - 4*c)} - (3*a^7 + 28*a^6*b + 130*a^5 \\ & *b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^{(-6*d*x - 6*c)} \end{aligned}$$

+ (3*a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^(-8*d*x - 8*c) + (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-10*d*x - 10*c) - (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b^5)*e^(-12*d*x - 12*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-14*d*x - 14*c))*d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6}{\sinh(c + dx)^4 (a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**3, x)

[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)

3.49 $\int \cosh^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=61

$$\frac{(3a + 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(3a + 4b) + \frac{a \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

[Out] $1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*\cosh(d*x+c)*\sinh(d*x+c)/d+1/4*a*\cosh(d*x+c)^3*\sinh(d*x+c)/d$

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4045, 2635, 8}

$$\frac{(3a + 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(3a + 4b) + \frac{a \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]`

[Out] $((3*a + 4*b)*x)/8 + ((3*a + 4*b)*\cosh[c + d*x]*\sinh[c + d*x])/(8*d) + (a*\cosh[c + d*x]^3*\sinh[c + d*x])/(4*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 4045

`Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]`

Rubi steps

$$\begin{aligned} \int \cosh^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{1}{4}(3a + 4b) \int \cosh^2(c + dx) dx \\ &= \frac{(3a + 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} \\ &= \frac{1}{8}(3a + 4b)x + \frac{(3a + 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a \cosh^3(c + dx) \sinh(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.74

$$\frac{4(3a + 4b)(c + dx) + 8(a + b) \sinh(2(c + dx)) + a \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] (4*(3*a + 4*b)*(c + d*x) + 8*(a + b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)])/ (32*d)

fricas [A] time = 0.40, size = 61, normalized size = 1.00

$$\frac{a \cosh(dx + c) \sinh(dx + c)^3 + (3a + 4b)dx + (a \cosh(dx + c)^3 + 4(a + b) \cosh(dx + c)) \sinh(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] 1/8*(a*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a + 4*b)*d*x + (a*cosh(d*x + c)^3 + 4*(a + b)*cosh(d*x + c))*sinh(d*x + c))/d

giac [B] time = 0.14, size = 116, normalized size = 1.90

$$\frac{8(dx + c)(3a + 4b) + ae^{4dx+4c} + 8ae^{2dx+2c} + 8be^{2dx+2c} - (18ae^{4dx+4c} + 24be^{4dx+4c} + 8ae^{2dx+2c} + 8be^{2dx+2c})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] 1/64*(8*(d*x + c)*(3*a + 4*b) + a*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 24*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c))/d

maple [A] time = 0.46, size = 66, normalized size = 1.08

$$\frac{a \left(\left(\frac{\cosh^3(dx+c)}{4} + \frac{3 \cosh(dx+c)}{8} \right) \sinh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right) + b \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2), x)

[Out] 1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 0.31, size = 97, normalized size = 1.59

$$\frac{1}{64} a \left(24x + \frac{e^{4dx+4c}}{d} + \frac{8e^{2dx+2c}}{d} - \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) + \frac{1}{8} b \left(4x + \frac{e^{2dx+2c}}{d} - \frac{e^{-2dx-2c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/8*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d)

mupad [B] time = 0.13, size = 50, normalized size = 0.82

$$\frac{\frac{a \sinh(2c+2dx)}{4} + \frac{a \sinh(4c+4dx)}{32} + \frac{b \sinh(2c+2dx)}{4}}{d} + \frac{3ax}{8} + \frac{bx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] ((a*sinh(2*c + 2*d*x))/4 + (a*sinh(4*c + 4*d*x))/32 + (b*sinh(2*c + 2*d*x))
/4)/d + (3*a*x)/8 + (b*x)/2
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \operatorname{sech}^2(c + dx)) \cosh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**4, x)
```

3.50 $\int \cosh^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=30

$$\frac{(a + b) \sinh(c + dx)}{d} + \frac{a \sinh^3(c + dx)}{3d}$$

[Out] (a+b)*sinh(d*x+c)/d+1/3*a*sinh(d*x+c)^3/d

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4044, 3013}

$$\frac{(a + b) \sinh(c + dx)}{d} + \frac{a \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] ((a + b)*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 4044

Int[csc[(e_.) + (f_.)*(x_.)]^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \cosh^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \int \cosh(c + dx) (b + a \cosh^2(c + dx)) dx \\ &= \frac{i \operatorname{Subst} \left(\int (a + b - ax^2) dx, x, -i \sinh(c + dx) \right)}{d} \\ &= \frac{(a + b) \sinh(c + dx)}{d} + \frac{a \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.67

$$\frac{a \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d} + \frac{b \sinh(c) \cosh(dx)}{d} + \frac{b \cosh(c) \sinh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + (a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)

fricas [A] time = 0.39, size = 41, normalized size = 1.37

$$\frac{a \sinh(dx + c)^3 + 3(a \cosh(dx + c)^2 + 3a + 4b) \sinh(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] $1/12*(a*\sinh(d*x + c)^3 + 3*(a*\cosh(d*x + c)^2 + 3*a + 4*b)*\sinh(d*x + c))/d$

giac [B] time = 0.16, size = 72, normalized size = 2.40

$$\frac{ae^{(3dx+3c)} + 9ae^{(dx+c)} + 12be^{(dx+c)} - (9ae^{(2dx+2c)} + 12be^{(2dx+2c)} + a)e^{(-3dx-3c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] $1/24*(a*e^{(3*d*x + 3*c)} + 9*a*e^{(d*x + c)} + 12*b*e^{(d*x + c)} - (9*a*e^{(2*d*x + 2*c)} + 12*b*e^{(2*d*x + 2*c)} + a)*e^{(-3*d*x - 3*c)})/d$

maple [A] time = 0.43, size = 34, normalized size = 1.13

$$\frac{a\left(\frac{2}{3} + \frac{\cosh^2(dx+c)}{3}\right)\sinh(dx+c) + b\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x)

[Out] $1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+b*sinh(d*x+c))$

maxima [B] time = 0.31, size = 85, normalized size = 2.83

$$\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d} + \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} - \frac{e^{(-3dx-3c)}}{d}\right) + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $1/24*a*(e^{(3*d*x + 3*c)}/d + 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d - e^{(-3*d*x - 3*c)}/d) + 1/2*b*(e^{(d*x + c)}/d - e^{(-d*x - c)}/d)$

mupad [B] time = 0.09, size = 34, normalized size = 1.13

$$\frac{3a\sinh(c+dx) + 3b\sinh(c+dx) + a\sinh(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c+d*x)^3*(a+b/cosh(c+d*x)^2),x)

[Out] $(3*a*\sinh(c+d*x) + 3*b*\sinh(c+d*x) + a*\sinh(c+d*x)^3)/(3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \cosh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2),x)

[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**3, x)

3.51 $\int \cosh^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=31

$$\frac{1}{2}x(a + 2b) + \frac{a \sinh(c + dx) \cosh(c + dx)}{2d}$$

[Out] 1/2*(a+2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4045, 8}

$$\frac{1}{2}x(a + 2b) + \frac{a \sinh(c + dx) \cosh(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m_*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cosh^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{1}{2}(a + 2b) \int 1 dx \\ &= \frac{1}{2}(a + 2b)x + \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.06

$$\frac{a(c + dx)}{2d} + \frac{a \sinh(2(c + dx))}{4d} + bx$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] b*x + (a*(c + d*x))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)

fricas [A] time = 0.41, size = 28, normalized size = 0.90

$$\frac{(a + 2b)dx + a \cosh(dx + c) \sinh(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] $1/2*((a + 2*b)*d*x + a*\cosh(d*x + c)*\sinh(d*x + c))/d$

giac [B] time = 0.15, size = 66, normalized size = 2.13

$$\frac{4(dx+c)(a+2b) + ae^{(2dx+2c)} - (2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)e^{(-2dx-2c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")`

[Out] $1/8*(4*(d*x + c)*(a + 2*b) + a*e^{(2*d*x + 2*c)} - (2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)*e^{(-2*d*x - 2*c)})/d$

maple [A] time = 0.31, size = 37, normalized size = 1.19

$$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + (dx+c)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x)`

[Out] $1/d*(a*(1/2*\cosh(d*x+c)*\sinh(d*x+c)+1/2*d*x+1/2*c)+(d*x+c)*b)$

maxima [A] time = 0.31, size = 38, normalized size = 1.23

$$\frac{1}{8}a\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $1/8*a*(4*x + e^{(2*d*x + 2*c)}/d - e^{(-2*d*x - 2*c)}/d) + b*x$

mupad [B] time = 0.08, size = 23, normalized size = 0.74

$$\frac{ax}{2} + bx + \frac{a \sinh(2c + 2dx)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)`

[Out] $(a*x)/2 + b*x + (a*\sinh(2*c + 2*d*x))/(4*d)$

sympy [A] time = 11.24, size = 60, normalized size = 1.94

$$a \left(\begin{cases} -\frac{x \sinh^2(c+dx)}{2} + \frac{x \cosh^2(c+dx)}{2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d} & \text{for } d \neq 0 \\ x \cosh^2(c) & \text{otherwise} \end{cases} \right) + b \left(\begin{cases} x & \text{for } \\ G_{2,2}^{1,1} \left(\begin{matrix} 1 & 2 \\ 1 & 0 \end{matrix} \middle| x \right) + G_{2,2}^{0,2} \left(\begin{matrix} 2, 1 \\ 1, 0 \end{matrix} \middle| x \right) & \text{other} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2),x)`

[Out] `a*Piecewise((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*cosh(c)**2, True)) + b*Piecewise((x, Abs(x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((1, 0)), x), True))`

3.52 $\int \cosh(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{a \sinh(c + dx)}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

[Out] b*arctan(sinh(d*x+c))/d+a*sinh(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4045, 3770}

$$\frac{a \sinh(c + dx)}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :- Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :- Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{a \sinh(c + dx)}{d} + b \int \operatorname{sech}(c + dx) dx \\ &= \frac{b \tan^{-1}(\sinh(c + dx))}{d} + \frac{a \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a \sinh(c) \cosh(dx)}{d} + \frac{a \cosh(c) \sinh(dx)}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d

fricas [B] time = 0.45, size = 93, normalized size = 3.88

$$\frac{a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + 4(b \cosh(dx + c) + b \sinh(dx + c)) \operatorname{arctan}(\sinh(dx + c))}{2(d \cosh(dx + c) + d \sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{2}*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c) + b*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - a)/(d*\cosh(d*x + c) + d*\sinh(d*x + c))$

giac [A] time = 0.15, size = 36, normalized size = 1.50

$$\frac{4 b \arctan \left(e^{(dx+c)} \right) + a e^{(dx+c)} - a e^{(-dx-c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{2}*(4*b*\arctan(e^{(d*x + c)}) + a*e^{(d*x + c)} - a*e^{(-d*x - c)})/d$

maple [A] time = 0.29, size = 26, normalized size = 1.08

$$\frac{a \sinh(dx + c)}{d} + \frac{2b \arctan \left(e^{dx+c} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x)

[Out] $a*\sinh(d*x+c)/d+2/d*b*\arctan(\exp(d*x+c))$

maxima [A] time = 0.41, size = 28, normalized size = 1.17

$$-\frac{2 b \arctan \left(e^{(-dx-c)} \right)}{d} + \frac{a \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $-2*b*\arctan(e^{(-d*x - c)})/d + a*\sinh(d*x + c)/d$

mupad [B] time = 1.44, size = 62, normalized size = 2.58

$$\frac{2 \operatorname{atan} \left(\frac{b e^{dx} e^c \sqrt{d^2}}{d \sqrt{b^2}} \right) \sqrt{b^2}}{\sqrt{d^2}} - \frac{a e^{-c-dx}}{2 d} + \frac{a e^{c+dx}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2),x)

[Out] $(2*\operatorname{atan}((b*\exp(d*x)*\exp(c)*(d^2)^{(1/2)})/(d*(b^2)^{(1/2)}))*(b^2)^{(1/2)})/(d^2)^{(1/2)} - (a*\exp(-c - d*x))/(2*d) + (a*\exp(c + d*x))/(2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2),x)

[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x), x)

3.53 $\int \operatorname{sech}(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=40

$$\frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c))/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4046, 3770}

$$\frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} + \frac{1}{2} (2a + b) \int \operatorname{sech}(c + dx) dx \\ &= \frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.20

$$\frac{a \tan^{-1}(\sinh(c + dx))}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

fricas [B] time = 0.41, size = 321, normalized size = 8.02

$$b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 + ((2a + b) \cosh(dx + c)^4 + 4(2a + b) \cosh(dx + c) \sinh(dx + c)^2 + 4(2a + b) \sinh(dx + c)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.12, size = 84, normalized size = 2.10

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)(2a + b) + \frac{4b\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) + 4*b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d

maple [A] time = 0.30, size = 45, normalized size = 1.12

$$\frac{2a \arctan\left(e^{dx+c}\right)}{d} + \frac{b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{b \arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2),x)

[Out] 2/d*a*arctan(exp(d*x+c))+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))

maxima [B] time = 0.40, size = 81, normalized size = 2.02

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d\left(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1\right)}\right) + \frac{a \arctan(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d

mupad [B] time = 0.16, size = 124, normalized size = 3.10

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c \left(2a \sqrt{d^2} + b \sqrt{d^2}\right)}{d \sqrt{4a^2 + 4ab + b^2}}\right) \sqrt{4a^2 + 4ab + b^2}}{\sqrt{d^2}} + \frac{b e^{c+dx}}{d \left(e^{2c+2dx} + 1\right)} - \frac{2b e^{c+dx}}{d \left(2e^{2c+2dx} + e^{4c+4dx} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x),x)

```
[Out] (atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2
+ b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) + (b*exp(c + d*x))
/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x), x)
```

3.54 $\int \operatorname{sech}^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=30

$$\frac{(a + b) \tanh(c + dx)}{d} - \frac{b \tanh^3(c + dx)}{3d}$$

[Out] (a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4046, 3767, 8}

$$\frac{(3a + 2b) \tanh(c + dx)}{3d} + \frac{b \tanh(c + dx) \operatorname{sech}^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] ((3*a + 2*b)*Tanh[c + d*x])/(3*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^(2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{b \operatorname{sech}^2(c + dx) \tanh(c + dx)}{3d} + \frac{1}{3}(3a + 2b) \int \operatorname{sech}^2(c + dx) dx \\ &= \frac{b \operatorname{sech}^2(c + dx) \tanh(c + dx)}{3d} + \frac{(i(3a + 2b)) \operatorname{Subst}(\int 1 dx, x, -i \tanh(c + dx))}{3d} \\ &= \frac{(3a + 2b) \tanh(c + dx)}{3d} + \frac{b \operatorname{sech}^2(c + dx) \tanh(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.30

$$\frac{a \tanh(c + dx)}{d} - \frac{b \tanh^3(c + dx)}{3d} + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] $(a*\text{Tanh}[c + d*x])/d + (b*\text{Tanh}[c + d*x])/d - (b*\text{Tanh}[c + d*x]^3)/(3*d)$

fricas [B] time = 0.38, size = 158, normalized size = 5.27

$$\frac{4\left((3a+b)\cosh(dx+c)^2 - 2b\cosh(dx+c)\sinh(dx+c)\right)}{3\left(d\cosh(dx+c)^4 + 4d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh(dx+c)^4 + 4d\cosh(dx+c)^2 + 2\left(3d\cosh(dx+c)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")`

[Out] $-4/3*((3*a + b)*\cosh(d*x + c)^2 - 2*b*\cosh(d*x + c)*\sinh(d*x + c) + (3*a + b)*\sinh(d*x + c)^2 + 3*a + 3*b)/(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 + 4*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c))^2 + 2*d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + 3*d)$

giac [B] time = 0.12, size = 61, normalized size = 2.03

$$\frac{2\left(3ae^{4dx+4c} + 6ae^{2dx+2c} + 6be^{2dx+2c} + 3a + 2b\right)}{3d\left(e^{2dx+2c} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")`

[Out] $-2/3*(3*a*e^{(4*d*x + 4*c)} + 6*a*e^{(2*d*x + 2*c)} + 6*b*e^{(2*d*x + 2*c)} + 3*a + 2*b)/(d*(e^{(2*d*x + 2*c)} + 1)^3)$

maple [A] time = 0.34, size = 34, normalized size = 1.13

$$\frac{a \tanh(dx+c) + b\left(\frac{2}{3} + \frac{\text{sech}(dx+c)^2}{3}\right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x)`

[Out] $1/d*(a*\tanh(d*x+c)+b*(2/3+1/3*\text{sech}(d*x+c)^2)*\tanh(d*x+c))$

maxima [B] time = 0.32, size = 112, normalized size = 3.73

$$\frac{4}{3}b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + \frac{2a}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $4/3*b*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 2*a/(d*(e^{(-2*d*x - 2*c)} + 1))$

mupad [B] time = 1.38, size = 61, normalized size = 2.03

$$\frac{2\left(3a + 2b + 6ae^{2c+2dx} + 3ae^{4c+4dx} + 6be^{2c+2dx}\right)}{3d\left(e^{2c+2dx} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^2,x)`

[Out] $-(2*(3*a + 2*b + 6*a*\exp(2*c + 2*d*x) + 3*a*\exp(4*c + 4*d*x) + 6*b*\exp(2*c + 2*d*x)))/(3*d*(\exp(2*c + 2*d*x) + 1)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2),x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**2, x)`

3.55 $\int \operatorname{sech}^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=70

$$\frac{(4a + 3b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(4a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c))/d+1/8*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4046, 3768, 3770}

$$\frac{(4a + 3b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(4a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{b \operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} + \frac{1}{4}(4a + 3b) \int \operatorname{sech}^3(c + dx) dx \\ &= \frac{(4a + 3b) \operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{b \operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} \\ &= \frac{(4a + 3b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(4a + 3b) \operatorname{sech}(c + dx) \tanh(c + dx)}{8d} \end{aligned}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 0.86

$$\frac{(4a + 3b) \tan^{-1}(\sinh(c + dx)) + (4a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx) + 2b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]] + (4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] + 2*b*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)

fricas [B] time = 0.42, size = 1112, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] 1/4*((4*a + 3*b)*cosh(d*x + c)^7 + 7*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (4*a + 3*b)*sinh(d*x + c)^7 + (4*a + 11*b)*cosh(d*x + c)^5 + (21*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 11*b)*sinh(d*x + c)^5 + 5*(7*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a + 11*b)*cosh(d*x + c)^3 + (35*(4*a + 3*b)*cosh(d*x + c)^4 + 10*(4*a + 11*b)*cosh(d*x + c)^2 - 4*a - 11*b)*sinh(d*x + c)^3 + (21*(4*a + 3*b)*cosh(d*x + c)^5 + 10*(4*a + 11*b)*cosh(d*x + c)^3 - 3*(4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a + 3*b)*cosh(d*x + c)^8 + 8*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + 3*b)*sinh(d*x + c)^8 + 4*(4*a + 3*b)*cosh(d*x + c)^6 + 4*(7*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^6 + 8*(7*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + 3*b)*cosh(d*x + c)^4 + 2*(35*(4*a + 3*b)*cosh(d*x + c)^4 + 30*(4*a + 3*b)*cosh(d*x + c)^2 + 12*a + 9*b)*sinh(d*x + c)^4 + 8*(7*(4*a + 3*b)*cosh(d*x + c)^5 + 10*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + 3*b)*cosh(d*x + c)^2 + 4*(7*(4*a + 3*b)*cosh(d*x + c)^6 + 15*(4*a + 3*b)*cosh(d*x + c)^4 + 9*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^2 + 8*((4*a + 3*b)*cosh(d*x + c)^7 + 3*(4*a + 3*b)*cosh(d*x + c)^5 + 3*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + 3*b)*cosh(d*x + c) + (7*(4*a + 3*b)*cosh(d*x + c)^6 + 5*(4*a + 11*b)*cosh(d*x + c)^4 - 3*(4*a + 11*b)*cosh(d*x + c)^2 - 4*a - 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.16, size = 156, normalized size = 2.23

$$\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)(4a + 3b) + \frac{4\left(4a\left(e^{dx+c} - e^{-dx-c}\right)^3 + 3b\left(e^{dx+c} - e^{-dx-c}\right)^3 + 16a\left(e^{dx+c} - e^{-dx-c}\right) + 20b\left(e^{dx+c} - e^{-dx-c}\right)\right)}{\left(\left(e^{dx+c} - e^{-dx-c}\right)^2 + 4\right)^2}$$

16d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + 3*b) + 4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b*(e^(d*x + c) - e^(-d*x - c))^3 + 16*a*(e^(d*x + c) - e^(-d*x - c)) + 20*b*(e^(d*x + c) - e^(-d*x - c)))/(e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d

maple [A] time = 0.42, size = 83, normalized size = 1.19

$$\frac{a \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a \arctan(e^{dx+c})}{d} + \frac{b \operatorname{sech}(dx+c)^3 \tanh(dx+c)}{4d} + \frac{3b \operatorname{sech}(dx+c) \tanh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2), x)

[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))+1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d+3/8*b*sech(d*x+c)*tanh(d*x+c)/d+3/4/d*b*arctan(exp(d*x+c))

maxima [B] time = 0.41, size = 184, normalized size = 2.63

$$-\frac{1}{4}b \left(\frac{3 \arctan(e^{-dx-c})}{d} - \frac{3e^{-dx-c} + 11e^{-3dx-3c} - 11e^{-5dx-5c} - 3e^{-7dx-7c}}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right) - a \left(\frac{\arctan(e^{(-dx-c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] -1/4*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))

mupad [B] time = 1.38, size = 283, normalized size = 4.04

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (4a \sqrt{d^2} + 3b \sqrt{d^2})}{d \sqrt{16a^2 + 24ab + 9b^2}}\right) \sqrt{16a^2 + 24ab + 9b^2}}{4\sqrt{d^2}} - \frac{\frac{ae^{5c+5dx}}{d} + \frac{2e^{3c+3dx}(a+2b)}{d} + \frac{ae^{c+dx}}{d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1}}{2d} (2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^3, x)

[Out] (atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + 3*b*(d^2)^(1/2)))/(d*(24*a*b + 16*a^2 + 9*b^2)^(1/2)))*(24*a*b + 16*a^2 + 9*b^2)^(1/2))/(4*(d^2)^(1/2)) - ((a*exp(5*c + 5*d*x))/d + (2*exp(3*c + 3*d*x)*(a + 2*b))/d + (a*exp(c + d*x))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a - b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(4*a + 3*b))/(4*d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2), x)

[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**3, x)

3.56 $\int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=50

$$-\frac{(a + 2b) \tanh^3(c + dx)}{3d} + \frac{(a + b) \tanh(c + dx)}{d} + \frac{b \tanh^5(c + dx)}{5d}$$

[Out] (a+b)*tanh(d*x+c)/d-1/3*(a+2*b)*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4046, 3767}

$$-\frac{(5a + 4b) \tanh^3(c + dx)}{15d} + \frac{(5a + 4b) \tanh(c + dx)}{5d} + \frac{b \tanh(c + dx) \operatorname{sech}^4(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] ((5*a + 4*b)*Tanh[c + d*x])/(5*d) + (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - ((5*a + 4*b)*Tanh[c + d*x]^3)/(15*d)

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{b \operatorname{sech}^4(c + dx) \tanh(c + dx)}{5d} + \frac{1}{5}(5a + 4b) \int \operatorname{sech}^4(c + dx) dx \\ &= \frac{b \operatorname{sech}^4(c + dx) \tanh(c + dx)}{5d} + \frac{(i(5a + 4b)) \operatorname{Subst}\left(\int (1 + x^2) dx, x, \tanh(c + dx)\right)}{5d} \\ &= \frac{(5a + 4b) \tanh(c + dx)}{5d} + \frac{b \operatorname{sech}^4(c + dx) \tanh(c + dx)}{5d} - \frac{(5a + 4b) \tanh^3(c + dx)}{15d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.42

$$-\frac{a \tanh^3(c + dx)}{3d} + \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^5(c + dx)}{5d} - \frac{2b \tanh^3(c + dx)}{3d} + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (2*b*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)

fricas [B] time = 0.38, size = 343, normalized size = 6.86

$$15 \left(d \cosh(dx+c)^7 + 7d \cosh(dx+c) \sinh(dx+c)^6 + d \sinh(dx+c)^7 + 5d \cosh(dx+c)^5 + (21d \cosh(dx+c) \sinh(dx+c)^4 + 15d \cosh(dx+c)^3 + 5d \cosh(dx+c) \sinh(dx+c)^2 + 5d \sinh(dx+c)^2 + 5d) \sinh(dx+c)^3 + 5d \cosh(dx+c)^2 + 5d \sinh(dx+c) \cosh(dx+c) + 5d \sinh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\frac{-8/15*(2*(5*a + b)*\cosh(d*x + c)^3 + 6*(5*a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (5*a - 2*b)*\sinh(d*x + c)^3 + 30*(a + b)*\cosh(d*x + c) + (3*(5*a - 2*b)*\cosh(d*x + c)^2 + 5*a + 10*b)*\sinh(d*x + c))/(d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + d*\sinh(d*x + c)^7 + 5*d*\cosh(d*x + c)^5 + (21*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^5 + 5*(7*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 11*d*\cosh(d*x + c)^3 + (35*d*\cosh(d*x + c)^4 + 50*d*\cosh(d*x + c)^2 + 9*d)*\sinh(d*x + c)^3 + (21*d*\cosh(d*x + c)^5 + 50*d*\cosh(d*x + c)^3 + 33*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 15*d*\cosh(d*x + c) + (7*d*\cosh(d*x + c)^6 + 25*d*\cosh(d*x + c)^4 + 27*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)}$$

giac [A] time = 0.15, size = 85, normalized size = 1.70

$$\frac{4 \left(15 a e^{(6dx+6c)} + 35 a e^{(4dx+4c)} + 40 b e^{(4dx+4c)} + 25 a e^{(2dx+2c)} + 20 b e^{(2dx+2c)} + 5 a + 4 b \right)}{15 d \left(e^{(2dx+2c)} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out]
$$\frac{-4/15*(15*a*e^{(6*d*x + 6*c)} + 35*a*e^{(4*d*x + 4*c)} + 40*b*e^{(4*d*x + 4*c)} + 25*a*e^{(2*d*x + 2*c)} + 20*b*e^{(2*d*x + 2*c)} + 5*a + 4*b)/(d*(e^{(2*d*x + 2*c)} + 1)^5)}$$

maple [A] time = 0.37, size = 56, normalized size = 1.12

$$\frac{a \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + b \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x)

[Out]
$$1/d*(a*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)+b*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))$$

maxima [B] time = 0.32, size = 300, normalized size = 6.00

$$\frac{16}{15} b \left(\frac{5 e^{(-2dx-2c)}}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{1}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out]
$$16/15*b*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 4/3*a*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)))$$

+ 3*e^{(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c) + 3*e^{(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))}}}

mupad [B] time = 1.43, size = 292, normalized size = 5.84

$$\frac{\frac{8(a+2b)}{15d} + \frac{4ae^{2c+2dx}}{5d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{8ae^{2c+2dx}}{5d} + \frac{8ae^{6c+6dx}}{5d} + \frac{16e^{4c+4dx}(a+2b)}{5d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1} - \frac{1}{4e^{2c+2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^4, x)

[Out] - ((8*(a + 2*b))/(15*d) + (4*a*exp(2*c + 2*d*x))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a*exp(2*c + 2*d*x))/(5*d) + (8*a*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*c + 4*d*x)*(a + 2*b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a)/(5*d) + (6*a*exp(4*c + 4*d*x))/(5*d) + (8*exp(2*c + 2*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a)/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2), x)

[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**4, x)

$$3.57 \quad \int \cosh^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$$

Optimal. Leaf size=82

$$\frac{1}{8}x(3a^2 + 8ab + 8b^2) + \frac{3a(a + 2b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{a \sinh(c + dx) \cosh^3(c + dx) (a - b \tanh^2(c + dx))}{4d}$$

[Out] $1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*\cosh(d*x+c)*\sinh(d*x+c)/d+1/4*a*\cosh(d*x+c)^3*\sinh(d*x+c)*(a+b-b*\tanh(d*x+c)^2)/d$

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 413, 385, 206}

$$\frac{1}{8}x(3a^2 + 8ab + 8b^2) + \frac{3a(a + 2b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{a \sinh(c + dx) \cosh^3(c + dx) (a - b \tanh^2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*\cosh[c + d*x]*\sinh[c + d*x])/(8*d) + (a*\cosh[c + d*x]^3*\sinh[c + d*x]*(a + b - b*\tanh[c + d*x]^2))/(4*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \cosh^4(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^2}{(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a \cosh^3(c+dx) \sinh(c+dx) (a+b-b \tanh^2(c+dx))}{4d} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{4d} \\
&= \frac{3a(a+2b) \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{a \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{4d} \\
&= \frac{1}{8} (3a^2 + 8ab + 8b^2) (c+dx) + \frac{3a(a+2b) \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{a \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.71

$$\frac{4(3a^2 + 8ab + 8b^2)(c+dx) + a^2 \sinh(4(c+dx)) + 8a(a+2b) \sinh(2(c+dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(c + d*x) + 8*a*(a + 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c + d*x)])/(32*d)

fricas [A] time = 0.40, size = 78, normalized size = 0.95

$$\frac{a^2 \cosh(dx+c) \sinh(dx+c)^3 + (3a^2 + 8ab + 8b^2)dx + (a^2 \cosh(dx+c)^3 + 4(a^2 + 2ab) \cosh(dx+c)) \sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*(a^2*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*d*x + (a^2*cosh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/d

giac [A] time = 0.15, size = 151, normalized size = 1.84

$$\frac{a^2 e^{(4dx+4c)} + 8a^2 e^{(2dx+2c)} + 16abe^{(2dx+2c)} + 8(3a^2 + 8ab + 8b^2)(dx+c) - (18a^2 e^{(4dx+4c)} + 48abe^{(4dx+4c)} + 48ab^2 e^{(4dx+4c)})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c) + 8*(3*a^2 + 8*a*b + 8*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) + 48*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c))/d

maple [A] time = 0.40, size = 79, normalized size = 0.96

$$\frac{a^2 \left(\left(\frac{\cosh^3(dx+c)}{4} + \frac{3 \cosh(dx+c)}{8} \right) \sinh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 2ab \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + b^2 (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b^2*(d*x+c))

maxima [A] time = 0.31, size = 105, normalized size = 1.28

$$\frac{1}{64} a^2 \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{4} ab \left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d} \right) + b^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/4*a*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + b^2*x

mupad [B] time = 1.37, size = 66, normalized size = 0.80

$$\frac{3a^2x}{8} + b^2x + abx + \frac{a^2 \sinh(2c + 2dx)}{4d} + \frac{a^2 \sinh(4c + 4dx)}{32d} + \frac{ab \sinh(2c + 2dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)

[Out] (3*a^2*x)/8 + b^2*x + a*b*x + (a^2*sinh(2*c + 2*d*x))/(4*d) + (a^2*sinh(4*c + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

3.58 $\int \cosh^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx)\right)^2 dx$

Optimal. Leaf size=49

$$\frac{a^2 \sinh^3(c + dx)}{3d} + \frac{a(a + 2b) \sinh(c + dx)}{d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d}$$

[Out] $b^2 \arctan(\sinh(d*x+c))/d + a*(a+2*b)*\sinh(d*x+c)/d + 1/3*a^2*\sinh(d*x+c)^3/d$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4147, 390, 203}

$$\frac{a^2 \sinh^3(c + dx)}{3d} + \frac{a(a + 2b) \sinh(c + dx)}{d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (a*(a + 2*b)*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cosh^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx)\right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{1+x^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a(a + 2b) + a^2x^2 + \frac{b^2}{1+x^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a(a + 2b) \sinh(c + dx)}{d} + \frac{a^2 \sinh^3(c + dx)}{3d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x\right)}{d} \\ &= \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d} + \frac{a(a + 2b) \sinh(c + dx)}{d} + \frac{a^2 \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.47

$$\frac{a^2 \sinh^3(c + dx)}{3d} + \frac{a^2 \sinh(c + dx)}{d} + \frac{2ab \sinh(c) \cosh(dx)}{d} + \frac{2ab \cosh(c) \sinh(dx)}{d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (2*a*b*Cosh[d*x]*Sinh[c])/d + (2*a*b*Cosh[c]*Sinh[d*x])/d + (a^2*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)

fricas [B] time = 0.41, size = 414, normalized size = 8.45

$$\frac{a^2 \cosh(dx + c)^6 + 6a^2 \cosh(dx + c) \sinh(dx + c)^5 + a^2 \sinh(dx + c)^6 + 3(3a^2 + 8ab) \cosh(dx + c)^4 + 3(5a^2 + 8ab) \cosh(dx + c)^2 \sinh(dx + c)^3 + 3b^2 \cosh(dx + c)^2 \sinh(dx + c) \operatorname{arctan}(\cosh(dx + c) + \sinh(dx + c))}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/24*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + 3*a^2 + 8*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 - 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 + 6*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 - 3*a^2 - 8*a*b)*sinh(d*x + c)^2 - a^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2*cosh(d*x + c)^5 + 2*(3*a^2 + 8*a*b)*cosh(d*x + c)^3 - (3*a^2 + 8*a*b)*cosh(d*x + c)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)

giac [A] time = 0.15, size = 94, normalized size = 1.92

$$\frac{48b^2 \arctan(e^{(dx+c)}) + a^2 e^{(3dx+3c)} + 9a^2 e^{(dx+c)} + 24abe^{(dx+c)} - (9a^2 e^{(2dx+2c)} + 24abe^{(2dx+2c)} + a^2) e^{(-3dx-3c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c) + 24*a*b*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) + 24*a*b*e^(2*d*x + 2*c) + a^2)*e^(-3*d*x - 3*c))/d

maple [A] time = 0.36, size = 66, normalized size = 1.35

$$\frac{2a^2 \sinh(dx + c)}{3d} + \frac{a^2 \sinh(dx + c) (\cosh^2(dx + c))}{3d} + \frac{2ab \sinh(dx + c)}{d} + \frac{2b^2 \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)

[Out] 2/3*a^2*sinh(d*x+c)/d+1/3/d*a^2*sinh(d*x+c)*cosh(d*x+c)^2+2*a*b*sinh(d*x+c)/d+2/d*b^2*arctan(exp(d*x+c))

maxima [B] time = 0.41, size = 105, normalized size = 2.14

$$\frac{1}{24} a^2 \left(\frac{e^{(3dx+3c)}}{d} + \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} - \frac{e^{(-3dx-3c)}}{d} \right) + ab \left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d} \right) - \frac{2b^2 \arctan(e^{(-dx-c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{24}a^2(e^{(3dx+3c)/d} + 9e^{(dx+c)/d} - 9e^{-(dx-c)/d} - e^{(-3dx-3c)/d}) + a*b*(e^{(dx+c)/d} - e^{-(dx-c)/d}) - 2*b^2*\arctan(e^{-(dx-c)})/d$

mupad [B] time = 0.17, size = 114, normalized size = 2.33

$$\frac{2 \operatorname{atan}\left(\frac{b^2 e^{dx} e^c \sqrt{d^2}}{d \sqrt{b^4}}\right) \sqrt{b^4}}{\sqrt{d^2}} - \frac{e^{-c-dx} (3a^2 + 8ba)}{8d} - \frac{a^2 e^{-3c-3dx}}{24d} + \frac{a^2 e^{3c+3dx}}{24d} + \frac{a e^{c+dx} (3a + 8b)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $(2*\operatorname{atan}((b^2*\exp(dx)*\exp(c)*(d^2)^{(1/2)})/(d*(b^4)^{(1/2)}))*(b^4)^{(1/2)})/(d^2)^{(1/2)} - (\exp(-c-dx)*(8*a*b + 3*a^2))/(8*d) - (a^2*\exp(-3*c - 3*d*x))/(24*d) + (a^2*\exp(3*c + 3*d*x))/(24*d) + (a*\exp(c + d*x)*(3*a + 8*b))/(8*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

3.59 $\int \cosh^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=47

$$\frac{a^2 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}ax(a + 4b) + \frac{b^2 \tanh(c + dx)}{d}$$

[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 390, 385, 206}

$$\frac{a^2 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}ax(a + 4b) + \frac{b^2 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (a*(a + 4*b)*x)/2 + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[(b*c - a*d)*x*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \cosh^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(b^2 + \frac{a(a+2b)-2abx^2}{(1-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2 \tanh(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a(a+2b)-2abx^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a^2 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2 \tanh(c+dx)}{d} + \frac{(a(a+4b)) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{1}{2} a(a+4b)x + \frac{a^2 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2 \tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 1.11

$$\frac{a^2(c+dx)}{2d} + \frac{a^2 \sinh(2(c+dx))}{4d} + 2abx + \frac{b^2 \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] 2*a*b*x + (a^2*(c + d*x))/(2*d) + (a^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tanh[c + d*x])/d

fricas [A] time = 0.39, size = 80, normalized size = 1.70

$$\frac{a^2 \sinh(dx+c)^3 + 4((a^2 + 4ab)dx - 2b^2) \cosh(dx+c) + (3a^2 \cosh(dx+c)^2 + a^2 + 8b^2) \sinh(dx+c)}{8d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*(a^2*sinh(d*x + c)^3 + 4*((a^2 + 4*a*b)*d*x - 2*b^2)*cosh(d*x + c) + (3*a^2*cosh(d*x + c)^2 + a^2 + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))

giac [B] time = 0.17, size = 128, normalized size = 2.72

$$\frac{a^2 e^{2(dx+2c)} + 4(a^2 + 4ab)(dx+c) - \frac{a^2 e^{4(dx+4c)} + 4abe^{4(dx+4c)} + 2a^2 e^{2(dx+2c)} + 4abe^{2(dx+2c)} + 16b^2 e^{2(dx+2c)} + a^2}{e^{4(dx+4c)} + e^{2(dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/8*(a^2*e^(2*d*x + 2*c) + 4*(a^2 + 4*a*b)*(d*x + c) - (a^2*e^(4*d*x + 4*c) + 4*a*b*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x + 2*c) + 16*b^2*e^(2*d*x + 2*c) + a^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c))/d

maple [A] time = 0.36, size = 51, normalized size = 1.09

$$\frac{a^2 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 2ab(dx+c) + b^2 \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)`

[Out] `1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(d*x+c)+b^2*tanh(d*x+c))`

maxima [A] time = 0.32, size = 63, normalized size = 1.34

$$\frac{1}{8} a^2 \left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d} \right) + 2abx + \frac{2b^2}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] `1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))`

mupad [B] time = 0.16, size = 65, normalized size = 1.38

$$\frac{a^2 e^{2c+2dx}}{8d} - \frac{a^2 e^{-2c-2dx}}{8d} - \frac{2b^2}{d(e^{2c+2dx} + 1)} + \frac{ax(a+4b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)`

[Out] `(a^2*exp(2*c + 2*d*x))/(8*d) - (a^2*exp(- 2*c - 2*d*x))/(8*d) - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) + (a*x*(a + 4*b))/2`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \cosh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**2, x)`

3.60 $\int \cosh(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=56

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{b(4a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^2 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $1/2*b*(4*a+b)*\arctan(\sinh(d*x+c))/d+a^2*\sinh(d*x+c)/d+1/2*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4147, 390, 385, 203}

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{b(4a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^2 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $(b*(4*a + b)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) + (a^2*\operatorname{Sinh}[c + d*x])/d + (b^2*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \cosh(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(a^2 + \frac{b(2a+b)+2abx^2}{(1+x^2)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{a^2 \sinh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{b(2a+b)+2abx^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{a^2 \sinh(c + dx)}{d} + \frac{b^2 \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} + \frac{(b(4a + b)) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{2d} \\
&= \frac{b(4a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^2 \sinh(c + dx)}{d} + \frac{b^2 \operatorname{sech}(c + dx) \tanh(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 1.43

$$\frac{a^2 \sinh(c) \cosh(dx)}{d} + \frac{a^2 \cosh(c) \sinh(dx)}{d} + \frac{2ab \tan^{-1}(\sinh(c + dx))}{d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^2 \tanh(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*Cosh[d*x]*Sinh[c])/d + (a^2*Cosh[c]*Sinh[d*x])/d + (b^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

fricas [B] time = 0.42, size = 653, normalized size = 11.66

$$\frac{a^2 \cosh(dx + c)^6 + 6 a^2 \cosh(dx + c) \sinh(dx + c)^5 + a^2 \sinh(dx + c)^6 + (a^2 + 2 b^2) \cosh(dx + c)^4 + (15 a^2 c + 2 b^2) \cosh(dx + c)^3 + (15 a^2 c + 2 b^2) \sinh(dx + c)^3 + (15 a^2 c + 2 b^2) \cosh(dx + c)^2 + (15 a^2 c + 2 b^2) \sinh(dx + c)^2 + (15 a^2 c + 2 b^2) \cosh(dx + c) + (15 a^2 c + 2 b^2) \sinh(dx + c) + (15 a^2 c + 2 b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/2*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 2*b^2)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 2*b^2)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 - a^2 + 2*((4*a*b + b^2)*cosh(d*x + c)^5 + 5*(4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a*b + b^2)*sinh(d*x + c)^5 + 2*(4*a*b + b^2)*cosh(d*x + c)^3 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c)^3 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + b^2)*cosh(d*x + c) + (5*(4*a*b + b^2)*cosh(d*x + c)^4 + 6*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))

giac [B] time = 0.14, size = 112, normalized size = 2.00

$$\frac{2a^2(e^{dx+c} - e^{-dx-c}) + \left(\pi + 2 \arctan\left(\frac{1}{2}(e^{2dx+2c} - 1)e^{-dx-c}\right)\right)(4ab + b^2) + \frac{4b^2(e^{dx+c} - e^{-dx-c})}{(e^{dx+c} - e^{-dx-c})^2 + 4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a*b + b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c)))/(e^(d*x + c) - e^(-d*x - c))^2 + 4)/d

maple [A] time = 0.36, size = 63, normalized size = 1.12

$$\frac{a^2 \sinh(dx+c)}{d} + \frac{4ab \arctan(e^{dx+c})}{d} + \frac{b^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{b^2 \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)

[Out] a^2*sinh(d*x+c)/d+4/d*a*b*arctan(exp(d*x+c))+1/2/d*b^2*sech(d*x+c)*tanh(d*x+c)+1/d*b^2*arctan(exp(d*x+c))

maxima [A] time = 0.42, size = 101, normalized size = 1.80

$$-b^2 \left(\frac{\arctan(e^{-dx-c})}{d} - \frac{e^{-dx-c} - e^{-3dx-3c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right) - \frac{4ab \arctan(e^{-dx-c})}{d} + \frac{a^2 \sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b*arctan(e^(-d*x - c))/d + a^2*sinh(d*x + c)/d

mupad [B] time = 1.44, size = 172, normalized size = 3.07

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (b^2 \sqrt{d^2 + 4ab\sqrt{d^2}})}{d \sqrt{16a^2 b^2 + 8ab^3 + b^4}}\right) \sqrt{16a^2 b^2 + 8ab^3 + b^4}}{\sqrt{d^2}} + \frac{a^2 e^{c+dx}}{2d} - \frac{a^2 e^{-c-dx}}{2d} + \frac{b^2 e^{c+dx}}{d(e^{2c+2dx} + 1)} - \frac{2b^2 e^{c+dx}}{d(2e^{2c+2dx} + e^{4c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)

[Out] (atan((exp(d*x)*exp(c)*(b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(8*a*b^3 + b^4 + 16*a^2*b^2)^(1/2)))*(8*a*b^3 + b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2) + (a^2*exp(c + d*x))/(2*d) - (a^2*exp(-c - d*x))/(2*d) + (b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x), x)

3.61 $\int \operatorname{sech}(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=90

$$\frac{(8a^2 + 8ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3b(2a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

[Out] $1/8*(8*a^2+8*a*b+3*b^2)*\arctan(\sinh(d*x+c))/d+3/8*b*(2*a+b)*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d+1/4*b*\operatorname{sech}(d*x+c)^3*(a+b+a*\sinh(d*x+c)^2)*\tanh(d*x+c)/d$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4147, 413, 385, 203}

$$\frac{(8a^2 + 8ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3b(2a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $((8*a^2 + 8*a*b + 3*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + (3*b*(2*a + b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + (b*\operatorname{Sech}[c + d*x]^3*(a + b + a*\operatorname{Sinh}[c + d*x]^2)*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{b\operatorname{sech}^3(c+dx) (a+b+a\sinh^2(c+dx)) \tanh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{a}{(1+x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{3b(2a+b)\operatorname{sech}(c+dx) \tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx) (a+b+a\sinh^2(c+dx)) \tanh(c+dx)}{4d} \\
&= \frac{(8a^2+8ab+3b^2) \tan^{-1}(\sinh(c+dx))}{8d} + \frac{3b(2a+b)\operatorname{sech}(c+dx) \tanh(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.79

$$\frac{(8a^2 + 8ab + 3b^2) \tan^{-1}(\sinh(c + dx)) + b(8a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx) + 2b^2 \tanh(c + dx) \operatorname{sech}^3(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2, x]

[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]] + b*(8*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] + 2*b^2*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)

fricas [B] time = 0.43, size = 1372, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/4*((8*a*b + 3*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + (8*a*b + 3*b^2)*sinh(d*x + c)^7 + (8*a*b + 11*b^2)*cosh(d*x + c)^5 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x + c)^5 + 5*(7*(8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (8*a*b + 11*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 3*b^2)*cosh(d*x + c)^4 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 11*b^2)*sinh(d*x + c)^3 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^3 - 3*(8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 3*b^2)*cosh(d*x + c) + (7*(8*a*b + 3*b^2)*cosh(d*x + c)^6 + 5*(8*a*b + 11*b^2)*cosh(d*x + c)^4 + 4*(8*a*b + 11*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x + c)

$$c)^4 - 3*(8*a*b + 11*b^2)*\cosh(d*x + c)^2 - 8*a*b - 3*b^2)*\sinh(d*x + c))/((d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 + 4*d*\cosh(d*x + c)^6 + 4*(7*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 8*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.14, size = 170, normalized size = 1.89

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8a^2 + 8ab + 3b^2\right) + \frac{4\left(8ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 32ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*b + 3*b^2) + 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 20*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2/d

maple [A] time = 0.38, size = 106, normalized size = 1.18

$$\frac{2a^2 \arctan\left(e^{dx+c}\right)}{d} + \frac{ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} + \frac{2ab \arctan\left(e^{dx+c}\right)}{d} + \frac{b^2 \tanh(dx+c) \operatorname{sech}(dx+c)^3}{4d} + \frac{3b^2 \operatorname{sech}(dx+c)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)

[Out] 2/d*a^2*arctan(exp(d*x+c))+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^2*arctan(exp(d*x+c))

maxima [B] time = 0.42, size = 201, normalized size = 2.23

$$-\frac{1}{4}b^2\left(\frac{3 \arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3e^{(-dx-c)} + 11e^{(-3dx-3c)} - 11e^{(-5dx-5c)} - 3e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right) - 2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right) + a^2\arctan(\sinh(d*x + c))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d

mupad [B] time = 1.52, size = 303, normalized size = 3.37

$$\frac{\operatorname{atan}\left(\frac{e^{dx}e^c\left(8a^2\sqrt{d^2}+3b^2\sqrt{d^2}+8ab\sqrt{d^2}\right)}{d\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}\right)}{4\sqrt{d^2}} + \frac{6b^2e^c}{d\left(3e^{2c+2dx}+3e^{4c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x),x)

[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2)^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (exp(c + d*x)*(8*a*b + 3*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) - (exp(c + d*x)*(8*a*b - b^2))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x), x)

3.62 $\int \operatorname{sech}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=53

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d} + \frac{(a+b)^2\tanh(c+dx)}{d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

[Out] (a+b)^2*tanh(d*x+c)/d-2/3*b*(a+b)*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4146, 194}

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d} + \frac{(a+b)^2\tanh(c+dx)}{d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Tanh[c + d*x])/d - (2*b*(a + b)*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^5)/(5*d)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst} \left(\int (a + b - bx^2)^2 dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(a^2 \left(1 + \frac{b(2a+b)}{a^2} \right) - 2ab \left(1 + \frac{b}{a} \right) x^2 + b^2 x^4 \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{(a+b)^2 \tanh(c+dx)}{d} - \frac{2b(a+b)\tanh^3(c+dx)}{3d} + \frac{b^2 \tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.75

$$\frac{a^2 \tanh(c + dx)}{d} - \frac{2ab \tanh^3(c + dx)}{3d} + \frac{2ab \tanh(c + dx)}{d} + \frac{b^2 \tanh^5(c + dx)}{5d} - \frac{2b^2 \tanh^3(c + dx)}{3d} + \frac{b^2 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^5)/(5*d)

fricas [B] time = 0.41, size = 404, normalized size = 7.62

$$\frac{4\left(\left(15a^2 + 10ab + 4b^2\right)\cosh(dx + c)^4 - 8\left(5ab + 2b^2\right)\cosh(dx + c)\sinh(dx + c)^3 + \left(15a^2 + 10ab + 4b^2\right)\sinh(dx + c)^4\right) + 20\left(3a^2 + 4ab + b^2\right)\cosh(dx + c)^2 + 2\left(3\left(15a^2 + 10ab + 4b^2\right)\cosh(dx + c)^2 + 30a^2 + 40ab + 10b^2\right)\sinh(dx + c)^2 + 45a^2 + 70ab + 40b^2 - 8\left(\left(5ab + 2b^2\right)\cosh(dx + c)^3 + 5\left(ab + b^2\right)\cosh(dx + c)\right)\sinh(dx + c)}{15\left(d\cosh(dx + c)^6 + 6d\cosh(dx + c)\sinh(dx + c)^5 + d\sinh(dx + c)^6 + 6d\cosh(dx + c)^4 + 3\left(5d\cosh(dx + c)\sinh(dx + c)^3 + 5d\sinh(dx + c)^4 + 4\left(5d\cosh(dx + c)^3 + 4d\cosh(dx + c)\right)\sinh(dx + c)^3 + 15d\cosh(dx + c)^2 + 3\left(5d\cosh(dx + c)^4 + 12d\cosh(dx + c)^2 + 5d\right)\sinh(dx + c)^2 + 2\left(3d\cosh(dx + c)^5 + 8d\cosh(dx + c)^3 + 5d\cosh(dx + c)\right)\sinh(dx + c) + 10d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -4/15*((15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^4 - 8*(5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (15*a^2 + 10*a*b + 4*b^2)*sinh(d*x + c)^4 + 20*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^2 + 30*a^2 + 40*a*b + 10*b^2)*sinh(d*x + c)^2 + 45*a^2 + 70*a*b + 40*b^2 - 8*((5*a*b + 2*b^2)*cosh(d*x + c)^3 + 5*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) + 10*d)

giac [B] time = 0.17, size = 156, normalized size = 2.94

$$\frac{2\left(15a^2e^{(8dx+8c)} + 60a^2e^{(6dx+6c)} + 60abe^{(6dx+6c)} + 90a^2e^{(4dx+4c)} + 140abe^{(4dx+4c)} + 80b^2e^{(4dx+4c)} + 60a^2e^{(2dx+2c)}\right)}{15d\left(e^{(2dx+2c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 140*a*b*e^(4*d*x + 4*c) + 80*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 100*a*b*e^(2*d*x + 2*c) + 40*b^2*e^(2*d*x + 2*c) + 15*a^2 + 20*a*b + 8*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)

maple [A] time = 0.42, size = 70, normalized size = 1.32

$$\frac{a^2 \tanh(dx + c) + 2ab\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right) \tanh(dx + c) + b^2\left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15}\right) \tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.33, size = 324, normalized size = 6.11

$$\frac{16}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1\right)} + \frac{1}{d\left(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 16/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))

$x - 2*c) + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 8/3*a*b*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 2*a^2/(d*(e^{(-2*d*x - 2*c)} + 1))$

mapad [B] time = 1.46, size = 452, normalized size = 8.53

$$\frac{\frac{2a(a+2b)}{5d} + \frac{2a^2 e^{2c+2dx}}{5d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{\frac{2a^2}{5d} + \frac{2a^2 e^{8c+8dx}}{5d} + \frac{4e^{4c+4dx}(3a^2+8ab+8b^2)}{5d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1} + \frac{8ae^{2c+2dx}(a+2b)}{5d} + \frac{8ae^{6c+6dx}(a+2b)}{5d} - \frac{2a(a+2b)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^2, x)

[Out] $-\left(\frac{2*a*(a + 2*b)}{5*d} + \frac{2*a^2*\exp(2*c + 2*d*x)}{5*d}\right)/\left(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1\right) - \left(\frac{2*a^2}{5*d} + \frac{2*a^2*\exp(8*c + 8*d*x)}{5*d}\right)/\left(5*d + (4*\exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/5*d + (8*a*\exp(2*c + 2*d*x)*(a + 2*b))/5*d + (8*a*\exp(6*c + 6*d*x)*(a + 2*b))/5*d\right)/\left(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1\right) - \left(\frac{2*a*(a + 2*b)}{5*d} + \frac{2*a^2*\exp(6*c + 6*d*x)}{5*d}\right)/\left(5*d + (2*\exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/5*d + (6*a*\exp(4*c + 4*d*x)*(a + 2*b))/5*d\right)/\left(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1\right) - \left(\frac{2*(8*a*b + 3*a^2 + 8*b^2)}{15*d}\right) + \frac{2*a^2*\exp(4*c + 4*d*x)}{5*d} + \frac{4*a*\exp(2*c + 2*d*x)*(a + 2*b)}{5*d} \right)/\left(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1\right) - \frac{2*a^2}{5*d*(\exp(2*c + 2*d*x) + 1)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**2, x)

3.63 $\int \operatorname{sech}^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=128

$$\frac{(8a^2 + 12ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 12ab + 5b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} + \frac{b(8a + 5b) \tanh(c + dx)}{24d}$$

[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+12*a*b+5*b^2)*sech(d*x+c)*tanh(d*x+c)/d+1/24*b*(8*a+5*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4147, 413, 385, 199, 203}

$$\frac{(8a^2 + 12ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 12ab + 5b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} + \frac{b(8a + 5b) \tanh(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]]/(16*d) + ((8*a^2 + 12*a*b + 5*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + (b*(8*a + 5*b)*Sech[c + d*x]^3*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(6*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^3(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{b \operatorname{sech}^5(c + dx) (a + b + a \sinh^2(c + dx)) \tanh(c + dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{b(8a + 5b) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{24d} + \frac{b \operatorname{sech}^5(c + dx) (a + b + a \sinh^2(c + dx)) \tanh(c + dx)}{6d} \\ &= \frac{(8a^2 + 12ab + 5b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{16d} + \frac{b(8a + 5b) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{6d} \\ &= \frac{(8a^2 + 12ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 12ab + 5b^2) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.81

$$\frac{3(8a^2 + 12ab + 5b^2) \tan^{-1}(\sinh(c + dx)) + 3(8a^2 + 12ab + 5b^2) \tanh(c + dx) \operatorname{sech}(c + dx) + 2b(12a + 5b) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 3*(8*a^2 + 12*a*b + 5*b^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b*(12*a + 5*b)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d)
```

fricas [B] time = 0.43, size = 2946, normalized size = 23.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^11 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 72*a^2 + 204*a*b + 85*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)*sinh(d*x + c)^8 + 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^7 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 6*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^2 + 8*a^2 + 28*a*b + 33*b^2)*sinh(d*x + c)^7 + 42*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^3 + (8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^4 + 21*
```

$$\begin{aligned}
& (8a^2 + 28ab + 33b^2) \cosh(dx + c)^2 - 8a^2 - 28ab - 33b^2) \sinh(dx + c)^5 + 6(165(8a^2 + 12ab + 5b^2) \cosh(dx + c)^7 + 21(72a^2 + 204ab + 85b^2) \cosh(dx + c)^5 + 35(8a^2 + 28ab + 33b^2) \cosh(dx + c)^3 - 5(8a^2 + 28ab + 33b^2) \cosh(dx + c)) \sinh(dx + c)^4 - (72a^2 + 204ab + 85b^2) \cosh(dx + c)^3 + (495(8a^2 + 12ab + 5b^2) \cosh(dx + c)^8 + 84(72a^2 + 204ab + 85b^2) \cosh(dx + c)^6 + 210(8a^2 + 28ab + 33b^2) \cosh(dx + c)^4 - 60(8a^2 + 28ab + 33b^2) \cosh(dx + c)^2 - 72a^2 - 204ab - 85b^2) \sinh(dx + c)^3 + 3(55(8a^2 + 12ab + 5b^2) \cosh(dx + c)^9 + 12(72a^2 + 204ab + 85b^2) \cosh(dx + c)^7 + 42(8a^2 + 28ab + 33b^2) \cosh(dx + c)^5 - 20(8a^2 + 28ab + 33b^2) \cosh(dx + c)^3 - (72a^2 + 204ab + 85b^2) \cosh(dx + c)) \sinh(dx + c)^2 + 3((8a^2 + 12ab + 5b^2) \cosh(dx + c)^12 + 12(8a^2 + 12ab + 5b^2) \cosh(dx + c) \sinh(dx + c)^11 + (8a^2 + 12ab + 5b^2) \sinh(dx + c)^12 + 6(8a^2 + 12ab + 5b^2) \cosh(dx + c)^10 + 6(11(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 8a^2 + 12ab + 5b^2) \sinh(dx + c)^10 + 20(11(8a^2 + 12ab + 5b^2) \cosh(dx + c)^3 + 3(8a^2 + 12ab + 5b^2) \cosh(dx + c)) \sinh(dx + c)^9 + 15(8a^2 + 12ab + 5b^2) \cosh(dx + c)^8 + 15(33(8a^2 + 12ab + 5b^2) \cosh(dx + c)^4 + 18(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 8a^2 + 12ab + 5b^2) \sinh(dx + c)^8 + 24(33(8a^2 + 12ab + 5b^2) \cosh(dx + c)^5 + 30(8a^2 + 12ab + 5b^2) \cosh(dx + c)^3 + 5(8a^2 + 12ab + 5b^2) \cosh(dx + c)) \sinh(dx + c)^7 + 20(8a^2 + 12ab + 5b^2) \cosh(dx + c)^6 + 4(231(8a^2 + 12ab + 5b^2) \cosh(dx + c)^6 + 315(8a^2 + 12ab + 5b^2) \cosh(dx + c)^4 + 105(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 40a^2 + 60ab + 25b^2) \sinh(dx + c)^6 + 24(33(8a^2 + 12ab + 5b^2) \cosh(dx + c)^7 + 63(8a^2 + 12ab + 5b^2) \cosh(dx + c)^5 + 35(8a^2 + 12ab + 5b^2) \cosh(dx + c)^3 + 5(8a^2 + 12ab + 5b^2) \cosh(dx + c)) \sinh(dx + c)^5 + 15(8a^2 + 12ab + 5b^2) \cosh(dx + c)^4 + 15(33(8a^2 + 12ab + 5b^2) \cosh(dx + c)^8 + 84(8a^2 + 12ab + 5b^2) \cosh(dx + c)^6 + 70(8a^2 + 12ab + 5b^2) \cosh(dx + c)^4 + 20(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 8a^2 + 12ab + 5b^2) \sinh(dx + c)^4 + 20(11(8a^2 + 12ab + 5b^2) \cosh(dx + c)^9 + 36(8a^2 + 12ab + 5b^2) \cosh(dx + c)^7 + 42(8a^2 + 12ab + 5b^2) \cosh(dx + c)^5 + 20(8a^2 + 12ab + 5b^2) \cosh(dx + c)^3 + 3(8a^2 + 12ab + 5b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 6(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 6(11(8a^2 + 12ab + 5b^2) \cosh(dx + c)^10 + 45(8a^2 + 12ab + 5b^2) \cosh(dx + c)^8 + 70(8a^2 + 12ab + 5b^2) \cosh(dx + c)^6 + 50(8a^2 + 12ab + 5b^2) \cosh(dx + c)^4 + 15(8a^2 + 12ab + 5b^2) \cosh(dx + c)^2 + 8a^2 + 12ab + 5b^2) \sinh(dx + c)^2 + 8a^2 + 12ab + 5b^2 + 12((8a^2 + 12ab + 5b^2) \cosh(dx + c)^11 + 5(8a^2 + 12ab + 5b^2) \cosh(dx + c)^9 + 10(8a^2 + 12ab + 5b^2) \cosh(dx + c)^7 + 10(8a^2 + 12ab + 5b^2) \cosh(dx + c)^5 + 5(8a^2 + 12ab + 5b^2) \cosh(dx + c)^3 + (8a^2 + 12ab + 5b^2) \cosh(dx + c)) \sinh(dx + c) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 3(8a^2 + 12ab + 5b^2) \cosh(dx + c) + 3(11(8a^2 + 12ab + 5b^2) \cosh(dx + c)^10 + 3(72a^2 + 204ab + 85b^2) \cosh(dx + c)^8 + 14(8a^2 + 28ab + 33b^2) \cosh(dx + c)^6 - 10(8a^2 + 28ab + 33b^2) \cosh(dx + c)^4 - (72a^2 + 204ab + 85b^2) \cosh(dx + c)^2 - 8a^2 - 12ab - 5b^2) \sinh(dx + c)) / (d \cosh(dx + c)^12 + 12d \cosh(dx + c) \sinh(dx + c)^11 + d \sinh(dx + c)^12 + 6d \cosh(dx + c)^10 + 6(11d \cosh(dx + c)^2 + d) \sinh(dx + c)^10 + 20(11d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^9 + 15d \cosh(dx + c)^8 + 15(33d \cosh(dx + c)^4 + 18d \cosh(dx + c)^2 + d) \sinh(dx + c)^8 + 24(33d \cosh(dx + c)^5 + 30d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^7 + 20d \cosh(dx + c)^6 + 4(231d \cosh(dx + c)^6 + 315d \cosh(dx + c)^4 + 105d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^6 + 24(33d \cosh(dx + c)^7 + 63d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^5 + 15d \cosh(dx + c)^4 + 15(33d \cosh(dx + c)^8 + 84d \cosh(dx + c)^6 + 70d \cosh(dx + c)^4 + 20d \cosh(dx + c)^2 + d) \sinh(dx + c)^4 + 20(11d \cosh(dx + c)^9 + 36d \cosh(dx + c)^7 + 42d \cosh(dx + c)^5 + 20d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 + 6d
\end{aligned}$$

*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.16, size = 293, normalized size = 2.29

$$3 \left(\pi + 2 \arctan \left(\frac{1}{2} \left(e^{(2dx+2c)} - 1 \right) e^{(-dx-c)} \right) \right) \left(8a^2 + 12ab + 5b^2 \right) + \frac{4 \left(24a^2 \left(e^{(dx+c)} - e^{(-dx-c)} \right)^5 + 36ab \left(e^{(dx+c)} - e^{(-dx-c)} \right)^5 + 15b^2 \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 12*a*b + 5*b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 36*a*b*(e^(d*x + c) - e^(-d*x - c))^5 + 15*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 160*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(e^(d*x + c) - e^(-d*x - c)) + 960*a*b*(e^(d*x + c) - e^(-d*x - c)) + 528*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3/d

maple [A] time = 0.46, size = 169, normalized size = 1.32

$$\frac{a^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^2 \arctan(e^{dx+c})}{d} + \frac{ab \tanh(dx+c) \operatorname{sech}(dx+c)^3}{2d} + \frac{3ab \operatorname{sech}(dx+c) \tanh(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))+1/2/d*a*b*tanh(d*x+c)*sech(d*x+c)^3+3/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+3/2/d*a*b*arctan(exp(d*x+c))+1/6/d*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+5/16/d*b^2*sech(d*x+c)*tanh(d*x+c)+5/8/d*b^2*arctan(exp(d*x+c))

maxima [B] time = 0.42, size = 348, normalized size = 2.72

$$-\frac{1}{24} b^2 \left(\frac{15 \arctan(e^{(-dx-c)})}{d} - \frac{15 e^{(-dx-c)} + 85 e^{(-3dx-3c)} + 198 e^{(-5dx-5c)} - 198 e^{(-7dx-7c)} - 85 e^{(-9dx-9c)} - 15 e^{(-11dx-11c)}}{d(6 e^{(-2dx-2c)} + 15 e^{(-4dx-4c)} + 20 e^{(-6dx-6c)} + 15 e^{(-8dx-8c)} + 6 e^{(-10dx-10c)} + e^{(-12dx-12c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/24*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 1/2*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))

mupad [B] time = 1.57, size = 569, normalized size = 4.45

$$\frac{\operatorname{atan} \left(\frac{e^{dx} e^c \left(8a^2 \sqrt{d^2} + 5b^2 \sqrt{d^2} + 12ab \sqrt{d^2} \right)}{d \sqrt{64a^4 + 192a^3b + 224a^2b^2 + 120ab^3 + 25b^4}} \right)}{8 \sqrt{d^2}} - \frac{\frac{2a^2 e^{c+dx}}{3d} + \frac{2a^2 e^{9c}}{3d}}{6 e^{2c+2dx} + 15 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^3, x)`

[Out] $(\operatorname{atan}((\exp(d*x)*\exp(c)*(8*a^2*(d^2)^{(1/2)} + 5*b^2*(d^2)^{(1/2)} + 12*a*b*(d^2)^{(1/2)}))/d*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^{(1/2)})*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^{(1/2)})/(8*(d^2)^{(1/2)}) - ((2*a^2*\exp(c + d*x))/(3*d) + (2*a^2*\exp(9*c + 9*d*x))/(3*d) + (4*\exp(5*c + 5*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(3*d) + (8*a*\exp(3*c + 3*d*x)*(a + 2*b))/(3*d) + (8*a*\exp(7*c + 7*d*x)*(a + 2*b))/(3*d))/(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1) + (2*\exp(c + d*x)*(4*a*b - 11*b^2))/(3*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (16*b^2*\exp(c + d*x))/(3*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) + (\exp(c + d*x)*(12*a*b + 8*a^2 + 5*b^2))/(8*d*(\exp(2*c + 2*d*x) + 1)) + (\exp(c + d*x)*(12*a*b - 16*a^2 + 5*b^2))/(12*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (\exp(c + d*x)*(20*a*b - b^2))/(3*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**2, x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**3, x)`

3.64 $\int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=80

$$\frac{b(2a + 3b) \tanh^5(c + dx)}{5d} - \frac{(a + b)(a + 3b) \tanh^3(c + dx)}{3d} + \frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

[Out] (a+b)^2*tanh(d*x+c)/d-1/3*(a+b)*(a+3*b)*tanh(d*x+c)^3/d+1/5*b*(2*a+3*b)*tanh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4146, 373}

$$\frac{b(2a + 3b) \tanh^5(c + dx)}{5d} - \frac{(a + b)(a + 3b) \tanh^3(c + dx)}{3d} + \frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Tanh[c + d*x])/d - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/(3*d) + (b*(2*a + 3*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int (1 - x^2) (a + b - bx^2)^2 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int ((a + b)^2 + (-a - 3b)(a + b)x^2 + b(2a + 3b)x^4 - b^2x^6) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{(a + b)(a + 3b) \tanh^3(c + dx)}{3d} + \frac{b(2a + 3b) \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^7(c + dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 144, normalized size = 1.80

$$-\frac{a^2 \tanh^3(c + dx)}{3d} + \frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^5(c + dx)}{5d} - \frac{4ab \tanh^3(c + dx)}{3d} + \frac{2ab \tanh(c + dx)}{d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $(a^2 \operatorname{Tanh}[c + dx])/d + (2ab \operatorname{Tanh}[c + dx])/d + (b^2 \operatorname{Tanh}[c + dx])/d - (a^2 \operatorname{Tanh}[c + dx]^3)/(3d) - (4ab \operatorname{Tanh}[c + dx]^3)/(3d) - (b^2 \operatorname{Tanh}[c + dx]^3)/d + (2ab \operatorname{Tanh}[c + dx]^5)/(5d) + (3b^2 \operatorname{Tanh}[c + dx]^5)/(5d) - (b^2 \operatorname{Tanh}[c + dx]^7)/(7d)$

fricas [B] time = 0.39, size = 677, normalized size = 8.46

$$\frac{8(2(35a^2 + 14ab + 6b^2) \cosh(dx+c)^9 + 9d \cosh(dx+c) \sinh(dx+c)^8 + d \sinh(dx+c)^9 + 7d \cosh(dx+c)^7 + (36d \cosh(dx+c)^5 + 126d \cosh(dx+c)^3 + 7d \cosh(dx+c)) \sinh(dx+c)^6 + 22d \cosh(dx+c)^5 + (126d \cosh(dx+c)^4 + 147d \cosh(dx+c)^2 + 20d) \sinh(dx+c)^5 + (126d \cosh(dx+c)^5 + 245d \cosh(dx+c)^3 + 110d \cosh(dx+c)) \sinh(dx+c)^4 + 42d \cosh(dx+c)^3 + (84d \cosh(dx+c)^6 + 245d \cosh(dx+c)^4 + 200d \cosh(dx+c)^2 + 28d) \sinh(dx+c)^3 + (36d \cosh(dx+c)^7 + 147d \cosh(dx+c)^5 + 220d \cosh(dx+c)^3 + 126d \cosh(dx+c)) \sinh(dx+c)^2 + 56d \cosh(dx+c) + (9d \cosh(dx+c)^8 + 49d \cosh(dx+c)^6 + 100d \cosh(dx+c)^4 + 84d \cosh(dx+c)^2 + 14d) \sinh(dx+c))}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(dx+c)^4*(a+b*sech(dx+c)^2)^2,x, algorithm="fricas")`

[Out] $-8/105*(2*(35a^2 + 14ab + 6b^2) \cosh(dx+c)^5 + 10*(35a^2 + 14ab + 6b^2) \cosh(dx+c) \sinh(dx+c)^4 + (35a^2 - 28ab - 12b^2) \sinh(dx+c)^5 + 14*(25a^2 + 34ab + 6b^2) \cosh(dx+c)^3 + (10*(35a^2 - 28ab - 12b^2) \cosh(dx+c)^2 + 105a^2 + 84ab - 84b^2) \sinh(dx+c)^3 + 2*(10*(35a^2 + 14ab + 6b^2) \cosh(dx+c)^3 + 21*(25a^2 + 34ab + 6b^2) \cosh(dx+c)) \sinh(dx+c)^2 + 28*(25a^2 + 46ab + 24b^2) \cosh(dx+c) + (5*(35a^2 - 28ab - 12b^2) \cosh(dx+c)^4 + 63*(5a^2 + 4ab - 4b^2) \cosh(dx+c)^2 + 70a^2 + 112ab + 168b^2) \sinh(dx+c))/(d \cosh(dx+c)^9 + 9d \cosh(dx+c) \sinh(dx+c)^8 + d \sinh(dx+c)^9 + 7d \cosh(dx+c)^7 + (36d \cosh(dx+c)^5 + 126d \cosh(dx+c)^3 + 7d \cosh(dx+c)) \sinh(dx+c)^6 + 22d \cosh(dx+c)^5 + (126d \cosh(dx+c)^4 + 147d \cosh(dx+c)^2 + 20d) \sinh(dx+c)^5 + (126d \cosh(dx+c)^5 + 245d \cosh(dx+c)^3 + 110d \cosh(dx+c)) \sinh(dx+c)^4 + 42d \cosh(dx+c)^3 + (84d \cosh(dx+c)^6 + 245d \cosh(dx+c)^4 + 200d \cosh(dx+c)^2 + 28d) \sinh(dx+c)^3 + (36d \cosh(dx+c)^7 + 147d \cosh(dx+c)^5 + 220d \cosh(dx+c)^3 + 126d \cosh(dx+c)) \sinh(dx+c)^2 + 56d \cosh(dx+c) + (9d \cosh(dx+c)^8 + 49d \cosh(dx+c)^6 + 100d \cosh(dx+c)^4 + 84d \cosh(dx+c)^2 + 14d) \sinh(dx+c))$

giac [B] time = 0.16, size = 197, normalized size = 2.46

$$\frac{4(105a^2e^{(10dx+10c)} + 455a^2e^{(8dx+8c)} + 560abe^{(8dx+8c)} + 770a^2e^{(6dx+6c)} + 1400abe^{(6dx+6c)} + 840b^2e^{(6dx+6c)} + 630a^2e^{(4dx+4c)} + 1176ab^2e^{(4dx+4c)} + 504b^2e^{(4dx+4c)} + 245a^2e^{(2dx+2c)} + 392ab^2e^{(2dx+2c)} + 168b^2e^{(2dx+2c)} + 35a^2 + 56ab + 24b^2)/(d(e^{(2dx+2c)} + 1)^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(dx+c)^4*(a+b*sech(dx+c)^2)^2,x, algorithm="giac")`

[Out] $-4/105*(105a^2e^{(10dx+10c)} + 455a^2e^{(8dx+8c)} + 560ab^2e^{(8dx+8c)} + 770a^2e^{(6dx+6c)} + 1400ab^2e^{(6dx+6c)} + 840b^2e^{(6dx+6c)} + 630a^2e^{(4dx+4c)} + 1176ab^2e^{(4dx+4c)} + 504b^2e^{(4dx+4c)} + 245a^2e^{(2dx+2c)} + 392ab^2e^{(2dx+2c)} + 168b^2e^{(2dx+2c)} + 35a^2 + 56ab + 24b^2)/(d*(e^{(2dx+2c)} + 1)^7)$

maple [A] time = 0.44, size = 102, normalized size = 1.28

$$\frac{a^2 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 2ab \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c) + b^2 \left(\frac{16}{35} + \frac{\operatorname{sech}(dx+c)^6}{7} + \frac{6\operatorname{sech}(dx+c)^4}{35} \right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(dx+c)^4*(a+b*sech(dx+c)^2)^2,x)`

[Out] $1/d*(a^2*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)+2*a*b*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)+b^2*(16/35+1/7*\operatorname{sech}(d*x+c)^6+6/35*\operatorname{sech}(d*x+c)^4+8/35*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))$

maxima [B] time = 0.33, size = 671, normalized size = 8.39

$$\frac{32}{35} b^2 \left(\frac{7 e^{(-2dx-2c)}}{d(7 e^{(-2dx-2c)} + 21 e^{(-4dx-4c)} + 35 e^{(-6dx-6c)} + 35 e^{(-8dx-8c)} + 21 e^{(-10dx-10c)} + 7 e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $32/35*b^2*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 32/15*a*b*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 4/3*a^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))$

mupad [B] time = 1.42, size = 692, normalized size = 8.65

$$\frac{\frac{32a(a+2b)}{105d} + \frac{8a^2e^{2c+2dx}}{21d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{8a^2e^{2c+2dx}}{7d} + \frac{8a^2e^{10c+10dx}}{7d} + \frac{16e^{6c+6dx}(3a^2+8ab+8b^2)}{7d}}{7e^{2c+2dx} + 21e^{4c+4dx} + 35e^{6c+6dx} + 35e^{8c+8dx} + 21e^{10c+10dx} + 7e^{12c+12dx} + e^{14c+14dx} + 1} + \frac{32ae^{4c+4dx}(a+2b)}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2)^2/cosh(c + d*x)^4,x)`

[Out] $-((32*a*(a + 2*b))/(105*d) + (8*a^2*\exp(2*c + 2*d*x))/(21*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - ((8*a^2*\exp(2*c + 2*d*x))/(7*d) + (8*a^2*\exp(10*c + 10*d*x))/(7*d) + (16*\exp(6*c + 6*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(7*d) + (32*a*\exp(4*c + 4*d*x)*(a + 2*b))/(7*d) + (32*a*\exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1) - ((4*a^2)/(21*d) + (20*a^2*\exp(8*c + 8*d*x))/(21*d) + (8*\exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(7*d) + (32*a*\exp(2*c + 2*d*x)*(a + 2*b))/(21*d) + (64*a*\exp(6*c + 6*d*x)*(a + 2*b))/(21*d))/(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1) - ((32*a*(a + 2*b))/(105*d) + (16*a^2*\exp(6*c + 6*d*x))/(21*d) + (16*\exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(35*d) + (64*a*\exp(4*c + 4*d*x)*(a + 2*b))/(35*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - ((4*(8*a*b + 3*a^2 + 8*b^2))/(35*d) + (4*a^2*\exp(4*c + 4*d*x))/(7*d) + (32*a*\exp(2*c + 2*d*x)*(a + 2*b))/(35*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - (4*a^2)/(21*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**4, x)

3.65 $\int \cosh^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=84

$$\frac{a^3 \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{3}{8} ax (a^2 + 4ab + 8b^2) + \frac{3a^2(a + 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{b^3 \tanh(c + dx)}{d}$$

[Out] $3/8*a*(a^2+4*a*b+8*b^2)*x+3/8*a^2*(a+4*b)*\cosh(d*x+c)*\sinh(d*x+c)/d+1/4*a^3*\cosh(d*x+c)^3*\sinh(d*x+c)/d+b^3*\tanh(d*x+c)/d$

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4146, 390, 1157, 385, 206}

$$\frac{3}{8} ax (a^2 + 4ab + 8b^2) + \frac{3a^2(a + 4b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{a^3 \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{b^3 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(3*a*(a^2 + 4*a*b + 8*b^2)*x)/8 + (3*a^2*(a + 4*b)*\cosh[c + d*x]*\sinh[c + d*x])/(8*d) + (a^3*\cosh[c + d*x]^3*\sinh[c + d*x])/(4*d) + (b^3*\tanh[c + d*x])/d$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S

```

ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]

```

Rubi steps

$$\begin{aligned}
\int \cosh^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(b^3 + \frac{a(a^2+3ab+3b^2)-3ab(a+2b)x^2+3ab^2x^4}{(1-x^2)^3}\right) dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^3 \tanh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a(a^2+3ab+3b^2)-3ab(a+2b)x^2+3ab^2x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{a^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{b^3 \tanh(c + dx)}{d} - \frac{\operatorname{Subst}\left(\int \frac{-3a(a+b-bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{3a^2(a + 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} \\
&= \frac{3}{8} a (a^2 + 4ab + 8b^2) x + \frac{3a^2(a + 4b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{a^3 \cosh^3(c + dx) \sinh(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 70, normalized size = 0.83

$$\frac{a^3 \sinh(4(c + dx)) + 12a(a^2 + 4ab + 8b^2)(c + dx) + 8a^2(a + 3b) \sinh(2(c + dx)) + 32b^3 \tanh(c + dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (12*a*(a^2 + 4*a*b + 8*b^2)*(c + d*x) + 8*a^2*(a + 3*b)*Sinh[2*(c + d*x)] + a^3*Sinh[4*(c + d*x)] + 32*b^3*Tanh[c + d*x])/(32*d)
```

fricas [A] time = 0.40, size = 153, normalized size = 1.82

$$\frac{a^3 \sinh(dx + c)^5 + (10a^3 \cosh(dx + c)^2 + 9a^3 + 24a^2b) \sinh(dx + c)^3 - 8(8b^3 - 3(a^3 + 4a^2b + 8ab^2)dx) \cosh(dx + c)}{64d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(a^3*sinh(d*x + c)^5 + (10*a^3*cosh(d*x + c)^2 + 9*a^3 + 24*a^2*b)*sinh(d*x + c)^3 - 8*(8*b^3 - 3*(a^3 + 4*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) + (5*a^3*cosh(d*x + c)^4 + 8*a^3 + 24*a^2*b + 64*b^3 + 9*(3*a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 0.21, size = 177, normalized size = 2.11

$$\frac{a^3 e^{4dx+4c} + 8a^3 e^{2dx+2c} + 24a^2 b e^{2dx+2c} + 24(a^3 + 4a^2b + 8ab^2)(dx + c) - \frac{128b^3}{e^{2dx+2c} + 1} - (18a^3 e^{4dx+4c} + 72a^3 e^{2dx+2c})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{64}*(a^3*e^{(4*d*x + 4*c)} + 8*a^3*e^{(2*d*x + 2*c)} + 24*a^2*b*e^{(2*d*x + 2*c)} + 24*(a^3 + 4*a^2*b + 8*a*b^2)*(d*x + c) - 128*b^3/(e^{(2*d*x + 2*c)} + 1) - (18*a^3*e^{(4*d*x + 4*c)} + 72*a^2*b*e^{(4*d*x + 4*c)} + 144*a*b^2*e^{(4*d*x + 4*c)} + 8*a^3*e^{(2*d*x + 2*c)} + 24*a^2*b*e^{(2*d*x + 2*c)} + a^3)*e^{(-4*d*x - 4*c)})/d$

maple [A] time = 0.48, size = 93, normalized size = 1.11

$$\frac{a^3 \left(\left(\frac{\cosh^3(dx+c)}{4} + \frac{3\cosh(dx+c)}{8} \right) \sinh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 3a^2b \left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 3ab^2(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)

[Out] $\frac{1}{d}*(a^3*((1/4*\cosh(d*x+c)^3+3/8*\cosh(d*x+c))*\sinh(d*x+c)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*\cosh(d*x+c)*\sinh(d*x+c)+1/2*d*x+1/2*c)+3*a*b^2*(d*x+c)+b^3*\tanh(d*x+c))$

maxima [A] time = 0.32, size = 130, normalized size = 1.55

$$\frac{1}{64}a^3\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{3}{8}a^2b\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) + 3ab^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{64}a^3*(24*x + e^{(4*d*x + 4*c)}/d + 8*e^{(2*d*x + 2*c)}/d - 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + \frac{3}{8}a^2*b*(4*x + e^{(2*d*x + 2*c)}/d - e^{(-2*d*x - 2*c)}/d) + 3*a*b^2*x + 2*b^3/(d*(e^{(-2*d*x - 2*c)} + 1))$

mupad [B] time = 1.53, size = 117, normalized size = 1.39

$$\frac{3ax(a^2 + 4ab + 8b^2)}{8} - \frac{2b^3}{d(e^{2c+2dx} + 1)} - \frac{a^3e^{-4c-4dx}}{64d} + \frac{a^3e^{4c+4dx}}{64d} - \frac{a^2e^{-2c-2dx}(a+3b)}{8d} + \frac{a^2e^{2c+2dx}(a+3b)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)

[Out] $\frac{(3*a*x*(4*a*b + a^2 + 8*b^2))/8 - (2*b^3)/(d*(\exp(2*c + 2*d*x) + 1)) - (a^3*\exp(-4*c - 4*d*x))/(64*d) + (a^3*\exp(4*c + 4*d*x))/(64*d) - (a^2*\exp(-2*c - 2*d*x)*(a + 3*b))/(8*d) + (a^2*\exp(2*c + 2*d*x)*(a + 3*b))/(8*d)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.66 $\int \cosh^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=81

$$\frac{a^3 \sinh^3(c + dx)}{3d} + \frac{a^2(a + 3b) \sinh(c + dx)}{d} + \frac{b^2(6a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $\frac{1}{2} b^2 (6a + b) \arctan(\sinh(dx + c)) / d + a^2 (a + 3b) \sinh(dx + c) / d + \frac{1}{3} a^3 \sinh(dx + c)^3 / d + \frac{1}{2} b^3 \operatorname{sech}(dx + c) \tanh(dx + c) / d$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4147, 390, 385, 203}

$$\frac{a^2(a + 3b) \sinh(c + dx)}{d} + \frac{a^3 \sinh^3(c + dx)}{3d} + \frac{b^2(6a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(b^2(6a + b) \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]) / (2*d) + (a^2(a + 3*b) \operatorname{Sinh}[c + d*x]) / d + (a^3 \operatorname{Sinh}[c + d*x]^3) / (3*d) + (b^3 \operatorname{Sech}[c + d*x] \operatorname{Tanh}[c + d*x]) / (2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \cosh^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bax^2)^3}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(a^2(a+3b) + a^3x^2 + \frac{b^2(3a+b)+3ab^2x^2}{(1+x^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{a^2(a+3b)\sinh(c+dx)}{d} + \frac{a^3\sinh^3(c+dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(3a+b)+3ab^2x^2}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{a^2(a+3b)\sinh(c+dx)}{d} + \frac{a^3\sinh^3(c+dx)}{3d} + \frac{b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \\
&= \frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a^2(a+3b)\sinh(c+dx)}{d} + \frac{a^3\sinh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.88, size = 483, normalized size = 5.96

$$\operatorname{coth}^3(c+dx)\operatorname{csch}^2(c+dx)(a\cosh(c+dx)+b\operatorname{sech}(c+dx))^3 \left(-256\sinh^8(c+dx)(a\sinh^2(c+dx)+a+b)^3 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (Coth[c + d*x]^3*Csch[c + d*x]^2*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 - 47*Sinh[c + d*x]^6) + 3*a^2*b*Cosh[c + d*x]^4*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*(2401 + 4276*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 148*Sinh[c + d*x]^6 + Sinh[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*(36015 + 16120*Sinh[c + d*x]^2 + 1473*Sinh[c + d*x]^4) + 3*a*b^2*(36015 + 52135*Sinh[c + d*x]^2 + 17593*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*(36015 + 88150*Sinh[c + d*x]^2 + 69728*Sinh[c + d*x]^4 + 19786*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(3780*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)

fricas [B] time = 0.42, size = 1409, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 + (11*a^3 + 36*a^2*b)*cosh(d*x + c)^8 + (45*a^3*cosh(d*x + c)^2 + 11*a^3 + 36*a^2*b)*sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 + (11*a^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^6 + 2*(105*a^3*cosh(d*x + c)^4 + 5*a^3 + 18*a^2*b + 12*b^3 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^3*cosh

$(dx + c)^5 + 14(11a^3 + 36a^2b) \cosh(dx + c)^3 + 3(5a^3 + 18a^2b + 12b^3) \cosh(dx + c) \sinh(dx + c)^5 - 2(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^4 + 2(105a^3 \cosh(dx + c)^6 + 35(11a^3 + 36a^2b) \cosh(dx + c)^4 - 5a^3 - 18a^2b - 12b^3 + 15(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(15a^3 \cosh(dx + c)^7 + 7(11a^3 + 36a^2b) \cosh(dx + c)^5 + 5(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^3 - (5a^3 + 18a^2b + 12b^3) \cosh(dx + c) \sinh(dx + c)^3 - a^3 - (11a^3 + 36a^2b) \cosh(dx + c)^2 + (45a^3 \cosh(dx + c)^8 + 28(11a^3 + 36a^2b) \cosh(dx + c)^6 + 30(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^4 - 11a^3 - 36a^2b - 12(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 24((6ab^2 + b^3) \cosh(dx + c)^7 + 7(6ab^2 + b^3) \cosh(dx + c) \sinh(dx + c)^6 + (6ab^2 + b^3) \sinh(dx + c)^7 + 2(6ab^2 + b^3) \cosh(dx + c)^5 + (12ab^2 + 2b^3 + 21(6ab^2 + b^3) \cosh(dx + c)^2) \sinh(dx + c)^5 + 5(7(6ab^2 + b^3) \cosh(dx + c)^3 + 2(6ab^2 + b^3) \cosh(dx + c)) \sinh(dx + c)^4 + (6ab^2 + b^3) \cosh(dx + c)^3 + (35(6ab^2 + b^3) \cosh(dx + c)^4 + 6ab^2 + b^3 + 20(6ab^2 + b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + (21(6ab^2 + b^3) \cosh(dx + c)^5 + 20(6ab^2 + b^3) \cosh(dx + c)^3 + 3(6ab^2 + b^3) \cosh(dx + c)) \sinh(dx + c)^2 + (7(6ab^2 + b^3) \cosh(dx + c)^6 + 10(6ab^2 + b^3) \cosh(dx + c)^4 + 3(6ab^2 + b^3) \cosh(dx + c)^2) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) + 2(5a^3 \cosh(dx + c)^9 + 4(11a^3 + 36a^2b) \cosh(dx + c)^7 + 6(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^5 - 4(5a^3 + 18a^2b + 12b^3) \cosh(dx + c)^3 - (11a^3 + 36a^2b) \cosh(dx + c) \sinh(dx + c)) / (d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 2d \cosh(dx + c)^5 + (21d \cosh(dx + c)^2 + 2d) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 2d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 20d \cosh(dx + c)^2 + d) \sinh(dx + c)^3 + (21d \cosh(dx + c)^5 + 20d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + (7d \cosh(dx + c)^6 + 10d \cosh(dx + c)^4 + 3d \cosh(dx + c)^2) \sinh(dx + c))$

giac [B] time = 0.21, size = 163, normalized size = 2.01

$$\frac{a^3(e^{(dx+c)} - e^{(-dx-c)})^3 + 12a^3(e^{(dx+c)} - e^{(-dx-c)}) + 36a^2b(e^{(dx+c)} - e^{(-dx-c)}) + \frac{24b^3(e^{(dx+c)} - e^{(-dx-c)})}{(e^{(dx+c)} - e^{(-dx-c)})^2 + 4} + 6(\pi + 2 \arctan(\frac{e^{(dx+c)} - e^{(-dx-c)}}{2}))}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^3*(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{24}(a^3(e^{(dx+c)} - e^{(-dx-c)})^3 + 12a^3(e^{(dx+c)} - e^{(-dx-c)}) + 36a^2b(e^{(dx+c)} - e^{(-dx-c)}) + 24b^3(e^{(dx+c)} - e^{(-dx-c)})) / ((e^{(dx+c)} - e^{(-dx-c)})^2 + 4) + 6(\pi + 2 \arctan(1/2(e^{(2dx+c)} - 1)e^{(-dx-c)})) * (6ab^2 + b^3)) / d$

maple [A] time = 0.45, size = 103, normalized size = 1.27

$$\frac{2a^3 \sinh(dx+c)}{3d} + \frac{a^3 \sinh(dx+c) (\cosh^2(dx+c))}{3d} + \frac{3a^2b \sinh(dx+c)}{d} + \frac{6ab^2 \arctan(e^{dx+c})}{d} + \frac{b^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(dx+c)^3*(a+b*sech(dx+c)^2)^3,x)

[Out] $\frac{2}{3}a^3 \sinh(dx+c) / d + \frac{1}{3}d a^3 \sinh(dx+c) \cosh(dx+c)^2 + \frac{3}{d} a^2 b \sinh(dx+c) + \frac{6}{d} a b^2 \arctan(\exp(dx+c)) + \frac{1}{2} d b^3 \operatorname{sech}(dx+c) \tanh(dx+c) + \frac{1}{d} b^3 \arctan(\exp(dx+c))$

maxima [B] time = 0.42, size = 179, normalized size = 2.21

$$-b^3 \left(\frac{\arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) + \frac{1}{24} a^3 \left(\frac{e^{(3dx+3c)}}{d} + \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} - \frac{e^{(-3dx-3c)}}{d} \right) + \frac{3}{2} b^3 \left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-b^3 \frac{\arctan(e^{-d*x - c})}{d} - \frac{e^{-d*x - c} - e^{-3*d*x - 3*c}}{d(2e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + 1)} + \frac{1}{24} a^3 \frac{e^{3*d*x + 3*c}}{d} + 9e^{-d*x + c} \frac{e^{-d*x - c}}{d} - 9e^{-d*x - c} \frac{e^{-3*d*x - 3*c}}{d} + \frac{3}{2} a^2 b \frac{e^{d*x + c}}{d} - e^{-d*x - c} \frac{e^{-d*x - c}}{d} - 6a^2 b^2 \frac{\arctan(e^{-d*x - c})}{d}$

mupad [B] time = 0.22, size = 218, normalized size = 2.69

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (b^3 \sqrt{d^2} + 6ab^2 \sqrt{d^2})}{d \sqrt{36a^2 b^4 + 12ab^5 + b^6}}\right) \sqrt{36a^2 b^4 + 12ab^5 + b^6}}{\sqrt{d^2}} - \frac{a^3 e^{-3c-3dx}}{24d} + \frac{a^3 e^{3c+3dx}}{24d} - \frac{3a^2 e^{-c-dx} (a+4b)}{8d} + \frac{3a^2 e^{c+dx}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)

[Out] $(\operatorname{atan}((\exp(d*x) \exp(c) (b^3 (d^2)^{1/2} + 6a^2 b^2 (d^2)^{1/2}))) / (d(12a^2 b^5 + b^6 + 36a^2 b^4)^{1/2})) * (12a^2 b^5 + b^6 + 36a^2 b^4)^{1/2} / (d^2)^{1/2} - (a^3 \exp(-3c - 3d*x)) / (24*d) + (a^3 \exp(3c + 3d*x)) / (24*d) - (3a^2 \exp(-c - d*x) (a + 4*b)) / (8*d) + (3a^2 \exp(c + d*x) (a + 4*b)) / (8*d) + (b^3 \exp(c + d*x)) / (d(\exp(2*c + 2*d*x) + 1)) - (2*b^3 \exp(c + d*x)) / (d(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.67 $\int \cosh^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=72

$$\frac{a^3 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2} a^2 x (a + 6b) + \frac{b^2 (3a + b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^3(c + dx)}{3d}$$

[Out] $1/2*a^2*(a+6*b)*x+1/2*a^3*\cosh(d*x+c)*\sinh(d*x+c)/d+b^2*(3*a+b)*\tanh(d*x+c)/d-1/3*b^3*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 390, 385, 206}

$$\frac{1}{2} a^2 x (a + 6b) + \frac{a^3 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{b^2 (3a + b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(a^2*(a + 6*b)*x)/2 + (a^3*\cosh[c + d*x]*\sinh[c + d*x])/(2*d) + (b^2*(3*a + b)*\tanh[c + d*x])/d - (b^3*\tanh[c + d*x]^3)/(3*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \cosh^2(c+dx) (a+b\operatorname{sech}^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^3}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(b^2(3a+b) - b^3x^2 + \frac{a^2(a+3b)-3a^2bx^2}{(1-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{a^2(a+3b)-3a^2bx^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d} \\
&= \frac{1}{2}a^2(a+6b)x + \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.51, size = 64, normalized size = 0.89

$$\frac{3a^3\sinh(2(c+dx)) + 6a^2(a+6b)(c+dx) + 4b^2\tanh(c+dx)(9a+b\operatorname{sech}^2(c+dx) + 2b)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (6*a^2*(a + 6*b)*(c + d*x) + 3*a^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 2*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)

fricas [B] time = 0.43, size = 270, normalized size = 3.75

$$\frac{3a^3\sinh(dx+c)^5 - 4(18ab^2 + 4b^3 - 3(a^3 + 6a^2b)dx)\cosh(dx+c)^3 - 12(18ab^2 + 4b^3 - 3(a^3 + 6a^2b)dx)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*(3*a^3*sinh(d*x + c)^5 - 4*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*a^3*cosh(d*x + c)^2 + 9*a^3 + 72*a*b^2 + 16*b^3)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c) + 3*(5*a^3*cosh(d*x + c)^4 + 2*a^3 + 24*a*b^2 + 16*b^3 + (9*a^3 + 72*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.20, size = 152, normalized size = 2.11

$$\frac{3a^3e^{(2dx+2c)} + 12(a^3 + 6a^2b)(dx+c) - 3(2a^3e^{(2dx+2c)} + 12a^2be^{(2dx+2c)} + a^3)e^{(-2dx-2c)} - \frac{16(9ab^2e^{(4dx+4c)} + 18ab^2e^{(2dx+2c)} + 16b^3e^{(2dx+2c)} + a^3e^{(-2dx-2c)})}{24d}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/24*(3*a^3*e^(2*d*x + 2*c) + 12*(a^3 + 6*a^2*b)*(d*x + c) - 3*(2*a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-2*d*x - 2*c) - 16*(9*a*b^2

$$\frac{e^{4dx+4c} + 18ab^2e^{2dx+2c} + 6b^3e^{2dx+2c} + 9a^2b^2 + 2b^3}{(e^{2dx+2c} + 1)^3} \cdot \frac{1}{d}$$

maple [A] time = 0.46, size = 77, normalized size = 1.07

$$\frac{a^3 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 3a^2b(dx+c) + 3ab^2 \tanh(dx+c) + b^3 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(d*x+c)+3*a*b^2*tanh(d*x+c)+b^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.32, size = 160, normalized size = 2.22

$$\frac{1}{8} a^3 \left(4x + \frac{e^{2dx+2c}}{d} - \frac{e^{-2dx-2c}}{d} \right) + 3a^2bx + \frac{4}{3} b^3 \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/8*a^3*(4*x + e^{(2*d*x + 2*c)}/d - e^{(-2*d*x - 2*c)}/d) + 3*a^2*b*x + 4/3*b^3*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 6*a*b^2/(d*(e^{(-2*d*x - 2*c)} + 1))

mupad [B] time = 0.17, size = 221, normalized size = 3.07

$$\frac{a^2 x (a + 6b)}{2} - \frac{\frac{2ab^2}{d} + \frac{4e^{2c+2dx}(2b^3+3ab^2)}{3d} + \frac{2ab^2e^{4c+4dx}}{d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{2(2b^3+3ab^2)}{3d} + \frac{2ab^2e^{2c+2dx}}{d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{a^3 e^{-2c-2dx}}{8d} + \frac{a^3 e^{2c+2dx}}{8d} - \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (a^2*x*(a + 6*b))/2 - ((2*a*b^2)/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 + 2*b^3))/(3*d) + (2*a*b^2*exp(2*c + 2*d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (a^3*exp(-2*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d) - (2*a*b^2)/(d*(exp(2*c + 2*d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.68 $\int \cosh(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=93

$$\frac{a^3 \sinh(c + dx)}{d} + \frac{3b(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3b^2(4a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b^3 \tanh(c + dx)}{4d}$$

[Out] $3/8*b*(8*a^2+4*a*b+b^2)*\arctan(\sinh(d*x+c))/d+a^3*\sinh(d*x+c)/d+3/8*b^2*(4*a+b)*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d+1/4*b^3*\operatorname{sech}(d*x+c)^3*\tanh(d*x+c)/d$

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4147, 390, 1157, 385, 203}

$$\frac{3b(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{3b^2(4a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{b^3 \tanh(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(3*b*(8*a^2 + 4*a*b + b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + (a^3*\operatorname{Sinh}[c + d*x])/d + (3*b^2*(4*a + b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + (b^3*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,

Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a^3 + \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(1+x^2)^3}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a^3 \sinh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a^3 \sinh(c + dx)}{d} + \frac{b^3 \operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{-3b(2a+b)}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a^3 \sinh(c + dx)}{d} + \frac{3b^2(4a + b) \operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{b^3 \operatorname{sech}^3(c + dx)}{4d} \\ &= \frac{3b(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{3b^2(4a + b) \operatorname{sech}(c + dx) \tanh(c + dx)}{8d} \end{aligned}$$

Mathematica [C] time = 7.88, size = 575, normalized size = 6.18

$$\cosh(c + dx) \operatorname{coth}^5(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 \left(256 \sinh^8(c + dx) (a \sinh^2(c + dx) + a + b)^3 {}_6F_5\left(\frac{3}{2}, 2, 2, 2, 2, 2; 1, 1, 1, 1, 11/2; -\sinh^2(c + dx)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out]
$$\frac{-1/7560 \cdot (\cosh[c + d*x] \operatorname{Coth}[c + d*x]^5 (a + b \operatorname{Sech}[c + d*x]^2)^3 (256 \operatorname{HypergeometricPFQ}[\{3/2, 2, 2, 2, 2\}, \{1, 1, 1, 1, 11/2\}, -\sinh^2[c + d*x]] \operatorname{Sinh}[c + d*x]^8 (a + b + a \operatorname{Sinh}[c + d*x]^2)^3 + 384 \operatorname{HypergeometricPFQ}[\{3/2, 2, 2, 2, 2\}, \{1, 1, 1, 11/2\}, -\sinh^2[c + d*x]] \operatorname{Sinh}[c + d*x]^8 (a + b + a \operatorname{Sinh}[c + d*x]^2)^2 (7b + a(7 + 5 \operatorname{Sinh}[c + d*x]^2)) + (315 \operatorname{ArcTanh}[\operatorname{Sqrt}[-\sinh^2[c + d*x]]]) (b^3 (16807 + 15000 \operatorname{Sinh}[c + d*x]^2 + 2187 \operatorname{Sinh}[c + d*x]^4 - 62 \operatorname{Sinh}[c + d*x]^6) + a^3 \operatorname{Cosh}[c + d*x]^4 (16807 + 24604 \operatorname{Sinh}[c + d*x]^2 + 11562 \operatorname{Sinh}[c + d*x]^4 + 1468 \operatorname{Sinh}[c + d*x]^6 + 7 \operatorname{Sinh}[c + d*x]^8) + 3a^2 b (16807 + 29406 \operatorname{Sinh}[c + d*x]^2 + 15312 \operatorname{Sinh}[c + d*x]^4 + 1858 \operatorname{Sinh}[c + d*x]^6 + 9 \operatorname{Sinh}[c + d*x]^8) + 3a^2 b (16807 + 43812 \operatorname{Sinh}[c + d*x]^2 + 40442 \operatorname{Sinh}[c + d*x]^4 + 14956 \operatorname{Sinh}[c + d*x]^6 + 1719 \operatorname{Sinh}[c + d*x]^8 + 8 \operatorname{Sinh}[c + d*x]^10))) / \operatorname{Sqrt}[-\sinh^2[c + d*x]] - 21 (b^3 (252105 + 140965 \operatorname{Sinh}[c + d*x]^2 + 8226 \operatorname{Sinh}[c + d*x]^4) + 3a^2 b (252105 + 357055 \operatorname{Sinh}[c + d*x]^2 + 133071 \operatorname{Sinh}[c + d*x]^4 + 6393 \operatorname{Sinh}[c + d*x]^6) + 3a^2 b (252105 + 573145 \operatorname{Sinh}[c + d*x]^2 + 437991 \operatorname{Sinh}[c + d*x]^4 + 120431 \operatorname{Sinh}[c + d*x]^6 + 5640 \operatorname{Sinh}[c + d*x]^8) + a^3 (252105 + 789235 \operatorname{Sinh}[c + d*x]^2 + 922986 \operatorname{Sinh}[c + d*x]^4 + 491574 \operatorname{Sinh}[c + d*x]^6 + 107725 \operatorname{Sinh}[c + d*x]^8 + 4887 \operatorname{Sinh}[c + d*x]^10))) / (d (a + 2b + a \operatorname{Cosh}[2c + 2d*x])^3)$$

fricas [B] time = 0.47, size = 1992, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (2a^3 \cosh(dx+c)^{10} + 20a^3 \cosh(dx+c) \sinh(dx+c)^9 + 2a^3 \sinh(dx+c)^{10} + 3(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^8 + 3(30a^3 \cosh(dx+c)^2 + 2a^3 + 4ab^2 + b^3) \sinh(dx+c)^8 + 24(10a^3 \cosh(dx+c)^3 + (2a^3 + 4ab^2 + b^3) \cosh(dx+c)) \sinh(dx+c)^7 + (4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^6 + (420a^3 \cosh(dx+c)^4 + 4a^3 + 12ab^2 + 11b^3 + 84(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^2) \sinh(dx+c)^6 + 6(84a^3 \cosh(dx+c)^5 + 28(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^3 + (4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)) \sinh(dx+c)^5 - (4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^4 + (420a^3 \cosh(dx+c)^6 + 210(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^4 - 4a^3 - 12ab^2 - 11b^3 + 15(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^2) \sinh(dx+c)^4 + 4(60a^3 \cosh(dx+c)^7 + 42(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^5 + 5(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^3 - (4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)) \sinh(dx+c)^3 - 2a^3 - 3(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^2 + 3(30a^3 \cosh(dx+c)^8 + 28(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^6 + 5(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^4 - 2a^3 - 4ab^2 - b^3 - 2(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^2) \sinh(dx+c)^2 + 3((8a^2b + 4ab^2 + b^3) \cosh(dx+c)^9 + 9(8a^2b + 4ab^2 + b^3) \cosh(dx+c) \sinh(dx+c)^8 + (8a^2b + 4ab^2 + b^3) \sinh(dx+c)^9 + 4(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^7 + 4(8a^2b + 4ab^2 + b^3 + 9(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^2) \sinh(dx+c)^7 + 28(3(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^3 + (8a^2b + 4ab^2 + b^3) \cosh(dx+c)) \sinh(dx+c)^6 + 6(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^5 + 6(21(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^4 + 8a^2b + 4ab^2 + b^3 + 14(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^2) \sinh(dx+c)^5 + 2(63(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^5 + 70(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^3 + 15(8a^2b + 4ab^2 + b^3) \cosh(dx+c)) \sinh(dx+c)^4 + 4(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^3 + 4(21(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^6 + 35(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^4 + 8a^2b + 4ab^2 + b^3 + 15(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^2) \sinh(dx+c)^3 + 12(3(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^7 + 7(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^5 + 5(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^3 + (8a^2b + 4ab^2 + b^3) \cosh(dx+c)) \sinh(dx+c)^2 + (8a^2b + 4ab^2 + b^3) \cosh(dx+c) + (9(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^8 + 28(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^6 + 30(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^4 + 8a^2b + 4ab^2 + b^3 + 12(8a^2b + 4ab^2 + b^3) \cosh(dx+c)^2) \sinh(dx+c)) \arctan(\cosh(dx+c) + \sinh(dx+c)) + 2(10a^3 \cosh(dx+c)^9 + 12(2a^3 + 4ab^2 + b^3) \cosh(dx+c)^7 + 3(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^5 - 2(4a^3 + 12ab^2 + 11b^3) \cosh(dx+c)^3 - 3(2a^3 + 4ab^2 + b^3) \cosh(dx+c)) \sinh(dx+c) / (d \cosh(dx+c)^9 + 9d \cosh(dx+c) \sinh(dx+c)^8 + d \sinh(dx+c)^9 + 4d \cosh(dx+c)^7 + 4(9d \cosh(dx+c)^2 + d) \sinh(dx+c)^7 + 28(3d \cosh(dx+c)^3 + d \cosh(dx+c)) \sinh(dx+c)^6 + 6d \cosh(dx+c)^5 + 6(21d \cosh(dx+c)^4 + 14d \cosh(dx+c)^2 + d) \sinh(dx+c)^5 + 2(63d \cosh(dx+c)^5 + 70d \cosh(dx+c)^3 + 15d \cosh(dx+c)) \sinh(dx+c)^4 + 4d \cosh(dx+c)^3 + 4(21d \cosh(dx+c)^6 + 35d \cosh(dx+c)^4 + 15d \cosh(dx+c)^2 + d) \sinh(dx+c)^3 + 12(3d \cosh(dx+c)^7 + 7d \cosh(dx+c)^5 + 5d \cosh(dx+c)^3 + d \cosh(dx+c)) \sinh(dx+c)^2 + d \cosh(dx+c) + (9d \cosh(dx+c)^8 + 28d \cosh(dx+c)^6 + 30d \cosh(dx+c)^4 + 12d \cosh(dx+c)^2 + d) \sinh(dx+c)$

giac [B] time = 0.19, size = 199, normalized size = 2.14

$$8a^3(e^{dx+c} - e^{-dx-c}) + 3\left(\pi + 2 \arctan\left(\frac{1}{2}(e^{2dx+2c} - 1)e^{-dx-c}\right)\right)(8a^2b + 4ab^2 + b^3) + \frac{4(12ab^2(e^{dx+c} - e^{-dx-c}))^3 + 3}{16d}$$

16d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2*b + 4*a*b^2 + b^3) + 4*(12*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 20*b^3*(e^(d*x + c) - e^(-d*x - c))))/(e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d

maple [A] time = 0.51, size = 125, normalized size = 1.34

$$\frac{a^3 \sinh(dx+c)}{d} + \frac{6a^2b \arctan(e^{dx+c})}{d} + \frac{3ab^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{3ab^2 \arctan(e^{dx+c})}{d} + \frac{b^3 \tanh(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)

[Out] a^3*sinh(d*x+c)/d+6/d*a^2*b*arctan(exp(d*x+c))+3/2/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+3/d*a*b^2*arctan(exp(d*x+c))+1/4/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^3*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^3*arctan(exp(d*x+c))

maxima [B] time = 0.41, size = 221, normalized size = 2.38

$$-\frac{1}{4}b^3\left(\frac{3 \arctan(e^{-dx-c})}{d} - \frac{3e^{-dx-c} + 11e^{-3dx-3c} - 11e^{-5dx-5c} - 3e^{-7dx-7c}}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)}\right) - 3ab^2\left(\frac{\arctan(e^{-dx-c})}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/4*b^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c)))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a*b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 6*a^2*b*arctan(e^(-d*x - c))/d + a^3*sinh(d*x + c)/d

mupad [B] time = 0.19, size = 344, normalized size = 3.70

$$\frac{a^3 e^{c+dx}}{2d} - \frac{a^3 e^{-c-dx}}{2d} + \frac{3 \operatorname{atan}\left(\frac{e^{dx} e^c (b^3 \sqrt{d^2} + 4ab^2 \sqrt{d^2} + 8a^2b \sqrt{d^2})}{d \sqrt{64a^4b^2 + 64a^3b^3 + 32a^2b^4 + 8ab^5 + b^6}}\right)}{4\sqrt{d^2}} \frac{\sqrt{64a^4b^2 + 64a^3b^3 + 32a^2b^4 + 8ab^5 + b^6}}{d(3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (a^3*exp(c + d*x))/(2*d) - (a^3*exp(-c - d*x))/(2*d) + (3*atan((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 4*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)^(1/2)))/(d*(8*a*b^5 + b^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2))))*(8*a*b^5 + b^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*b^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +

1)) - (exp(c + d*x)*(12*a*b^2 - b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (4*b^3*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*exp(c + d*x)*(4*a*b^2 + b^3))/(4*d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*cosh(c + d*x), x)

3.69 $\int \operatorname{sech}(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=147

$$\frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{b(44a^2 + 44ab + 15b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{48d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{6d}$$

[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*b*(44*a^2+44*a*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*b*(2*a+b)*sech(d*x+c)^3*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d+1/6*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)^2*tanh(d*x+c)/d

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4147, 413, 526, 385, 203}

$$\frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{b(44a^2 + 44ab + 15b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{48d} + \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]]/(16*d) + (b*(44*a^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*b*(2*a + b)*Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 526

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*(e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{b \operatorname{sech}^5(c + dx) (a + b + a \sinh^2(c + dx))^2 \tanh(c + dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{5b(2a + b) \operatorname{sech}^3(c + dx) (a + b + a \sinh^2(c + dx)) \tanh(c + dx)}{24d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{b(44a^2 + 44ab + 15b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d} + \frac{5b(2a + b) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d}$$

$$= \frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{b(44a^2 + 44ab + 15b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d}$$

Mathematica [C] time = 9.84, size = 1430, normalized size = 9.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3, x]
```

```
[Out] (Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(117228825*(a + b)^3*Sinh[c + d*x]^2 + 274542345*a*(a + b)^2*Sinh[c + d*x]^4 + 70189350*(a + b)^3*Sinh[c + d*x]^4 + 215549775*a^2*(a + b)*Sinh[c + d*x]^6 + 168951510*a*(a + b)^2*Sinh[c + d*x]^6 + 4093425*(a + b)^3*Sinh[c + d*x]^6 + 58009455*a^3*Sinh[c + d*x]^8 + 135323370*a^2*(a + b)*Sinh[c + d*x]^8 + 9514449*a*(a + b)^2*Sinh[c + d*x]^8 + 36772890*a^3*Sinh[c + d*x]^10 + 7808535*a^2*(a + b)*Sinh[c + d*x]^10 - 75520*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 13824*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 2160711*a^3*Sinh[c + d*x]^12 - 189696*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 38400*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 3072*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 158976*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 35328*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 3072*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 44800*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 10752*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16)
```

$$\begin{aligned}
& d*x]^{16} - 1024*a^3*HypergeometricPFQ[\{3/2, 2, 2, 2, 2, 2\}, \{1, 1, 1, 1, \\
& 1, 11/2\}, -\text{Sinh}[c + d*x]^2]*\text{Sinh}[c + d*x]^{16} + 117228825*(a + b)^3*\text{ArcTanh} \\
& [\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 215549775*a^2*(a + b)*\text{Arc} \\
& \text{Tanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^4*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 26046 \\
& 5625*a*(a + b)^2*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^4*\text{Sqrt}[-\text{Sinh} \\
& [c + d*x]^2] + 17069535*(a + b)^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + \\
& d*x]^4*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 58009455*a^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2] \\
&]*\text{Sinh}[c + d*x]^6*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 207173295*a^2*(a + b)*\text{ArcTanh}[\text{Sqr} \\
& \text{rt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^6*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 41427855*a*(\\
& a + b)^2*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^6*\text{Sqrt}[-\text{Sinh}[c + d*x] \\
&]^2] + 56109375*a^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^8*\text{Sqrt}[-\text{S} \\
& \text{inh}[c + d*x]^2] + 33756345*a^2*(a + b)*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh} \\
& [c + d*x]^8*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 210735*a*(a + b)^2*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[\\
& c + d*x]^2]]*\text{Sinh}[c + d*x]^8*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 9261945*a^3*\text{ArcTanh}[S \\
& \text{qrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^{10}*\text{Sqrt}[-\text{Sinh}[c + d*x]^2] + 174825*a^2 \\
& *(a + b)*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^{10}*\text{Sqrt}[-\text{Sinh}[c + d* \\
& x]^2] + 48825*a^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*\text{Sinh}[c + d*x]^{12}*\text{Sqrt}[-\text{Si} \\
& \text{nh}[c + d*x]^2] - 274542345*a*(a + b)^2*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]*(-\text{Si} \\
& \text{nh}[c + d*x]^2)^{(3/2)} - 109265625*(a + b)^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]]* \\
& (-\text{Sinh}[c + d*x]^2)^{(3/2)} + 142065*(a + b)^3*\text{ArcTanh}[\text{Sqrt}[-\text{Sinh}[c + d*x]^2]] \\
& *\text{Sinh}[c + d*x]^4*(-\text{Sinh}[c + d*x]^2)^{(3/2)))/(90720*d*(a + 2*b + a*\text{Cosh}[2*c \\
& + 2*d*x])^3)
\end{aligned}$$

fricas [B] time = 0.47, size = 3465, normalized size = 23.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $1/24*(3*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^{11} + 33*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{10} + 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*\sinh(d*x + c)^{11} + (216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^9 + (216*a^2*b + 306*a*b^2 + 85*b^3 + 165*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^8 + 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 8*a^2*b + 14*a*b^2 + 11*b^3 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^3 + 3*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 - 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^4 - 8*a^2*b - 14*a*b^2 - 11*b^3 + 21*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^5 + 35*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^3 - 5*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - (216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^3 + (495*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 84*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^6 + 630*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^4 - 216*a^2*b - 306*a*b^2 - 85*b^3 - 180*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + 12*(216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c)^7 + 126*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^5 - 60*(8*a^2*b + 14*a*b^2 + 11*b^3)*\cosh(d*x + c)^3 - (216*a^2*b + 306*a*b^2 + 85*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 3*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^12 + 12*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^11 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\sinh(d*x + c)^12 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2$

$$\begin{aligned}
&) * \sinh(dx + c)^{10} + 20 * (11 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c) \\
& + c)^3 + 3 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c) * \sinh(dx + c)^9 + 15 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^8 + 15 * (33 \\
& * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^4 + 16 * a^3 + 24 * a^2 * b \\
& + 18 * a * b^2 + 5 * b^3 + 18 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + \\
& c)^2) * \sinh(dx + c)^8 + 24 * (33 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^5 + 30 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^3 + 5 * \\
& (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c) * \sinh(dx + c)^7 + 20 * \\
& (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^6 + 4 * (231 * (16 * a^3 + 2 \\
& 4 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^6 + 315 * (16 * a^3 + 24 * a^2 * b + 18 * a \\
& * b^2 + 5 * b^3) * \cosh(dx + c)^4 + 80 * a^3 + 120 * a^2 * b + 90 * a * b^2 + 25 * b^3 + 10 \\
& 5 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + \\
& 24 * (33 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^7 + 63 * (16 * a^3 \\
& + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^5 + 35 * (16 * a^3 + 24 * a^2 * b + 1 \\
& 8 * a * b^2 + 5 * b^3) * \cosh(dx + c)^3 + 5 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) \\
& * \cosh(dx + c) * \sinh(dx + c)^5 + 15 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) \\
& * \cosh(dx + c)^4 + 15 * (33 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + \\
& c)^8 + 84 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^6 + 70 * (16 * \\
& a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^4 + 16 * a^3 + 24 * a^2 * b + 18 \\
& * a * b^2 + 5 * b^3 + 20 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^2) \\
& * \sinh(dx + c)^4 + 20 * (11 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + \\
& c)^9 + 36 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^7 + 42 * (16 * \\
& a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^5 + 20 * (16 * a^3 + 24 * a^2 * b \\
& + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^3 + 3 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b \\
& ^3) * \cosh(dx + c) * \sinh(dx + c)^3 + 16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3 + \\
& 6 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^2 + 6 * (11 * (16 * a^3 + \\
& 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^10 + 45 * (16 * a^3 + 24 * a^2 * b + 18 \\
& * a * b^2 + 5 * b^3) * \cosh(dx + c)^8 + 70 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) \\
& * \cosh(dx + c)^6 + 50 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^4 \\
& + 16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3 + 15 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 \\
& + 5 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 12 * ((16 * a^3 + 24 * a^2 * b + 18 * a * \\
& b^2 + 5 * b^3) * \cosh(dx + c)^11 + 5 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh \\
& (dx + c)^9 + 10 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^7 + \\
& 10 * (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^5 + 5 * (16 * a^3 + 24 \\
& * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c)^3 + (16 * a^3 + 24 * a^2 * b + 18 * a * b^2 \\
& + 5 * b^3) * \cosh(dx + c) * \sinh(dx + c) * \arctan(\cosh(dx + c) + \sinh(dx + c) \\
&) - 3 * (24 * a^2 * b + 18 * a * b^2 + 5 * b^3) * \cosh(dx + c) + 3 * (11 * (24 * a^2 * b + 18 * a * \\
& b^2 + 5 * b^3) * \cosh(dx + c)^10 + 3 * (216 * a^2 * b + 306 * a * b^2 + 85 * b^3) * \cosh(dx + \\
& c)^8 + 42 * (8 * a^2 * b + 14 * a * b^2 + 11 * b^3) * \cosh(dx + c)^6 - 30 * (8 * a^2 * b + \\
& 14 * a * b^2 + 11 * b^3) * \cosh(dx + c)^4 - 24 * a^2 * b - 18 * a * b^2 - 5 * b^3 - (216 * a^2 \\
& * b + 306 * a * b^2 + 85 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c) / (d * \cosh(dx + c)^1 \\
& 2 + 12 * d * \cosh(dx + c) * \sinh(dx + c)^11 + d * \sinh(dx + c)^12 + 6 * d * \cosh(dx + \\
& c)^10 + 6 * (11 * d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^10 + 20 * (11 * d * \cosh(dx + \\
& c)^3 + 3 * d * \cosh(dx + c)) * \sinh(dx + c)^9 + 15 * d * \cosh(dx + c)^8 + 15 * (\\
& 33 * d * \cosh(dx + c)^4 + 18 * d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^8 + 24 * (33 * d \\
& * \cosh(dx + c)^5 + 30 * d * \cosh(dx + c)^3 + 5 * d * \cosh(dx + c)) * \sinh(dx + c)^7 \\
& + 20 * d * \cosh(dx + c)^6 + 4 * (231 * d * \cosh(dx + c)^6 + 315 * d * \cosh(dx + c)^4 \\
& + 105 * d * \cosh(dx + c)^2 + 5 * d) * \sinh(dx + c)^6 + 24 * (33 * d * \cosh(dx + c)^7 \\
& + 63 * d * \cosh(dx + c)^5 + 35 * d * \cosh(dx + c)^3 + 5 * d * \cosh(dx + c)) * \sinh(dx + \\
& c)^5 + 15 * d * \cosh(dx + c)^4 + 15 * (33 * d * \cosh(dx + c)^8 + 84 * d * \cosh(dx + \\
& c)^6 + 70 * d * \cosh(dx + c)^4 + 20 * d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^4 + \\
& 20 * (11 * d * \cosh(dx + c)^9 + 36 * d * \cosh(dx + c)^7 + 42 * d * \cosh(dx + c)^5 + 20 \\
& * d * \cosh(dx + c)^3 + 3 * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 6 * d * \cosh(dx + c) \\
& ^2 + 6 * (11 * d * \cosh(dx + c)^10 + 45 * d * \cosh(dx + c)^8 + 70 * d * \cosh(dx + c)^6 \\
& + 50 * d * \cosh(dx + c)^4 + 15 * d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^2 + 12 * (d \\
& * \cosh(dx + c)^11 + 5 * d * \cosh(dx + c)^9 + 10 * d * \cosh(dx + c)^7 + 10 * d * \cosh(dx + \\
& c)^5 + 5 * d * \cosh(dx + c)^3 + d * \cosh(dx + c)) * \sinh(dx + c) + d)
\end{aligned}$$

giac [B] time = 0.19, size = 310, normalized size = 2.11

$$3 \left(\pi + 2 \arctan \left(\frac{1}{2} \left(e^{2dx+2c} - 1 \right) e^{-dx-c} \right) \right) (16a^3 + 24a^2b + 18ab^2 + 5b^3) + \frac{4 \left(72a^2b \left(e^{dx+c} - e^{-dx-c} \right)^5 + 54ab^2 \left(e^{dx+c} - e^{-dx-c} \right)^5 \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{96} (3(\pi + 2 \arctan(\frac{1}{2}(e^{2dx+2c} - 1)e^{-dx-c})) (16a^3 + 24a^2b + 18ab^2 + 5b^3) + 4(72a^2b(e^{dx+c} - e^{-dx-c})^5 + 54ab^2(e^{dx+c} - e^{-dx-c})^5) + 15b^3(e^{dx+c} - e^{-dx-c})^5 + 576a^2b(e^{dx+c} - e^{-dx-c})^3 + 576ab^2(e^{dx+c} - e^{-dx-c})^3 + 160b^3(e^{dx+c} - e^{-dx-c})^3 + 1152a^2b(e^{dx+c} - e^{-dx-c}) + 1440ab^2(e^{dx+c} - e^{-dx-c}) + 528b^3(e^{dx+c} - e^{-dx-c})) / ((e^{dx+c} - e^{-dx-c})^2 + 4)^3) / d$

maple [A] time = 0.46, size = 193, normalized size = 1.31

$$\frac{2a^3 \arctan(e^{dx+c})}{d} + \frac{3a^2b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{3a^2b \arctan(e^{dx+c})}{d} + \frac{3ab^2 \tanh(dx+c) \operatorname{sech}(dx+c)^3}{4d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)

[Out] $\frac{2}{d} a^3 \arctan(\exp(dx+c)) + \frac{3}{2} a^2 b \operatorname{sech}(dx+c) \tanh(dx+c) + \frac{3}{d} a^2 b \arctan(\exp(dx+c)) + \frac{3}{4} a b^2 \tanh(dx+c) \operatorname{sech}(dx+c)^3 + \frac{9}{8} a b^2 \operatorname{sech}(dx+c) \tanh(dx+c) + \frac{9}{4} a b^2 \arctan(\exp(dx+c)) + \frac{1}{6} b^3 \tanh(dx+c) \operatorname{sech}(dx+c)^5 + \frac{5}{24} b^3 \tanh(dx+c) \operatorname{sech}(dx+c)^3 + \frac{5}{16} b^3 \operatorname{sech}(dx+c) \tanh(dx+c) + \frac{5}{8} b^3 \arctan(\exp(dx+c))$

maxima [B] time = 0.41, size = 365, normalized size = 2.48

$$-\frac{1}{24} b^3 \left(\frac{15 \arctan(e^{-dx-c})}{d} - \frac{15 e^{-dx-c} + 85 e^{-3dx-3c} + 198 e^{-5dx-5c} - 198 e^{-7dx-7c} - 85 e^{-9dx-9c} - 15 e^{-11dx-11c}}{d(6 e^{-2dx-2c} + 15 e^{-4dx-4c} + 20 e^{-6dx-6c} + 15 e^{-8dx-8c} + 6 e^{-10dx-10c} + e^{-12dx-12c})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-\frac{1}{24} b^3 (15 \arctan(e^{-dx-c}) / d - (15 e^{-dx-c} + 85 e^{-3dx-3c} - 3 e^{-5dx-5c} + 198 e^{-7dx-7c} - 198 e^{-9dx-9c} - 85 e^{-11dx-11c}) / (d(6 e^{-2dx-2c} + 15 e^{-4dx-4c} + 20 e^{-6dx-6c} + 15 e^{-8dx-8c} + e^{-10dx-10c} + 1))) - \frac{3}{4} a b^2 (3 \arctan(e^{-dx-c}) / d - (3 e^{-dx-c} + 11 e^{-3dx-3c} - 11 e^{-5dx-5c} - 3 e^{-7dx-7c}) / (d(4 e^{-2dx-2c} + 6 e^{-4dx-4c} + 4 e^{-6dx-6c} + e^{-8dx-8c} + 1))) - 3 a^2 b (\arctan(e^{-dx-c}) / d - (e^{-dx-c} - e^{-3dx-3c}) / (d(2 e^{-2dx-2c} + e^{-4dx-4c} + 1))) + a^3 \arctan(\sinh(dx+c)) / d$

mupad [B] time = 1.47, size = 535, normalized size = 3.64

$$\frac{\operatorname{atan} \left(\frac{e^{dx+c} (16a^3 \sqrt{d^2} + 5b^3 \sqrt{d^2} + 18ab^2 \sqrt{d^2} + 24a^2b \sqrt{d^2})}{d \sqrt{256a^6 + 768a^5b + 1152a^4b^2 + 1024a^3b^3 + 564a^2b^4 + 180ab^5 + 25b^6}} \right)}{8 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x))^2)^3/cosh(c + d*x),x)

[Out] (atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*(d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 + 768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 - b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (80*b^3*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2 - 3*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 + 24*a^2*b + 5*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 - 72*a^2*b + 5*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x), x)

3.70 $\int \operatorname{sech}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=74

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d} - \frac{b(a+b)^2\tanh^3(c+dx)}{d} + \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

[Out] $(a+b)^3 \tanh(d*x+c)/d - b*(a+b)^2 \tanh(d*x+c)^3/d + 3/5*b^2*(a+b)*\tanh(d*x+c)^5/d - 1/7*b^3*\tanh(d*x+c)^7/d$

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4146, 194}

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d} - \frac{b(a+b)^2\tanh^3(c+dx)}{d} + \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3, x]

[Out] $((a+b)^3 \operatorname{Tanh}[c+d*x])/d - (b*(a+b)^2 \operatorname{Tanh}[c+d*x]^3)/d + (3*b^2*(a+b)*\operatorname{Tanh}[c+d*x]^5)/(5*d) - (b^3*\operatorname{Tanh}[c+d*x]^7)/(7*d)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst}\left(\int (a + b - bx^2)^3 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a^3 \left(1 + \frac{b(3a^2 + 3ab + b^2)}{a^3}\right) - 3b(a + b)^2 x^2 + 3b^2(a + b)x^4 - b^3 x^6\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a+b)^3 \tanh(c+dx)}{d} - \frac{b(a+b)^2 \tanh^3(c+dx)}{d} + \frac{3b^2(a+b) \tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [B] time = 1.49, size = 319, normalized size = 4.31

$$\frac{\operatorname{sech}(c)\operatorname{sech}(c + dx) \left(525a^3 \sinh(2c + 3dx) - 210a^3 \sinh(4c + 3dx) + 210a^3 \sinh(4c + 5dx) - 35a^3 \sinh(6c + 5dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3, x]


```
[Out] (Sech[c]*Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(140*(5*a^3 + 11*a^2*b + 10*a*b^2 + 4*b^3)*Sinh[d*x] - 35*a*(15*a^2 + 26*a*b + 16*b^2)*Sinh[2*c + d*x] + 525*a^3*Sinh[2*c + 3*d*x] + 1260*a^2*b*Sinh[2*c + 3*d*x] + 1176*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] - 210*a^3*Sinh[4*c + 3*d*x] - 210*a^2*b*Sinh[4*c + 3*d*x] + 210*a^3*Sinh[4*c + 5*d*x] + 490*a^2*b*Sinh[4*c + 5*d*x] + 392*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] - 35*a^3*Sinh[6*c + 5*d*x] + 35*a^3*Sinh[6*c + 7*d*x] + 70*a^2*b*Sinh[6*c + 7*d*x] + 56*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(280*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.44, size = 816, normalized size = 11.03

$$\frac{4\left((35a^3 + 35a^2b + 28ab^2 + 8b^3)\cosh(dx + c)^6 - 6(35a^2b + 28ab^2 + 8b^3)\cosh(dx + c)\sinh(dx + c)^5 + \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^6 - 6*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + (210*a^3 + 350*a^2*b + 196*a*b^2 + 56*b^3 + 15*(35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 350*a^3 + 770*a^2*b + 700*a*b^2 + 280*b^3 + 7*(75*a^3 + 155*a^2*b + 124*a*b^2 + 24*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 525*a^3 + 1085*a^2*b + 868*a*b^2 + 168*b^3 + 84*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 7*(25*a^2*b + 44*a*b^2 + 24*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) + 35*d)
```

giac [B] time = 0.19, size = 302, normalized size = 4.08

$$\frac{2\left(35a^3e^{(12dx+12c)} + 210a^3e^{(10dx+10c)} + 210a^2be^{(10dx+10c)} + 525a^3e^{(8dx+8c)} + 910a^2be^{(8dx+8c)} + 560ab^2e^{(8dx+8c)} + \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 210*a^2*b*e^(10*d*x + 10*c) + 525*a^3*e^(8*d*x + 8*c) + 910*a^2*b*e^(8*d*x + 8*c) + 560*a*b^2*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 1540*a^2*b*e^(6*d*x + 6*c) + 1400*a*b^2*e^(6*d*x + 6*c) + 560*b^3*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 1260*a^2*b*e^(4*d*x + 4*c) + 1176*a*b^2*e^(4*d*x + 4*c) + 336*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x + 2*c) + 490*a^2*b*e^(2*d*x + 2*c) + 392*a*b^2*e^(2*d*x + 2*c) + 112*b^3*e^(2*d*x + 2*c) + 35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

maple [A] time = 0.57, size = 116, normalized size = 1.57

$$\frac{a^3 \tanh(dx+c) + 3a^2b \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 3ab^2 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c) + b^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)`

[Out] $1/d*(a^3*\tanh(d*x+c)+3*a^2*b*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)+3*a*b^2*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)+b^3*(16/35+1/7*\operatorname{sech}(d*x+c)^6+6/35*\operatorname{sech}(d*x+c)^4+8/35*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))$

maxima [B] time = 0.33, size = 695, normalized size = 9.39

$$\frac{32}{35} b^3 \left(\frac{7e^{(-2dx-2c)}}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-14dx-14c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $32/35*b^3*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 16/5*a*b^2*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 4*a^2*b*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 2*a^3/(d*(e^{(-2*d*x - 2*c)} + 1)))$

mupad [B] time = 1.50, size = 978, normalized size = 13.22

$$\frac{\frac{2(5a^3+18a^2b+24ab^2+16b^3)}{35d} + \frac{2a^3e^{6c+6dx}}{7d} + \frac{6ae^{2c+2dx}(5a^2+16ab+16b^2)}{35d} + \frac{6a^2e^{4c+4dx}(a+2b)}{7d} - \frac{2a^2(a+2b)}{7d} + \frac{2a^3e^{2c+2dx}}{7d} - \frac{2}{7d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{2}{2e^{2c+2dx} + e^{4c+4dx} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^3/cosh(c + d*x)^2,x)`

[Out] $-((2*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*\exp(6*c + 6*d*x))/(7*d) + (6*a*\exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (6*a^2*\exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - ((2*a^2*(a + 2*b))/(7*d) + (2*a^3*\exp(2*c + 2*d*x))/(7*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*\exp(2*c + 2*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*\exp(8*c + 8*d*x))/(7*d) + (12*a*\exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*a^2*\exp(6*c + 6*d*x)*(a + 2*b))/(7*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)$

$$\begin{aligned}
 & p(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - ((2*a^3)/(7*d) + (8*\exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d) + (2*a^3*\exp(12*c + 12*d*x))/(7*d) + (6*a*\exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (6*a*\exp(8*c + 8*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (12*a^2*\exp(2*c + 2*d*x)*(a + 2*b))/(7*d) + (12*a^2*\exp(10*c + 10*d*x)*(a + 2*b))/(7*d))/(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (2*a^3*\exp(4*c + 4*d*x))/(7*d) + (4*a^2*\exp(2*c + 2*d*x)*(a + 2*b))/(7*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - ((2*a^2*(a + 2*b))/(7*d) + (4*\exp(4*c + 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d) + (2*a^3*\exp(10*c + 10*d*x))/(7*d) + (2*a*\exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (4*a*\exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (10*a^2*\exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1) - (2*a^3)/(7*d*(\exp(2*c + 2*d*x) + 1))
 \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**2, x)

3.71 $\int \operatorname{sech}^3(c + dx) (a + b\operatorname{sech}^2(c + dx))^3 dx$

Optimal. Leaf size=196

$$\frac{b(72a^2 + 92ab + 35b^2) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{192d} + \frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \tan^{-1}(\sinh(c + dx))}{128d} + \frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \tan^{-1}(\sinh(c + dx))}{128d}$$

[Out] 1/128*(64*a^3+144*a^2*b+120*a*b^2+35*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3+144*a^2*b+120*a*b^2+35*b^3)*sech(d*x+c)*tanh(d*x+c)/d+1/192*b*(72*a^2+92*a*b+35*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d+1/48*b*(12*a+7*b)*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d+1/8*b*sech(d*x+c)^7*(a+b+a*sinh(d*x+c)^2)^2*tanh(d*x+c)/d

Rubi [A] time = 0.23, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4147, 413, 526, 385, 199, 203}

$$\frac{(144a^2b + 64a^3 + 120ab^2 + 35b^3) \tan^{-1}(\sinh(c + dx))}{128d} + \frac{b(72a^2 + 92ab + 35b^2) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{192d} + \frac{(144a^2b + 64a^3 + 120ab^2 + 35b^3) \tan^{-1}(\sinh(c + dx))}{128d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d) + (b*(72*a^2 + 92*a*b + 35*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) + (b*(12*a + 7*b)*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(48*d) + (b*Sech[c + d*x]^7*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + 1) + 1)), x], x]

```
+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{b \operatorname{sech}^7(c + dx) (a + b + a \sinh^2(c + dx))^2 \tanh(c + dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{b(12a + 7b) \operatorname{sech}^5(c + dx) (a + b + a \sinh^2(c + dx)) \tanh(c + dx)}{48d} + \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{b(72a^2 + 92ab + 35b^2) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{192d} + \frac{b(12a + 7b) \operatorname{sech}^2(c + dx) \tanh(c + dx)}{48d} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \operatorname{sech}(c + dx) \tanh(c + dx)}{128d} + \frac{b(12a + 7b) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d} + \frac{(64a^3 + 144a^2b + 120ab^2 + 35b^3) \tan^{-1}(\sinh(c + dx))}{128d} + \frac{(64a^3 - 144a^2b + 120ab^2 + 35b^3) \operatorname{sech}(c + dx) \tanh(c + dx)}{128d}$$

Mathematica [A] time = 9.40, size = 297, normalized size = 1.52

$$\frac{\operatorname{sech}^8(c + dx) (a \cosh^2(c + dx) + b)^3 (2b(144a^2 + 120ab + 35b^2) \tanh(c) \cosh^5(c + dx) + 2b(144a^2 + 120ab + 35b^2) \operatorname{sech}^2(c + dx) \cosh^3(c + dx) + 2b(144a^2 + 120ab + 35b^2) \operatorname{sech}^4(c + dx) \cosh(c + dx) + 2b(144a^2 + 120ab + 35b^2) \operatorname{sech}^6(c + dx) \cosh(c + dx) + 2b(144a^2 + 120ab + 35b^2) \operatorname{sech}^8(c + dx))}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^8*(6*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]]*Cosh[c + d*x]^8 + 48*b^3*Sech[c]*Sin
h[d*x] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 2*b*(144*a^2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] + 48*b^3*Cosh[
```

$$c + d*x]*\text{Tanh}[c] + 8*b^2*(24*a + 7*b)*\text{Cosh}[c + d*x]^3*\text{Tanh}[c] + 2*b*(144*a^2 + 120*a*b + 35*b^2)*\text{Cosh}[c + d*x]^5*\text{Tanh}[c] + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\text{Cosh}[c + d*x]^7*\text{Tanh}[c]))/(48*d*(a + 2*b + a*\text{Cosh}[2*(c + d*x)])^3)$$

fricas [B] time = 0.50, size = 6114, normalized size = 31.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/192*(3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^15 + 45*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^15 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^13 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(105*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^11 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3 + 78*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(819*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3 + (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^9 + (15015*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 715*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^4 + 960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 429*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^5 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 + 3*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - (960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 + (19305*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 1716*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^6 + 330*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 960*a^3 - 4464*a^2*b - 6792*a*b^2 - 5053*b^3 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^7 + 462*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^5 + 84*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^3 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^5 + (9009*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 1287*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^8 + 462*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^6 + 126*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 1728*a^3 - 7344*a^2*b - 9192*a*b^2 - 2681*b^3 - 21*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 715*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^9 + 330*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^7 + 126*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^5 - 35*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^3 - 5*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3 + (1365*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^10 + 165*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 + 44

$$\begin{aligned}
& 64a^2b + 6792ab^2 + 5053b^3) \cosh(dx + c)^6 - 35(960a^3 + 4464a^2b + 6792ab^2 + 5053b^3) \cosh(dx + c)^4 - 960a^3 - 3312a^2b - 2760ab^2 - 805b^3 - 10(1728a^3 + 7344a^2b + 9192ab^2 + 2681b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + (315(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{13} + 78(960a^3 + 3312a^2b + 2760ab^2 + 805b^3) \cosh(dx + c)^{11} + 55(1728a^3 + 7344a^2b + 9192ab^2 + 2681b^3) \cosh(dx + c)^9 + 36(960a^3 + 4464a^2b + 6792ab^2 + 5053b^3) \cosh(dx + c)^7 - 21(960a^3 + 4464a^2b + 6792ab^2 + 5053b^3) \cosh(dx + c)^5 - 10(1728a^3 + 7344a^2b + 9192ab^2 + 2681b^3) \cosh(dx + c)^3 - 3(960a^3 + 3312a^2b + 2760ab^2 + 805b^3) \cosh(dx + c)) \sinh(dx + c)^2 + 3((64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{16} + 16(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c) \sinh(dx + c)^{15} + (64a^3 + 144a^2b + 120ab^2 + 35b^3) \sinh(dx + c)^{16} + 8(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{14} + 8(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^{14} + 112(5(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^3 + (64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)) \sinh(dx + c)^{13} + 28(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{12} + 28(65(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 64a^3 + 144a^2b + 120ab^2 + 35b^3 + 26(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^{12} + 112(39(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^5 + 26(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^3 + 3(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)) \sinh(dx + c)^{11} + 56(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{10} + 56(143(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^6 + 143(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 64a^3 + 144a^2b + 120ab^2 + 35b^3 + 33(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^{10} + 16(715(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^7 + 1001(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^5 + 385(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^3 + 35(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)) \sinh(dx + c)^9 + 70(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^8 + 2(6435(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^8 + 12012(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^6 + 6930(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 2240a^3 + 5040a^2b + 4200ab^2 + 1225b^3 + 1260(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^8 + 16(715(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^9 + 1716(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^7 + 1386(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^5 + 420(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^3 + 35(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)) \sinh(dx + c)^7 + 56(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^6 + 56(143(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{10} + 429(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^8 + 462(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^6 + 210(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 64a^3 + 144a^2b + 120ab^2 + 35b^3 + 35(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^6 + 112(39(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{11} + 143(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^9 + 198(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^7 + 126(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^5 + 35(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^3 + 3(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)) \sinh(dx + c)^5 + 28(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 28(65(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{12} + 286(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^{10} + 495(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^8 + 420(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^6 + 175(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^4 + 64a^3 + 144a^2b + 120ab^2 + 35b^3 + 30(64a^3 + 144a^2b + 120ab^2 + 35b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 112
\end{aligned}$$

$$\begin{aligned}
&*(5*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^{13} + 26*(64*a^3 \\
&+ 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^{11} + 55*(64*a^3 + 144*a^2* \\
&b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^9 + 60*(64*a^3 + 144*a^2*b + 120*a*b^ \\
&2 + 35*b^3)*\cosh(d*x + c)^7 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)* \\
&\cosh(d*x + c)^5 + 10*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c \\
&)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^3 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 8*(64*a^3 + 144*a^2*b + 120 \\
&*a*b^2 + 35*b^3)*\cosh(d*x + c)^2 + 8*(15*(64*a^3 + 144*a^2*b + 120*a*b^2 + \\
&35*b^3)*\cosh(d*x + c)^{14} + 91*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cos \\
&h(d*x + c)^{12} + 231*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c) \\
&)^{10} + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^8 + 245*(\\
&64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^6 + 105*(64*a^3 + 14 \\
&4*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a* \\
&b^2 + 35*b^3 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^2 \\
&))*\sinh(d*x + c)^2 + 16*((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x \\
&+ c)^{15} + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^{13} + 21 \\
&)*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^{11} + 35*(64*a^3 + \\
&144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c)^9 + 35*(64*a^3 + 144*a^2*b + \\
&120*a*b^2 + 35*b^3)*\cosh(d*x + c)^7 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 + \\
&35*b^3)*\cosh(d*x + c)^5 + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(\\
&d*x + c)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*\cosh(d*x + c))*\sinh(\\
&d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 3*(64*a^3 + 144*a^2*b + 1 \\
&20*a*b^2 + 35*b^3)*\cosh(d*x + c) + (45*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35 \\
&*b^3)*\cosh(d*x + c)^{14} + 13*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*\c \\
&osh(d*x + c)^{12} + 11*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*\cosh(d \\
&*x + c)^{10} + 9*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*\cosh(d*x + c) \\
&)^8 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*\cosh(d*x + c)^6 - 5*(\\
&1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*\cosh(d*x + c)^4 - 192*a^3 - \\
&432*a^2*b - 360*a*b^2 - 105*b^3 - 3*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 80 \\
&5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^{16} + 16*d*\cosh(d*x \\
&+ c)*\sinh(d*x + c)^{15} + d*\sinh(d*x + c)^{16} + 8*d*\cosh(d*x + c)^{14} + 8*(15*d \\
&)*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^{14} + 112*(5*d*\cosh(d*x + c)^3 + d*\cosh(\\
&d*x + c))*\sinh(d*x + c)^{13} + 28*d*\cosh(d*x + c)^{12} + 28*(65*d*\cosh(d*x + c) \\
&)^4 + 26*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^{12} + 112*(39*d*\cosh(d*x + c)^5 \\
&+ 26*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^{11} + 56*d*\cosh(d \\
&*x + c)^{10} + 56*(143*d*\cosh(d*x + c)^6 + 143*d*\cosh(d*x + c)^4 + 33*d*\cosh(\\
&d*x + c)^2 + d)*\sinh(d*x + c)^{10} + 16*(715*d*\cosh(d*x + c)^7 + 1001*d*\cosh(\\
&d*x + c)^5 + 385*d*\cosh(d*x + c)^3 + 35*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + \\
&70*d*\cosh(d*x + c)^8 + 2*(6435*d*\cosh(d*x + c)^8 + 12012*d*\cosh(d*x + c)^6 \\
&+ 6930*d*\cosh(d*x + c)^4 + 1260*d*\cosh(d*x + c)^2 + 35*d)*\sinh(d*x + c)^8 + \\
&16*(715*d*\cosh(d*x + c)^9 + 1716*d*\cosh(d*x + c)^7 + 1386*d*\cosh(d*x + c)^ \\
&5 + 420*d*\cosh(d*x + c)^3 + 35*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 56*d*\cosh \\
&(d*x + c)^6 + 56*(143*d*\cosh(d*x + c)^{10} + 429*d*\cosh(d*x + c)^8 + 462*d*\co \\
&sh(d*x + c)^6 + 210*d*\cosh(d*x + c)^4 + 35*d*\cosh(d*x + c)^2 + d)*\sinh(d*x \\
&+ c)^6 + 112*(39*d*\cosh(d*x + c)^{11} + 143*d*\cosh(d*x + c)^9 + 198*d*\cosh(d \\
&x + c)^7 + 126*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c) \\
&))*\sinh(d*x + c)^5 + 28*d*\cosh(d*x + c)^4 + 28*(65*d*\cosh(d*x + c)^{12} + 286* \\
&d*\cosh(d*x + c)^{10} + 495*d*\cosh(d*x + c)^8 + 420*d*\cosh(d*x + c)^6 + 175*d* \\
&\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 112*(5*d*\cosh \\
&(d*x + c)^{13} + 26*d*\cosh(d*x + c)^{11} + 55*d*\cosh(d*x + c)^9 + 60*d*\cosh(d*x \\
&+ c)^7 + 35*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\si \\
&>nh(d*x + c)^3 + 8*d*\cosh(d*x + c)^2 + 8*(15*d*\cosh(d*x + c)^{14} + 91*d*\cosh(\\
&d*x + c)^{12} + 231*d*\cosh(d*x + c)^{10} + 315*d*\cosh(d*x + c)^8 + 245*d*\cosh(d \\
&>*x + c)^6 + 105*d*\cosh(d*x + c)^4 + 21*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c) \\
&)^2 + 16*(d*\cosh(d*x + c)^{15} + 7*d*\cosh(d*x + c)^{13} + 21*d*\cosh(d*x + c)^{11} \\
&+ 35*d*\cosh(d*x + c)^9 + 35*d*\cosh(d*x + c)^7 + 21*d*\cosh(d*x + c)^5 + 7*d* \\
&\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)
\end{aligned}$$

giac [B] time = 0.18, size = 485, normalized size = 2.47

$$3 \left(\pi + 2 \arctan \left(\frac{1}{2} \left(e^{(2dx+2c)} - 1 \right) e^{(-dx-c)} \right) \right) (64a^3 + 144a^2b + 120ab^2 + 35b^3) + \frac{4 \left(192a^3 \left(e^{(dx+c)} - e^{(-dx-c)} \right)^7 + 432a^2b \left(e^{(dx+c)} - e^{(-dx-c)} \right)^6 + \dots \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7 + 432*a^2*b*(e^(d*x + c) - e^(-d*x - c))^6 + 360*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 105*b^3*(e^(d*x + c) - e^(-d*x - c))^4 + 2304*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 6336*a^2*b*(e^(d*x + c) - e^(-d*x - c))^2 + 5280*a*b^2*(e^(d*x + c) - e^(-d*x - c))^1 + 1540*b^3*(e^(d*x + c) - e^(-d*x - c))^0 + 9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) - e^(-d*x - c))^2 + 28032*a*b^2*(e^(d*x + c) - e^(-d*x - c))^1 + 8176*b^3*(e^(d*x + c) - e^(-d*x - c))^0 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) + 46080*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 50688*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 17856*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^4/d

maple [A] time = 0.47, size = 280, normalized size = 1.43

$$\frac{a^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^3 \arctan(e^{dx+c})}{d} + \frac{3a^2b \tanh(dx+c) \operatorname{sech}(dx+c)^3}{4d} + \frac{9a^2b \operatorname{sech}(dx+c) \tanh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))+3/4/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+9/4/d*a^2*b*arctan(exp(d*x+c))+1/2/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/8/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+15/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+15/8/d*a*b^2*arctan(exp(d*x+c))+1/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+7/48/d*b^3*tanh(d*x+c)*sech(d*x+c)^5+35/192/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+35/128/d*b^3*sech(d*x+c)*tanh(d*x+c)+35/64/d*b^3*arctan(exp(d*x+c))

maxima [B] time = 0.42, size = 556, normalized size = 2.84

$$-\frac{1}{192} b^3 \left(\frac{105 \arctan(e^{(-dx-c)})}{d} - \frac{105 e^{(-dx-c)} + 805 e^{(-3dx-3c)} + 2681 e^{(-5dx-5c)} + 5053 e^{(-7dx-7c)} - 5053 e^{(-9dx-9c)} - 2681 e^{(-11dx-11c)} - 805 e^{(-13dx-13c)} - 105 e^{(-15dx-15c)}}{d(8 e^{(-2dx-2c)} + 28 e^{(-4dx-4c)} + 56 e^{(-6dx-6c)} + 70 e^{(-8dx-8c)} + 56 e^{(-10dx-10c)} + 28 e^{(-12dx-12c)} + 8 e^{(-14dx-14c)} + e^{(-16dx-16c)} + 1)} \right) - \frac{1}{8} a b^2 (15 \arctan(e^{(-dx-c)})/d - (15 e^{(-dx-c)} + 85 e^{(-3dx-3c)} + 198 e^{(-5dx-5c)} - 198 e^{(-7dx-7c)} - 85 e^{(-9dx-9c)} - 15 e^{(-11dx-11c)})/d(6 e^{(-2dx-2c)} + 15 e^{(-4dx-4c)} + 20 e^{(-6dx-6c)} + 15 e^{(-8dx-8c)} + 6 e^{(-10dx-10c)} + e^{(-12dx-12c)} + 1)) - \frac{3}{4} a^2 b (3 \arctan(e^{(-dx-c)})/d - (3 e^{(-dx-c)} + 11 e^{(-3dx-3c)} - 11 e^{(-5dx-5c)} + 11 e^{(-7dx-7c)} - 11 e^{(-9dx-9c)} + 11 e^{(-11dx-11c)} - 11 e^{(-13dx-13c)} + 11 e^{(-15dx-15c)} - 11 e^{(-17dx-17c)} + 11 e^{(-19dx-19c)} - 11 e^{(-21dx-21c)} + 11 e^{(-23dx-23c)} - 11 e^{(-25dx-25c)} + 11 e^{(-27dx-27c)} - 11 e^{(-29dx-29c)} + 11 e^{(-31dx-31c)} - 11 e^{(-33dx-33c)} + 11 e^{(-35dx-35c)} - 11 e^{(-37dx-37c)} + 11 e^{(-39dx-39c)} - 11 e^{(-41dx-41c)} + 11 e^{(-43dx-43c)} - 11 e^{(-45dx-45c)} + 11 e^{(-47dx-47c)} - 11 e^{(-49dx-49c)} + 11 e^{(-51dx-51c)} - 11 e^{(-53dx-53c)} + 11 e^{(-55dx-55c)} - 11 e^{(-57dx-57c)} + 11 e^{(-59dx-59c)} - 11 e^{(-61dx-61c)} + 11 e^{(-63dx-63c)} - 11 e^{(-65dx-65c)} + 11 e^{(-67dx-67c)} - 11 e^{(-69dx-69c)} + 11 e^{(-71dx-71c)} - 11 e^{(-73dx-73c)} + 11 e^{(-75dx-75c)} - 11 e^{(-77dx-77c)} + 11 e^{(-79dx-79c)} - 11 e^{(-81dx-81c)} + 11 e^{(-83dx-83c)} - 11 e^{(-85dx-85c)} + 11 e^{(-87dx-87c)} - 11 e^{(-89dx-89c)} + 11 e^{(-91dx-91c)} - 11 e^{(-93dx-93c)} + 11 e^{(-95dx-95c)} - 11 e^{(-97dx-97c)} + 11 e^{(-99dx-99c)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d - (105*e^(-d*x - c) + 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x - 5*c) + 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) - 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) - 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 11*e^(-7*d*x - 7*c) - 11*e^(-9*d*x - 9*c) + 11*e^(-11*d*x - 11*c) - 11*e^(-13*d*x - 13*c) + 11*e^(-15*d*x - 15*c) - 11*e^(-17*d*x - 17*c) + 11*e^(-19*d*x - 19*c) - 11*e^(-21*d*x - 21*c) + 11*e^(-23*d*x - 23*c) - 11*e^(-25*d*x - 25*c) + 11*e^(-27*d*x - 27*c) - 11*e^(-29*d*x - 29*c) + 11*e^(-31*d*x - 31*c) - 11*e^(-33*d*x - 33*c) + 11*e^(-35*d*x - 35*c) - 11*e^(-37*d*x - 37*c) + 11*e^(-39*d*x - 39*c) - 11*e^(-41*d*x - 41*c) + 11*e^(-43*d*x - 43*c) - 11*e^(-45*d*x - 45*c) + 11*e^(-47*d*x - 47*c) - 11*e^(-49*d*x - 49*c) + 11*e^(-51*d*x - 51*c) - 11*e^(-53*d*x - 53*c) + 11*e^(-55*d*x - 55*c) - 11*e^(-57*d*x - 57*c) + 11*e^(-59*d*x - 59*c) - 11*e^(-61*d*x - 61*c) + 11*e^(-63*d*x - 63*c) - 11*e^(-65*d*x - 65*c) + 11*e^(-67*d*x - 67*c) - 11*e^(-69*d*x - 69*c) + 11*e^(-71*d*x - 71*c) - 11*e^(-73*d*x - 73*c) + 11*e^(-75*d*x - 75*c) - 11*e^(-77*d*x - 77*c) + 11*e^(-79*d*x - 79*c) - 11*e^(-81*d*x - 81*c) + 11*e^(-83*d*x - 83*c) - 11*e^(-85*d*x - 85*c) + 11*e^(-87*d*x - 87*c) - 11*e^(-89*d*x - 89*c) + 11*e^(-91*d*x - 91*c) - 11*e^(-93*d*x - 93*c) + 11*e^(-95*d*x - 95*c) - 11*e^(-97*d*x - 97*c) + 11*e^(-99*d*x - 99*c))

$- 5*c) - 3*e^{(-7*d*x - 7*c)}/(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) - a^3*(\arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))$

mupad [B] time = 1.60, size = 931, normalized size = 4.75

$$\operatorname{atan}\left(\frac{e^{d x} e^c \left(64 a^3 \sqrt{d^2}+35 b^3 \sqrt{d^2}+120 a b^2 \sqrt{d^2}+144 a^2 b \sqrt{d^2}\right)}{d \sqrt{4096 a^6+18432 a^5 b+36096 a^4 b^2+39040 a^3 b^3+24480 a^2 b^4+8400 a b^5+1225 b^6}}\right) \frac{\sqrt{4096 a^6+18432 a^5 b+36096 a^4 b^2+39040 a^3 b^3+24480 a^2 b^4+8400 a b^5+1225 b^6}}{64 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^3,x)`

[Out] $(\operatorname{atan}((\exp(d*x)*\exp(c)*(64*a^3*(d^2)^{(1/2)} + 35*b^3*(d^2)^{(1/2)} + 120*a*b^2*(d^2)^{(1/2)} + 144*a^2*b*(d^2)^{(1/2)}))/(d*(8400*a*b^5 + 18432*a^5*b + 4096*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^3 + 36096*a^4*b^2)^{(1/2)})))*(8400*a*b^5 + 18432*a^5*b + 4096*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^3 + 36096*a^4*b^2)^{(1/2)})/(64*(d^2)^{(1/2)}) - ((a^3*\exp(c + d*x))/(2*d) + (2*\exp(7*c + 7*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/d + (a^3*\exp(13*c + 13*d*x))/(2*d) + (3*a*\exp(5*c + 5*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d) + (3*a*\exp(9*c + 9*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d) + (3*a^2*\exp(3*c + 3*d*x)*(a + 2*b))/d + (3*a^2*\exp(11*c + 11*d*x)*(a + 2*b))/d)/(8*\exp(2*c + 2*d*x) + 28*\exp(4*c + 4*d*x) + 56*\exp(6*c + 6*d*x) + 70*\exp(8*c + 8*d*x) + 56*\exp(10*c + 10*d*x) + 28*\exp(12*c + 12*d*x) + 8*\exp(14*c + 14*d*x) + \exp(16*c + 16*d*x) + 1) + (2*\exp(c + d*x)*(48*a*b^2 - 37*b^3))/(3*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) + (\exp(c + d*x)*(24*a^2*b - 120*a*b^2 + b^3))/(4*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - (16*b^3*\exp(c + d*x))/(d*(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1)) + (\exp(c + d*x)*(120*a*b^2 + 144*a^2*b + 64*a^3 + 35*b^3))/(64*d*(\exp(2*c + 2*d*x) + 1)) - (4*\exp(c + d*x)*(6*a*b^2 - 29*b^3))/(3*d*(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1)) + (\exp(c + d*x)*(120*a*b^2 + 144*a^2*b - 144*a^3 + 35*b^3))/(96*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) + (\exp(c + d*x)*(24*a*b^2 - 288*a^2*b + 7*b^3))/(24*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**3, x)`

3.72 $\int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx$

Optimal. Leaf size=108

$$-\frac{b^2(3a + 4b) \tanh^7(c + dx)}{7d} + \frac{3b(a + b)(a + 2b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^2(a + 4b) \tanh^3(c + dx)}{3d} + \frac{(a + b)^3 \tanh(c + dx)}{d}$$

[Out] $(a+b)^3 \tanh(d*x+c)/d - 1/3*(a+b)^2*(a+4*b)*\tanh(d*x+c)^3/d + 3/5*b*(a+b)*(a+2*b)*\tanh(d*x+c)^5/d - 1/7*b^2*(3*a+4*b)*\tanh(d*x+c)^7/d + 1/9*b^3*\tanh(d*x+c)^9/d$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4146, 373}

$$-\frac{b^2(3a + 4b) \tanh^7(c + dx)}{7d} + \frac{3b(a + b)(a + 2b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^2(a + 4b) \tanh^3(c + dx)}{3d} + \frac{(a + b)^3 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $((a + b)^3 \operatorname{Tanh}[c + d*x])/d - ((a + b)^2(a + 4*b) \operatorname{Tanh}[c + d*x]^3)/(3*d) + (3*b*(a + b)*(a + 2*b) \operatorname{Tanh}[c + d*x]^5)/(5*d) - (b^2*(3*a + 4*b) \operatorname{Tanh}[c + d*x]^7)/(7*d) + (b^3 \operatorname{Tanh}[c + d*x]^9)/(9*d)$

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int (1 - x^2) (a + b - bx^2)^3 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int ((a + b)^3 - (a + b)^2(a + 4b)x^2 + 3b(a + b)(a + 2b)x^4 - b^2(a + 4b)x^6 + b^3x^8) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b)^3 \tanh(c + dx)}{d} - \frac{(a + b)^2(a + 4b) \tanh^3(c + dx)}{3d} + \frac{3b(a + b)(a + 2b) \tanh^5(c + dx)}{5d} - \frac{b^2(a + 4b) \tanh^7(c + dx)}{7d} + \frac{b^3 \tanh^9(c + dx)}{9d} \end{aligned}$$

Mathematica [B] time = 1.76, size = 348, normalized size = 3.22

$$\frac{\operatorname{sech}(c) \operatorname{sech}^9(c + dx) (6825a^3 \sinh(2c + 3dx) - 1995a^3 \sinh(4c + 3dx) + 3465a^3 \sinh(4c + 5dx) - 315a^3 \sinh(4c + 7dx) + 15a^3 \sinh(4c + 9dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (Sech[c]*Sech[c + d*x]^9*(63*(125*a^3 + 324*a^2*b + 312*a*b^2 + 128*b^3)*Sinh[d*x] - 315*a*(17*a^2 + 36*a*b + 24*b^2)*Sinh[2*c + d*x] + 6825*a^3*Sinh[2*c + 3*d*x] + 18648*a^2*b*Sinh[2*c + 3*d*x] + 18144*a*b^2*Sinh[2*c + 3*d*x] + 5376*b^3*Sinh[2*c + 3*d*x] - 1995*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] + 3465*a^3*Sinh[4*c + 5*d*x] + 9072*a^2*b*Sinh[4*c + 5*d*x] + 7776*a*b^2*Sinh[4*c + 5*d*x] + 2304*b^3*Sinh[4*c + 5*d*x] - 315*a^3*Sinh[6*c + 5*d*x] + 945*a^3*Sinh[6*c + 7*d*x] + 2268*a^2*b*Sinh[6*c + 7*d*x] + 1944*a*b^2*Sinh[6*c + 7*d*x] + 576*b^3*Sinh[6*c + 7*d*x] + 105*a^3*Sinh[8*c + 9*d*x] + 252*a^2*b*Sinh[8*c + 9*d*x] + 216*a*b^2*Sinh[8*c + 9*d*x] + 64*b^3*Sinh[8*c + 9*d*x]))/(40320*d)

fricas [B] time = 0.42, size = 1190, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -8/315*(2*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^7 + 14*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3 + 7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^3 + 3*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 18*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x + c)^3 + (35*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 945*a^3 + 1134*a^2*b - 108*a*b^2 - 1152*b^3 + 30*(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 93*a^2*b + 90*a*b^2 + 32*b^3)*cosh(d*x + c) + (7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 15*(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^4 + 525*a^3 + 882*a^2*b + 756*a*b^2 + 1344*b^3 + 27*(105*a^3 + 126*a^2*b - 12*a*b^2 - 128*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*cosh(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^5 + 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^2 + 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5 + 1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*cosh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cosh(d*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d*x + c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x + c)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*cosh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))

giac [B] time = 0.18, size = 360, normalized size = 3.33

$$\frac{4(315 a^3 e^{(14 dx+14 c)} + 1995 a^3 e^{(12 dx+12 c)} + 2520 a^2 b e^{(12 dx+12 c)} + 5355 a^3 e^{(10 dx+10 c)} + 11340 a^2 b e^{(10 dx+10 c)} + 7560 a^3 e^{(8 dx+8 c)} + 11340 a^2 b e^{(8 dx+8 c)} + 5355 a^3 e^{(6 dx+6 c)} + 1995 a^3 e^{(4 dx+4 c)} + 315 a^3 e^{(2 dx+2 c)})}{40320 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\frac{-4/315*(315*a^3*e^{(14*d*x + 14*c)} + 1995*a^3*e^{(12*d*x + 12*c)} + 2520*a^2*b*e^{(12*d*x + 12*c)} + 5355*a^3*e^{(10*d*x + 10*c)} + 11340*a^2*b*e^{(10*d*x + 10*c)} + 7560*a*b^2*e^{(10*d*x + 10*c)} + 7875*a^3*e^{(8*d*x + 8*c)} + 20412*a^2*b*e^{(8*d*x + 8*c)} + 19656*a*b^2*e^{(8*d*x + 8*c)} + 8064*b^3*e^{(8*d*x + 8*c)} + 6825*a^3*e^{(6*d*x + 6*c)} + 18648*a^2*b*e^{(6*d*x + 6*c)} + 18144*a*b^2*e^{(6*d*x + 6*c)} + 5376*b^3*e^{(6*d*x + 6*c)} + 3465*a^3*e^{(4*d*x + 4*c)} + 9072*a^2*b*e^{(4*d*x + 4*c)} + 7776*a*b^2*e^{(4*d*x + 4*c)} + 2304*b^3*e^{(4*d*x + 4*c)} + 945*a^3*e^{(2*d*x + 2*c)} + 2268*a^2*b*e^{(2*d*x + 2*c)} + 1944*a*b^2*e^{(2*d*x + 2*c)} + 576*b^3*e^{(2*d*x + 2*c)} + 105*a^3 + 252*a^2*b + 216*a*b^2 + 64*b^3)/(d*(e^{(2*d*x + 2*c)} + 1)^9)}$$

maple [A] time = 0.52, size = 158, normalized size = 1.46

$$a^3 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 3a^2b \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c) + 3ab^2 \left(\frac{16}{35} + \frac{\operatorname{sech}(dx+c)^6}{7} \right) \tanh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$\frac{1}{d} \left(a^3 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 3a^2b \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c) + 3ab^2 \left(\frac{16}{35} + \frac{\operatorname{sech}(dx+c)^6}{7} \right) \tanh(dx+c) \right)$$

maxima [B] time = 0.34, size = 1245, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\frac{256}{315} b^3 \frac{(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{36e^{(-4*d*x - 4*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{84e^{(-6*d*x - 6*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{126e^{(-8*d*x - 8*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{126e^{(-10*d*x - 10*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{84e^{(-12*d*x - 12*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{36e^{(-14*d*x - 14*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{9e^{(-16*d*x - 16*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{e^{(-18*d*x - 18*c)}}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{1}{(d(9e^{(-2*d*x - 2*c)} + 36e^{(-4*d*x - 4*c)} + 84e^{(-6*d*x - 6*c)} + 126e^{(-8*d*x - 8*c)} + 126e^{(-10*d*x - 10*c)} + 84e^{(-12*d*x - 12*c)} + 36e^{(-14*d*x - 14*c)} + 9e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))} + \frac{96}{35} a^2 b^2 \frac{(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{21e^{(-4*d*x - 4*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{35e^{(-6*d*x - 6*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{35e^{(-8*d*x - 8*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{21e^{(-10*d*x - 10*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{7e^{(-12*d*x - 12*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{e^{(-14*d*x - 14*c)}}{(d(7e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))} + \frac{16}{5} a^2 b \frac{(5e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))}{(d(5e^{(-2*d*x - 2*c)} + 21e^{(-4*d*x - 4*c)} + 35e^{(-6*d*x - 6*c)} + 35e^{(-8*d*x - 8*c)} + 21e^{(-10*d*x - 10*c)} + 7e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))}$$

```
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-
2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

mupad [B] time = 1.47, size = 1333, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^4, x)
```

```
[Out] - ((16*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(315*d) + (4*a^3*exp(6*c + 6
*d*x))/(9*d) + (4*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (8
*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)
- ((32*exp(8*c + 8*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(9*d) + (8
*a^3*exp(2*c + 2*d*x))/(9*d) + (8*a^3*exp(14*c + 14*d*x))/(9*d) + (8*a*exp(
6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(3*d) + (8*a*exp(10*c + 10*d*x)*(16
*a*b + 5*a^2 + 16*b^2))/(3*d) + (16*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(3*d) +
(16*a^2*exp(12*c + 12*d*x)*(a + 2*b))/(3*d))/(9*exp(2*c + 2*d*x) + 36*exp(
4*c + 4*d*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c +
10*d*x) + 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d
*x) + exp(18*c + 18*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (2*a^3*exp(2*c
+ 2*d*x))/(9*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) + 1) - ((a*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (16*exp(2*c + 2*d*x)*(24*a
*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(63*d) + (5*a^3*exp(8*c + 8*d*x))/(9*d)
+ (10*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (40*a^2*exp(6*
c + 6*d*x)*(a + 2*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 2
0*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - (a^3/(9*d) + (16*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5
*a^3 + 16*b^3))/(9*d) + (7*a^3*exp(12*c + 12*d*x))/(9*d) + (a*exp(4*c + 4*d
*x)*(16*a*b + 5*a^2 + 16*b^2))/d + (5*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 +
16*b^2))/(3*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(3*d) + (4*a^2*exp(10*c
+ 10*d*x)*(a + 2*b))/d)/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp
(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c +
12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((a*(16*a*b + 5
*a^2 + 16*b^2))/(21*d) + (a^3*exp(4*c + 4*d*x))/(3*d) + (4*a^2*exp(2*c + 2*
d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (16*exp(4*c
+ 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(21*d) + (2*a^3*exp(10*c
+ 10*d*x))/(3*d) + (2*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) +
(20*a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (20*a^2*exp(8*c
+ 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*
exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
+ 12*d*x) + exp(14*c + 14*d*x) + 1) - a^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4
*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**3, x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

$$3.73 \quad \int \frac{\cosh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=117

$$-\frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3 d \sqrt{a+b}} + \frac{(3a-4b) \sinh(c+dx) \cosh(c+dx)}{8a^2 d} + \frac{x(3a^2-4ab+8b^2)}{8a^3} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4ad}$$

[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/d/(a+b)^(1/2)

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4146, 414, 527, 522, 206, 208}

$$-\frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3 d \sqrt{a+b}} + \frac{x(3a^2-4ab+8b^2)}{8a^3} + \frac{(3a-4b) \sinh(c+dx) \cosh(c+dx)}{8a^2 d} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) + ((3*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p+1)*(c +

$d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

Rule 4146

$\text{Int}[\text{sec}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\text{sec}[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 + ff^2*x^2)^{(m/2 - 1)}*\text{ExpandToSum}[a + b*(1 + ff^2*x^2)^{(n/2)}, x]^p, x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n/2]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{4ad} \\ &= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a^2-ab}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{4ad} \\ &= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{b^3 \text{Subst}\left(\int \frac{1}{a+b-x^2} dx, x, \tanh(c+dx)\right)}{4ad} \\ &= \frac{(3a^2-4ab+8b^2)x}{8a^3} - \frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}d} + \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} \end{aligned}$$

Mathematica [A] time = 0.50, size = 95, normalized size = 0.81

$$\frac{4(3a^2 - 4ab + 8b^2)(c + dx) + a^2 \sinh(4(c + dx)) - \frac{32b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 8a(a-b) \sinh(2(c + dx))}{32a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(c + d*x) - (32*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c + d*x)]/(32*a^3*d)

fricas [B] time = 0.46, size = 1713, normalized size = 14.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^

$6 + 8*(7*a^2*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5$
 $+ 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*$
 $b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 -$
 $4*a*b + 8*b^2)*d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*$
 $x + c)^3 - 8*(a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3$
 $*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4$
 $- 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x$
 $+ c)^3)*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*b^2*cosh(d$
 $x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/(a + b))*log((a^2*co$
 $sh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 +$
 $2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*$
 $sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a$
 $*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2$
 $+ a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*$
 $a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d$
 $*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d$
 $*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cos$
 $h(d*x + c))*sinh(d*x + c) + a) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 -$
 $4*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^$
 $2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos$
 $h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^$
 $3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(d$
 $x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*$
 $(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)*cosh(d*x + c)^6$
 $+ 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cos$
 $h(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cos$
 $h(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*b)*cosh(d*x + c)$
 $^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + 8*b^2)*$
 $d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^$
 $2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 - 4*a*b + 8$
 $*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4 - 2*a^2 + 2*a*b)$
 $*sinh(d*x + c)^2 - 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3)*sinh(d*x$
 $+ c) + 6*b^2*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh(d*$
 $x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)$
 $)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-$
 $b/(a + b))/b) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*$
 $x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^2 - a*b)*cosh(d*x$
 $+ c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(d$
 $x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)$
 $*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]$

giac [B] time = 2.62, size = 208, normalized size = 1.78

$$\frac{64b^3 \arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right) - \frac{8(3a^2-4ab+8b^2)(dx+c)}{a^3} - \frac{ae^{4dx+4c}+8ae^{2dx+2c}-8be^{2dx+2c}}{a^2} + \frac{(18a^2e^{4dx+4c}-24abe^{4dx+4c}+48b^2e^{4dx+4c})}{64d}}{\sqrt{-ab-b^2}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] $-1/64*(64*b^3*arctan(1/2*(a*e^{(2*d*x + 2*c)} + a + 2*b)/sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^3) - 8*(3*a^2 - 4*a*b + 8*b^2)*(d*x + c)/a^3 - (a*e^{(4*d*x + 4*c)} + 8*a*e^{(2*d*x + 2*c)} - 8*b*e^{(2*d*x + 2*c)})/a^2 + (18*a^2*e^{(4*d*x + 4*c)} - 24*a*b*e^{(4*d*x + 4*c)} + 48*b^2*e^{(4*d*x + 4*c)} + 8*a^2*e^{(2*d*x + 2*c)} - 8*a*b*e^{(2*d*x + 2*c)} + a^2)*e^{(-4*d*x - 4*c)}/a^3)/d$

maple [B] time = 0.54, size = 493, normalized size = 4.21

$$\frac{1}{4da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^4} + \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} + \frac{7}{8da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{b}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)`

[Out] $\frac{1}{4} \frac{d}{da} \left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^{-4} + \frac{1}{2} \frac{d}{da} \left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^{-3} + \frac{7}{8} \frac{d}{da} \left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^{-2} - \frac{b}{2} \frac{d}{da^2} \left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^{-2} + \dots$

maxima [B] time = 0.46, size = 526, normalized size = 4.50

$$\frac{3b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16\sqrt{(a+b)b}ad} + \frac{3(dx+c)}{8ad} - \frac{(8be^{(-2dx-2c)}-a)e^{(4dx+4c)}}{64a^2d} + \frac{e^{(2dx+2c)}}{8ad} - \frac{e^{(-2dx-2c)}}{8ad} - \frac{b \log(ae^{(4dx+4c)})}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $\frac{3}{16} b \log\left(\frac{(a e^{-2d*x - 2c} + a + 2b - 2\sqrt{(a+b)b})}{(a e^{-2d*x - 2c} + a + 2b + 2\sqrt{(a+b)b})}\right) / (\sqrt{(a+b)b} a d) + \frac{3}{8} (d*x + c) / (a*d) - \frac{1}{64} (8*b*e^{-2*d*x - 2*c} - a)*e^{(4*d*x + 4*c)} / (a^2*d) + \frac{1}{8} e^{(2*d*x + 2*c)} / (a*d) - \frac{1}{8} e^{(-2*d*x - 2*c)} / (a*d) - \frac{1}{4} b \log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a) / (a^2*d) + \frac{1}{4} b \log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a) / (a^2*d) + \frac{1}{8} (a*b + 2*b^2) \log\left(\frac{(a e^{(2*d*x + 2*c)} + a + 2b - 2\sqrt{(a+b)b})}{(a e^{(2*d*x + 2*c)} + a + 2b + 2\sqrt{(a+b)b})}\right) / (\sqrt{(a+b)b} a^2*d) - \frac{1}{8} (a*b + 2*b^2) \log\left(\frac{(a e^{(-2*d*x - 2*c)} + a + 2b - 2\sqrt{(a+b)b})}{(a e^{(-2*d*x - 2*c)} + a + 2b + 2\sqrt{(a+b)b})}\right) / (\sqrt{(a+b)b} a^2*d) + \frac{1}{2} (a*b + 2*b^2) (d*x + c) / (a^3*d) + \frac{1}{64} (8*b*e^{(-2*d*x - 2*c)} - a*e^{(-4*d*x - 4*c)}) / (a^2*d) + \frac{1}{16} (a^2*b + 8*a*b^2 + 8*b^3) \log\left(\frac{(a e^{(-2*d*x - 2*c)} + a + 2b - 2\sqrt{(a+b)b})}{(a e^{(-2*d*x - 2*c)} + a + 2b + 2\sqrt{(a+b)b})}\right) / (\sqrt{(a+b)b} a^3*d)$

mupad [B] time = 2.00, size = 260, normalized size = 2.22

$$\frac{x(3a^2 - 4ab + 8b^2)}{8a^3} - \frac{e^{-4c-4dx}}{64ad} + \frac{e^{4c+4dx}}{64ad} - \frac{e^{-2c-2dx}(a-b)}{8a^2d} + \frac{e^{2c+2dx}(a-b)}{8a^2d} + \frac{b^{5/2} \ln\left(\frac{4b^3 e^{2c+2dx}}{a^4} - \frac{2b^{5/2}(ad+ade)}{a}\right)}{2a^3 d \sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)`

[Out] $(x*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3) - \exp(-4*c - 4*d*x)/(64*a*d) + \exp(4*c + 4*d*x)/(64*a*d) - (\exp(-2*c - 2*d*x)*(a - b))/(8*a^2*d) + (\exp(2*c + 2*d*x)*(a - b))/(8*a^2*d)$

$$\frac{d*x*(a - b)}{(8*a^2*d) + (b^{(5/2)}*\log((4*b^3*\exp(2*c + 2*d*x))/a^4 - (2*b^{(5/2)}*(a*d + a*d*\exp(2*c + 2*d*x) + 2*b*d*\exp(2*c + 2*d*x)))/(a^4*d*(a + b)^{(1/2)})))/(2*a^3*d*(a + b)^{(1/2)})} - \frac{b^{(5/2)}*\log((4*b^3*\exp(2*c + 2*d*x))/a^4 + (2*b^{(5/2)}*(a*d + a*d*\exp(2*c + 2*d*x) + 2*b*d*\exp(2*c + 2*d*x)))/(a^4*d*(a + b)^{(1/2)})))/(2*a^3*d*(a + b)^{(1/2)})}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4/(a+b*sech(d*x+c)**2), x)

[Out] Integral(cosh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)

$$3.74 \quad \int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=76

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}} + \frac{(a-b) \sinh(c+dx)}{a^2d} + \frac{\sinh^3(c+dx)}{3ad}$$

[Out] (a-b)*sinh(d*x+c)/a^2/d+1/3*sinh(d*x+c)^3/a/d+b^2*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/d/(a+b)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4147, 390, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}} + \frac{(a-b) \sinh(c+dx)}{a^2d} + \frac{\sinh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] (b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*d) + ((a - b)*Sinh[c + d*x])/(a^2*d) + Sinh[c + d*x]^3/(3*a*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a-b}{a^2} + \frac{x^2}{a} + \frac{b^2}{a^2(a+b+ax^2)}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a-b)\sinh(c+dx)}{a^2d} + \frac{\sinh^3(c+dx)}{3ad} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{a^2d} \\
&= \frac{b^2 \tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}d} + \frac{(a-b)\sinh(c+dx)}{a^2d} + \frac{\sinh^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 79, normalized size = 1.04

$$\frac{a^{3/2} \sinh(3(c+dx)) - \frac{12b^2 \tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 3\sqrt{a}(3a-4b)\sinh(c+dx)}{12a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] ((-12*b^2*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(5/2)*d)

fricas [B] time = 0.47, size = 1616, normalized size = 21.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 3*a^3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)]/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d*sinh(d*x + c)^3), 1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4

```

*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 -
3*a^3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d
*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh
(d*x + c))/(a + b)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b -
4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d
*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)
^2*sinh(d*x + c) + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 +
a^3*b)*d*sinh(d*x + c)^3]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[36,86]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[-59,-45]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[15,66]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[55,80]Undef/Unsigned Inf encountered in limitEvaluation ti
me: 1.25Limit: Max order reached or unable to make series expansion Error:
Bad Argument Value

maple [B] time = 0.49, size = 256, normalized size = 3.37

$$\frac{1}{3da \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^3} - \frac{1}{2da \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^2} - \frac{1}{da \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)} + \frac{b}{da^2 \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)} - \frac{1}{3da \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)

[Out] $-1/3/d/a/(\tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(\tanh(1/2*d*x+1/2*c)-1)^2-1/d/a/(\tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)*b-1/3/d/a/(\tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/a/(\tanh(1/2*d*x+1/2*c)+1)^2-1/d/a/(\tanh(1/2*d*x+1/2*c)+1)+1/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)*b+1/d*b^2/a^{(5/2)}/(a+b)^{(1/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)-2*b^{(1/2)})/a^{(1/2)})+1/d*b^2/a^{(5/2)}/(a+b)^{(1/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)+2*b^{(1/2)})/a^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 \left(3 a e^{4c} - 4 b e^{4c} \right) e^{4dx} - 3 \left(3 a e^{2c} - 4 b e^{2c} \right) e^{2dx} + a e^{6dx+6c} - a e^{(-3dx-3c)}}{24 a^2 d} + \frac{1}{8} \int \frac{16 \left(b^2 e^{3dx} \right)}{a^3 e^{4dx+4c} + a^3 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $1/24*(3*(3*a*e^{(4*c)} - 4*b*e^{(4*c)})*e^{(4*d*x)} - 3*(3*a*e^{(2*c)} - 4*b*e^{(2*c)})*e^{(2*d*x)} + a*e^{(6*d*x + 6*c)} - a)*e^{(-3*d*x - 3*c)}/(a^2*d) + 1/8*\integrate(16*(b^2*e^{(3*d*x + 3*c)} + b^2*e^{(d*x + c)})/(a^3*e^{(4*d*x + 4*c)} + a^3 + 2*(a^3*e^{(2*c)} + 2*a^2*b*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [B] time = 2.21, size = 332, normalized size = 4.37

$$\frac{e^{3c+3dx}}{24ad} - \frac{e^{-3c-3dx}}{24ad} - \frac{\sqrt{b^4} \left(2 \operatorname{atan} \left(\left(e^{dx} e^c \left(\frac{2b^2}{a^8 d (a+b)^2 \sqrt{b^4}} - \frac{4 \left(2a^2 b^4 d \sqrt{b^4} + 2a^3 b^3 d \sqrt{b^4} \right)}{a^6 b^5 (a+b) \sqrt{a^6 d^2 + b a^5 d^2} \sqrt{a^5 d^2 (a+b)}} \right) - \frac{2b^2 e^{3c} e^{3dx}}{a^8 d (a+b)^2 \sqrt{b^4}} \right) \right)}{2 \sqrt{a^6 d^2 + b a^5 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)

[Out] $\exp(3c + 3d*x)/(24*a*d) - \exp(-3c - 3d*x)/(24*a*d) - ((b^4)^{(1/2)}*(2*a*\tan((\exp(d*x)*\exp(c))*((2*b^2)/(a^8*d*(a + b)^2*(b^4)^{(1/2)}) - (4*(2*a^2*b^4*d*(b^4)^{(1/2)} + 2*a^3*b^3*d*(b^4)^{(1/2)}))/(a^6*b^5*(a + b)*(a^6*d^2 + a^5*b*d^2)^{(1/2)}*(a^5*d^2*(a + b))^{(1/2)})) - (2*b^2*\exp(3c)*\exp(3d*x))/(a^8*d*(a + b)^2*(b^4)^{(1/2)}))*((a^7*(a^6*d^2 + a^5*b*d^2)^{(1/2)})/4 + (a^6*b*(a^6*d^2 + a^5*b*d^2)^{(1/2)})/4) - 2*\operatorname{atan}((b^2*\exp(d*x)*\exp(c)*(a^5*d^2*(a + b))^{(1/2)})/(2*a^2*d*(a + b)*(b^4)^{(1/2)})))/(2*(a^6*d^2 + a^5*b*d^2)^{(1/2)}) + (\exp(c + d*x)*(3*a - 4*b))/(8*a^2*d) - (\exp(-c - d*x)*(3*a - 4*b))/(8*a^2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)

[Out] Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)

$$3.75 \quad \int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=75

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2 d \sqrt{a+b}} + \frac{x(a-2b)}{2a^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2ad}$$

[Out] 1/2*(a-2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+b^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, number of rules / integrand size = 0.217, Rules used = {4146, 414, 522, 206, 208}

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2 d \sqrt{a+b}} + \frac{x(a-2b)}{2a^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^2*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4146

Int[sec[(e_) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[

m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{\operatorname{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{2ad} \\
 &= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2a^2d} + \frac{b^2\operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, \tanh(c+dx)\right)}{2a^2d} \\
 &= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 67, normalized size = 0.89

$$\frac{\frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2(a-2b)(c+dx) + a\sinh(2(c+dx))}{4a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] (2*(a - 2*b)*(c + d*x) + (4*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b] + a*Sinh[2*(c + d*x)]/(4*a^2*d)

fricas [B] time = 0.46, size = 829, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(d*x + c)^3)*sinh(d*x + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), 1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/(a + b))*arctan(1/

$$2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b)*\sqrt{-b/(a + b)}/b + 4*(2*(a - 2*b)*d*x*\cosh(d*x + c) + a*\cosh(d*x + c)^3)*\sinh(d*x + c) - a/(a^2*d*\cosh(d*x + c)^2 + 2*a^2*d*\cosh(d*x + c)*\sinh(d*x + c) + a^2*d*\sinh(d*x + c)^2)]$$

giac [A] time = 1.57, size = 125, normalized size = 1.67

$$\frac{8b^2 \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2} + \frac{4(dx+c)(a-2b)}{a^2} + \frac{e^{(2dx+2c)}}{a} - \frac{(2ae^{(2dx+2c)}-4be^{(2dx+2c)+a})e^{(-2dx-2c)}}{a^2}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{8}*(8*b^2*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + a + 2*b)/\sqrt{-a*b - b^2}))/(\sqrt{-a*b - b^2}*a^2) + 4*(d*x + c)*(a - 2*b)/a^2 + e^{(2*d*x + 2*c)}/a - (2*a*e^{(2*d*x + 2*c)} - 4*b*e^{(2*d*x + 2*c)} + a)*e^{(-2*d*x - 2*c)}/a^2)/d$

maple [B] time = 0.46, size = 278, normalized size = 3.71

$$\frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b}{da^2} - \frac{1}{2da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)

[Out] $\frac{1}{2}/d/a/(\tanh(1/2*d*x+1/2*c)-1)^2 + 1/2/d/a/(\tanh(1/2*d*x+1/2*c)-1) - 1/2/d/a*\ln(\tanh(1/2*d*x+1/2*c)-1) + 1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)*b - 1/2/d/a/(\tanh(1/2*d*x+1/2*c)+1)^2 + 1/2/d/a/(\tanh(1/2*d*x+1/2*c)+1) + 1/2/d/a*\ln(\tanh(1/2*d*x+1/2*c)+1) - 1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)*b - 1/2/d*b^{(3/2)}/a^2/(a+b)^{(1/2)}*\ln(-(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c) - (a+b)^{(1/2)}) + 1/2/d/a^2*b^{(3/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)})$

maxima [B] time = 0.42, size = 352, normalized size = 4.69

$$\frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}ad} + \frac{dx+c}{2ad} + \frac{e^{(2dx+2c)}}{8ad} - \frac{e^{(-2dx-2c)}}{8ad} - \frac{b \log\left(ae^{(4dx+4c)} + 2(a+2b)e^{(2dx+2c)} + a\right)}{4a^2d} + \frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{4}*b*\log\left(\frac{(a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})}{(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b})}\right)/(\sqrt{(a + b)*b}*a*d) + 1/2*(d*x + c)/(a*d) + 1/8*e^{(2*d*x + 2*c)}/(a*d) - 1/8*e^{(-2*d*x - 2*c)}/(a*d) - 1/4*b*\log\left(\frac{a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a}{a^2*d}\right) + 1/4*b*\log\left(\frac{(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a}{a^2*d}\right) + 1/8*(a*b + 2*b^2)*\log\left(\frac{(a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})}{(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b})}\right)/(\sqrt{(a + b)*b}*a^2*d) - 1/8*(a*b + 2*b^2)*\log\left(\frac{(a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})}{(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b})}\right)/(\sqrt{(a + b)*b}*a^2*d)$

mupad [B] time = 1.97, size = 206, normalized size = 2.75

$$\frac{x(a-2b)}{2a^2} - \frac{e^{-2c-2dx}}{8ad} + \frac{e^{2c+2dx}}{8ad} + \frac{b^{3/2} \ln\left(-\frac{4b^2 e^{2c+2dx}}{a^3} - \frac{2b^{3/2}(ad+ade^{2c+2dx}+2bde^{2c+2dx})}{a^3 d \sqrt{a+b}}\right)}{2a^2 d \sqrt{a+b}} - \frac{b^{3/2} \ln\left(\frac{2b^{3/2}(ad+ade^{2c+2dx}+2bde^{2c+2dx})}{a^3 d \sqrt{a+b}}\right)}{2a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2), x)`

[Out] $(x*(a - 2*b))/(2*a^2) - \exp(-2*c - 2*d*x)/(8*a*d) + \exp(2*c + 2*d*x)/(8*a*d) + (b^{3/2}*\log(- (4*b^2*\exp(2*c + 2*d*x))/a^3 - (2*b^{3/2}*(a*d + a*d*\exp(2*c + 2*d*x) + 2*b*d*\exp(2*c + 2*d*x)))/(a^3*d*(a + b)^{1/2}))))/(2*a^2*d*(a + b)^{1/2}) - (b^{3/2}*\log((2*b^{3/2}*(a*d + a*d*\exp(2*c + 2*d*x) + 2*b*d*\exp(2*c + 2*d*x)))/(a^3*d*(a + b)^{1/2}) - (4*b^2*\exp(2*c + 2*d*x))/a^3))/(2*a^2*d*(a + b)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2), x)`

[Out] `Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)`

$$3.76 \quad \int \frac{\cosh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{ad} - \frac{b \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

[Out] $\sinh(d*x+c)/a/d-b*\arctan(\sinh(d*x+c)*a^{(1/2)/(a+b)^{(1/2)})/a^{(3/2)}/d/(a+b)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4147, 388, 205}

$$\frac{\sinh(c+dx)}{ad} - \frac{b \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $-(b*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sinh}[c + d*x])/(\text{Sqrt}[a + b])])/(a^{(3/2)*\text{Sqrt}[a + b]*d}) + \text{Sinh}[c + d*x]/(a*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\sinh(c+dx)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{ad} \\ &= -\frac{b \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{ad} \end{aligned}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 1.00

$$\frac{\sqrt{a} \sinh(c + dx) - \frac{b \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $-\left(\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + d*x]}{\sqrt{a+b}}\right]}{\sqrt{a+b}}\right) / \sqrt{a+b} + \sqrt{a} \operatorname{Sinh}[c + d*x] / (a^{3/2} * d)$

fricas [B] time = 0.46, size = 718, normalized size = 13.81

$$\left[\frac{(a^2 + ab) \cosh(dx + c)^2 + 2(a^2 + ab) \cosh(dx + c) \sinh(dx + c) + (a^2 + ab) \sinh(dx + c)^2 - \sqrt{-a^2 - ab} (b \cosh(dx + c) + b \sinh(dx + c))}{a^3 d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] $\left[\frac{1}{2} * ((a^2 + a*b) * \cosh(d*x + c)^2 + 2 * (a^2 + a*b) * \cosh(d*x + c) * \sinh(d*x + c) + (a^2 + a*b) * \sinh(d*x + c)^2 - \sqrt{-a^2 - a*b} * (b * \cosh(d*x + c) + b * \sinh(d*x + c))) * \log((a * \cosh(d*x + c)^4 + 4 * a * \cosh(d*x + c) * \sinh(d*x + c)^3 + a * \sinh(d*x + c)^4 - 2 * (3 * a + 2 * b) * \cosh(d*x + c)^2 + 2 * (3 * a * \cosh(d*x + c)^2 - 3 * a - 2 * b) * \sinh(d*x + c)^2 + 4 * (a * \cosh(d*x + c)^3 - (3 * a + 2 * b) * \cosh(d*x + c) * \sinh(d*x + c) + 4 * (\cosh(d*x + c)^3 + 3 * \cosh(d*x + c) * \sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3 * \cosh(d*x + c)^2 - 1) * \sinh(d*x + c) - \cosh(d*x + c)) * \sqrt{-a^2 - a*b} + a) / (a * \cosh(d*x + c)^4 + 4 * a * \cosh(d*x + c) * \sinh(d*x + c)^3 + a * \sinh(d*x + c)^4 + 2 * (a + 2 * b) * \cosh(d*x + c)^2 + 2 * (3 * a * \cosh(d*x + c)^2 + a + 2 * b) * \sinh(d*x + c)^2 + 4 * (a * \cosh(d*x + c)^3 + (a + 2 * b) * \cosh(d*x + c) * \sinh(d*x + c) + a)) - a^2 - a*b) / ((a^3 + a^2 * b) * d * \cosh(d*x + c) + (a^3 + a^2 * b) * d * \sinh(d*x + c)), \frac{1}{2} * ((a^2 + a*b) * \cosh(d*x + c)^2 + 2 * (a^2 + a*b) * \cosh(d*x + c) * \sinh(d*x + c) + (a^2 + a*b) * \sinh(d*x + c)^2 - 2 * \sqrt{a^2 + a*b} * (b * \cosh(d*x + c) + b * \sinh(d*x + c))) * \arctan(1/2 * (a * \cosh(d*x + c)^3 + 3 * a * \cosh(d*x + c) * \sinh(d*x + c)^2 + a * \sinh(d*x + c)^3 + (3 * a + 4 * b) * \cosh(d*x + c) + (3 * a * \cosh(d*x + c)^2 + 3 * a + 4 * b) * \sinh(d*x + c))) / \sqrt{a^2 + a*b} - 2 * \sqrt{a^2 + a*b} * (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \arctan(1/2 * \sqrt{a^2 + a*b} * (\cosh(d*x + c) + \sinh(d*x + c)) / (a + b)) - a^2 - a*b) / ((a^3 + a^2 * b) * d * \cosh(d*x + c) + (a^3 + a^2 * b) * d * \sinh(d*x + c)) \right]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-65,-82]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]

Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[80,44] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[22,73] Undefined/Unsigned Inf encountered in limitEvaluation time: 0.84 Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.40, size = 128, normalized size = 2.46

$$\frac{1}{da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{b \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\sqrt{b}}{2\sqrt{a}}\right)}{d a^{\frac{3}{2}} \sqrt{a+b}} - \frac{b \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\sqrt{b}}{2\sqrt{a}}\right)}{d a^{\frac{3}{2}} \sqrt{a+b}} - \frac{1}{da \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2), x)

[Out] -1/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/d/a^(3/2)*b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-1/d/a^(3/2)*b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/a/(tanh(1/2*d*x+1/2*c)-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(e^{2dx+2c} - 1)e^{-dx-c}}{2ad} - \frac{1}{2} \int \frac{4(b e^{3dx+3c} + b e^{dx+c})}{a^2 e^{4dx+4c} + a^2 + 2(a^2 e^{2c} + 2 a b e^{2c}) e^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 1.74, size = 277, normalized size = 5.33

$$\frac{e^{c+dx}}{2ad} - \frac{e^{-c-dx}}{2ad} - \frac{\left(2 \operatorname{atan}\left(\frac{b^3 e^{dx} e^c \sqrt{a^3 d^2 (a+b)}}{2 a d (a+b) (b^2)^{3/2}}\right) - 2 \operatorname{atan}\left(\left(e^{dx} e^c \left(\frac{2 b^3}{a^5 d (a+b)^2 (b^2)^{3/2}} - \frac{4(2 a^2 d (b^2)^{3/2} + 2 a b d (b^2)^{3/2})}{a^4 b^3 (a+b) \sqrt{a^4 d^2 + b a^3 d^2} \sqrt{a^3 d^2 (a+b)}}\right) \right) \right)}{2 \sqrt{a^4 d^2 + b a^3 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2), x)

[Out] exp(c + d*x)/(2*a*d) - exp(- c - d*x)/(2*a*d) - ((2*atan((b^3*exp(d*x)*exp(c)*(a^3*d^2*(a + b))^(1/2))/(2*a*d*(a + b)*(b^2)^(3/2)))) - 2*atan((exp(d*x)*exp(c)*((2*b^3)/(a^5*d*(a + b)^2*(b^2)^(3/2)) - (4*(2*a^2*d*(b^2)^(3/2) + 2*a*b*d*(b^2)^(3/2)))/(a^4*b^3*(a + b)*(a^4*d^2 + a^3*b*d^2)^(1/2)*(a^3*d^2*(a + b))^(1/2)))) - (2*b^3*exp(3*c)*exp(3*d*x))/(a^5*d*(a + b)^2*(b^2)^(3/2)))*((a^5*(a^4*d^2 + a^3*b*d^2)^(1/2))/4 + (a^4*b*(a^4*d^2 + a^3*b*d^2)^(1/2))/4))*((b^2)^(1/2))/(2*(a^4*d^2 + a^3*b*d^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2), x)

[Out] Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2), x)

$$3.77 \quad \int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

[Out] arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/d/a^(1/2)/(a+b)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4147, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a} \sqrt{a+b} d} \end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)

fricas [B] time = 0.42, size = 487, normalized size = 13.53

$$\left[\frac{\sqrt{-a^2 - ab} \log \left(\frac{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 - 2(3a+2b) \cosh(dx+c)^2 + 2(3a \cosh(dx+c)^2 - 3a - 2b) \sinh(dx+c)}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a + b) \cosh(dx+c) \sinh(dx+c)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)))/((a^2 + a*b)*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-13,-93]Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.28, size = 82, normalized size = 2.28

$$\frac{\arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\sqrt{b}}{2\sqrt{a}}\right)}{d\sqrt{a+b} \sqrt{a}} + \frac{\arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\sqrt{b}}{2\sqrt{a}}\right)}{d\sqrt{a+b} \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2),x)

[Out] 1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{b \operatorname{sech}(dx+c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(sech(d*x + c)/(b*sech(d*x + c)^2 + a), x)

mupad [B] time = 1.90, size = 108, normalized size = 3.00

$$\frac{\ln\left(-\frac{4(b-b e^{2c+2dx})}{a^2(a+b)} - \frac{8b e^{c+dx}}{(-a)^{5/2}\sqrt{a+b}}\right) - \ln\left(\frac{8b e^{c+dx}}{(-a)^{5/2}\sqrt{a+b}} - \frac{4(b-b e^{2c+2dx})}{a^2(a+b)}\right)}{2\sqrt{-a} d \sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)

[Out] -(log(-(4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b)) - (8*b*exp(c + d*x))/((-a)^(5/2)*(a + b)^(1/2)))) - log((8*b*exp(c + d*x))/((-a)^(5/2)*(a + b)^(1/2)) - (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b))))/(2*(-a)^(1/2)*d*(a + b)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2), x)

$$3.78 \quad \int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

[Out] $\operatorname{arctanh}(b^{(1/2)}\tanh(d*x+c)/(a+b)^{(1/2)})/d/b^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4146, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] `Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]`

[Out] `ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 4146

`Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]`

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d} \end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]`

[Out] `ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)`

fricas [B] time = 0.43, size = 411, normalized size = 11.42

$$\log \left(\frac{a^2 \cosh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c)^3 + a^2 \sinh(dx+c)^4 + 2(a^2 + 2ab) \cosh(dx+c)^2 + 2(3a^2 \cosh(dx+c)^2 + a^2 + 2ab) \sinh(dx+c)^2 + a^2 + 2ab}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a+2b) \cosh(dx+c)^2 + 2(a+2b) \sinh(dx+c)^2 + a + 2b} \right) \frac{1}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2))/((a*b + b^2)*d)]

giac [A] time = 0.64, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*d)

maple [B] time = 0.25, size = 104, normalized size = 2.89

$$\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\sqrt{b}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{a+b}\right)}{2d\sqrt{b}\sqrt{a+b}} + \frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{a+b}\right)}{2d\sqrt{b}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x)

[Out] -1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))

maxima [B] time = 0.42, size = 66, normalized size = 1.83

$$\frac{\log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2\sqrt{(a+b)b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] -1/2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d)

mupad [B] time = 0.57, size = 125, normalized size = 3.47

$$\frac{\operatorname{atan}\left(\frac{d(a+2b)}{2\sqrt{-bd^2(a+b)}} + \frac{ae^{2c}e^{2dx}\left(\frac{4}{a^2d} + \frac{(a+2b)(ad+2bd)}{a^2\sqrt{-bd^2-ad^2}\sqrt{-bd^2(a+b)}}\right)\sqrt{-bd^2-ad^2}}{2}\right)}{\sqrt{-bd^2-ad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)), x)`

[Out] `atan((d*(a + 2*b))/(2*(-b*d^2*(a + b))^(1/2)) + (a*exp(2*c)*exp(2*d*x)*(4/(a^2*d) + ((a + 2*b)*(a*d + 2*b*d))/(a^2*(- b^2*d^2 - a*b*d^2)^(1/2))*(-b*d^2*(a + b))^(1/2)))*(- b^2*d^2 - a*b*d^2)^(1/2))/2)/(- b^2*d^2 - a*b*d^2)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2), x)`

[Out] `Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2), x)`

$$3.79 \quad \int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

[Out] arctan(sinh(d*x+c))/b/d-arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/d/(a+b)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4147, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] ArcTan[Sinh[c + d*x]]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(b*Sqrt[a + b]*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bax^2)} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{bd} - \frac{a \operatorname{Subst}\left(\int \frac{1}{a+bax^2} dx, x, \sinh(c+dx)\right)}{bd}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}d}$$

Mathematica [B] time = 0.71, size = 194, normalized size = 3.53

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(2\sqrt{a+b} \sqrt{(\cosh(c) - \sinh(c))^2} \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a} \cosh(c+dx)\right)}{2bd\sqrt{a+b} \sqrt{(\cosh(c) - \sinh(c))^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*Cosh[c] + 2*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] - Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*Sinh[c]))/(2*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])

fricas [B] time = 0.44, size = 526, normalized size = 9.56

$$\left[\frac{\sqrt{-\frac{a}{a+b}} \log\left(\frac{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 - 2(3a+2b) \cosh(dx+c)^2 + 2(3a \cosh(dx+c)^2 - 3a - 2b) \sinh(dx+c)^2 + 4(a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4)}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), -(sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + sqrt(a/(a + b))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*sqrt(a/(a + b))/a - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
 lynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomi
 al with parameters. This might be wrong.The choice was done assuming [a,b]=[
 -65,-82]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
 Undef/Unsigned Inf encountered in limitEvaluation time: 0.57Limit: Max orde
 r reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.25, size = 108, normalized size = 1.96

$$\frac{\sqrt{a} \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\sqrt{b}}{2\sqrt{a}}\right) - \sqrt{a} \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\sqrt{b}}{2\sqrt{a}}\right)}{db\sqrt{a+b}} + \frac{2 \arctan\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x)

[Out] -1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*
 b^(1/2))/a^(1/2))-1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(
 1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+2/d/b*arctan(tanh(1/2*d*x+1/2*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \arctan\left(e^{(dx+c)}\right)}{bd} - 8 \int \frac{ae^{(3dx+3c)} + ae^{(dx+c)}}{4\left(abe^{(4dx+4c)} + ab + 2\left(abe^{(2c)} + 2b^2e^{(2c)}\right)e^{(2dx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] 2*arctan(e^(d*x + c))/(b*d) - 8*integrate(1/4*(a*e^(3*d*x + 3*c) + a*e^(d*x
 + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*
 x)), x)

mupad [B] time = 1.81, size = 307, normalized size = 5.58

$$\frac{2 \operatorname{atan}\left(\frac{e^{dx} e^c \left(9 a^2 \sqrt{b^2 d^2} + 16 b^2 \sqrt{b^2 d^2} + 24 a b \sqrt{b^2 d^2}\right)}{9 d a^2 b + 24 d a b^2 + 16 d b^3}\right)}{\sqrt{b^2 d^2}} + \frac{\sqrt{a} \left(2 \operatorname{atan}\left(\frac{\sqrt{a} e^{dx} e^c \sqrt{b^2 d^2} (a+b)}{2 b d (a+b)}\right) + 2 \operatorname{atan}\left(\frac{4 b^4 d^2 e^{dx} e^c + 4 a^2 b^2 d^2}{\dots}\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)

[Out] (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + 16*b^2*(b^2*d^2)^(1/2) +
 24*a*b*(b^2*d^2)^(1/2)))/(16*b^3*d + 24*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^(1
 /2) - (a^(1/2)*(2*atan((a^(1/2)*exp(d*x)*exp(c)*(b^2*d^2*(a + b))^(1/2))/(2
 *b*d*(a + b))) + 2*atan((4*b^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*exp(d*x)
 *exp(c) - a*exp(d*x)*exp(c)*(b^3*d^2 + a*b^2*d^2)^(1/2)*(b^2*d^2*(a + b))^(
 1/2) + 8*a*b^3*d^2*exp(d*x)*exp(c) + a*exp(3*c)*exp(3*d*x)*(b^3*d^2 + a*b^2
 d^2)^(1/2)(b^2*d^2*(a + b))^(1/2)))/(a^(1/2)*d*(2*a*b + 2*b^2)*(b^2*d^2*(a
 + b))^(1/2)))))/(2*(b^3*d^2 + a*b^2*d^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2), x)

[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2), x)

$$3.80 \quad \int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=52

$$\frac{\tanh(c+dx)}{bd} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

[Out] $-a*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})/b^{(3/2)}/d/(a+b)^{(1/2)}+\tanh(d*x+c)/b/d$

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4146, 388, 208}

$$\frac{\tanh(c+dx)}{bd} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] $-\left(\frac{a*\operatorname{ArcTanh}\left[\frac{\sqrt{b}*\operatorname{Tanh}[c+d*x]}{\sqrt{a+b}}\right]}{b^{3/2}*\operatorname{Sqrt}[a+b]*d}\right) + \operatorname{Tanh}[c+d*x]/(b*d)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1)/(b*(n*(p+1)+1)) - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tanh(c+dx)}{bd} - \frac{a \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{bd} \\ &= -\frac{a \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}d} + \frac{\tanh(c+dx)}{bd} \end{aligned}$$

Mathematica [B] time = 0.63, size = 182, normalized size = 3.50

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(\sqrt{a+b} \operatorname{sech}(c) \sinh(dx) \sqrt{b(\cosh(c) - \sinh(c))^4} \operatorname{sech}(c+dx) + a(\sinh(c) - \cosh(c)) \right)}{2bd\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))^4} (a + b \operatorname{sech}^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(a*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*Sech[c]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x])/(2*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.43, size = 645, normalized size = 12.40

$$\left[\frac{(a \cosh(dx+c)^2 + 2a \cosh(dx+c) \sinh(dx+c) + a \sinh(dx+c)^2 + a) \sqrt{ab+b^2} \log\left(\frac{a^2 \cosh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c) + a^2 \sinh(dx+c)^4 + a^2}{2((ab^2+b^3))}\right)}{2((ab^2+b^3))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 4*a*b - 4*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x + c)^2 + (a*b^2 + b^3)*d), -(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 2*a*b + 2*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x + c)^2 + (a*b^2 + b^3)*d)]

giac [A] time = 0.62, size = 72, normalized size = 1.38

$$\frac{a \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b} + \frac{2}{b(e^{(2dx+2c)+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -(a*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b) + 2/(b*(e^(2*d*x + 2*c) + 1))/d

maple [B] time = 0.22, size = 141, normalized size = 2.71

$$\frac{a \ln\left(-\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{a+b}\right)}{2db^2\sqrt{a+b}} - \frac{a \ln\left(\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{a+b}\right)}{2db^2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x)`

[Out] $\frac{1}{2} \frac{d a / b^{3/2}}{(a+b)^{1/2}} \ln(- (a+b)^{1/2} \tanh(1/2 d x + 1/2 c)^2 + 2 b^{1/2} \tanh(1/2 d x + 1/2 c) - (a+b)^{1/2}) - \frac{1}{2} \frac{d a / b^{3/2}}{(a+b)^{1/2}} \ln((a+b)^{1/2} \tanh(1/2 d x + 1/2 c)^2 + 2 b^{1/2} \tanh(1/2 d x + 1/2 c) + (a+b)^{1/2}) + \frac{2}{d b \tanh(1/2 d x + 1/2 c) (\tanh(1/2 d x + 1/2 c)^2 + 1)}$

maxima [B] time = 0.43, size = 91, normalized size = 1.75

$$\frac{a \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2\sqrt{(a+b)b}bd} + \frac{2}{(be^{(-2dx-2c)+b})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")`

[Out] $\frac{1}{2} a \log\left(\frac{a e^{-2 d x - 2 c} + a + 2 b - 2 \sqrt{(a + b) b}}{a e^{-2 d x - 2 c} + a + 2 b + 2 \sqrt{(a + b) b}}\right) / (\sqrt{(a + b) b} b d) + \frac{2}{(b e^{-2 d x - 2 c} + b) d}$

mupad [B] time = 0.48, size = 166, normalized size = 3.19

$$\frac{a \ln\left(\frac{4 e^{2 c + 2 d x}}{b} - \frac{2(a d + a d e^{2 c + 2 d x} + 2 b d e^{2 c + 2 d x})}{b^{3/2} d \sqrt{a + b}}\right)}{2 b^{3/2} d \sqrt{a + b}} - \frac{2}{b d (e^{2 c + 2 d x} + 1)} - \frac{a \ln\left(\frac{4 e^{2 c + 2 d x}}{b} + \frac{2(a d + a d e^{2 c + 2 d x} + 2 b d e^{2 c + 2 d x})}{b^{3/2} d \sqrt{a + b}}\right)}{2 b^{3/2} d \sqrt{a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)`

[Out] $(a \log((4 \exp(2c + 2dx))/b - (2(a*d + a*d*\exp(2c + 2dx)) + 2*b*d*\exp(2c + 2dx))))/(b^{3/2}*d*(a + b)^{1/2})) / (2*b^{3/2}*d*(a + b)^{1/2}) - 2 / (b*d*(\exp(2c + 2dx) + 1)) - (a \log((4 \exp(2c + 2dx))/b + (2(a*d + a*d*\exp(2c + 2dx)) + 2*b*d*\exp(2c + 2dx)))) / (b^{3/2}*d*(a + b)^{1/2})) / (2*b^{3/2}*d*(a + b)^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2),x)`

[Out] `Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2), x)`

$$3.81 \quad \int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=86

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2 d \sqrt{a+b}} - \frac{(2a-b) \tan^{-1}(\sinh(c+dx))}{2b^2 d} + \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2bd}$$

[Out] $-1/2*(2*a-b)*\arctan(\sinh(d*x+c))/b^2/d+a^{(3/2)*\arctan(\sinh(d*x+c))*a^{(1/2)/(a+b)^{(1/2))}/b^2/d/(a+b)^{(1/2)+1/2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/b/d}$

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4147, 414, 522, 203, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2 d \sqrt{a+b}} - \frac{(2a-b) \tan^{-1}(\sinh(c+dx))}{2b^2 d} + \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2), x]

[Out] $-\left(\frac{(2a-b)\operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{2b^2d} + \frac{a^{3/2}\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Sinh}[c+d*x]}{\sqrt{a+b}}\right]}{b^2\sqrt{a+b}d} + \frac{\operatorname{Sech}[c+d*x]\operatorname{Tanh}[c+d*x]}{2b^2d}\right)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4147

Int[sec[(e_) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^(m +

$n \cdot p + 1)/2$), x], x , $\text{Sin}[e + f \cdot x]/ff$], x] /; $\text{FreeQ}[\{a, b, e, f\}, x]$ && $\text{IntegerQ}[(m - 1)/2]$ && $\text{IntegerQ}[n/2]$ && $\text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{\text{sech}^5(c + dx)}{a + b \text{sech}^2(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{sech}(c + dx) \tanh(c + dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a-b-ax^2}{(1+x^2)(a+b+ax^2)} dx, x, \sinh(c + dx)\right)}{2bd} \\ &= \frac{\text{sech}(c + dx) \tanh(c + dx)}{2bd} + \frac{a^2 \text{Subst}\left(\int \frac{1}{a+b+ax^2} dx, x, \sinh(c + dx)\right)}{b^2 d} - \frac{(2a - b) \text{Sinh}^{-1}\left(\frac{\sinh(c + dx)}{\sqrt{a+b}}\right)}{b^2 d} \\ &= -\frac{(2a - b) \tan^{-1}(\sinh(c + dx))}{2b^2 d} + \frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \sinh(c + dx)}{\sqrt{a+b}}\right)}{b^2 \sqrt{a+b} d} + \frac{\text{sech}(c + dx) \tanh(c + dx)}{2bd} \end{aligned}$$

Mathematica [B] time = 1.84, size = 213, normalized size = 2.48

$$\frac{\cosh(c) \text{sech}^2(c + dx) (a \cosh(2(c + dx)) + a + 2b) \left(2a^{3/2} (\tanh(c) - 1) \tan^{-1}\left(\frac{\sqrt{a+b} \sqrt{(\cosh(c) - \sinh(c))^2 (\sinh(c) + \cosh(c))}}{\sqrt{a}}\right)\right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2), x]

[Out] (Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*Sqrt[a + b]*Sech[c]^2*Sech[c + d*x]^2*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^(3/2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(-1 + Tanh[c]) - Sqrt[a + b]*Sech[c]*Sqrt[(Cosh[c] - Sinh[c])^2]*(2*(2*a - b)*ArcTan[Tanh[(c + d*x)/2]] - b*Sech[c + d*x]*Tanh[c])))/(4*b^2*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])

fricas [B] time = 0.49, size = 1518, normalized size = 17.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d*x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 2*((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a - b)*si

```

nh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^
2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(cosh(d*x + c) + sinh(d*x + c))
- 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(b^2*d*cos
h(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^
4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*s
inh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x +
c)), (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x +
c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b
))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + (a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b))*arctan(1/2*(a*c
osh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3
*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*
sqrt(a/(a + b))/a - ((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a - b)*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*(2*a - b)*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(
cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x +
c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d
*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*
d*cosh(d*x + c))*sinh(d*x + c))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Undef/Unsigned Inf encountered in limitEvaluation time: 0.48Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.28, size = 189, normalized size = 2.20

$$\frac{a^{\frac{3}{2}} \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\sqrt{b}}{2\sqrt{a}}\right)}{db^2\sqrt{a+b}} + \frac{a^{\frac{3}{2}} \arctan\left(\frac{2\sqrt{a+b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\sqrt{b}}{2\sqrt{a}}\right)}{db^2\sqrt{a+b}} - \frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{db\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{\tanh\left(\frac{dx}{2}\right)}{db\left(\tanh^2\left(\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2
*b^(1/2))/a^(1/2))+1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2*tan
```

$h(1/2*d*x+1/2*c)^3+1/d/b/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)+1/d/b*\arctan(\tanh(1/2*d*x+1/2*c))-2/d/b^2*\arctan(\tanh(1/2*d*x+1/2*c))*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3dx+3c)} - e^{(dx+c)}}{bde^{(4dx+4c)} + 2bde^{(2dx+2c)} + bd} - \frac{(2ae^c - be^c) \arctan(e^{(dx+c)}) e^{(-c)}}{b^2d} + 32 \int \frac{a^2e^{(3dx+3c)} + a^2e^{(dx+c)}}{16(ab^2e^{(4dx+4c)} + ab^2 + 2(ab^2e^{(2c)}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $(e^{(3*d*x + 3*c)} - e^{(d*x + c)})/(b*d*e^{(4*d*x + 4*c)} + 2*b*d*e^{(2*d*x + 2*c)} + b*d) - (2*a*e^c - b*e^c)*\arctan(e^{(d*x + c)})*e^{(-c)}/(b^2*d) + 32*\integrate(1/16*(a^2*e^{(3*d*x + 3*c)} + a^2*e^{(d*x + c)})/(a*b^2*e^{(4*d*x + 4*c)} + a*b^2 + 2*(a*b^2*e^{(2*c)} + 2*b^3*e^{(2*c)}))*e^{(2*d*x)}, x)$

mupad [B] time = 3.50, size = 946, normalized size = 11.00

$$\sqrt{a^3} \left(2 \operatorname{atan} \left(\left(e^{dx} e^c \left(\frac{64(6b^3d(a^3)^{3/2} + 2b^6d\sqrt{a^3} + 6ab^2d(a^3)^{3/2} - 4ab^5d\sqrt{a^3} - 6a^2b^4d\sqrt{a^3})}{a^4b^4(a+b)(b^2+ab)\sqrt{b^5d^2+ab^4d^2}\sqrt{b^4d^2(a+b)}(3a^3-3ab^2+b^3)} \right) - \frac{32(3a^5\sqrt{b^5d^2+ab^4d^2} + a^2b^3\sqrt{b^5d^2+ab^4d^2})}{a^2b^6d(a+b)^2(b^2+ab)\sqrt{b^5d^2+ab^4d^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)), x)

[Out] $((a^3)^{(1/2)}*(2*\operatorname{atan}((\exp(d*x)*\exp(c))*((64*(6*b^3*d*(a^3)^{(3/2)} + 2*b^6*d*(a^3)^{(1/2)} + 6*a*b^2*d*(a^3)^{(3/2)} - 4*a*b^5*d*(a^3)^{(1/2)} - 6*a^2*b^4*d*(a^3)^{(1/2)}))/((a^4*b^4*(a+b)*(a*b+b^2)*(b^5*d^2+a*b^4*d^2)^{(1/2)}*(b^4*d^2*(a+b))^{(1/2)}*(3*a^3-3*a*b^2+b^3)) - (32*(3*a^5*(b^5*d^2+a*b^4*d^2)^{(1/2)} + a^2*b^3*(b^5*d^2+a*b^4*d^2)^{(1/2)} - 3*a^3*b^2*(b^5*d^2+a*b^4*d^2)^{(1/2)}))/((a^2*b^6*d*(a+b)^2*(a*b+b^2)*(b^5*d^2+a*b^4*d^2)^{(1/2)}*(a^3)^{(1/2)}*(3*a^3-3*a*b^2+b^3)))) + (32*\exp(3*c)*\exp(3*d*x)*(3*a^5*(b^5*d^2+a*b^4*d^2)^{(1/2)} + a^2*b^3*(b^5*d^2+a*b^4*d^2)^{(1/2)} - 3*a^3*b^2*(b^5*d^2+a*b^4*d^2)^{(1/2)}))/((a^2*b^6*d*(a+b)^2*(a*b+b^2)*(b^5*d^2+a*b^4*d^2)^{(1/2)}*(a^3)^{(1/2)}*(3*a^3-3*a*b^2+b^3))))*((a^2*b^7*(b^5*d^2+a*b^4*d^2)^{(1/2)})/64 + (a^3*b^6*(b^5*d^2+a*b^4*d^2)^{(1/2)})/32 + (a^4*b^5*(b^5*d^2+a*b^4*d^2)^{(1/2)})/64)) + 2*\operatorname{atan}((a^2*\exp(d*x)*\exp(c)*(b^4*d^2*(a+b))^{(1/2)})/(2*b^2*d*(a+b)*(a^3)^{(1/2)})))/((2*(b^5*d^2+a*b^4*d^2)^{(1/2)}) - (\operatorname{atan}((\exp(d*x)*\exp(c))*((18*a^7*(b^4*d^2)^{(1/2)} - b^7*(b^4*d^2)^{(1/2)} - 21*a^2*b^5*(b^4*d^2)^{(1/2)} + 12*a^3*b^4*(b^4*d^2)^{(1/2)} + 30*a^4*b^3*(b^4*d^2)^{(1/2)} - 36*a^5*b^2*(b^4*d^2)^{(1/2)} + 8*a*b^6*(b^4*d^2)^{(1/2)} - 9*a^6*b*(b^4*d^2)^{(1/2)}))/((b^8*d*(4*a^2-4*a*b+b^2)^{(1/2)} + 9*a^2*b^6*d*(4*a^2-4*a*b+b^2)^{(1/2)} + 6*a^3*b^5*d*(4*a^2-4*a*b+b^2)^{(1/2)} - 18*a^4*b^4*d*(4*a^2-4*a*b+b^2)^{(1/2)} + 9*a^6*b^2*d*(4*a^2-4*a*b+b^2)^{(1/2)} - 6*a*b^7*d*(4*a^2-4*a*b+b^2)^{(1/2)}))*(4*a^2-4*a*b+b^2)^{(1/2)}))/((b^4*d^2)^{(1/2)} + \exp(c+d*x)/(b*d*(\exp(2*c+2*d*x)+1)) - (2*\exp(c+d*x))/(b*d*(2*\exp(2*c+2*d*x)+\exp(4*c+4*d*x)+1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2), x)

[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2), x)

$$3.82 \quad \int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=77

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}} - \frac{(a-b) \tanh(c+dx)}{b^2d} - \frac{\tanh^3(c+dx)}{3bd}$$

[Out] $a^2 \arctan\left(\frac{b^{1/2} \tanh(d*x+c)}{(a+b)^{1/2}}\right) / b^{5/2} / d / (a+b)^{1/2} - (a-b) \tanh(d*x+c) / b^2 / d - 1/3 \tanh(d*x+c)^3 / b / d$

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4146, 390, 208}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}} - \frac{(a-b) \tanh(c+dx)}{b^2d} - \frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2), x]

[Out] $(a^2 \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] \operatorname{Tanh}[c + d*x]) / \operatorname{Sqrt}[a + b]]) / (b^{5/2} \operatorname{Sqrt}[a + b] * d) - ((a - b) \operatorname{Tanh}[c + d*x]) / (b^2 * d) - \operatorname{Tanh}[c + d*x]^3 / (3 * b * d)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{a-b}{b^2} - \frac{x^2}{b} + \frac{a^2}{b^2(a+b-bx^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a-b)\tanh(c+dx)}{b^2d} - \frac{\tanh^3(c+dx)}{3bd} + \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{b^2d} \\
&= \frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}d} - \frac{(a-b)\tanh(c+dx)}{b^2d} - \frac{\tanh^3(c+dx)}{3bd}
\end{aligned}$$

Mathematica [B] time = 2.18, size = 214, normalized size = 2.78

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(3a^2(\cosh(2c) - \sinh(2c)) \tanh^{-1}\left(\frac{(\cosh(2c) - \sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(c))}{2\sqrt{a+b}\sqrt{b(\cosh(c) - \sinh(c))}}\right)\right)}{6b^2d\sqrt{a+b}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a^2*ArcTanh[(Sech[d*x]*Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*sqrt[a + b]*sqrt[b*(Cosh[c] - Sinh[c])^4]))*(Cosh[2*c] - Sinh[2*c]) + sqrt[a + b]*Sech[c + d*x]*sqrt[b*(Cosh[c] - Sinh[c])^4]*(Sech[c]*(-3*a + 2*b + b*Sech[c + d*x]^2)*Sinh[d*x] + b*Sech[c + d*x]*Tanh[c]))/(6*b^2*sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.49, size = 1905, normalized size = 24.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 4*a*b^2 - 8*b^3 + 24*(a^2*b - b^3)*cosh(d*x + c)^2 + 24*(a^2*b - b^3 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c)]/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3 +

$b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3 + b^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 24*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 6*a^2*b + 2*a*b^2 - 4*b^3 + 12*(a^2*b - b^3)*cosh(d*x + c)^2 + 12*(a^2*b - b^3 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)/sqrt(-a*b - b^2)/(a*b + b^2)) + 24*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3 + b^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))]$

giac [A] time = 0.64, size = 118, normalized size = 1.53

$$\frac{3a^2 \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^2} + \frac{2(3ae^{(4dx+4c)}+6ae^{(2dx+2c)}-6be^{(2dx+2c)}+3a-2b)}{b^2(e^{(2dx+2c)}+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] 1/3*(3*a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 3*a - 2*b)/(b^2*(e^(2*d*x + 2*c) + 1)^3))/d

maple [B] time = 0.28, size = 316, normalized size = 4.10

$$\frac{a^2 \ln\left(-\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{a+b}\right)}{2db^2\sqrt{a+b}} + \frac{a^2 \ln\left(\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{a+b}\right)}{2db^2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x)

[Out] -1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-2/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a+2/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5-4/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a+4/3/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)

$*c)^3 - 2/d/b^2 / (\tanh(1/2*d*x+1/2*c)^2 + 1)^3 * \tanh(1/2*d*x+1/2*c) * a + 2/d/b / (\tanh(1/2*d*x+1/2*c)^2 + 1)^3 * \tanh(1/2*d*x+1/2*c)$

maxima [B] time = 0.46, size = 160, normalized size = 2.08

$$\frac{a^2 \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2\sqrt{(a+b)b}b^2d} - \frac{2\left(6(a-b)e^{(-2dx-2c)} + 3ae^{(-4dx-4c)} + 3a - 2b\right)}{3\left(3b^2e^{(-2dx-2c)} + 3b^2e^{(-4dx-4c)} + b^2e^{(-6dx-6c)} + b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/2*a^2*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*b^2*d) - 2/3*(6*(a - b)*e^{(-2*d*x - 2*c)} + 3*a*e^{(-4*d*x - 4*c)} + 3*a - 2*b)/((3*b^2*e^{(-2*d*x - 2*c)} + 3*b^2*e^{(-4*d*x - 4*c)} + b^2*e^{(-6*d*x - 6*c)} + b^2)*d)$

mupad [B] time = 1.99, size = 334, normalized size = 4.34

$$\frac{8}{3bd(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{4}{bd(2e^{2c+2dx} + e^{4c+4dx} + 1)} + \frac{2a}{b^2d(e^{2c+2dx} + 1)} - \frac{a^2 \ln\left(\frac{4a^2(2e^{2c+2dx} + 1)}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1}\right)}{3bd(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)),x)

[Out] $8/(3*b*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - 4/(b*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) + (2*a)/(b^2*d*(\exp(2*c + 2*d*x) + 1)) - (a^2*\log((4*a^2*(2*a*b + a^2 + a^2*\exp(2*c + 2*d*x) + 8*b^2*\exp(2*c + 2*d*x) + 8*a*b*\exp(2*c + 2*d*x)))/(b^5*(a + b)) - (8*a^2*(a + 2*a*\exp(2*c + 2*d*x) + 4*b*\exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2))))/(2*b^(5/2)*d*(a + b)^(1/2)) + (a^2*\log((8*a^2*(a + 2*a*\exp(2*c + 2*d*x) + 4*b*\exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2)) + (4*a^2*(2*a*b + a^2 + a^2*\exp(2*c + 2*d*x) + 8*b^2*\exp(2*c + 2*d*x) + 8*a*b*\exp(2*c + 2*d*x)))/(b^5*(a + b))))/(2*b^(5/2)*d*(a + b)^(1/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2), x)

$$3.83 \quad \int \frac{\cosh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=125

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}} - \frac{b^3\sinh(c+dx)}{2a^3d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d}$$

[Out] $1/2*b^2*(6*a+5*b)*\arctan(\sinh(d*x+c)*a^{(1/2)}/(a+b)^{(1/2)})/a^{(7/2)}/(a+b)^{(3/2)}/d+(a-2*b)*\sinh(d*x+c)/a^3/d+1/3*\sinh(d*x+c)^3/a^2/d-1/2*b^3*\sinh(d*x+c)/a^3/(a+b)/d/(a+b+a*\sinh(d*x+c)^2)$

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4147, 390, 385, 205}

$$-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}} + \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]`

[Out] $(b^2*(6*a + 5*b)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sinh}[c + d*x])/\text{Sqrt}[a + b]])/(2*a^{(7/2)}*(a + b)^{(3/2)*d} + ((a - 2*b)*\text{Sinh}[c + d*x])/(a^3*d) + \text{Sinh}[c + d*x]^3/(3*a^2*d) - (b^3*\text{Sinh}[c + d*x])/(2*a^3*(a + b)*d*(a + b + a*\text{Sinh}[c + d*x]^2))$

Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 385

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])`

Rule 390

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]`

Rule 4147

`Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]`

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a-2b}{a^3} + \frac{x^2}{a^2} + \frac{b^2(3a+2b)+3ab^2x^2}{a^3(a+b+ax^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(3a+2b)+3ab^2x^2}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{a^3d} \\
&= \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} - \frac{b^3\sinh(c+dx)}{2a^3(a+b)d(a+b+a\sinh^2(c+dx))} \\
&= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d} + \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} - \frac{b^3\sinh(c+dx)}{2a^3(a+b)d(a+b+a\sinh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 113, normalized size = 0.90

$$\frac{a^{3/2}\sinh(3(c+dx)) + 3\sqrt{a}\sinh(c+dx)\left(-\frac{4b^3}{(a+b)(a\cosh(2(c+dx))+a+2b)} + 3a - 8b\right) - \frac{6b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}}{12a^{7/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((-6*b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))))*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)]/(12*a^(7/2)*d)

fricas [B] time = 0.54, size = 5842, normalized size = 46.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/24*((a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^10 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^3 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^6 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*a^2*b^3 + 60*a*b^4 + 35*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cos

$$\begin{aligned}
& h(dx + c)^2 * \sinh(dx + c)^4 + 8 * (15 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^7 + 7 * (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^5 + 5 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^3 - (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)) * \sinh(dx + c)^3 - (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^2 + (45 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^8 + 28 * (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^6 - 11 * a^5 - 2 * a^4 * b + 29 * a^3 * b^2 + 20 * a^2 * b^3 + 30 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^4 - 12 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 - 6 * ((6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^7 + 7 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c) * \sinh(dx + c)^6 + (6 * a^2 * b^2 + 5 * a * b^3) * \sinh(dx + c)^7 + 2 * (6 * a^2 * b^2 + 17 * a * b^3 + 10 * b^4) * \cosh(dx + c)^5 + (12 * a^2 * b^2 + 34 * a * b^3 + 20 * b^4 + 21 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^5 + 5 * (7 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^3 + 2 * (6 * a^2 * b^2 + 17 * a * b^3 + 10 * b^4) * \cosh(dx + c)) * \sinh(dx + c)^4 + (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^3 + (35 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^4 + 6 * a^2 * b^2 + 5 * a * b^3 + 20 * (6 * a^2 * b^2 + 17 * a * b^3 + 10 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^3 + (21 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^5 + 20 * (6 * a^2 * b^2 + 17 * a * b^3 + 10 * b^4) * \cosh(dx + c)^3 + 3 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)) * \sinh(dx + c)^2 + (7 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^6 + 10 * (6 * a^2 * b^2 + 17 * a * b^3 + 10 * b^4) * \cosh(dx + c)^4 + 3 * (6 * a^2 * b^2 + 5 * a * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)) * \sqrt{-a^2 - a * b} * \log((a * \cosh(dx + c)^4 + 4 * a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 - 2 * (3 * a + 2 * b) * \cosh(dx + c)^2 + 2 * (3 * a * \cosh(dx + c)^2 - 3 * a - 2 * b) * \sinh(dx + c)^2 + 4 * (a * \cosh(dx + c)^3 - (3 * a + 2 * b) * \cosh(dx + c)) * \sinh(dx + c) - 4 * (\cosh(dx + c)^3 + 3 * \cosh(dx + c) * \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 * \cosh(dx + c)^2 - 1) * \sinh(dx + c) - \cosh(dx + c)) * \sqrt{-a^2 - a * b} + a) / (a * \cosh(dx + c)^4 + 4 * a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 + 2 * (a + 2 * b) * \cosh(dx + c)^2 + 2 * (3 * a * \cosh(dx + c)^2 + a + 2 * b) * \sinh(dx + c)^2 + 4 * (a * \cosh(dx + c)^3 + (a + 2 * b) * \cosh(dx + c)) * \sinh(dx + c) + a)) + 2 * (5 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^9 + 4 * (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^7 + 6 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^5 - 4 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^3 - (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^7 + 7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c) * \sinh(dx + c)^6 + (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \sinh(dx + c)^7 + 2 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d * \cosh(dx + c)^5 + (21 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^2 + 2 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d) * \sinh(dx + c)^5 + (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^3 + 5 * (7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^3 + 2 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^4 + (35 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^4 + 20 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d * \cosh(dx + c)^2 + (a^7 + 2 * a^6 * b + a^5 * b^2) * d) * \sinh(dx + c)^3 + (21 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^5 + 20 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d * \cosh(dx + c)^3 + 3 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^2 + (7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^6 + 10 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * d * \cosh(dx + c)^4 + 3 * (a^7 + 2 * a^6 * b + a^5 * b^2) * d * \cosh(dx + c)^2) * \sinh(dx + c)), 1/24 * ((a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^10 + 10 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c) * \sinh(dx + c)^9 + (a^5 + 2 * a^4 * b + a^3 * b^2) * \sinh(dx + c)^10 + (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^8 + (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3 + 45 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 8 * (15 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^3 + (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)) * \sinh(dx + c)^7 + 2 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4) * \cosh(dx + c)^6 + 2 * (5 * a^5 + 16 * a^4 * b - 31 * a^3 * b^2 - 102 * a^2 * b^3 - 60 * a * b^4 + 105 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^4 + 14 * (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 4 * (63 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(dx + c)^5 + 14 * (11 * a^5 + 2 * a^4 * b - 29 * a^3 * b^2 - 20 * a^2 * b^3) * \cosh(dx + c)^3 + 3 * (5 * a^5
\end{aligned}$$

$$\begin{aligned}
& + 16a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c) \sinh(dx \\
& + c)^5 - a^5 - 2a^4b - a^3b^2 - 2(5a^5 + 16a^4b - 31a^3b^2 - 102a \\
& ^2b^3 - 60ab^4) \cosh(dx + c)^4 + 2(105(a^5 + 2a^4b + a^3b^2) \cosh(\\
& dx + c)^6 - 5a^5 - 16a^4b + 31a^3b^2 + 102a^2b^3 + 60ab^4 + 35(1 \\
& 1a^5 + 2a^4b - 29a^3b^2 - 20a^2b^3) \cosh(dx + c)^4 + 15(5a^5 + 16 \\
& a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^2) \sinh(dx + c \\
&)^4 + 8(15(a^5 + 2a^4b + a^3b^2) \cosh(dx + c)^7 + 7(11a^5 + 2a^4b \\
& - 29a^3b^2 - 20a^2b^3) \cosh(dx + c)^5 + 5(5a^5 + 16a^4b - 31a^3 \\
& b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^3 - (5a^5 + 16a^4b - 31a^3 \\
& b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)) \sinh(dx + c)^3 - (11a^5 + 2 \\
& a^4b - 29a^3b^2 - 20a^2b^3) \cosh(dx + c)^2 + (45(a^5 + 2a^4b + a^3 \\
& b^2) \cosh(dx + c)^8 + 28(11a^5 + 2a^4b - 29a^3b^2 - 20a^2b^3) \cos \\
& h(dx + c)^6 - 11a^5 - 2a^4b + 29a^3b^2 + 20a^2b^3 + 30(5a^5 + 16 \\
& a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^4 - 12(5a^5 + \\
& 16a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^2) \sinh(dx + \\
& c)^2 + 12((6a^2b^2 + 5ab^3) \cosh(dx + c)^7 + 7(6a^2b^2 + 5ab^3) \\
&) \cosh(dx + c) \sinh(dx + c)^6 + (6a^2b^2 + 5ab^3) \sinh(dx + c)^7 + 2 \\
& (6a^2b^2 + 17ab^3 + 10b^4) \cosh(dx + c)^5 + (12a^2b^2 + 34ab^3 + \\
& 20b^4 + 21(6a^2b^2 + 5ab^3) \cosh(dx + c)^2) \sinh(dx + c)^5 + 5(7(\\
& 6a^2b^2 + 5ab^3) \cosh(dx + c)^3 + 2(6a^2b^2 + 17ab^3 + 10b^4) \cos \\
& h(dx + c)) \sinh(dx + c)^4 + (6a^2b^2 + 5ab^3) \cosh(dx + c)^3 + (35 \\
& (6a^2b^2 + 5ab^3) \cosh(dx + c)^4 + 6a^2b^2 + 5ab^3 + 20(6a^2b^2 \\
& + 17ab^3 + 10b^4) \cosh(dx + c)^2) \sinh(dx + c)^3 + (21(6a^2b^2 + 5 \\
& ab^3) \cosh(dx + c)^5 + 20(6a^2b^2 + 17ab^3 + 10b^4) \cosh(dx + c)^ \\
& 3 + 3(6a^2b^2 + 5ab^3) \cosh(dx + c)) \sinh(dx + c)^2 + (7(6a^2b^2 \\
& + 5ab^3) \cosh(dx + c)^6 + 10(6a^2b^2 + 17ab^3 + 10b^4) \cosh(dx + \\
& c)^4 + 3(6a^2b^2 + 5ab^3) \cosh(dx + c)^2) \sinh(dx + c)) \sqrt{a^2 + a \\
& b} \arctan(1/2(a \cosh(dx + c)^3 + 3a \cosh(dx + c) \sinh(dx + c)^2 + a \sin \\
& h(dx + c)^3 + (3a + 4b) \cosh(dx + c) + (3a \cosh(dx + c)^2 + 3a + 4 \\
& b) \sinh(dx + c)) / \sqrt{a^2 + ab}) + 12((6a^2b^2 + 5ab^3) \cosh(dx + \\
& c)^7 + 7(6a^2b^2 + 5ab^3) \cosh(dx + c) \sinh(dx + c)^6 + (6a^2b^2 + \\
& 5ab^3) \sinh(dx + c)^7 + 2(6a^2b^2 + 17ab^3 + 10b^4) \cosh(dx + c) \\
& ^5 + (12a^2b^2 + 34ab^3 + 20b^4 + 21(6a^2b^2 + 5ab^3) \cosh(dx + \\
& c)^2) \sinh(dx + c)^5 + 5(7(6a^2b^2 + 5ab^3) \cosh(dx + c)^3 + 2(6a \\
& ^2b^2 + 17ab^3 + 10b^4) \cosh(dx + c)) \sinh(dx + c)^4 + (6a^2b^2 + 5 \\
& ab^3) \cosh(dx + c)^3 + (35(6a^2b^2 + 5ab^3) \cosh(dx + c)^4 + 6a^2 \\
& b^2 + 5ab^3 + 20(6a^2b^2 + 17ab^3 + 10b^4) \cosh(dx + c)^2) \sinh(dx \\
& + c)^3 + (21(6a^2b^2 + 5ab^3) \cosh(dx + c)^5 + 20(6a^2b^2 + 17 \\
& ab^3 + 10b^4) \cosh(dx + c)^3 + 3(6a^2b^2 + 5ab^3) \cosh(dx + c)) \sin \\
& h(dx + c)^2 + (7(6a^2b^2 + 5ab^3) \cosh(dx + c)^6 + 10(6a^2b^2 + \\
& 17ab^3 + 10b^4) \cosh(dx + c)^4 + 3(6a^2b^2 + 5ab^3) \cosh(dx + c)^ \\
& 2) \sinh(dx + c)) \sqrt{a^2 + ab} \arctan(1/2 \sqrt{a^2 + ab} (\cosh(dx + c) \\
& + \sinh(dx + c)) / (a + b)) + 2(5(a^5 + 2a^4b + a^3b^2) \cosh(dx + c)^9 \\
& + 4(11a^5 + 2a^4b - 29a^3b^2 - 20a^2b^3) \cosh(dx + c)^7 + 6(5a^5 \\
& + 16a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^5 - 4(5 \\
& a^5 + 16a^4b - 31a^3b^2 - 102a^2b^3 - 60ab^4) \cosh(dx + c)^3 - (11 \\
& a^5 + 2a^4b - 29a^3b^2 - 20a^2b^3) \cosh(dx + c)) \sinh(dx + c)) / ((a \\
& ^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^7 + 7(a^7 + 2a^6b + a^5b^2) d \c \\
& osh(dx + c) \sinh(dx + c)^6 + (a^7 + 2a^6b + a^5b^2) d \sinh(dx + c)^7 \\
& + 2(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^5 + (21(a^7 + \\
& 2a^6b + a^5b^2) d \cosh(dx + c)^2 + 2(a^7 + 4a^6b + 5a^5b^2 + 2a^4 \\
& b^3) d) \sinh(dx + c)^5 + (a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^3 + 5 \\
& (7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^3 + 2(a^7 + 4a^6b + 5a^5b \\
& ^2 + 2a^4b^3) d \cosh(dx + c)) \sinh(dx + c)^4 + (35(a^7 + 2a^6b + a^5 \\
& b^2) d \cosh(dx + c)^4 + 20(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh \\
& (dx + c)^2 + (a^7 + 2a^6b + a^5b^2) d) \sinh(dx + c)^3 + (21(a^7 + 2a \\
& ^6b + a^5b^2) d \cosh(dx + c)^5 + 20(a^7 + 4a^6b + 5a^5b^2 + 2a^4b \\
& ^3) d \cosh(dx + c)^3 + 3(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)) \sinh(dx \\
& + c)^2 + (7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^6 + 10(a^7 + 4a^
\end{aligned}$$

$6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*\cosh(d*x + c)^4 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(d*x + c)^2*\sinh(d*x + c))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[12,-32]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-37,-59]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-67,22]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[62,70]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[50,35]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[43,41]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[37,80]Undef/Unsigned Inf encountered in limitEvaluation time: 1.64Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.53, size = 517, normalized size = 4.14

$$\frac{1}{3da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{1}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{1}{da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{2b}{da^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{1}{3da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} - \frac{1}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{1}{da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{2b}{da^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/3/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)^2-1/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)+2/d/a^3/(\tanh(1/2*d*x+1/2*c)-1)*b-1/3/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)^2-1/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)+2/d/a^3/(\tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^3*b^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)^3-1/d/a^3*b^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)+3/d/a^(5/2)*b^2/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+3/d/a^(5/2)*b^2/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+5/2/d/a^(7/2)*b^3/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+5/2/d/a^(7/2)*b^3/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 + a^2b - (a^3e^{10c} + a^2be^{10c})e^{10dx} - (11a^3e^{8c} - 9a^2be^{8c} - 20ab^2e^{8c})e^{8dx} - 2(5a^3e^{6c} + 11a^2be^{6c})e^{6dx} - 2(5a^3e^{4c} + 11a^2be^{4c})e^{4dx} - 2(5a^3e^{2c} + 11a^2be^{2c})e^{2dx}}{24((a^5de^{7c} + a^4bde^{7c})e^{7dx} + 2(a^5de^{5c} + a^4bde^{5c})e^{5dx} + 2(a^5de^{3c} + a^4bde^{3c})e^{3dx} + 2(a^5de^{c} + a^4bde^{c})e^{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/24*(a^3 + a^2*b - (a^3*e^(10*c) + a^2*b*e^(10*c))*e^(10*d*x) - (11*a^3*e^(8*c) - 9*a^2*b*e^(8*c) - 20*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c) + 11*a^2*b*e^(6*c) - 42*a*b^2*e^(6*c) - 60*b^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 11*a^2*b*e^(4*c) - 42*a*b^2*e^(4*c) - 60*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) - 9*a^2*b*e^(2*c) - 20*a*b^2*e^(2*c))*e^(2*d*x))/(a^5*d*e^(7*c) + a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 3*a^4*b*d*e^(5*c) + 2*a^3*b^2*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + a^4*b*d*e^(3*c))*e^(3*d*x) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x) + (6*a*b^2*e^c + 5*b^3*e^c)*e^(d*x))/(a^5 + a^4*b + (a^5*e^(4*c) + a^4*b*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 2*a^3*b^2*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^3}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.84 \quad \int \frac{\cosh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=144

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}} + \frac{x(a-4b)}{2a^3} + \frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)(a-b\tanh^2(c+dx)+b)} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad(a-b\tanh^2(c+dx))}$$

[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)+1/2*b*(a+2*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}} + \frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)(a-b\tanh^2(c+dx)+b)} + \frac{x(a-4b)}{2a^3} + \frac{\sinh(c+dx)\cosh(c+dx)}{2ad(a-b\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)) + (b*(a + 2*b)*Tanh[c + d*x])/(2*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{b(a + 2b) \tanh(c + dx)}{2a^2(a + b)d(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a-4bx^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{b(a + 2b) \tanh(c + dx)}{2a^2(a + b)d(a + b - b \tanh^2(c + dx))} + \frac{(a - 4b) \operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{2a^3}$$

$$= \frac{(a - 4b)x}{2a^3} + \frac{b^{3/2}(5a + 4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{2a^3(a + b)^{3/2}d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))}$$

Mathematica [A] time = 1.38, size = 103, normalized size = 0.72

$$\frac{2b^{3/2}(5a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{\sinh(2(c+dx)) \left(\frac{2ab^2}{(a+b)(a \cosh(2(c+dx))+a+2b)} + a \right) + 2(a-4b)(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]
```

```
[Out] (2*(a - 4*b)*(c + d*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])))*Sinh[2*(c + d*x)]/(4*a^3*d)
```

fricas [B] time = 0.49, size = 3739, normalized size = 25.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a^2*b - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*((5*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3 + 15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 5*a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 4*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(2*(a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^4*b)*d*cosh(d*x + c)^6 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + a^4*b)*d*sinh(d*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^4 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d)*sinh(d*x + c)^4 + (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a^4*b)*d*cosh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5 + a^4*b)*d*cosh(d*x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3
```

$$\begin{aligned}
& - (a^3 - a^2b - 10ab^2 - 8b^3)d^2x \cosh(dx + c) \sinh(dx + c)^3 - a^3 \\
& - a^2b - 2(a^3 + 3a^2b + 6ab^2 - 2(a^3 - 3a^2b - 4ab^2)d^2x) \cosh(dx + c)^2 + 2(14(a^3 + a^2b) \cosh(dx + c)^6 + 15(a^3 + 3a^2b + \\
& 2ab^2 + 2(a^3 - 3a^2b - 4ab^2)d^2x) \cosh(dx + c)^4 - a^3 - 3a^2b - 6ab^2 + 2(a^3 - 3a^2b - 4ab^2)d^2x - 24(ab^2 + 2b^3 - (a^3 - a^2b - \\
& 10ab^2 - 8b^3)d^2x) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((5a^2b + 4ab^2) \cosh(dx + c)^6 + 6(5a^2b + 4ab^2) \cosh(dx + c) \sinh(dx + c)^5 + \\
& (5a^2b + 4ab^2) \sinh(dx + c)^6 + 2(5a^2b + 14ab^2 + 8b^3) \cosh(dx + c)^4 + (10a^2b + 28ab^2 + 16b^3 + 15(5a^2b + 4ab^2) \cosh(dx + c)^2) \sinh(dx + c)^4 + \\
& 4(5(5a^2b + 4ab^2) \cosh(dx + c)^3 + 2(5a^2b + 14ab^2 + 8b^3) \cosh(dx + c) \sinh(dx + c)^3 + (5a^2b + 4ab^2) \cosh(dx + c)^2 + (15(5a^2b + 4ab^2) \cosh(dx + c)^4 + 5a^2b + 4ab^2 + 12(5a^2b + 14ab^2 + 8b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(3(5a^2b + 4ab^2) \cosh(dx + c)^5 + 4(5a^2b + 14ab^2 + 8b^3) \cosh(dx + c)^3 + (5a^2b + 4ab^2) \cosh(dx + c) \sinh(dx + c)) \sqrt{-b/(a + b)} \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-b/(a + b)})/b) + 4(2(a^3 + a^2b) \cosh(dx + c)^7 + 3(a^3 + 3a^2b + 2ab^2 + 2(a^3 - 3a^2b - 4ab^2)d^2x) \cosh(dx + c)^5 - 8(ab^2 + 2b^3 - (a^3 - a^2b - 10ab^2 - 8b^3)d^2x) \cosh(dx + c)^3 - (a^3 + 3a^2b + 6ab^2 - 2(a^3 - 3a^2b - 4ab^2)d^2x) \cosh(dx + c)) \sinh(dx + c) / ((a^5 + a^4b) d \cosh(dx + c)^6 + 6(a^5 + a^4b) d \cosh(dx + c) \sinh(dx + c)^5 + (a^5 + a^4b) d \sinh(dx + c)^6 + 2(a^5 + 3a^4b + 2a^3b^2) d \cosh(dx + c)^4 + (15(a^5 + a^4b) d \cosh(dx + c)^2 + 2(a^5 + 3a^4b + 2a^3b^2) d) \sinh(dx + c)^4 + (a^5 + a^4b) d \cosh(dx + c)^2 + 4(5(a^5 + a^4b) d \cosh(dx + c)^3 + 2(a^5 + 3a^4b + 2a^3b^2) d \cosh(dx + c) \sinh(dx + c)^3 + (15(a^5 + a^4b) d \cosh(dx + c)^4 + 12(a^5 + 3a^4b + 2a^3b^2) d \cosh(dx + c)^2 + (a^5 + a^4b) d) \sinh(dx + c)^2 + 2(3(a^5 + a^4b) d \cosh(dx + c)^5 + 4(a^5 + 3a^4b + 2a^3b^2) d \cosh(dx + c)^3 + (a^5 + a^4b) d \cosh(dx + c)) \sinh(dx + c))]]
\end{aligned}$$

giac [B] time = 2.02, size = 323, normalized size = 2.24

$$\frac{12(5ab^2+4b^3) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^4+a^3b)\sqrt{-ab-b^2}} - \frac{2a^3e^{(6dx+6c)} - 6a^2be^{(6dx+6c)} - 8ab^2e^{(6dx+6c)} + 7a^3e^{(4dx+4c)} - a^2be^{(4dx+4c)} - 16ab^2e^{(4dx+4c)} + 16b^3e^{(4dx+4c)}}{(a^4+a^3b)(ae^{(6dx+6c)} + 2ae^{(4dx+4c)} + 4be^{(4dx+4c)})}$$

$24d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& 1/24*(12*(5a^2b^2 + 4b^3) \arctan(1/2(ae^{(2dx+2c)} + a + 2b)/\sqrt{-ab-b^2}) / ((a^4 + a^3b) \sqrt{-ab-b^2}) - (2a^3e^{(6dx+6c)} - 6a^2b \\
& e^{(6dx+6c)} - 8a^2b^2e^{(6dx+6c)} + 7a^3e^{(4dx+4c)} - a^2be^{(4dx+4c)} - 16a^2b^2e^{(4dx+4c)} + 16b^3e^{(4dx+4c)} + 8a^3e^{(2dx+2c)} + 12a^2b^2e^{(2dx+2c)} + 28a^2b^2e^{(2dx+2c)} + 3a^3 + 3a^2b) / ((a^4 + a^3b) (ae^{(6dx+6c)} + 2ae^{(4dx+4c)} + 4be^{(4dx+4c)} + 4b^2e^{(4dx+4c)} + ae^{(2dx+2c)})) + 12(dx+c)(a-4b)/a^3 + 3e^{(2dx+2c)}/a^2)/d
\end{aligned}$$

maple [B] time = 0.51, size = 557, normalized size = 3.87

$$\frac{1}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{1}{2da^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2da^2} + \frac{2\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b}{da^3}$$

2

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(dx+c)^2/(a+b*sech(dx+c)^2)^2,x)

```
[Out] 1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-5/4/d/a^2*b^(3/2)/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+5/4/d/a^2*b^(3/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/a^3*b^(5/2)/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/d/a^3*b^(5/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.47, size = 696, normalized size = 4.83

$$\frac{(3a^2b + 12ab^2 + 8b^3) \log\left(\frac{ae^{(2dx+2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(2dx+2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^4 + a^3b)\sqrt{(a+b)bd}} - \frac{(3a^2b + 12ab^2 + 8b^3) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^4 + a^3b)\sqrt{(a+b)bd}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/8*(3*a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x - 2*c))/((a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (a^5 + a^4*b)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) - 1/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^2}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

$$3.85 \quad \int \frac{\cosh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=100

$$-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}} + \frac{b^2\sinh(c+dx)}{2a^2d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\sinh(c+dx)}{a^2d}$$

[Out] $-1/2*b*(4*a+3*b)*\arctan(\sinh(d*x+c)*a^{(1/2)/(a+b)^{(1/2)})}/a^{(5/2)/(a+b)^{(3/2)}/d+\sinh(d*x+c)/a^2/d+1/2*b^2*\sinh(d*x+c)/a^2/(a+b)/d/(a+b+a*\sinh(d*x+c)^2)$

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4147, 390, 385, 205}

$$\frac{b^2\sinh(c+dx)}{2a^2d(a+b)(a\sinh^2(c+dx)+a+b)} - \frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}} + \frac{\sinh(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $-(b*(4*a+3*b)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sinh}[c+d*x])/\text{Sqrt}[a+b]])/(2*a^{(5/2)*(a+b)^{(3/2)*d}+\text{Sinh}[c+d*x]/(a^2*d)+(b^2*\text{Sinh}[c+d*x])/(2*a^2*(a+b)*d*(a+b+a*\text{Sinh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m+n*p+1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m-1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{a^2} - \frac{b(2a+b)+2abx^2}{a^2(a+b+ax^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{a^2d} - \frac{\operatorname{Subst}\left(\int \frac{b(2a+b)+2abx^2}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{a^2d} \\
&= \frac{\sinh(c+dx)}{a^2d} + \frac{b^2 \sinh(c+dx)}{2a^2(a+b)d(a+b+a\sinh^2(c+dx))} - \frac{(b(4a+3b)) \operatorname{Subst}\left(\int \frac{1}{a+b}\right)}{2a^2(a+b)d} \\
&= -\frac{b(4a+3b) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{a^2d} + \frac{b^2 \sinh(c+dx)}{2a^2(a+b)d(a+b+a\sinh^2(c+dx))}
\end{aligned}$$

Mathematica [B] time = 1.81, size = 234, normalized size = 2.34

$$\operatorname{sech}^3(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(\frac{2\sqrt{a}b^2 \tanh(c+dx)}{a+b} + 2\sqrt{a} \sinh(c) \cosh(dx) \operatorname{sech}(c+dx)(a \cosh(2(c+dx)) + a + 2b) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/(a + b)^(3/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 2*Sqrt[a]*Cosh[d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[c] + 2*Sqrt[a]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[d*x] + (2*Sqrt[a]*b^2*Tanh[c + d*x])/(a + b))/(8*a^(5/2)*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.49, size = 3154, normalized size = 31.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^6 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b - 11*a^2*b^2 - 6*a*b^3 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b +

$$\begin{aligned}
& 3*a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*\sinh(d*x + c) \\
& ^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 \\
& + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(5*(4 \\
& *a^2*b + 3*a*b^2)*\cosh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^2 + (4*a^2*b + 3*a*b^2)*\cosh(d*x + c) + (5*(4*a^2*b + \\
& 3*a*b^2)*\cosh(d*x + c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^ \\
& 3)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\sqrt{-a^2 - a*b}*\log((a*\cosh(d*x + c)^4 \\
& + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 - 2*(3*a + 2*b)*\cos \\
& h(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 - 3*a - 2*b)*\sinh(d*x + c)^2 + 4*(a*c \\
& osh(d*x + c)^3 - (3*a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(\cosh(d*x + c \\
&)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^ \\
& 2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a^2 - a*b} + a)/(a*\cosh(d*x + c \\
&)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*c \\
& osh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*c \\
& osh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 4*(3*(a^4 + \\
& 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b \\
& ^3)*\cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)) \\
& *\sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d*x + c)^5 + 5*(a^6 + 2*a \\
& ^5*b + a^4*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^6 + 2*a^5*b + a^4*b^2) \\
& *d*\sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*\cosh(d*x + \\
& c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d*x + c)^2 + (a^6 + 4*a^5*b + \\
& 5*a^4*b^2 + 2*a^3*b^3)*d)*\sinh(d*x + c)^3 + (a^6 + 2*a^5*b + a^4*b^2)*d*\co \\
& sh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d*x + c)^3 + 3*(a^6 + 4 \\
& *a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + (5*(a^6 \\
& + 2*a^5*b + a^4*b^2)*d*\cosh(d*x + c)^4 + 6*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a \\
& ^3*b^3)*d*\cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2)*d)*\sinh(d*x + c)), 1/ \\
& 2*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)* \\
& \cosh(d*x + c)*\sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(d*x + c)^6 + \\
& (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^4 + (a^4 + 6*a^3*b + \\
& 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d \\
& *x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d \\
& *x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2 + (15*(a^4 \\
& + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^4 - a^4 - 6*a^3*b - 11*a^2*b^2 - 6*a*b^3 \\
& + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
& 2 - ((4*a^2*b + 3*a*b^2)*\cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + \\
& c)*\sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*\sinh(d*x + c)^5 + 2*(4*a^2*b + 11 \\
& *a*b^2 + 6*b^3)*\cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b \\
& + 3*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*co \\
& sh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^2 + (4*a^2*b + 3*a*b^2)*\cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + \\
& c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x \\
& + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (3*a + 4*b)*\cosh(d*x + c) + (3* \\
& a*\cosh(d*x + c)^2 + 3*a + 4*b)*\sinh(d*x + c))/\sqrt{a^2 + a*b}) - ((4*a^2*b \\
& + 3*a*b^2)*\cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^4 + (4*a^2*b + 3*a*b^2)*\sinh(d*x + c)^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 \\
&)*\cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*c \\
& osh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + c)^3 \\
& + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (4*a^2*b \\
& + 3*a*b^2)*\cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*\cosh(d*x + c)^4 + 4*a^2*b \\
& + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)) \\
& *\sqrt{a^2 + a*b}*\arctan(1/2*\sqrt{a^2 + a*b}*(\cosh(d*x + c) + \sinh(d*x + c)) \\
& /(a + b)) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^5 + 2*(a^4 + 6*a^3 \\
& *b + 11*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + \\
& 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d* \\
& x + c)^5 + 5*(a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a \\
& ^6 + 2*a^5*b + a^4*b^2)*d*\sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 + \\
& 2*a^3*b^3)*d*\cosh(d*x + c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*\cosh(d*x +
\end{aligned}$$

$$c)^2 + (a^6 + 4a^5b + 5a^4b^2 + 2a^3b^3)d) \sinh(dx + c)^3 + (a^6 + 2a^5b + a^4b^2)d \cosh(dx + c) + 2(5(a^6 + 2a^5b + a^4b^2)d \cosh(dx + c)^3 + 3(a^6 + 4a^5b + 5a^4b^2 + 2a^3b^3)d \cosh(dx + c)) \sinh(dx + c)^2 + (5(a^6 + 2a^5b + a^4b^2)d \cosh(dx + c)^4 + 6(a^6 + 4a^5b + 5a^4b^2 + 2a^3b^3)d \cosh(dx + c)^2 + (a^6 + 2a^5b + a^4b^2)d) \sinh(dx + c))]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[12,-32]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[67,31]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-88,66]Undef/Unsigned Inf encountered in limitEvaluation time: 0.99Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.51, size = 385, normalized size = 3.85

$$\frac{1}{d a^2 \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)} - \frac{1}{d a^2 \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)} - \frac{b^2 \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d a^2 \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/d/a^2/(\tanh(1/2*d*x+1/2*c)-1)-1/d/a^2/(\tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)-2/d/a^{(3/2)}*b/(a+b)^{(3/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)-2*b^{(1/2)})/a^{(1/2)})-2/d/a^{(3/2)}*b/(a+b)^{(3/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)+2*b^{(1/2)})/a^{(1/2)})-3/2/d/a^{(5/2)}*b^2/(a+b)^{(3/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)-2*b^{(1/2)})/a^{(1/2)})-3/2/d/a^{(5/2)}*b^2/(a+b)^{(3/2)}*\arctan(1/2*(2*(a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)+2*b^{(1/2)})/a^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 + ab - (a^2 e^{6c} + ab e^{6c}) e^{6dx} - (a^2 e^{4c} + 5 ab e^{4c} + 6 b^2 e^{4c}) e^{4dx} + (a^2 e^{2c} + 5 ab e^{2c} + 6 b^2 e^{2c}) e^{2dx}}{2 \left((a^4 d e^{5c} + a^3 b d e^{5c}) e^{5dx} + 2 (a^4 d e^{3c} + 3 a^3 b d e^{3c} + 2 a^2 b^2 d e^{3c}) e^{3dx} + (a^4 d e^c + a^3 b d e^c) e^{dx} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

```
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) + 5*
a*b*e^(4*c) + 6*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) + 5*a*b*e^(2*c) + 6*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + a^3*b*d*e^(5*c))*e^(5*d*x) + 2*(a^
4*d*e^(3*c) + 3*a^3*b*d*e^(3*c) + 2*a^2*b^2*d*e^(3*c))*e^(3*d*x) + (a^4*d*e
^c + a^3*b*d*e^c)*e^(d*x)) - 1/2*integrate(2*((4*a*b*e^(3*c) + 3*b^2*e^(3*c)
))*e^(3*d*x) + (4*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^4 + a^3*b + (a^4*e^(4*c)
+ a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 2*a^2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2, x)
```

```
[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**2, x)
```

```
[Out] Timed out
```

$$3.86 \quad \int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=82

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}} - \frac{b\sinh(c+dx)}{2ad(a+b)(a\sinh^2(c+dx)+a+b)}$$

[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/d-1/2*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4147, 385, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}} - \frac{b\sinh(c+dx)}{2ad(a+b)(a\sinh^2(c+dx)+a+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] ((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^(3/2)*d) - (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b \sinh(c+dx)}{2a(a+b)d(a+b+a\sinh^2(c+dx))} + \frac{(2a+b) \operatorname{Subst}\left(\int \frac{1}{a+b+ax^2} dx, x, \sinh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{(2a+b) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d} - \frac{b \sinh(c+dx)}{2a(a+b)d(a+b+a\sinh^2(c+dx))}$$

Mathematica [A] time = 0.31, size = 124, normalized size = 1.51

$$\frac{(2a^2 + 3ab + b^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right) - \sqrt{a} b \sqrt{a+b} \sinh(c+dx) + a(2a+b) \sinh^2(c+dx) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d(a+b)^{3/2}(a \cosh(2(c+dx)) + a + 2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2, x]

[Out] ((2*a^2 + 3*a*b + b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] - Sqrt[a]*b*Sqrt[a + b]*Sinh[c + d*x] + a*(2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]*Sinh[c + d*x]^2)/(a^(3/2)*(a + b)^(3/2)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))

fricas [B] time = 0.45, size = 1856, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2, x, algorithm="fricas")

[Out] [-1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 + ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 4*(a^2*b + a*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*

```
(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[89,-63]Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.34, size = 331, normalized size = 4.04

$$\frac{b \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right) a (a + b) d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/(a+b)^(3/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/d/(a+b)^(3/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{be^{(3dx+3c)} - be^{(dx+c)}}{a^3d + a^2bd + (a^3de^{(4c)} + a^2bde^{(4c)})e^{(4dx)} + 2(a^3de^{(2c)} + 3a^2bde^{(2c)} + 2ab^2de^{(2c)})e^{(2dx)}} + 2 \int \frac{1}{2(a^3 + a^2b + (a^3e^{(4c)} + a^2be^{(4c)})e^{(4dx)} + 2(a^3e^{(2c)} + 3a^2be^{(2c)} + 2ab^2e^{(2c)})e^{(2dx)})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $-(b \cdot e^{(3d \cdot x + 3c)} - b \cdot e^{(d \cdot x + c)}) / (a^3 \cdot d + a^2 \cdot b \cdot d + (a^3 \cdot d \cdot e^{(4c)} + a^2 \cdot b \cdot d \cdot e^{(4c)}) \cdot e^{(4d \cdot x)} + 2 \cdot (a^3 \cdot d \cdot e^{(2c)} + 3 \cdot a^2 \cdot b \cdot d \cdot e^{(2c)} + 2 \cdot a \cdot b^2 \cdot d \cdot e^{(2c)}) \cdot e^{(2d \cdot x)}) + 2 \cdot \int (1/2 \cdot ((2 \cdot a \cdot e^{(3c)} + b \cdot e^{(3c)}) \cdot e^{(3d \cdot x)} + (2 \cdot a \cdot e^{(c)} + b \cdot e^{(c)}) \cdot e^{(d \cdot x)}) / (a^3 + a^2 \cdot b + (a^3 \cdot e^{(4c)} + a^2 \cdot b \cdot e^{(4c)}) \cdot e^{(4d \cdot x)} + 2 \cdot (a^3 \cdot e^{(2c)} + 3 \cdot a^2 \cdot b \cdot e^{(2c)} + 2 \cdot a \cdot b^2 \cdot e^{(2c)}) \cdot e^{(2d \cdot x)})) dx$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx) \left(a + \frac{b}{\cosh(c + dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)

$$3.87 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}} + \frac{\tanh(c+dx)}{2d(a+b)(a-b\tanh^2(c+dx)+b)}$$

[Out] $1/2*\arctanh(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})/(a+b)^{(3/2)}/d/b^{(1/2)}+1/2*\tanh(d*x+c)/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4146, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}} + \frac{\tanh(c+dx)}{2d(a+b)(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) + Tanh[c + d*x]/(2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2(a+b)d(a+b-b\tanh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{2(a+b)d(a+b-b\tanh^2(c+dx))}$$

Mathematica [B] time = 1.01, size = 187, normalized size = 2.53

$$\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b) \frac{\left(\cosh(2c)-\sinh(2c)\right)(a\cosh(2(c+dx))+a+2b)\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(2c))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))}}\right)}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

$$8d(a+b)(a+b\operatorname{sech}^2(c+dx))^2$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*Sinh[2*d*x] - ((a + 2*b)*Tanh[2*c])/a)/(8*(a + b)*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.44, size = 1489, normalized size = 20.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b + 4*a^3*b^2

+ 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c))^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)))]

giac [A] time = 0.74, size = 130, normalized size = 1.76

$$\frac{\arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}(a+b)} - \frac{2(ae^{(2dx+2c)+2be^{(2dx+2c)+a}})}{(a^2+ab)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}}})} }{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)) - 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a^2 + a*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)))/d

maple [B] time = 0.25, size = 263, normalized size = 3.55

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a + b\right)(a + b)} + \frac{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a + b\right)(a + b)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a + b\right)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/(a+b)^(3/2)/b^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))

maxima [B] time = 0.46, size = 150, normalized size = 2.03

$$\frac{(a + 2b)e^{(-2dx-2c)} + a}{(a^3 + a^2b + 2(a^3 + 3a^2b + 2ab^2)e^{(-2dx-2c)} + (a^3 + a^2b)e^{(-4dx-4c)})d} - \frac{\log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

```
[Out] ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2)
)*e^(-2*d*x - 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) - 1/4*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2
*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^2 \left(a + \frac{b}{\cosh(c + dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**2, x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

$$3.88 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=73

$$\frac{\sinh(c+dx)}{2d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}d(a+b)^{3/2}}$$

[Out] 1/2*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d/a^(1/2)

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4147, 199, 205}

$$\frac{\sinh(c+dx)}{2d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*d) + Sinh[c + d*x]/(2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bax^2)^2} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2(a+b)d(a+b+a\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+bax^2} dx, x, \sinh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{2(a+b)d(a+b+a\sinh^2(c+dx))}$$

Mathematica [A] time = 0.23, size = 108, normalized size = 1.48

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2 \left(\frac{\sinh(c+dx)}{(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}} \right)}{8d(a+b\operatorname{sech}^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2, x]

[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + Sinh[c + d*x]/((a + b)*(a + b + a*Sinh[c + d*x]^2))))/(8*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.48, size = 1570, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)]/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*cosh(d*x + c)^3 + 6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b)*sinh(d*x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)

$$\begin{aligned} &^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + \\ &a*\sqrt{a^2 + a*b}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (3*a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + 3*a + 4*b)*\sinh(d*x + c))/\sqrt{a^2 + a*b}) + (a*\cosh(d*x + c)^4 + \\ &4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)*\sqrt{a^2 + a*b}*\arctan(1/2*\sqrt{a^2 + a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/(a + b)) - 2*(a^2 + a*b)*\cosh(d*x + c) + 2*(3*(a^2 + a*b)*\cosh(d*x + c)^2 - a^2 - a*b)*\sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*\cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*\sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*\cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*\cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*\sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*\cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[12,-32]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Undefined/Unsigned Inf encountered in limitEvaluation time: 0.58Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.27, size = 241, normalized size = 3.30

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a + b\right)(a + b)} + \frac{1}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a + b\right)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$\begin{aligned} &-1/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)^3+1/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)+1/2/d/(a+b)^(3/2)/a^(1/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/d/(a+b)^(3/2)/a^(1/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3dx+3c)} - e^{(dx+c)}}{a^2d + abd + (a^2de^{(4c)} + abde^{(4c)})e^{(4dx)} + 2(a^2de^{(2c)} + 3abde^{(2c)} + 2b^2de^{(2c)})e^{(2dx)}} + 8 \int \frac{1}{8(a^2 + ab + (a^2e^{(4c)} + a^2e^{(2c)} + abe^{(2c)} + b^2e^{(2c)}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $(e^{(3*d*x + 3*c)} - e^{(d*x + c)})/(a^2*d + a*b*d + (a^2*d*e^{(4*c)} + a*b*d*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*d*e^{(2*c)} + 3*a*b*d*e^{(2*c)} + 2*b^2*d*e^{(2*c)})*e^{(2*d*x)}) + 8*integrate(1/8*(e^{(3*d*x + 3*c)} + e^{(d*x + c)})/(a^2 + a*b + (a^2*e^{(4*c)} + a*b*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*e^{(2*c)} + 3*a*b*e^{(2*c)} + 2*b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^3 \left(a + \frac{b}{\cosh(c+dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)

$$3.89 \quad \int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=83

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}} - \frac{a\tanh(c+dx)}{2bd(a+b)(a-b\tanh^2(c+dx)+b)}$$

[Out] 1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d-1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4146, 385, 208}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}} - \frac{a\tanh(c+dx)}{2bd(a+b)(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(3/2)*d) - (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 4146

Int[sec[(e_) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a \tanh(c+dx)}{2b(a+b)d(a+b-b \tanh^2(c+dx))} + \frac{(a+2b) \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{(a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} - \frac{a \tanh(c+dx)}{2b(a+b)d(a+b-b \tanh^2(c+dx))}$$

Mathematica [A] time = 0.22, size = 88, normalized size = 1.06

$$4 \left(\frac{(a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{3/2}} - \frac{a \sinh(2(c+dx))}{8bd(a+b)(a \cosh(2(c+dx)) + a + 2b)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] 4*((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(8*b^(3/2)*(a + b)^(3/2)*d) - (a*Sinh[2*(c + d*x)]/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])))

fricas [B] time = 0.45, size = 1569, normalized size = 18.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(d*x + c), 1/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c)^2 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))]

$b^3) \cosh(dx + c) \sinh(dx + c) + 2(a^2b + 3ab^2 + 2b^3) \sinh(dx + c)^2 + ((a^2 + 2ab) \cosh(dx + c)^4 + 4(a^2 + 2ab) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 2ab) \sinh(dx + c)^4 + 2(a^2 + 4ab + 4b^2) \cosh(dx + c)^2 + 2(3(a^2 + 2ab) \cosh(dx + c)^2 + a^2 + 4ab + 4b^2) \sinh(dx + c)^2 + a^2 + 2ab + 4((a^2 + 2ab) \cosh(dx + c)^3 + (a^2 + 4ab + 4b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{-ab - b^2} \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-ab - b^2} / (ab + b^2))) / ((a^3b^2 + 2a^2b^3 + ab^4) d \cosh(dx + c)^4 + 4(a^3b^2 + 2a^2b^3 + ab^4) d \cosh(dx + c) \sinh(dx + c)^3 + (a^3b^2 + 2a^2b^3 + ab^4) d \sinh(dx + c)^4 + 2(a^3b^2 + 4a^2b^3 + 5ab^4 + 2b^5) d \cosh(dx + c)^2 + 2(3(a^3b^2 + 2a^2b^3 + ab^4) d \cosh(dx + c)^2 + (a^3b^2 + 4a^2b^3 + 5ab^4 + 2b^5) d) \sinh(dx + c)^2 + (a^3b^2 + 2a^2b^3 + ab^4) d + 4((a^3b^2 + 2a^2b^3 + ab^4) d \cosh(dx + c)^3 + (a^3b^2 + 4a^2b^3 + 5ab^4 + 2b^5) d \cosh(dx + c)) \sinh(dx + c))]$

giac [A] time = 0.77, size = 139, normalized size = 1.67

$$\frac{(a+2b) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(ab+b^2)\sqrt{-ab-b^2}} + \frac{2(ae^{(2dx+2c)+2be^{(2dx+2c)+a}})}{(ab+b^2)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}}})}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^4/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out] $1/2 * ((a + 2b) \arctan(1/2 * (a * e^{(2 * dx + 2 * c)} + a + 2b) / \sqrt{-a * b - b^2})) / ((a * b + b^2) \sqrt{-a * b - b^2}) + 2 * (a * e^{(2 * dx + 2 * c)} + 2 * b * e^{(2 * dx + 2 * c)} + a) / ((a * b + b^2) * (a * e^{(4 * dx + 4 * c)} + 2 * a * e^{(2 * dx + 2 * c)} + 4 * b * e^{(2 * dx + 2 * c)} + a)) / d$

maple [B] time = 0.29, size = 374, normalized size = 4.51

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right) b (a + b) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^4/(a+b*sech(dx+c)^2)^2,x)

[Out] $-1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + b * \tanh(1/2 * dx + 1/2 * c)^4 + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a - 2 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a + b) * a / b / (a + b) * \tanh(1/2 * dx + 1/2 * c)^3 - 1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + b * \tanh(1/2 * dx + 1/2 * c)^4 + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a - 2 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a + b) * a / b / (a + b) * \tanh(1/2 * dx + 1/2 * c) - 1/4 / d / (a + b)^{(3/2)} / b^{(3/2)} * a * \ln((a + b)^{(1/2)} * \tanh(1/2 * dx + 1/2 * c)^2 - 2 * b^{(1/2)} * \tanh(1/2 * dx + 1/2 * c) + (a + b)^{(1/2)}) + 1/4 / d / (a + b)^{(3/2)} / b^{(3/2)} * a * \ln((a + b)^{(1/2)} * \tanh(1/2 * dx + 1/2 * c)^2 + 2 * b^{(1/2)} * \tanh(1/2 * dx + 1/2 * c) + (a + b)^{(1/2)}) - 1/2 / d / (a + b)^{(3/2)} / b^{(1/2)} * \ln((a + b)^{(1/2)} * \tanh(1/2 * dx + 1/2 * c)^2 - 2 * b^{(1/2)} * \tanh(1/2 * dx + 1/2 * c) + (a + b)^{(1/2)}) + 1/2 / d / (a + b)^{(3/2)} / b^{(1/2)} * \ln((a + b)^{(1/2)} * \tanh(1/2 * dx + 1/2 * c)^2 + 2 * b^{(1/2)} * \tanh(1/2 * dx + 1/2 * c) + (a + b)^{(1/2)})$

maxima [B] time = 0.48, size = 165, normalized size = 1.99

$$\frac{(a + 2b) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}(ab+b^2)d} - \frac{(a+2b)e^{(-2dx-2c)} + a}{(a^2b + ab^2 + 2(a^2b + 3ab^2 + 2b^3)e^{(-2dx-2c)} + (a^2b + ab^2)e^{(-4dx-4c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^4/(a+b*sech(dx+c)^2)^2,x, algorithm="maxima")

```
[Out] -1/4*(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)*d) - ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^4 \left(a + \frac{b}{\cosh(c + dx)^2}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**2, x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

$$3.90 \quad \int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=101

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}} - \frac{a\sinh(c+dx)}{2bd(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

[Out] arctan(sinh(d*x+c))/b^2/d-1/2*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
-1/2*(2*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3/2)/d

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4147, 414, 522, 203, 205}

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}} - \frac{a\sinh(c+dx)}{2bd(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*d) - (a*Sinh[c + d*x])/(2*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)^2} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{a \sinh(c + dx)}{2b(a + b)d(a + b + a \sinh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-ax^2}{(1+x^2)(a+b+ax^2)} dx, x, \sinh(c + dx)\right)}{2b(a + b)d}$$

$$= -\frac{a \sinh(c + dx)}{2b(a + b)d(a + b + a \sinh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{b^2d}$$

$$= \frac{\tan^{-1}(\sinh(c + dx))}{b^2d} - \frac{\sqrt{a}(2a + 3b) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c + dx)}{\sqrt{a+b}}\right)}{2b^2(a + b)^{3/2}d} - \frac{a \sinh(c + dx)}{2b(a + b)d(a + b + a \sinh^2(c + dx))}$$

Mathematica [B] time = 2.53, size = 282, normalized size = 2.79

$$\frac{\operatorname{sech}^3(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(-2ab\sqrt{a+b} \sqrt{(\cosh(c) - \sinh(c))^2} \tanh(c + dx) + \sqrt{a}(2a + 3b) \operatorname{ArcTan}\left(\frac{\sqrt{a} \sinh(c + dx)}{\sqrt{a+b}}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c])])/Sqrt[a]]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x] - (a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(-4*(a + b)^(3/2)*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] + Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c])])/Sqrt[a]]*Sinh[c] - 2*a*b*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8*b^2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])

fricas [B] time = 0.50, size = 2069, normalized size = 20.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)

```

*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3
*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh
(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a +
b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) - 8*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sinh
(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3 + (a^2 + 3*a*b + 2
*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) +
4*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d
*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2
+ a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c)^
2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*
d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a*b*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d
*x + c)^3 - 2*a*b*cosh(d*x + c) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a
^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4
+ 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*
x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a
^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d
*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d
*x + c))) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 7*a*b +
6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a
*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(d*x +
c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b
))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*si
nh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*
b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 4*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*
(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 +
a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x
+ c)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*
x + c) + sinh(d*x + c)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3
+ 2*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^
2*b^2 + 3*a*b^3 + 2*b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2
*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[1
```

2,-32]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Underf/Unsigned Inf encountered in limitEvaluation time: 0.62Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.27, size = 361, normalized size = 3.57

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right) b (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)-1/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-1/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+2/d/b^2*arctan(tanh(1/2*d*x+1/2*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^{(3dx+3c)} - ae^{(dx+c)}}{a^2bd + ab^2d + (a^2bde^{(4c)} + ab^2de^{(4c)})e^{(4dx)} + 2(a^2bde^{(2c)} + 3ab^2de^{(2c)} + 2b^3de^{(2c)})e^{(2dx)}} + \frac{2 \arctan(e^{(dx+c)})}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -(a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(a^2*b*d + a*b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c) + 3*a*b^2*d*e^(2*c) + 2*b^3*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(1/3*2*((2*a^2*e^(3*c) + 3*a*b*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + 3*a*b*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) + 3*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^5 \left(a + \frac{b}{\cosh(c + dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c + dx)}{\left(a + b \operatorname{sech}^2(c + dx) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```


$$3.91 \quad \int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=101

$$\frac{a^2 \tanh(c+dx)}{2b^2d(a+b)(a-b\tanh^2(c+dx)+b)} - \frac{a(3a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}} + \frac{\tanh(c+dx)}{b^2d}$$

[Out] $-1/2*a*(3*a+4*b)*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})/b^{(5/2)}/(a+b)^{(3/2)}/d+\tanh(d*x+c)/b^2/d+1/2*a^2*\tanh(d*x+c)/b^2/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 390, 385, 208}

$$\frac{a^2 \tanh(c+dx)}{2b^2d(a+b)(a-b\tanh^2(c+dx)+b)} - \frac{a(3a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}} + \frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2, x]

[Out] $-(a*(3*a+4*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c+d*x])/(\operatorname{Sqrt}[a+b])]/(2*b^{(5/2)}*(a+b)^{(3/2)}*d)+\operatorname{Tanh}[c+d*x]/(b^2*d)+(a^2*\operatorname{Tanh}[c+d*x])/(2*b^2*(a+b)*d*(a+b-b*\operatorname{Tanh}[c+d*x]^2))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a(a+2b)-2abx^2}{b^2(a+b-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{b^2d} - \frac{\operatorname{Subst}\left(\int \frac{a(a+2b)-2abx^2}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{b^2d} \\
&= \frac{\tanh(c+dx)}{b^2d} + \frac{a^2 \tanh(c+dx)}{2b^2(a+b)d(a+b-b\tanh^2(c+dx))} - \frac{(a(3a+4b)) \operatorname{Subst}\left(\int \frac{1}{a+b-x^2} dx, x, \tanh(c+dx)\right)}{2b^2(a+b)d} \\
&= -\frac{a(3a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{a^2 \tanh(c+dx)}{2b^2(a+b)d(a+b-b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [B] time = 3.81, size = 229, normalized size = 2.27

$$\frac{\operatorname{sech}^4(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(2 \operatorname{sech}(c) \sinh(dx) \operatorname{sech}(c+dx)(a \cosh(2(c+dx)) + a + 2b) + \frac{a \operatorname{asech}(2(c+dx))}{\sqrt{a+b}} \right)}{8b^2d(a+b\operatorname{sech}^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-((a*(3*a + 4*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(a + b)^(3/2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]*Sinh[d*x] + (a*(a*Sech[2*c]*Sinh[2*d*x] - (a + 2*b)*Tanh[2*c]))/(a + b))/(8*b^2*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.48, size = 2958, normalized size = 29.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 20*a^2*b^2 + 8*a*b^3 + 8*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c)^2 + 8*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 4*a^2*b)*cosh(d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3

$$\begin{aligned}
& + 3a^3 + 4a^2b + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^2 + (15(3a^3 + 4a^2b)\cosh(dx + c)^4 + 9a^3 + 24a^2b + 16ab^2 + 6(9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 2(3(3a^3 + 4a^2b)b)\cosh(dx + c)^5 + 2(9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^3 + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)\sinh(dx + c)\sqrt{ab + b^2}\log \\
& ((a^2\cosh(dx + c)^4 + 4a^2\cosh(dx + c)\sinh(dx + c)^3 + a^2\sinh(dx + c)^4 + 2(a^2 + 2ab)\cosh(dx + c)^2 + 2(3a^2\cosh(dx + c)^2 + a^2 + 2ab)\sinh(dx + c)^2 + a^2 + 8ab + 8b^2 + 4(a^2\cosh(dx + c)^3 + (a^2 + 2ab)\cosh(dx + c))\sinh(dx + c) + 4(a\cosh(dx + c)^2 + 2a\cosh(dx + c)\sinh(dx + c) + a\sinh(dx + c)^2 + a + 2b)\sqrt{ab + b^2}))/ (a\cosh(dx + c)^4 + 4a\cosh(dx + c)\sinh(dx + c)^3 + a\sinh(dx + c)^4 + 2(a + 2b)\cosh(dx + c)^2 + 2(3a\cosh(dx + c)^2 + a + 2b)\sinh(dx + c)^2 + 4(a\cosh(dx + c)^3 + (a + 2b)\cosh(dx + c))\sinh(dx + c) + a)) + \\
& 16((3a^3b + 7a^2b^2 + 4ab^3)\cosh(dx + c)^3 + (3a^3b + 10a^2b^2 + 11ab^3 + 4b^4)\cosh(dx + c))\sinh(dx + c))/((a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^6 + 6(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)\sinh(dx + c)^5 + (a^3b^3 + 2a^2b^4 + ab^5)d\sinh(dx + c)^6 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^4 + (15(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^2 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d)\sinh(dx + c)^4 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^2 + 4(5(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^3 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c))\sinh(dx + c)^3 + (15(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^4 + 6(3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^2 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d)\sinh(dx + c)^2 + (a^3b^3 + 2a^2b^4 + ab^5)d + 2(3(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^5 + 2(3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^3 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c))\sinh(dx + c)), -1/2(2(3a^3b + 7a^2b^2 + 4ab^3)\cosh(dx + c)^4 + 8(3a^3b + 7a^2b^2 + 4ab^3)\cosh(dx + c)\sinh(dx + c)^3 + 2(3a^3b + 7a^2b^2 + 4ab^3)\sinh(dx + c)^4 + 6a^3b + 10a^2b^2 + 4ab^3 + 4(3a^3b + 10a^2b^2 + 11ab^3 + 4b^4)\cosh(dx + c)^2 + 4(3a^3b + 10a^2b^2 + 11ab^3 + 4b^4 + 3(3a^3b + 7a^2b^2 + 4ab^3)\cosh(dx + c)^2)\sinh(dx + c)^2 + ((3a^3 + 4a^2b)\cosh(dx + c)^6 + 6(3a^3 + 4a^2b)\cosh(dx + c)\sinh(dx + c)^5 + (3a^3 + 4a^2b)\sinh(dx + c)^6 + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^4 + (9a^3 + 24a^2b + 16ab^2 + 15(3a^3 + 4a^2b)\cosh(dx + c)^2)\sinh(dx + c)^4 + 4(5(3a^3 + 4a^2b)\cosh(dx + c)^3 + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c))\sinh(dx + c)^3 + 3a^3 + 4a^2b + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^2 + (15(3a^3 + 4a^2b)\cosh(dx + c)^4 + 9a^3 + 24a^2b + 16ab^2 + 6(9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 2(3(3a^3 + 4a^2b)\cosh(dx + c)^5 + 2(9a^3 + 24a^2b + 16ab^2)\cosh(dx + c)^3 + (9a^3 + 24a^2b + 16ab^2)\cosh(dx + c))\sinh(dx + c)\sqrt{-ab - b^2}\arctan(1/2(a\cosh(dx + c)^2 + 2a\cosh(dx + c)\sinh(dx + c) + a\sinh(dx + c)^2 + a + 2b)\sqrt{-ab - b^2})/(ab + b^2)) + 8((3a^3b + 7a^2b^2 + 4ab^3)\cosh(dx + c)^3 + (3a^3b + 10a^2b^2 + 11ab^3 + 4b^4)\cosh(dx + c))\sinh(dx + c))/((a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^6 + 6(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)\sinh(dx + c)^5 + (a^3b^3 + 2a^2b^4 + ab^5)d\sinh(dx + c)^6 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^4 + (15(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^2 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d)\sinh(dx + c)^4 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^2 + 4(5(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^3 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c))\sinh(dx + c)^3 + (15(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^4 + 6(3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^2 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d)\sinh(dx + c)^2 + (a^3b^3 + 2a^2b^4 + ab^5)d + 2(3(a^3b^3 + 2a^2b^4 + ab^5)d\cosh(dx + c)^5 + 2(3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c)^3 + (3a^3b^3 + 10a^2b^4 + 11ab^5 + 4b^6)d\cosh(dx + c))\sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.79, size = 225, normalized size = 2.23

$$\frac{(3a^2+4ab)\arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(ab^2+b^3)\sqrt{-ab-b^2}} + \frac{2(3a^2e^{(4dx+4c)}+4abe^{(4dx+4c)}+6a^2e^{(2dx+2c)}+14abe^{(2dx+2c)}+8b^2e^{(2dx+2c)}+3a^2+2ab)}{(ab^2+b^3)(ae^{(6dx+6c)}+3ae^{(4dx+4c)}+4be^{(4dx+4c)}+3ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2*((3*a^2 + 4*a*b)*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + a + 2*b)/\sqrt{-a*b - b^2}))/((a*b^2 + b^3)*\sqrt{-a*b - b^2}) + 2*(3*a^2*e^{(4*d*x + 4*c)} + 4*a*b*e^{(4*d*x + 4*c)} + 6*a^2*e^{(2*d*x + 2*c)} + 14*a*b*e^{(2*d*x + 2*c)} + 8*b^2*e^{(2*d*x + 2*c)} + 3*a^2 + 2*a*b)/((a*b^2 + b^3)*(a*e^{(6*d*x + 6*c)} + 3*a*e^{(4*d*x + 4*c)} + 4*b*e^{(4*d*x + 4*c)} + 3*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a))/d$

maple [B] time = 0.28, size = 419, normalized size = 4.15

$$\frac{a^2 \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{db^2 \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right) (a + b) + db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x)

[Out] $1/d*a^2/b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)^3+1/d*a^2/b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)+3/4/d*a^2/b^(5/2)/(a+b)^(3/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-3/4/d*a^2/b^(5/2)/(a+b)^(3/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/d*a/b^(3/2)/(a+b)^(3/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/d/(a+b)^(3/2)/b^(3/2)*a*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+2/d/b^2*\tanh(1/2*d*x+1/2*c)/(\tanh(1/2*d*x+1/2*c)^2+1)$

maxima [B] time = 0.55, size = 244, normalized size = 2.42

$$\frac{(3a+4b)a \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4(ab^2+b^3)\sqrt{(a+b)b}d} + \frac{3a^2+2ab+2(3a^2+7ab+4b^2)e^{(-2dx-2c)}+(3a^2b^2+ab^3+(3a^2b^2+7ab^3+4b^4)e^{(-2dx-2c)}+(3a^2b^2+7ab^3+4b^4)e^{(-4dx-4c)}+(a^2b^2+ab^3)*e^{(-6dx-6c)})*d}{(a^2b^2+ab^3+(3a^2b^2+7ab^3+4b^4)e^{(-2dx-2c)}+(3a^2b^2+7ab^3+4b^4)e^{(-4dx-4c)}+(a^2b^2+ab^3)*e^{(-6dx-6c)})*d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/4*(3*a + 4*b)*a*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b}))/((a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a*b^2 + b^3)*\sqrt{(a + b)*b})*d + (3*a^2 + 2*a*b + 2*(3*a^2 + 7*a*b + 4*b^2)*e^{(-2*d*x - 2*c)} + (3*a^2 + 4*a*b)*e^{(-4*d*x - 4*c)})/((a^2*b^2 + a*b^3 + (3*a^2*b^2 + 7*a*b^3 + 4*b^4)*e^{(-2*d*x - 2*c)} + (3*a^2*b^2 + 7*a*b^3 + 4*b^4)*e^{(-4*d*x - 4*c)} + (a^2*b^2 + a*b^3)*e^{(-6*d*x - 6*c)})*d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+dx)^6 \left(a + \frac{b}{\cosh(c+dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2), x)`

[Out] `int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**2, x)`

[Out] `Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**2, x)`

$$3.92 \quad \int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=153

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}} - \frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)(a\sinh^2(c+dx)+a+b)} + \frac{\tanh(c+dx)}{2bd(a+b)}$$

[Out] $-1/2*(4*a-b)*\arctan(\sinh(d*x+c))/b^3/d+1/2*a^{(3/2)}*(4*a+5*b)*\arctan(\sinh(d*x+c)*a^{(1/2)/(a+b)^{(1/2)})}/b^3/(a+b)^{(3/2)}/d+1/2*a*(2*a+b)*\sinh(d*x+c)/b^2/(a+b)/d/(a+b+a*\sinh(d*x+c)^2)+1/2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/b/d/(a+b+a*\sinh(d*x+c)^2)$

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4147, 414, 527, 522, 203, 205}

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}} + \frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)(a\sinh^2(c+dx)+a+b)} - \frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{\tanh(c+dx)}{2bd(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]

[Out] $-((4*a - b)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*b^3*d) + (a^{(3/2)}*(4*a + 5*b)*\operatorname{ArcTan}[\operatorname{Sqrt}[a]*\operatorname{Sinh}[c + d*x]]/\operatorname{Sqrt}[a + b])/(2*b^3*(a + b)^{(3/2)*d} + (a*(2*a + b)*\operatorname{Sinh}[c + d*x])/(2*b^2*(a + b)*d*(a + b + a*\operatorname{Sinh}[c + d*x]^2)) + (\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(2*b*d*(a + b + a*\operatorname{Sinh}[c + d*x]^2))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)^2} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + b + a \sinh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a-b-3ax^2}{(1+x^2)(a+b+ax^2)^2} dx, x, \sinh(c + dx)\right)}{2bd}$$

$$= \frac{a(2a + b) \sinh(c + dx)}{2b^2(a + b)d(a + b + a \sinh^2(c + dx))} + \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + b + a \sinh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a-b-3ax^2}{(1+x^2)(a+b+ax^2)^2} dx, x, \sinh(c + dx)\right)}{2bd}$$

$$= \frac{a(2a + b) \sinh(c + dx)}{2b^2(a + b)d(a + b + a \sinh^2(c + dx))} + \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + b + a \sinh^2(c + dx))} - \frac{(4a - b) \tan^{-1}(\sinh(c + dx))}{2b^3d} + \frac{a^{3/2}(4a + 5b) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c + dx)}{\sqrt{a+b}}\right)}{2b^3(a + b)^{3/2}d} + \frac{a}{2b^2(a + b)}$$

Mathematica [B] time = 5.19, size = 489, normalized size = 3.20

$$\frac{\operatorname{sech}(c) \operatorname{sech}^3(c + dx) (a \cosh(2(c + dx)) + a + 2b) \left(-a^{3/2} (4a + 5b) \cosh^2(c) \operatorname{sech}(c + dx) (a \cosh(2(c + dx)) + a + 2b) \right)}{2b^3(a + b)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]^3*(-(a^(3/2)*(4*a +
5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
+ Sinh[c]))/Sqrt[a]]*Cosh[c]^2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*
x]) + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Sqrt[
(Cosh[c] - Sinh[c])^2]*Sinh[c] - Cosh[c]*Sech[c + d*x]*(2*Sqrt[a + b]*(4*a^
2 + 3*a*b - b^2)*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])*
Sqrt[(Cosh[c] - Sinh[c])^2] - a^(5/2)*b*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*S
qrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(13 + 5*Cosh[2*(c
```

$$+ dx)) * \text{Sinh}[c]) + a^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[a + b] * \text{Csch}[c + dx] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * (\text{Cosh}[c] + \text{Sinh}[c])) / \text{Sqrt}[a]] * (2 * a^2 + 5 * b^2 + 2 * a^2 * \text{Cosh}[2 * (c + dx)]) * \text{Sech}[c + dx] * \text{Sinh}[2 * c] + b * (a + b)^{(3/2)} * (a + 2 * b + a * \text{Cosh}[2 * (c + dx)]) * \text{Sech}[c + dx]^3 * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Sinh}[dx] + 2 * a^2 * b * \text{Sqrt}[a + b] * \text{Cosh}[c] * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2] * \text{Tanh}[c + dx])) / (8 * b^3 * (a + b)^{(3/2)} * d * (a + b * \text{Sech}[c + dx]^2)^2 * \text{Sqrt}[(\text{Cosh}[c] - \text{Sinh}[c])^2])$$

fricas [B] time = 0.58, size = 6499, normalized size = 42.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^7/(a+b*sech(dx+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(2*a^2*b + a*b^2)*cosh(dx + c)^7 + 28*(2*a^2*b + a*b^2)*cosh(dx + c)*sinh(dx + c)^6 + 4*(2*a^2*b + a*b^2)*sinh(dx + c)^7 + 4*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c)^5 + 4*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b + a*b^2)*cosh(dx + c)^2)*sinh(dx + c)^5 + 20*(7*(2*a^2*b + a*b^2)*cosh(dx + c)^3 + (2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c))*sinh(dx + c)^4 - 4*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c)^3 + 4*(35*(2*a^2*b + a*b^2)*cosh(dx + c)^4 - 2*a^2*b - 5*a*b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c)^2)*sinh(dx + c)^3 + 4*(21*(2*a^2*b + a*b^2)*cosh(dx + c)^5 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(dx + c))*sinh(dx + c)^2 + ((4*a^3 + 5*a^2*b)*cosh(dx + c)^8 + 8*(4*a^3 + 5*a^2*b)*cosh(dx + c)*sinh(dx + c)^7 + (4*a^3 + 5*a^2*b)*sinh(dx + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^6 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(dx + c)^2)*sinh(dx + c)^6 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(dx + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c))*sinh(dx + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(dx + c)^4 + 2*(35*(4*a^3 + 5*a^2*b)*cosh(dx + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^2)*sinh(dx + c)^4 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(dx + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(dx + c))*sinh(dx + c)^3 + 4*a^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^2 + 4*(7*(4*a^3 + 5*a^2*b)*cosh(dx + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^4 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(dx + c)^2)*sinh(dx + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(dx + c)^7 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(dx + c)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(dx + c))*sinh(dx + c)*sqrt(-a/(a + b))*log((a*cosh(dx + c)^4 + 4*a*cosh(dx + c)*sinh(dx + c)^3 + a*sinh(dx + c)^4 - 2*(3*a + 2*b)*cosh(dx + c)^2 + 2*(3*a*cosh(dx + c)^2 - 3*a - 2*b)*sinh(dx + c)^2 + 4*(a*cosh(dx + c)^3 - (3*a + 2*b)*cosh(dx + c))*sinh(dx + c) + 4*((a + b)*cosh(dx + c)^3 + 3*(a + b)*cosh(dx + c)*sinh(dx + c)^2 + (a + b)*sinh(dx + c)^3 - (a + b)*cosh(dx + c) + (3*(a + b)*cosh(dx + c)^2 - a - b)*sinh(dx + c))*sqrt(-a/(a + b)) + a)/(a*cosh(dx + c)^4 + 4*a*cosh(dx + c)*sinh(dx + c)^3 + a*sinh(dx + c)^4 + 2*(a + 2*b)*cosh(dx + c)^2 + 2*(3*a*cosh(dx + c)^2 + a + 2*b)*sinh(dx + c)^2 + 4*(a*cosh(dx + c)^3 + (a + 2*b)*cosh(dx + c))*sinh(dx + c) + a)) - 4*((4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)*sinh(dx + c)^7 + (4*a^3 + 3*a^2*b - a*b^2)*sinh(dx + c)^8 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(dx + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)^2)*sinh(dx + c)^6 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(dx + c))*sinh(dx + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(dx + c)^4 + 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)^4 + 12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(dx + c)^2)*sinh(dx + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(dx + c)^5 + 10*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(dx + c)^3 + (12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(dx + c))*sinh(dx + c)^3 + 4*a^3 + 3*a^2*b - a*b^2 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(dx + c)^2 + 4*(7*(4*a^3 + 3*a

$$\begin{aligned}
& ^2*b - a*b^2)*\cosh(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d \\
& *x + c)^4 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^2 \\
& - 4*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)* \\
& \cosh(d*x + c)^7 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^5 + (12 \\
& *a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*\cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a \\
& *b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + \\
& c)) - 4*(2*a^2*b + a*b^2)*\cosh(d*x + c) + 4*(7*(2*a^2*b + a*b^2)*\cosh(d*x \\
& + c)^6 + 5*(2*a^2*b + 5*a*b^2 + 4*b^3)*\cosh(d*x + c)^4 - 2*a^2*b - a*b^2 - \\
& 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^2*b^3 + a \\
& *b^4)*d*\cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^7 + (a^2*b^3 + a*b^4)*d*\sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*co \\
& sh(d*x + c)^6 + 4*(7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b \\
& ^4 + b^5)*d)*\sinh(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*\cosh(d*x + \\
& c)^4 + 8*(7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b \\
& ^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*\cosh(d*x + \\
& c)^4 + 30*(a^2*b^3 + 2*a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b \\
& ^4 + 4*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*\cosh(d*x + c \\
&)^2 + 8*(7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^ \\
& 5)*d*\cosh(d*x + c)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + 4*(7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b \\
& ^4 + b^5)*d*\cosh(d*x + c)^4 + 3*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*\cosh(d*x + \\
& c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*\sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d + \\
& 8*((a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^7 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cos \\
& h(d*x + c)^5 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*\cosh(d*x + c)^3 + (a^2*b^3 + \\
& 2*a*b^4 + b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/2*(2*(2*a^2*b + a*b^2)*c \\
& osh(d*x + c)^7 + 14*(2*a^2*b + a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(2* \\
& a^2*b + a*b^2)*\sinh(d*x + c)^7 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3)*\cosh(d*x + c \\
&)^5 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b + a*b^2)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^5 + 10*(7*(2*a^2*b + a*b^2)*\cosh(d*x + c)^3 + (2*a^2*b + 5*a* \\
& b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 2*(2*a^2*b + 5*a*b^2 + 4*b^3) \\
& *\cosh(d*x + c)^3 + 2*(35*(2*a^2*b + a*b^2)*\cosh(d*x + c)^4 - 2*a^2*b - 5*a* \\
& b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\
& ^3 + 2*(21*(2*a^2*b + a*b^2)*\cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 + 4*b^ \\
& 3)*\cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^2 + ((4*a^3 + 5*a^2*b)*\cosh(d*x + c)^8 + 8*(4*a^3 + 5*a^2*b)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*\sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2 \\
& *b + 5*a*b^2)*\cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5 \\
& *a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a^3 + 5*a^2*b)*\cosh(d*x \\
& + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(\\
& 12*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 5*a^2*b)*cos \\
& h(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30*(4*a^3 + 9*a^2*b + 5*a*b^2) \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^5 \\
& + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^3 + (12*a^3 + 31*a^2*b + 20 \\
& *a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2 \\
& *b + 5*a*b^2)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^6 + 15 \\
& *(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 5*a*b^2 + \\
& 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4*a \\
& ^3 + 5*a^2*b)*\cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c) \\
& ^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 5* \\
& a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a/(a + b))*\arctan(1/2*\sqrt{a/(a + \\
& b))*(\cosh(d*x + c) + \sinh(d*x + c))} + ((4*a^3 + 5*a^2*b)*\cosh(d*x + c)^8 \\
& + 8*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sin \\
& h(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^6 + 4*(4*a^3 + 9 \\
& *a^2*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8 \\
& *(7*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(\\
& d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^ \\
& 4 + 2*(35*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 \\
& + 30*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4 \\
& *a^3 + 5*a^2*b)*\cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^3 + (12a^3 + 31a^2b + 20ab^2) \cosh(dx + c) \sinh(dx + c)^3 + 4a^3 \\
& + 5a^2b + 4(4a^3 + 9a^2b + 5ab^2) \cosh(dx + c)^2 + 4(7(4a^3 + 5a^2b) \cosh(dx + c)^6 + 15(4a^3 + 9a^2b + 5ab^2) \cosh(dx + c)^4 \\
& + 4a^3 + 9a^2b + 5ab^2 + 3(12a^3 + 31a^2b + 20ab^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((4a^3 + 5a^2b) \cosh(dx + c)^7 + 3(4a^3 + 9 \\
& a^2b + 5ab^2) \cosh(dx + c)^5 + (12a^3 + 31a^2b + 20ab^2) \cosh(dx + c)^3 + (4a^3 + 9a^2b + 5ab^2) \cosh(dx + c) \sinh(dx + c)) \sqrt{a/ \\
& (a + b)} \arctan(1/2(a \cosh(dx + c)^3 + 3a \cosh(dx + c) \sinh(dx + c)^2 + a \sinh(dx + c)^3 + (3a + 4b) \cosh(dx + c) + (3a \cosh(dx + c)^2 + 3a \\
& + 4b) \sinh(dx + c)) \sqrt{a/(a + b)})/a - 2((4a^3 + 3a^2b - ab^2) \cosh(dx + c)^8 + 8(4a^3 + 3a^2b - ab^2) \cosh(dx + c) \sinh(dx + c)^7 \\
& + (4a^3 + 3a^2b - ab^2) \sinh(dx + c)^8 + 4(4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)^6 + 4(4a^3 + 7a^2b + 2ab^2 - b^3 + 7(4a^3 + 3a \\
& a^2b - ab^2) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(4a^3 + 3a^2b - ab^2) \cosh(dx + c)^3 + 3(4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)) \sinh(dx + c)^5 + 2(12a^3 + 25a^2b + 9ab^2 - 4b^3) \cosh(dx + c)^4 + \\
& 2(35(4a^3 + 3a^2b - ab^2) \cosh(dx + c)^4 + 12a^3 + 25a^2b + 9ab^2 - 4b^3 + 30(4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(7(4a^3 + 3a^2b - ab^2) \cosh(dx + c)^5 + 10(4a^3 + 7a \\
& ^2b + 2ab^2 - b^3) \cosh(dx + c)^3 + (12a^3 + 25a^2b + 9ab^2 - 4b^3) \cosh(dx + c)) \sinh(dx + c)^3 + 4a^3 + 3a^2b - ab^2 + 4(4a^3 + 7a \\
& ^2b + 2ab^2 - b^3) \cosh(dx + c)^2 + 4(7(4a^3 + 3a^2b - ab^2) \cosh(dx + c)^6 + 15(4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)^4 + 4a^3 \\
& + 7a^2b + 2ab^2 - b^3 + 3(12a^3 + 25a^2b + 9ab^2 - 4b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((4a^3 + 3a^2b - ab^2) \cosh(dx + c)^7 + \\
& 3(4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)^5 + (12a^3 + 25a^2b + 9ab^2 - 4b^3) \cosh(dx + c)^3 + (4a^3 + 7a^2b + 2ab^2 - b^3) \cosh(dx + c)) \sinh(dx + c) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 2(2a^2b \\
& + ab^2) \cosh(dx + c) + 2(7(2a^2b + ab^2) \cosh(dx + c)^6 + 5(2a^2b + 5ab^2 + 4b^3) \cosh(dx + c)^4 - 2a^2b - ab^2 - 3(2a^2b + 5ab^2 + 4b^3) \cosh(dx + c)^2) \sinh(dx + c) / ((a^2b^3 + ab^4) d \cosh(dx + c)^8 + 8(a^2b^3 + ab^4) d \cosh(dx + c) \sinh(dx + c)^7 + (a^2b^3 + ab^4) d \sinh(dx + c)^8 + 4(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^6 + 4(7(a^2b^3 + ab^4) d \cosh(dx + c)^2 + (a^2b^3 + 2ab^4 + b^5) d) \sinh(dx + c)^6 + 2(3a^2b^3 + 7ab^4 + 4b^5) d \cosh(dx + c)^4 + 8(7(a^2b^3 + ab^4) d \cosh(dx + c)^3 + 3(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^2b^3 + ab^4) d \cosh(dx + c)^4 + 30(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^2 + (3a^2b^3 + 7ab^4 + 4b^5) d) \sinh(dx + c)^4 + 4(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^2 + 8(7(a^2b^3 + ab^4) d \cosh(dx + c)^5 + 10(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^3 + (3a^2b^3 + 7ab^4 + 4b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^2b^3 + ab^4) d \cosh(dx + c)^6 + 15(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^4 + 3(3a^2b^3 + 7ab^4 + 4b^5) d \cosh(dx + c)^2 + (a^2b^3 + 2ab^4 + b^5) d) \sinh(dx + c)^2 + (a^2b^3 + ab^4) d + 8((a^2b^3 + ab^4) d \cosh(dx + c)^7 + 3(a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)^5 + (3a^2b^3 + 7ab^4 + 4b^5) d \cosh(dx + c)^3 + (a^2b^3 + 2ab^4 + b^5) d \cosh(dx + c)) \sinh(dx + c)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^7/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
UT:sage2:=int(sage0,x); OUTPUT: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[6,-20] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming

[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[12,-32]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Undefined/Unsigned Inf encountered in limitEvaluation time: 0.71Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.29, size = 448, normalized size = 2.93

$$\frac{a^2 \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{db^2 \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right) (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/d*a^2/b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)^3+1/d*a^2/b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*\tanh(1/2*d*x+1/2*c)+2/d*a^(5/2)/b^3/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+2/d*a^(5/2)/b^3/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a+b)^(3/2)*\arctan(1/2*(2*(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/b^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)+1/d/b^2*\arctan(\tanh(1/2*d*x+1/2*c))-4/d/b^3*\arctan(\tanh(1/2*d*x+1/2*c))*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2 a^2 e^{7c} + a b e^{7c} \right) e^{7dx} + \left(2 a^2 e^{5c} + 5 a b e^{5c} + 4 b^2 e^{5c} \right) e^{5dx} - \left(2 a^2 e^{3c} + 5 a^2 b^2 d + a b^3 d + \left(a^2 b^2 d e^{8c} + a b^3 d e^{8c} \right) e^{8dx} + 4 \left(a^2 b^2 d e^{6c} + 2 a b^3 d e^{6c} + b^4 d e^{6c} \right) e^{6dx} + 2 \left(3 a^2 b^2 d e^{4c} + 5 a^2 b^2 d e^{4c} + 4 a b^3 d e^{4c} + 2 a b^4 d e^{4c} \right) e^{4dx} + 2 \left(3 a^2 b^2 d e^{2c} + 4 a b^3 d e^{2c} + 2 a b^4 d e^{2c} \right) e^{2dx}}{\left(a^2 b^2 d + a b^3 d + \left(a^2 b^2 d e^{8c} + a b^3 d e^{8c} \right) e^{8dx} + 4 \left(a^2 b^2 d e^{6c} + 2 a b^3 d e^{6c} + b^4 d e^{6c} \right) e^{6dx} + 2 \left(3 a^2 b^2 d e^{4c} + 5 a^2 b^2 d e^{4c} + 4 a b^3 d e^{4c} + 2 a b^4 d e^{4c} \right) e^{4dx} + 2 \left(3 a^2 b^2 d e^{2c} + 4 a b^3 d e^{2c} + 2 a b^4 d e^{2c} \right) e^{2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $((2*a^2*e^{7*c} + a*b*e^{7*c})*e^{7*d*x} + (2*a^2*e^{5*c} + 5*a*b*e^{5*c} + 4*b^2*e^{5*c})*e^{5*d*x} - (2*a^2*e^{3*c} + 5*a*b*e^{3*c} + 4*b^2*e^{3*c})*e^{3*d*x} - (2*a^2*e^c + a*b*e^c)*e^{d*x})/(a^2*b^2*d + a*b^3*d + (a^2*b^2*d*e^{8*c} + a*b^3*d*e^{8*c})*e^{8*d*x} + 4*(a^2*b^2*d*e^{6*c} + 2*a*b^3*d*e^{6*c} + b^4*d*e^{6*c})*e^{6*d*x} + 2*(3*a^2*b^2*d*e^{4*c} + 7*a*b^3*d*e^{4*c} + 4*b^4*d*e^{4*c})*e^{4*d*x} + 4*(a^2*b^2*d*e^{2*c} + 2*a*b^3*d*e^{2*c} + b^4*d*e^{2*c})*e^{2*d*x}) - (4*a*e^c - b*e^c)*\arctan(e^{d*x + c})*e^{-c}/(b^3*d) + 128*integrate(1/128*((4*a^3*e^{3*c} + 5*a^2*b*e^{3*c})*e^{3*d*x} + (4*a^3*e^c + 5*a^2*b*e^c)*e^{d*x})/(a^2*b^3 + a*b^4 + (a^2*b^3*e^{4*c} + a*b^4*e^{4*c})*e^{4*d*x} + 2*(a^2*b^3*e^{2*c} + 3*a*b^4*e^{2*c} + 2*b^5*e^{2*c})*e^{2*d*x}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^7 \left(a + \frac{b}{\cosh(c + dx)^2} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2),x)

[Out] `int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**2, x)`

$$3.93 \quad \int \frac{\cosh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=204

$$\frac{x(a-6b)}{2a^4} + \frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2(a-b\tanh^2(c+dx)+b)} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2d(a+b)(a-b\tanh^2(c+dx)+b)^2} + \frac{b^{3/2}(35a^2+56ab+24b^2)\operatorname{arctanh}(b^{1/2}\tanh(d*x+c)/(a+b)^{1/2})/a^4/(a+b)^{5/2}/d+1/2*\cosh(d*x+c)*\sinh(d*x+c)/a/d/(a+b-b*\tanh(d*x+c)^2)^2+1/4*b*(2*a+3*b)*\tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)^2+1/8*b*(4*a+3*b)*(a+4*b)*\tanh(d*x+c)/a^3/(a+b)^2/d/(a+b-b*\tanh(d*x+c)^2)}{8a^4d(a+b)^{5/2}}$$

[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)^2+1/4*b*(2*a+3*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*b*(4*a+3*b)*(a+4*b)*tanh(d*x+c)/a^3/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.38, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, number of rules / integrand size = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}(35a^2+56ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{5/2}} + \frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2(a-b\tanh^2(c+dx)+b)} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2d(a+b)(a-b\tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*d) + (Cosh[c + d*x]*Sin h[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(2*a + 3*b)*Tanh[c + d*x])/(4*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*Tanh[c + d*x])/(8*a^3*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,

c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{b(2a + 3b) \tanh(c + dx)}{4a^2(a + b)d(a + b - b \tanh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{8a^3(a + b)}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{b(2a + 3b) \tanh(c + dx)}{4a^2(a + b)d(a + b - b \tanh^2(c + dx))^2} + \frac{b(4a^2 - 3ab + 3b^2)}{8a^3(a + b)}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2} + \frac{b(2a + 3b) \tanh(c + dx)}{4a^2(a + b)d(a + b - b \tanh^2(c + dx))^2} + \frac{b(4a^2 - 3ab + 3b^2)}{8a^3(a + b)}$$

$$= \frac{(a - 6b)x}{2a^4} + \frac{b^{3/2}(35a^2 + 56ab + 24b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a + b}}\right)}{8a^4(a + b)^{5/2}d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))^2}$$

Mathematica [A] time = 4.23, size = 156, normalized size = 0.76

$$\frac{b^{3/2}(35a^2 + 56ab + 24b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a + b}}\right)}{(a + b)^{5/2}} + a \sinh(2(c + dx)) \left(\frac{2b^3(5a \cosh(2(c + dx)) + 3a + 8b)}{(a + b)^2(a \cosh(2(c + dx)) + a + 2b)^2} + \frac{13ab^2}{(a + b)^2(a \cosh(2(c + dx)) + a + 2b)} + 2 \right)$$

$$8a^4d$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

```
[Out] (4*(a - 6*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cosh[2*(c + d*x)]))) + (2*b^3*(3*a + 8*b + 5*a*Cosh[2*(c + d*
x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)]^2))*Sinh[2*(c + d*x)]/(8*
a^4*d)
```

fricas [B] time = 0.66, size = 11740, normalized size = 57.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^12 + 24*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 2*a^4*b + a^3*b^2)*sinh(d
*x + c)^12 + 8*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^5 + 8*a^4*b + 10*a^3*b
^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 +
2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 +
(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*
a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5
+ 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*
b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^5 +
2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2
*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(
39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3
*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^5
+ 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 39*a^3*b^2 - 134*a^2*b^3 - 184*a*b^
4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*
b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x + 14*(5*a^5 + 26*a^4*b + 27*a^3
*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3
- 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 2*a^4*b +
a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a
^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 + 2
6*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b
^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*b^2 + 134*a^2*
b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 -
136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^5 - 4*a^4*b -
2*a^3*b^2 - 2*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 -
16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^
4 + 2*(495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 840*(a^5 + 4*a^4*b +
5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh
(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4 + 70
*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b
- 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^5 - 2*a
^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x - 30*(39*a^3*b^2 + 134*a^2*b
^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 1
36*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^
3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 + 1
34*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2
```

$$\begin{aligned}
& *b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c)^3 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(2*a^5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^2 + 4*(33*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^10 + 90*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*a^2*b^3 - 15*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c)^4 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\sinh(d*x + c)^10 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^8 + (140*a^4*b + 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^6 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^2*b^3 + 96*a*b^4 + 140*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^4 + 15*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^7 + 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^5 + 5*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^2 + (45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^8 + 112*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^6 + 35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/(a + b))*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^11 + 10*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^7 - 3*(39*a^3*
\end{aligned}$$

$$\begin{aligned}
& b^2 + 134a^2b^3 + 184a^3b^4 + 80b^5 - 4(3a^5 - 4a^4b - 57a^3b^2 - 138a^2b^3 - 136a^3b^4 - 48b^5)d^*x) * \cosh(dx + c)^5 - (5a^5 + 26a^4b + 131a^3b^2 + 256a^2b^3 + 128a^3b^4 - 16(a^5 - 2a^4b - 19a^3b^2 - 28a^2b^3 - 12a^3b^4)d^*x) * \cosh(dx + c)^3 - (2a^5 + 8a^4b + 23a^3b^2 + 14a^2b^3 - 2(a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c) * \sinh(dx + c) / ((a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^{10} + 10(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c) * \sinh(dx + c)^9 + (a^8 + 2a^7b + a^6b^2) * d * \sinh(dx + c)^{10} + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^8 + (45(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^2 + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d) * \sinh(dx + c)^8 + 2(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^6 + 8(15(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^3 + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 2(105(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^4 + 56(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^2 + (3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d) * \sinh(dx + c)^6 + 4(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^4 + 4(63(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^5 + 56(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^3 + 3(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2(105(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^6 + 140(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^4 + 15(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^2 + 2(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d) * \sinh(dx + c)^4 + (a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^2 + 8(15(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^7 + 28(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^5 + 5(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^3 + 2(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^8 + 112(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^6 + 30(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^4 + 24(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^2 + (a^8 + 2a^7b + a^6b^2) * d) * \sinh(dx + c)^2 + 2(5(a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)^9 + 16(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^7 + 6(3a^8 + 14a^7b + 27a^6b^2 + 24a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^5 + 8(a^8 + 4a^7b + 5a^6b^2 + 2a^5b^3) * d * \cosh(dx + c)^3 + (a^8 + 2a^7b + a^6b^2) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/8((a^5 + 2a^4b + a^3b^2) * \cosh(dx + c)^{12} + 12(a^5 + 2a^4b + a^3b^2) * \cosh(dx + c) * \sinh(dx + c)^{11} + (a^5 + 2a^4b + a^3b^2) * \sinh(dx + c)^{12} + 4(a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3 + (a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c)^{10} + 2(2a^5 + 8a^4b + 10a^3b^2 + 4a^2b^3 + 2(a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \sinh(dx + c)^{10} + 20(11(a^5 + 2a^4b + a^3b^2) * \cosh(dx + c)^3 + 2(a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3 + (a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c)) * \sinh(dx + c)^9 + (5a^5 + 26a^4b + 27a^3b^2 - 32a^2b^3 - 32a^3b^4 + 16(a^5 - 2a^4b - 19a^3b^2 - 28a^2b^3 - 12a^3b^4)d^*x) * \cosh(dx + c)^8 + (5a^5 + 26a^4b + 27a^3b^2 - 32a^2b^3 - 32a^3b^4 + 495(a^5 + 2a^4b + a^3b^2) * \cosh(dx + c)^4 + 16(a^5 - 2a^4b - 19a^3b^2 - 28a^2b^3 - 12a^3b^4)d^*x + 180(a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3 + (a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 8(99(a^5 + 2a^4b + a^3b^2) * \cosh(dx + c)^5 + 60(a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3 + (a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c)^3 + (5a^5 + 26a^4b + 27a^3b^2 - 32a^2b^3 - 32a^3b^4 + 16(a^5 - 2a^4b - 19a^3b^2 - 28a^2b^3 - 12a^3b^4)d^*x) * \cosh(dx + c)) * \sinh(dx + c)^7 - 2(39a^3b^2 + 134a^2b^3 + 184a^3b^4 + 80b^5 - 4(3a^5 - 4a^4b - 57a^3b^2 - 138a^2b^3 - 136a^3b^4 - 48b^5) * d^*x) * \cosh(dx + c)^6 + 2(462(a^5 + 2a^4b + a^3b^2) * \cosh(dx + c)^6 - 39a^3b^2 - 134a^2b^3 - 184a^3b^4 - 80b^5 + 420(a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3 + (a^5 - 4a^4b - 11a^3b^2 - 6a^2b^3)d^*x) * \cosh(dx + c)^4 + 4(3a^5 - 4a^4b - 57a^3b^2 - 138a^2b^3 - 136a^3b^4 - 48b^5) * d^*x + 14(5a^5 + 26a^4b + 27a^3b^2 - 32a^2b^3 - 32a^3b^4 + 16(a^5 -
\end{aligned}$$

$$\begin{aligned}
& 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(198*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^7 + 252*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^5 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^3 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^4 + (495*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^8 + 840*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4 + 70*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x - 30*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(55*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^9 + 120*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^7 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^5 - 10*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c)^3 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*(2*a^5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^2 + 2*(33*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(d*x + c)^10 + 90*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*\cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*a^2*b^3 - 15*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*\cosh(d*x + c)^4 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\sinh(d*x + c)^10 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^8 + (140*a^4*b + 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^6 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^2*b^3 + 96*a*b^4 + 140*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^4 + 15*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*\cosh(d*x + c)^7 + 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c)^5 + 5*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*\cosh(d*x + c)^3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3
\end{aligned}$$

$$\begin{aligned}
&) * \cosh(dx + c)^2 + (45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3) * \cosh(dx + c)^8 \\
& + 112*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4) * \cosh(dx + c)^6 + \\
& 35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2 \\
& *b^3 + 640*a*b^4 + 192*b^5) * \cosh(dx + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 + \\
& 136*a^2*b^3 + 48*a*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 2*(5*(35*a^4*b + \\
& 56*a^3*b^2 + 24*a^2*b^3) * \cosh(dx + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 13 \\
& 6*a^2*b^3 + 48*a*b^4) * \cosh(dx + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^ \\
& 2*b^3 + 640*a*b^4 + 192*b^5) * \cosh(dx + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 + \\
& 136*a^2*b^3 + 48*a*b^4) * \cosh(dx + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b \\
& ^3) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{-b/(a + b)} * \arctan(1/2*(a * \cosh(dx + \\
& c)^2 + 2*a * \cosh(dx + c) * \sinh(dx + c) + a * \sinh(dx + c)^2 + a + 2*b) * \sqrt{ \\
& (-b/(a + b))/b) + 4*(3*(a^5 + 2*a^4*b + a^3*b^2) * \cosh(dx + c)^11 + 10*(a^5 \\
& + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^ \\
& 3) * dx) * \cosh(dx + c)^9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 3 \\
& 2*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4) * dx) * \cosh \\
& (dx + c)^7 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - \\
& 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5) * dx) * \cosh(dx + c \\
&)^5 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - \\
& 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4) * dx) * \cosh(dx + c)^3 - (2*a^ \\
& 5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a \\
& ^2*b^3) * dx) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^8 + 2*a^7*b + a^6*b^2) * d * \cos \\
& h(dx + c)^10 + 10*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c) * \sinh(dx + c)^ \\
& 9 + (a^8 + 2*a^7*b + a^6*b^2) * d * \sinh(dx + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6 \\
& *b^2 + 2*a^5*b^3) * d * \cosh(dx + c)^8 + (45*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(\\
& dx + c)^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d) * \sinh(dx + c)^8 + \\
& 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4) * d * \cosh(dx + c) \\
& ^6 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c)^3 + 4*(a^8 + 4*a^7*b + \\
& 5*a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 2*(105*(a^8 + 2* \\
& a^7*b + a^6*b^2) * d * \cosh(dx + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5* \\
& b^3) * d * \cosh(dx + c)^2 + (3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^ \\
& 4*b^4) * d) * \sinh(dx + c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d * \cos \\
& h(dx + c)^4 + 4*(63*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c)^5 + 56*(a^8 \\
& + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + c)^3 + 3*(3*a^8 + 14*a^7*b \\
& + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 \\
& *(105*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c)^6 + 140*(a^8 + 4*a^7*b + 5* \\
& a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + c)^4 + 15*(3*a^8 + 14*a^7*b + 27*a^6*b^2 \\
& + 24*a^5*b^3 + 8*a^4*b^4) * d * \cosh(dx + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 \\
& + 2*a^5*b^3) * d) * \sinh(dx + c)^4 + (a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c) \\
& ^2 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c)^7 + 28*(a^8 + 4*a^7*b \\
& + 5*a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + c)^5 + 5*(3*a^8 + 14*a^7*b + 27*a^6*b \\
& ^2 + 24*a^5*b^3 + 8*a^4*b^4) * d * \cosh(dx + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b \\
& ^2 + 2*a^5*b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45*(a^8 + 2*a^7*b + a^6 \\
& *b^2) * d * \cosh(dx + c)^8 + 112*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d * \cos \\
& h(dx + c)^6 + 30*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4) * \\
& d * \cosh(dx + c)^4 + 24*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + \\
& c)^2 + (a^8 + 2*a^7*b + a^6*b^2) * d) * \sinh(dx + c)^2 + 2*(5*(a^8 + 2*a^7*b \\
& + a^6*b^2) * d * \cosh(dx + c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d \\
& * \cosh(dx + c)^7 + 6*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^ \\
& 4) * d * \cosh(dx + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3) * d * \cosh(dx + \\
& c)^3 + (a^8 + 2*a^7*b + a^6*b^2) * d * \cosh(dx + c)) * \sinh(dx + c))]
\end{aligned}$$

giac [B] time = 3.67, size = 395, normalized size = 1.94

$$\frac{(35a^2b^2 + 56ab^3 + 24b^4) \arctan\left(\frac{ae^{2dx+2c} + a + 2b}{2\sqrt{-ab-b^2}}\right)}{(a^6 + 2a^5b + a^4b^2)\sqrt{-ab-b^2}} - \frac{2(13a^3b^2e^{6dx+6c} + 40a^2b^3e^{6dx+6c} + 24ab^4e^{6dx+6c} + 39a^3b^2e^{4dx+4c} + 134a^2b^3e^{4dx+4c})}{(a^6 + 2a^5b + a^4b^2)(ae^{4dx+4c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*sech(dx+c))^2)^3,x, algorithm="giac")

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2
*b)/sqrt(-a*b - b^2))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a*b - b^2)) - 2*(13*
a^3*b^2*e^(6*d*x + 6*c) + 40*a^2*b^3*e^(6*d*x + 6*c) + 24*a*b^4*e^(6*d*x +
6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 134*a^2*b^3*e^(4*d*x + 4*c) + 184*a*b^4
*e^(4*d*x + 4*c) + 80*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 10
4*a^2*b^3*e^(2*d*x + 2*c) + 56*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 10*a^2*
b^3)/((a^6 + 2*a^5*b + a^4*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
4*b*e^(2*d*x + 2*c) + a)^2) + 4*(d*x + c)*(a - 6*b)/a^4 + e^(2*d*x + 2*c)/a
^3 - (2*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/a^4)
/d
```

maple [B] time = 0.59, size = 1435, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/2/d
/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^
3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a^3*ln(
tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+13/4/d/a^2*b^2/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+2/d/a^3*b^3/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+39/4/d/a*b^2/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+19/4/d/a^2*b^3/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-2/d/a^3*b^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+39/4/d/a*b^2/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+19/4/d/a^2*b
^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-2/d/a^3*b
^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+13/4/d/a^
2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+2/d/a^3*b^
3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-35/16/d/a^2*b^
(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b
^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-7/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(
a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2
*c)-(a+b)^(1/2))-3/2/d/a^4*b^(7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+35/16/
d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+7/2/d/a^3*b^(5/2)/(a^2+2*a*b
+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))+3/2/d/a^4*b^(7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.55, size = 1373, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b
)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt((a + b)*b)*d) + 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/16*(
9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 +
32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b
^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160
*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b
^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x +
6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x
+ 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) + 1/
16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b
^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 +
832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b
^3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*
b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*
b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*
c))*d) - 1/8*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-
2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c
) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a
^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b +
a^5*b^2)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/(a^3*d) + 1/8*e^(2*d*x + 2*c
)/(a^3*d) - 1/8*e^(-2*d*x - 2*c)/(a^3*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*
(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^4*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)^2}{\left(a + \frac{b}{\cosh(c + dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3, x)
```

```
[Out] Timed out
```

$$3.94 \quad \int \frac{\cosh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=154

$$\frac{3b(4(a+b)^2 + (2a+b)^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}d(a+b)^{5/2}} - \frac{b^3 \sinh(c+dx)}{4a^3d(a+b)(a \sinh^2(c+dx) + a+b)^2} + \frac{3b^2(4a+3b) \sinh(c+dx)}{8a^3d(a+b)^2(a \sinh^2(c+dx) + a+b)}$$

[Out] $-3/8*b*(4*(a+b)^2+(2*a+b)^2)*\arctan(\sinh(d*x+c)*a^{(1/2)/(a+b)^{(1/2)})}/a^{(7/2)/(a+b)^{(5/2)}/d+\sinh(d*x+c)/a^3/d-1/4*b^3*\sinh(d*x+c)/a^3/(a+b)/d/(a+b+a*\sinh(d*x+c)^2)^2+3/8*b^2*(4*a+3*b)*\sinh(d*x+c)/a^3/(a+b)^2/d/(a+b+a*\sinh(d*x+c)^2)$

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4147, 390, 1157, 385, 205}

$$\frac{b^3 \sinh(c+dx)}{4a^3d(a+b)(a \sinh^2(c+dx) + a+b)^2} + \frac{3b^2(4a+3b) \sinh(c+dx)}{8a^3d(a+b)^2(a \sinh^2(c+dx) + a+b)} - \frac{3b(4(a+b)^2 + (2a+b)^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] $(-3*b*(4*(a+b)^2+(2*a+b)^2)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sinh}[c+d*x])/\text{Sqrt}[a+b]])/(8*a^{(7/2)*(a+b)^{(5/2)*d})} + \text{Sinh}[c+d*x]/(a^3*d) - (b^3*\text{Sinh}[c+d*x])/(4*a^3*(a+b)*d*(a+b+a*\text{Sinh}[c+d*x]^2)^2) + (3*b^2*(4*a+3*b)*\text{Sinh}[c+d*x])/(8*a^3*(a+b)^2*d*(a+b+a*\text{Sinh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q+1))/(2*d*(q+1)), x] + Dist[1/(2*d*(q+1)), Int[(d + e*x^2)^(q+1)*ExpandToSum[2*d*(q+1)*Qx + R*(2*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -

$b*d*e + a*e^2, 0]$ && IGtQ[p, 0] && LtQ[q, -1]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{(a+bx^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{a^3} - \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{a^3(a+bx^2)^3}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\sinh(c + dx)}{a^3 d} - \frac{\operatorname{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(a+bx^2)^3} dx, x, \sinh(c + dx)\right)}{a^3 d} \\ &= \frac{\sinh(c + dx)}{a^3 d} - \frac{b^3 \sinh(c + dx)}{4a^3(a + b)d(a + b + a \sinh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-3b(2a+b)^2-12abx}{(a+bx^2)^2} dx, x, \sinh(c + dx)\right)}{4a^3 d} \\ &= \frac{\sinh(c + dx)}{a^3 d} - \frac{b^3 \sinh(c + dx)}{4a^3(a + b)d(a + b + a \sinh^2(c + dx))^2} + \frac{3b^2(4a + 3b) \sinh(c + dx)}{8a^3(a + b)^2 d(a + b + a \sinh^2(c + dx))} \\ &= -\frac{3b(4(a + b)^2 + (2a + b)^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c + dx)}{\sqrt{a + b}}\right)}{8a^{7/2}(a + b)^{5/2}d} + \frac{\sinh(c + dx)}{a^3 d} - \frac{b^3 \sinh(c + dx)}{4a^3(a + b)d(a + b + a \sinh^2(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 3.65, size = 292, normalized size = 1.90

$$\operatorname{sech}^5(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{3b(8a^2 + 12ab + 5b^2)(\cosh(c) - \sinh(c)) \operatorname{sech}(c + dx)(a \cosh(2(c + dx)) + a + 2b)^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c + dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2} \sqrt{(\cosh(c) - \sinh(c))^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*Cosh[d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[c] + 8*Sqrt[a]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[d*x] - (8*Sqrt[a]*b^3*Tanh[c + d*x])/(a + b) + (6*Sqrt[a]*b^2*(4*a + 3*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Tanh[c + d*x])/(a + b)^2)/(64*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.64, size = 9856, normalized size = 64.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^8 + 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 8*a^6 - 24*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^3 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 + 4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a^5*b - 104*a^4*b^2 - 209*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^5 + 5*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 4*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b - 78*a^4*b^2 - 75*a^3*b^3 - 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a

$$\begin{aligned}
& ^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^3 + 3* \\
& (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c) + (9*(8*a^4*b + 12*a^ \\
& 3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 \\
& + 10*a*b^4)*\cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4 \\
& *b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^4 + 12*(\\
& 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c \\
&))*\sqrt{-a^2 - a*b)*\log((a*\cosh(d*x + c))^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c \\
&)^3 + a*\sinh(d*x + c)^4 - 2*(3*a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + \\
& c)^2 - 3*a - 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 - (3*a + 2*b)*\cos \\
& h(d*x + c))*\sinh(d*x + c) + 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + \\
& c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x \\
& + c))*\sqrt{-a^2 - a*b) + a)/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x \\
& + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x \\
& + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(\\
& d*x + c))*\sinh(d*x + c) + a)) + 8*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) \\
& *\cosh(d*x + c)^9 + 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b \\
& ^4)*\cosh(d*x + c)^7 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189 \\
& *a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + \\
& 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^3 - (6*a^6 + 34*a^5*b + \\
& 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^9 \\
& + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*\cosh(d*x + c)^9 + 9*(a^9 + 3*a^8*b + 3*a \\
& ^7*b^2 + a^6*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^8 + (a^9 + 3*a^8*b + 3*a^7* \\
& b^2 + a^6*b^3)*d*\sinh(d*x + c)^9 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 \\
& + 2*a^5*b^4)*d*\cosh(d*x + c)^7 + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3 \\
&)*d*\cosh(d*x + c)^2 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d \\
&)*\sinh(d*x + c)^7 + 2*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5* \\
& b^4 + 8*a^4*b^5)*d*\cosh(d*x + c)^5 + 28*(3*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6 \\
& *b^3)*d*\cosh(d*x + c)^3 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^ \\
& 4)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^ \\
& 6*b^3)*d*\cosh(d*x + c)^4 + 42*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^ \\
& 5*b^4)*d*\cosh(d*x + c)^2 + (3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32 \\
& *a^5*b^4 + 8*a^4*b^5)*d)*\sinh(d*x + c)^5 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7 \\
& *a^6*b^3 + 2*a^5*b^4)*d*\cosh(d*x + c)^3 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 \\
& + a^6*b^3)*d*\cosh(d*x + c)^5 + 70*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + \\
& 2*a^5*b^4)*d*\cosh(d*x + c)^3 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^ \\
& 3 + 32*a^5*b^4 + 8*a^4*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*(a^9 + \\
& 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*\cosh(d*x + c)^6 + 35*(a^9 + 5*a^8*b + 9*a \\
& ^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*\cosh(d*x + c)^4 + 5*(3*a^9 + 17*a^8*b + 4 \\
& 1*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*\cosh(d*x + c)^2 + (a^9 + \\
& 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*\sinh(d*x + c)^3 + (a^9 + 3 \\
& *a^8*b + 3*a^7*b^2 + a^6*b^3)*d*\cosh(d*x + c) + 4*(9*(a^9 + 3*a^8*b + 3*a^7 \\
& *b^2 + a^6*b^3)*d*\cosh(d*x + c)^7 + 21*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b \\
& ^3 + 2*a^5*b^4)*d*\cosh(d*x + c)^5 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a \\
& ^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*\cosh(d*x + c)^3 + 3*(a^9 + 5*a^8*b + 9*a \\
& ^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + (9*(a^9 \\
& + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*\cosh(d*x + c)^8 + 28*(a^9 + 5*a^8*b + 9* \\
& a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*\cosh(d*x + c)^6 + 10*(3*a^9 + 17*a^8*b + \\
& 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*\cosh(d*x + c)^4 + 12*(\\
& a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*\cosh(d*x + c)^2 + (a^9 \\
& + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d)*\sinh(d*x + c)), 1/8*(4*(a^6 + 3*a^5*b \\
& + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^10 + 40*(a^6 + 3*a^5*b + 3*a^4*b^2 + a \\
& ^3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^9 + 4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3* \\
& b^3)*\sinh(d*x + c)^10 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25* \\
& a^2*b^4)*\cosh(d*x + c)^8 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + \\
& 25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^8 + 16*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^3 \\
& + (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c))* \\
& \sinh(d*x + c)^7 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^4 + 60*a*b^5)*\cosh(d*x + c)^6 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) \\
&)*\cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4) * \cosh(d*x + c)^2 * \sinh(d*x + c)^6 - 4*a^6 - 12*a^5*b - 12*a^4*b^2 - 4*a^3*b^3 + 4*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^5 + 28 \\
& *(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c)^3 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^4 + 2*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) \\
& *\cosh(d*x + c)^6 - 4*a^6 - 28*a^5*b - 104*a^4*b^2 - 209*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c)^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 \\
& + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^2 * \sinh(d*x + c)^4 + 8*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^7 + 14*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c)^5 + 5*(4*a^6 \\
& + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)*\sinh(d*x + c)^3 - 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75 \\
& *a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c)^2 + 2*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*\cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b - 78*a^4*b^2 - 75*a^3*b^3 \\
& - 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*\cosh(d*x + c)^2 * \sinh(d*x + c)^2 - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5 \\
& *a^2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^2 * \sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5) * \cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^2 * \sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)*\sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^2 * \sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c) + (9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*\cosh(d*x + c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^2 * \sinh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (3*a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + 3*a + 4*b)*\sinh(d*x + c))/\sqrt{a^2 + a*b}) - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*\cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^2 * \sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 3 + (8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c) \sinh(dx + c)^6 + 2(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5) \cosh(dx + c)^5 \\
& + 2(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5 + 63(8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c)^4 + 42(8a^4b + 28a^3b^2 \\
& + 29a^2b^3 + 10ab^4) \cosh(dx + c)^2) \sinh(dx + c)^5 + 2(63(8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c)^5 + 70(8a^4b + 28a^3b^2 \\
& + 29a^2b^3 + 10ab^4) \cosh(dx + c)^3 + 5(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5) \cosh(dx + c)) \sinh(dx + c)^4 + 4(8a^4b \\
& + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)^3 + 4(21(8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c)^6 + 8a^4b + 28a^3b^2 + 29a^2b^3 \\
& + 10ab^4 + 35(8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)^4 + 5(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5) \cosh(dx + c)^2) \sinh(dx + c)^3 \\
& + 4(9(8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c)^7 + 21(8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)^5 + 5(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5) \cosh(dx + c)^3 \\
& + 3(8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)) \sinh(dx + c)^2 + (8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c) + (9(8a^4b + 12a^3b^2 + 5a^2b^3) \cosh(dx + c)^8 + 28(8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)^6 + 8a^4b + 12a^3b^2 + 5a^2b^3 + 10(24a^4b + 100a^3b^2 + 175a^2b^3 + 136ab^4 + 40b^5) \cosh(dx + c)^4 + 12(8a^4b + 28a^3b^2 + 29a^2b^3 + 10ab^4) \cosh(dx + c)^2) \sinh(dx + c)) \sqrt{a^2 + ab} \arctan(1/2 \sqrt{a^2 + ab} (\cosh(dx + c) + \sinh(dx + c)) / (a + b)) + 4(10(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) \cosh(dx + c)^9 + 4(6a^6 + 34a^5b + 78a^4b^2 + 75a^3b^3 + 25a^2b^4) \cosh(dx + c)^7 + 3(4a^6 + 28a^5b + 104a^4b^2 + 209a^3b^3 + 189a^2b^4 + 60ab^5) \cosh(dx + c)^5 - 2(4a^6 + 28a^5b + 104a^4b^2 + 209a^3b^3 + 189a^2b^4 + 60ab^5) \cosh(dx + c)^3 - (6a^6 + 34a^5b + 78a^4b^2 + 75a^3b^3 + 25a^2b^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^9 + 9(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c) \sinh(dx + c)^8 + (a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \sinh(dx + c)^9 + 4(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^7 + 4(9(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^2 + (a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d) \sinh(dx + c)^7 + 2(3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d \cosh(dx + c)^5 + 28(3(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^3 + (a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)) \sinh(dx + c)^6 + 2(63(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^4 + 42(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^2 + (3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d) \sinh(dx + c)^5 + 4(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^3 + 2(63(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^5 + 70(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^3 + 5(3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d \cosh(dx + c)) \sinh(dx + c)^4 + 4(21(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^6 + 35(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^4 + 5(3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d \cosh(dx + c)^2 + (a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d) \sinh(dx + c)^3 + (a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c) + 4(9(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^7 + 21(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^5 + 5(3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d \cosh(dx + c)^3 + 3(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)) \sinh(dx + c)^2 + (9(a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d \cosh(dx + c)^8 + 28(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^6 + 10(3a^9 + 17a^8b + 41a^7b^2 + 51a^6b^3 + 32a^5b^4 + 8a^4b^5) d \cosh(dx + c)^4 + 12(a^9 + 5a^8b + 9a^7b^2 + 7a^6b^3 + 2a^5b^4) d \cosh(dx + c)^2 + (a^9 + 3a^8b + 3a^7b^2 + a^6b^3) d) \sinh(dx + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [a,b]=[-82,81]Precision problem choosing root in commo
n_EXT, current precision 14Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.The choice was done assu
ming [a,b]=[-60,-34]Undef/Unsigned Inf encountered in limitEvaluation time:
2.08Limit: Max order reached or unable to make series expansion Error: Bad
Argument Value
```

maple [B] time = 0.56, size = 1238, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-3/d/a^2*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-7/4/d/a^3*b^
3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/d/a*b^2/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d/a^2*b^3
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d/a^3*
b^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+3/d/a*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d/a^2
*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d/
a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/d/
a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+7/4/d/a^
3*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
```

$$\begin{aligned} &)^2 a - 2 \tanh(1/2 d x + 1/2 c)^2 b + a + b)^2 / (a + b) \tanh(1/2 d x + 1/2 c) - 3/d/a^{(3/2)} \\ &) * b / (a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan(1/2 * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) \\ & c) - 2 * b^{(1/2)}) / a^{(1/2)}) - 9/2/d/a^{(5/2)} * b^2 / (a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan \\ & (1/2 * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) - 2 * b^{(1/2)}) / a^{(1/2)}) - 15/8/d/a^{(7/2)} * \\ & b^3 / (a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan(1/2 * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) \\ & c) - 2 * b^{(1/2)}) / a^{(1/2)}) - 3/d/a^{(3/2)} * b / (a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan(1/2 \\ & * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) + 2 * b^{(1/2)}) / a^{(1/2)}) - 9/2/d/a^{(5/2)} * b^2 / (\\ & a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan(1/2 * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) + 2 * \\ & b^{(1/2)}) / a^{(1/2)}) - 15/8/d/a^{(7/2)} * b^3 / (a^2 + 2 a * b + b^2) / (a + b)^{(1/2)} * \arctan(1/2 \\ & * (2 * (a + b)^{(1/2)} * \tanh(1/2 d x + 1/2 c) + 2 * b^{(1/2)}) / a^{(1/2)}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2a^4 + 4a^3b + 2a^2b^2 - 2(a^4e^{10c} + 2a^3be^{10c} + a^2b^2e^{10c})e^{10dx} - (6a^4e^{8c} + 28a^3be^{8c} + 50a^2b^2e^{8c} + 2a^2b^2e^{8c} + 2a^2b^2e^{8c})e^{8dx} - (4a^4e^{6c} + 24a^3be^{6c} + 80a^2b^2e^{6c} + 129a^2b^2e^{6c} + 60b^4e^{6c})e^{6dx} + (4a^4e^{4c} + 24a^3be^{4c} + 80a^2b^2e^{4c} + 129a^2b^2e^{4c} + 60b^4e^{4c})e^{4dx} + (6a^4e^{2c} + 28a^3be^{2c} + 50a^2b^2e^{2c} + 25a^2b^2e^{2c} + 60b^4e^{2c})e^{2dx}}{4((a^7de^{9c} + 2a^6bde^{9c} + a^5b^2de^{9c})e^{9dx} + 4(a^7de^{7c} + 4a^6bde^{7c} + 5a^5b^2de^{7c} + 4a^5b^2de^{7c} + 4a^5b^2de^{7c} + 4a^5b^2de^{7c})e^{7dx} + 2(3a^7d^2e^{5c} + 14a^6b^2d^2e^{5c} + 27a^5b^2d^2e^{5c} + 24a^4b^3d^2e^{5c} + 8a^3b^4d^2e^{5c})e^{5dx} + 4(a^7d^2e^{3c} + 4a^6b^2d^2e^{3c} + 5a^5b^2d^2e^{3c} + 2a^4b^3d^2e^{3c})e^{3dx} + (a^7d^2e^c + 2a^6b^2d^2e^c + a^5b^2d^2e^c)e^c * e^{dx}} - 1/2 \int (3/2 * ((8a^2b^2e^{3c} + 12a^2b^2e^{3c} + 5b^3e^{3c})e^{3dx} + (8a^2b^2e^c + 12a^2b^2e^c + 5b^3e^c)e^c * e^{dx}) / (a^6 + 2a^5b + a^4b^2 + (a^6e^{4c} + 2a^5b^2e^{4c} + a^4b^2e^{4c})e^{4dx} + 2(a^6e^{2c} + 4a^5b^2e^{2c} + 5a^4b^2e^{2c} + 2a^3b^3e^{2c})e^{2dx})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/4 * (2 * a^4 + 4 * a^3 * b + 2 * a^2 * b^2 - 2 * (a^4 * e^{(10 * c)} + 2 * a^3 * b * e^{(10 * c)} + a^2 * b^2 * e^{(10 * c)}) * e^{(10 * d * x)} - (6 * a^4 * e^{(8 * c)} + 28 * a^3 * b * e^{(8 * c)} + 50 * a^2 * b^2 * e^{(8 * c)} + 25 * a * b^3 * e^{(8 * c)}) * e^{(8 * d * x)} - (4 * a^4 * e^{(6 * c)} + 24 * a^3 * b * e^{(6 * c)} + 80 * a^2 * b^2 * e^{(6 * c)} + 129 * a * b^3 * e^{(6 * c)} + 60 * b^4 * e^{(6 * c)}) * e^{(6 * d * x)} + (4 * a^4 * e^{(4 * c)} + 24 * a^3 * b * e^{(4 * c)} + 80 * a^2 * b^2 * e^{(4 * c)} + 129 * a * b^3 * e^{(4 * c)} + 60 * b^4 * e^{(4 * c)}) * e^{(4 * d * x)} + (6 * a^4 * e^{(2 * c)} + 28 * a^3 * b * e^{(2 * c)} + 50 * a^2 * b^2 * e^{(2 * c)} + 25 * a * b^3 * e^{(2 * c)}) * e^{(2 * d * x)}) / ((a^7 * d * e^{(9 * c)} + 2 * a^6 * b * d * e^{(9 * c)} + a^5 * b^2 * d * e^{(9 * c)}) * e^{(9 * d * x)} + 4 * (a^7 * d * e^{(7 * c)} + 4 * a^6 * b * d * e^{(7 * c)} + 5 * a^5 * b^2 * d * e^{(7 * c)} + 2 * a^4 * b^3 * d * e^{(7 * c)}) * e^{(7 * d * x)} + 2 * (3 * a^7 * d * e^{(5 * c)} + 14 * a^6 * b^2 * d * e^{(5 * c)} + 27 * a^5 * b^2 * d * e^{(5 * c)} + 24 * a^4 * b^3 * d * e^{(5 * c)} + 8 * a^3 * b^4 * d * e^{(5 * c)}) * e^{(5 * d * x)} + 4 * (a^7 * d * e^{(3 * c)} + 4 * a^6 * b * d * e^{(3 * c)} + 5 * a^5 * b^2 * d * e^{(3 * c)} + 2 * a^4 * b^3 * d * e^{(3 * c)}) * e^{(3 * d * x)} + (a^7 * d * e^c + 2 * a^6 * b * d * e^c + a^5 * b^2 * d * e^c) * e^c * e^{(d * x)}) - 1/2 * \int (3/2 * ((8 * a^2 * b^2 * e^{(3 * c)} + 12 * a^2 * b^2 * e^{(3 * c)} + 5 * b^3 * e^{(3 * c)}) * e^{(3 * d * x)} + (8 * a^2 * b^2 * e^c + 12 * a^2 * b^2 * e^c + 5 * b^3 * e^c) * e^c * e^{(d * x)}) / (a^6 + 2 * a^5 * b + a^4 * b^2 + (a^6 * e^{(4 * c)} + 2 * a^5 * b^2 * e^{(4 * c)} + a^4 * b^2 * e^{(4 * c)}) * e^{(4 * d * x)} + 2 * (a^6 * e^{(2 * c)} + 4 * a^5 * b^2 * e^{(2 * c)} + 5 * a^4 * b^2 * e^{(2 * c)} + 2 * a^3 * b^3 * e^{(2 * c)}) * e^{(2 * d * x)}), x) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{\left(a + \frac{b}{\cosh(c + dx)^2}\right)^3 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.95 \quad \int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=142

$$\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2(a\sinh^2(c+dx)+a+b)} + \frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}} - \frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)(a\sinh^2(c+dx)+a+b)}$$

[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(5/2)/d-1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2-3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)

Rubi [A] time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4147, 413, 385, 205}

$$\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}} - \frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2(a\sinh^2(c+dx)+a+b)} - \frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)(a\sinh^2(c+dx)+a+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^(5/2)*(a + b)^(5/2)*d) - (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+ax^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+(4a+3b)x^2}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{4a(a+b)d} \\
&= -\frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2} - \frac{3b(2a+b) \sinh(c+dx)}{8a^2(a+b)^2d(a+b+a\sinh^2(c+dx))} \\
&= \frac{(8a^2+8ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d} - \frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 2.94, size = 214, normalized size = 1.51

$$\operatorname{sech}^5(c+dx)(a \cosh(2(c+dx)) + a + 2b) \frac{\left((8a^2+8ab+3b^2)(\sinh(c)-\cosh(c))\operatorname{sech}(c+dx)(a \cosh(2(c+dx))+a+2b)^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sqrt{\cosh(c)-\sinh(c)}}{\sqrt{a+b}}\right) \right)}{\sqrt{a+b} \sqrt{(\cosh(c)-\sinh(c))^2}}$$

$64a^{5/2}d(a+b)$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(-Cosh[c] + Sinh[c]))/(Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*b^2*(a + b)*Tanh[c + d*x] - 2*Sqrt[a]*b*(8*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Tanh[c + d*x]))/(64*a^(5/2)*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.54, size = 6806, normalized size = 47.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 28*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3

$$\begin{aligned}
& - 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^2 + ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*b \\
& + 3*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2) \\
&)*\sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x + \\
& c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3* \\
& a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b \\
& ^2)*\cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24* \\
& b^4)*\cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^4 \\
& + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*b \\
& b + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*a^4 + 8*a^3*b \\
& + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^5 + 10*(8* \\
& a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^3 + (24*a^4 + 88*a^3*b \\
& + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a \\
& ^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3 \\
& *b + 3*a^2*b^2)*\cosh(d*x + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b \\
& ^3)*\cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 + \\
& 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\
& ^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b \\
& + 19*a^2*b^2 + 6*a*b^3)*\cosh(d*x + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 \\
& + 88*a*b^3 + 24*b^4)*\cosh(d*x + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6* \\
& a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 - a*b}*\log((a*\cosh(d*x + c)^ \\
& 4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 - 2*(3*a + 2*b)*c \\
& osh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 - 3*a - 2*b)*\sinh(d*x + c)^2 + 4*(a \\
& *\cosh(d*x + c)^3 - (3*a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + \\
& c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c) \\
&)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a^2 - a*b} + a)/(a*\cosh(d*x + \\
& c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b) \\
& *\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a \\
& *\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) - 4*(8*a^4*b \\
& + 13*a^3*b^2 + 5*a^2*b^3)*\cosh(d*x + c) + 4*(7*(8*a^4*b + 13*a^3*b^2 + 5* \\
& a^2*b^3)*\cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3 + 5*(8*a^4*b + \\
& 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^4 - 3*(8*a^4*b + 37*a^3*b \\
& ^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^8 + 3*a^7*b \\
& + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 + \\
& a^5*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5 \\
& *b^3)*d*\sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b \\
& ^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(\\
& d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d* \\
& x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a \\
& ^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*co \\
& sh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cos \\
& h(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d \\
& *\cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d \\
& *\cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 \\
& + 8*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 \\
& + 2*a^4*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3) \\
&)*d*\cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4) \\
&)*d*\cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b \\
& ^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3 \\
& *a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a \\
& ^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + \\
& 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9 \\
& *a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a \\
& ^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + \\
& c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + \\
& c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5) \\
&)*d*\cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d \\
& *\cosh(d*x + c))*\sinh(d*x + c)), -1/8*(2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*
\end{aligned}$$

$$\begin{aligned}
& \cosh(dx + c)^7 + 14*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*\cosh(dx + c)*\sinh(dx + c)^6 + 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*\sinh(dx + c)^7 + 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c)^5 + 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^5 + 10*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c)^3 + (8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c))*\sinh(dx + c)^4 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c)^3 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c)^2*\sinh(dx + c)^3 + 2*(21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c)^3 - 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(dx + c))*\sinh(dx + c)^2 - ((8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)*\sinh(dx + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2))*\sinh(dx + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^2*\sinh(dx + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^2*\sinh(dx + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c))*\sinh(dx + c))*\sqrt{a^2 + a*b}*\arctan(1/2*(a*\cosh(dx + c)^3 + 3*a*\cosh(dx + c)*\sinh(dx + c)^2 + a*\sinh(dx + c)^3 + (3*a + 4*b))*\cosh(dx + c) + (3*a*\cosh(dx + c)^2 + 3*a + 4*b))*\sinh(dx + c))/\sqrt{a^2 + a*b}) - ((8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)*\sinh(dx + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2))*\sinh(dx + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^2*\sinh(dx + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^2)*\sinh(dx + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2))*\cosh(dx + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4))*\cosh(dx + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3))*\cosh(dx + c))*\sinh(dx + c))*\sqrt{a^2 + a*b}*\arctan(1/2*\sqrt{a^2 + a*b}*(\cosh(dx + c) + \sinh(dx + c))/(a + b)) - 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c) + 2*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3))*\cosh(dx + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3
\end{aligned}$$

$$\begin{aligned}
& + 5*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^4 - 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c))/ \\
& ((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-43,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-35,-99]Undef/Unsigned Inf encountered in limitEvaluation time: 1.18Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.40, size = 1172, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)

[Out] $2/d/(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*\tanh(1/2*d*x+1/2*c)^7+3/4/d/$

$$\frac{a^2 b^2}{(\tanh(1/2 dx + 1/2 c))^4} + \frac{b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b}{(a+b) \tanh(1/2 dx + 1/2 c)^7} + \frac{2}{d} \frac{\tanh(1/2 dx + 1/2 c)^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^5} - \frac{13}{4} \frac{4}{d} \frac{a b^2}{(\tanh(1/2 dx + 1/2 c))^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b} + \frac{2}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^5} - \frac{9}{4} \frac{4}{d} \frac{a^2 b}{\tanh(1/2 dx + 1/2 c)^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b} + \frac{2}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^5} - \frac{2}{d} \frac{\tanh(1/2 dx + 1/2 c)^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^3} + \frac{13}{4} \frac{4}{d} \frac{a b^2}{(\tanh(1/2 dx + 1/2 c))^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b} + \frac{2}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^3} + \frac{9}{4} \frac{4}{d} \frac{a^2 b^3}{(\tanh(1/2 dx + 1/2 c))^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b} + \frac{2}{(a+b)^2 \tanh(1/2 dx + 1/2 c)^3} - \frac{2}{d} \frac{\tanh(1/2 dx + 1/2 c)^4 + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b}{(a+b) \tanh(1/2 dx + 1/2 c)} + \frac{1}{d} \frac{a^2 + 2 a b + b^2}{(a+b)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) - 2 b^{1/2}))}{a^{1/2}} + \frac{1}{d} \frac{a^{3/2} b}{(a^2 + 2 a b + b^2)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) - 2 b^{1/2}))}{a^{1/2}} + \frac{3}{8} \frac{4}{d} \frac{a^{5/2} b^2}{(a^2 + 2 a b + b^2)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) - 2 b^{1/2}))}{a^{1/2}} + \frac{1}{d} \frac{a^2 + 2 a b + b^2}{(a+b)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) + 2 b^{1/2}))}{a^{1/2}} + \frac{1}{d} \frac{a^{3/2} b}{(a^2 + 2 a b + b^2)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) + 2 b^{1/2}))}{a^{1/2}} + \frac{3}{8} \frac{4}{d} \frac{a^{5/2} b^2}{(a^2 + 2 a b + b^2)^{1/2}} \frac{\arctan(1/2 (2(a+b)^{1/2} \tanh(1/2 dx + 1/2 c) + 2 b^{1/2}))}{a^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(8 a^2 b e^{7c} + 5 a b^2 e^{7c}) e^{7dx} + (8 a^2 b e^{5c} + 29 a b^2 e^{5c} + 12 b^3 e^{5c}) e^{5dx} - (8 a^2 b e^{3c} + 29 a b^2 e^{3c} + 12 b^3 e^{3c}) e^{3dx} - (8 a^2 b e^c + 5 a b^2 e^c) e^{dx}}{4 (a^6 d + 2 a^5 b d + a^4 b^2 d + (a^6 d e^{8c} + 2 a^5 b d e^{8c} + a^4 b^2 d e^{8c}) e^{8dx} + 4 (a^6 d e^{6c} + 4 a^5 b d e^{6c} + 5 a^4 b^2 d e^{6c}) e^{6dx} + 4 (a^6 d e^{4c} + 4 a^5 b d e^{4c} + 5 a^4 b^2 d e^{4c}) e^{4dx} + 4 (a^6 d e^{2c} + 4 a^5 b d e^{2c} + 5 a^4 b^2 d e^{2c}) e^{2dx}} + 2 \int \frac{1}{8} ((8 a^2 e^{3c} + 8 a b e^c + 3 b^2 e^c) e^{3dx} + (8 a^2 e^c + 8 a b e^c + 3 b^2 e^c) e^{dx}) / (a^5 + 2 a^4 b + a^3 b^2 + (a^5 e^{4c} + 2 a^4 b e^{4c} + a^3 b^2 e^{4c}) e^{4dx} + 2 (a^5 e^{2c} + 4 a^4 b e^{2c} + 5 a^3 b^2 e^{2c}) e^{2dx}), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/4 * ((8 a^2 b e^{7c} + 5 a b^2 e^{7c}) e^{7dx} + (8 a^2 b e^{5c} + 29 a b^2 e^{5c} + 12 b^3 e^{5c}) e^{5dx} - (8 a^2 b e^{3c} + 29 a b^2 e^{3c} + 12 b^3 e^{3c}) e^{3dx} - (8 a^2 b e^c + 5 a b^2 e^c) e^{dx}) / (a^6 d + 2 a^5 b d + a^4 b^2 d + (a^6 d e^{8c} + 2 a^5 b d e^{8c} + a^4 b^2 d e^{8c}) e^{8dx} + 4 (a^6 d e^{6c} + 4 a^5 b d e^{6c} + 5 a^4 b^2 d e^{6c}) e^{6dx} + 4 (a^6 d e^{4c} + 4 a^5 b d e^{4c} + 5 a^4 b^2 d e^{4c}) e^{4dx} + 4 (a^6 d e^{2c} + 4 a^5 b d e^{2c} + 5 a^4 b^2 d e^{2c}) e^{2dx}) + 2 \int \frac{1}{8} ((8 a^2 e^{3c} + 8 a b e^c + 3 b^2 e^c) e^{3dx} + (8 a^2 e^c + 8 a b e^c + 3 b^2 e^c) e^{dx}) / (a^5 + 2 a^4 b + a^3 b^2 + (a^5 e^{4c} + 2 a^4 b e^{4c} + a^3 b^2 e^{4c}) e^{4dx} + 2 (a^5 e^{2c} + 4 a^4 b e^{2c} + 5 a^3 b^2 e^{2c}) e^{2dx}), x$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx) \left(a + \frac{b}{\cosh(c + dx)^2} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3), x)

[Out] `int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)`

[Out] `Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)`

$$3.96 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=108

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}d(a+b)^{5/2}} + \frac{3 \tanh(c+dx)}{8d(a+b)^2(a-b \tanh^2(c+dx)+b)} + \frac{\tanh(c+dx)}{4d(a+b)(a-b \tanh^2(c+dx)+b)^2}$$

[Out] 3/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d/b^(1/2)+1/4*tanh(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4146, 199, 208}

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}d(a+b)^{5/2}} + \frac{3 \tanh(c+dx)}{8d(a+b)^2(a-b \tanh^2(c+dx)+b)} + \frac{\tanh(c+dx)}{4d(a+b)(a-b \tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3, x]

[Out] (3*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*d) + Tanh[c + d*x]/(4*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\
&= \frac{\tanh(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{3\tanh(c+dx)}{8(a+b)^2d(a+b-b\tanh^2(c+dx))} + \frac{3S}{8(a+b)^2d(a+b-b\tanh^2(c+dx))} \\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}d} + \frac{\tanh(c+dx)}{4(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{3\tan}{8(a+b)^2d(a+b-b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [B] time = 2.17, size = 258, normalized size = 2.39

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-\frac{\operatorname{sech}(2c)((5a^2+16ab+8b^2)\sinh(2c)-a(5a+2b)\sinh(2dx))(a\cosh(2(c+dx))+a+2b)}{a^2} + \frac{4b(a+2b)}{a^2}\right)}{64d(a+b)^2(a+b-b\tanh^2(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((3*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*b*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a^2 - ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[2*c]*((5*a^2 + 16*a*b + 8*b^2)*Sinh[2*c] - a*(5*a + 2*b)*Sinh[2*d*x]))/a^2)/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.51, size = 5109, normalized size = 47.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 2*4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 20*a^4*b + 28*a^3*b^2 + 8*a^2*b^3 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^2

$$\begin{aligned}
&) * \sinh(dx + c)^2 - 3*(a^4 * \cosh(dx + c)^8 + 8*a^4 * \cosh(dx + c) * \sinh(dx + c)^7 + a^4 * \sinh(dx + c)^8 + 4*(a^4 + 2*a^3*b) * \cosh(dx + c)^6 + 4*(7*a^4 * \cosh(dx + c)^2 + a^4 + 2*a^3*b) * \sinh(dx + c)^6 + 8*(7*a^4 * \cosh(dx + c)^3 + 3*(a^4 + 2*a^3*b) * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^4 + 2*(35*a^4 * \cosh(dx + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + a^4 + 8*(7*a^4 * \cosh(dx + c)^5 + 10*(a^4 + 2*a^3*b) * \cosh(dx + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(a^4 + 2*a^3*b) * \cosh(dx + c)^2 + 4*(7*a^4 * \cosh(dx + c)^6 + 15*(a^4 + 2*a^3*b) * \cosh(dx + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8*(a^4 * \cosh(dx + c)^7 + 3*(a^4 + 2*a^3*b) * \cosh(dx + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^3 + (a^4 + 2*a^3*b) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a*b + b^2} * \log((a^2 * \cosh(dx + c)^4 + 4*a^2 * \cosh(dx + c) * \sinh(dx + c)^3 + a^2 * \sinh(dx + c)^4 + 2*(a^2 + 2*a*b) * \cosh(dx + c)^2 + 2*(3*a^2 * \cosh(dx + c)^2 + a^2 + 2*a*b) * \sinh(dx + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2 * \cosh(dx + c)^3 + (a^2 + 2*a*b) * \cosh(dx + c)) * \sinh(dx + c) - 4*(a * \cosh(dx + c)^2 + 2*a * \cosh(dx + c) * \sinh(dx + c) + a * \sinh(dx + c)^2 + a + 2*b) * \sqrt{a*b + b^2})) / (a * \cosh(dx + c)^4 + 4*a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 + 2*(a + 2*b) * \cosh(dx + c)^2 + 2*(3*a * \cosh(dx + c)^2 + a + 2*b) * \sinh(dx + c)^2 + 4*(a * \cosh(dx + c)^3 + (a + 2*b) * \cosh(dx + c)) * \sinh(dx + c) + a)) + 8*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5) * \cosh(dx + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \sinh(dx + c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d) * \sinh(dx + c)^6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6) * d * \cosh(dx + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6) * d) * \sinh(dx + c)^4 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^2 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^3 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^6 + 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6) * d * \cosh(dx + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d) * \sinh(dx + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d + 8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4) * d * \cosh(dx + c)^7 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)^5 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6) * d * \cosh(dx + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5) * d * \cosh(dx + c)) * \sinh(dx + c)), -1/8*(2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)^6 + 12*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \cosh(dx + c) * \sinh(dx + c)^5 + 2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \sinh(dx + c)^6 + 10*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5) * \cosh(dx + c)^4 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 8*(5*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)^3 + (15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5) * \cosh(dx + c)) * \sinh(dx + c)^3 + 2*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4) * \cosh(dx + c)^2 + 2*(15*a^4*b + 47*a
\end{aligned}$$

```

^3*b^2 + 40*a^2*b^3 + 8*a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^4 + 6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*
cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cos
h(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 +
8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c
)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x
+ c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^
3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*c
osh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a
^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x
+ c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*c
osh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x
+ c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
+ (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*
b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a
^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 +
2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4
*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6*
b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a^
5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
+ 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c
)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^
2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a
^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b +
5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7*
b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d
*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(
d*x + c))*sinh(d*x + c))]

```

giac [B] time = 1.75, size = 282, normalized size = 2.61

$$\frac{3 \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^2+2ab+b^2)\sqrt{-ab-b^2}} - \frac{2(5a^3e^{(6dx+6c)}+16a^2be^{(6dx+6c)}+8ab^2e^{(6dx+6c)}+15a^3e^{(4dx+4c)}+46a^2be^{(4dx+4c)}+56ab^2e^{(4dx+4c)}+16b^3e^{(4dx+4c)}+15a^4e^{(4dx+4c)}+46a^2be^{(4dx+4c)}+56ab^2e^{(4dx+4c)}+16b^3e^{(4dx+4c)}+15a^4e^{(4dx+4c)})}{(a^4+2a^3b+a^2b^2)(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)^2} \cdot 8d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c))^3,x, algorithm="giac")

[Out] 1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + 2*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(5*a^3*e^(6*d*x + 6*c) + 16*a^2*b*e^(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 46*a^2*b*e^(4

$*dx + 4*c) + 56*a*b^2*e^{(4*d*x + 4*c)} + 16*b^3*e^{(4*d*x + 4*c)} + 15*a^3*e^{(2*d*x + 2*c)} + 32*a^2*b*e^{(2*d*x + 2*c)} + 8*a*b^2*e^{(2*d*x + 2*c)} + 5*a^3 + 2*a^2*b)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a^2))/d$

maple [B] time = 0.34, size = 615, normalized size = 5.69

$$\frac{5 \left(\tanh^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right)^2 (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)

[Out] $5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^7+15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5+a+3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5+15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3*a+3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)-3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$

maxima [B] time = 0.51, size = 353, normalized size = 3.27

$$\frac{5a^3 + 2a^2b + (15a^3 + 32a^2b + 8ab^2)e^{(-2dx-2c)} + (15a^3 + 46a^2b + 56ab^2)}{4(a^6 + 2a^5b + a^4b^2 + 4(a^6 + 4a^5b + 5a^4b^2 + 2a^3b^3)e^{(-2dx-2c)} + 2(3a^6 + 14a^5b + 27a^4b^2 + 24a^3b^3 + 8a^2b^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $1/4*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^{(-2*d*x - 2*c)} + (15*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^{(-4*d*x - 4*c)} + (5*a^3 + 16*a^2*b + 8*a*b^2)*e^{(-6*d*x - 6*c)})/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*e^{(-2*d*x - 2*c)} + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2 + 24*a^3*b^3 + 8*a^2*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*e^{(-6*d*x - 6*c)} + (a^6 + 2*a^5*b + a^4*b^2)*e^{(-8*d*x - 8*c)})*d - 3/16*log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*sqrt((a + b)*b))/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^2 \left(a + \frac{b}{\cosh(c + dx)^2} \right)^3 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)

[Out] `int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)`

[Out] `Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)`

$$3.97 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=123

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}} + \frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2(a\sinh^2(c+dx)+a+b)} - \frac{b\sinh(c+dx)}{4ad(a+b)(a\sinh^2(c+dx)+a+b)^2}$$

[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/8*(4*a+b)*sinh(d*x+c)/a/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4147, 385, 199, 205}

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}} + \frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2(a\sinh^2(c+dx)+a+b)} - \frac{b\sinh(c+dx)}{4ad(a+b)(a\sinh^2(c+dx)+a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((4*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(8*a^(3/2)*(a + b)^(5/2)*d) - (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + ((4*a + b)*Sinh[c + d*x])/(8*a*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(a+b+ax^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{(4a+b) \operatorname{Subst}\left(\int \frac{1}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{4a(a+b)d} \\
&= -\frac{b \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{(4a+b) \sinh(c+dx)}{8a(a+b)^2 d(a+b+a\sinh^2(c+dx))} + \frac{(4a+b) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{b \sinh(c+dx)}{4a(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{(4a+b) \operatorname{Subst}\left(\int \frac{1}{(a+b+ax^2)^2} dx, x, \sinh(c+dx)\right)}{4a(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.79, size = 159, normalized size = 1.29

$$\frac{\operatorname{sech}^6(c+dx)(a \cosh(2(c+dx)) + a + 2b)^3 \left(\frac{8 \sinh(c+dx)}{(a \sinh^2(c+dx) + a + b)^2} - (4a+b) \left(\frac{5(a+b) \sinh(c+dx) + 3a \sinh^3(c+dx)}{(a+b)^2(a \sinh^2(c+dx) + a + b)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} \right) \right)}{192ad(a+b\operatorname{sech}^2(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] -1/192*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((8*Sinh[c + d*x])/(a + b + a*Sinh[c + d*x]^2)^2 - (4*a + b)*((3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2))))/(a*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.53, size = 6037, normalized size = 49.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^4 + 5*a^3*b + a^2*b^2)*sinh(d*x + c)^7 + 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2))*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4*(35*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2*b^2 + 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a

$$\begin{aligned}
& *b^2) * \cosh(dx + c) * \sinh(dx + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8* \\
& b^3) * \cosh(dx + c)^4 + 2*(35*(4*a^3 + a^2*b) * \cosh(dx + c)^4 + 12*a^3 + 35* \\
& a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2) * \cosh(dx + c)^2) * \\
& \sinh(dx + c)^4 + 8*(7*(4*a^3 + a^2*b) * \cosh(dx + c)^5 + 10*(4*a^3 + 9*a^2* \\
& b + 2*a*b^2) * \cosh(dx + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3) * \cosh(\\
& dx + c)) * \sinh(dx + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2) * c \\
& osh(dx + c)^2 + 4*(7*(4*a^3 + a^2*b) * \cosh(dx + c)^6 + 15*(4*a^3 + 9*a^2*b \\
& + 2*a*b^2) * \cosh(dx + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^ \\
& 2*b + 40*a*b^2 + 8*b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8*((4*a^3 + a^2* \\
& b) * \cosh(dx + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2) * \cosh(dx + c)^5 + (12*a^ \\
& 3 + 35*a^2*b + 40*a*b^2 + 8*b^3) * \cosh(dx + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b \\
& ^2) * \cosh(dx + c) * \sinh(dx + c)) * \sqrt{-a^2 - a*b} * \log((a * \cosh(dx + c)^4 + \\
& 4*a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 - 2*(3*a + 2*b) * \cosh \\
& (dx + c)^2 + 2*(3*a * \cosh(dx + c)^2 - 3*a - 2*b) * \sinh(dx + c)^2 + 4*(a * co \\
& sh(dx + c)^3 - (3*a + 2*b) * \cosh(dx + c)) * \sinh(dx + c) - 4*(\cosh(dx + c) \\
& ^3 + 3 * \cosh(dx + c) * \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 * \cosh(dx + c)^2 \\
& - 1) * \sinh(dx + c) - \cosh(dx + c)) * \sqrt{-a^2 - a*b} + a) / (a * \cosh(dx + c) \\
& ^4 + 4*a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 + 2*(a + 2*b) * co \\
& sh(dx + c)^2 + 2*(3*a * \cosh(dx + c)^2 + a + 2*b) * \sinh(dx + c)^2 + 4*(a * co \\
& sh(dx + c)^3 + (a + 2*b) * \cosh(dx + c)) * \sinh(dx + c) + a) - 4*(4*a^4 + 5 \\
& *a^3*b + a^2*b^2) * \cosh(dx + c) + 4*(7*(4*a^4 + 5*a^3*b + a^2*b^2) * \cosh(dx \\
& + c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)^4 - 4*a^ \\
& 4 - 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx \\
& + c)^2) * \sinh(dx + c)) / ((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + \\
& c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c) * \sinh(dx + \\
& c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \sinh(dx + c)^8 + 4*(a^7 + 5 \\
& *a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^6 + 4*(7*(a^7 + \\
& 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^2 + (a^7 + 5*a^6*b + 9*a^5* \\
& b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d) * \sinh(dx + c)^6 + 2*(3*a^7 + 17*a^6*b + 41* \\
& a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5) * d * \cosh(dx + c)^4 + 8*(7*(a^ \\
& 7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^3 + 3*(a^7 + 5*a^6*b + 9 \\
& *a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(35* \\
& (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^4 + 30*(a^7 + 5*a^6*b \\
& + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^2 + (3*a^7 + 17*a^6*b \\
& + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5) * d) * \sinh(dx + c)^4 + 4 \\
& *(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^2 + 8* \\
& (7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^5 + 10*(a^7 + 5*a^ \\
& 6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^3 + (3*a^7 + 17*a^ \\
& 6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5) * d * \cosh(dx + c)) * si \\
& nh(dx + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^ \\
& 6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^ \\
& 4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5) \\
& * d * \cosh(dx + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d) \\
& * \sinh(dx + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) * d + 8*((a^7 + 3*a^ \\
& 6*b + 3*a^5*b^2 + a^4*b^3) * d * \cosh(dx + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 \\
& + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^ \\
& 2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5) * d * \cosh(dx + c)^3 + (a^7 + 5*a^6*b \\
& + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/8* \\
& (2*(4*a^4 + 5*a^3*b + a^2*b^2) * \cosh(dx + c)^7 + 14*(4*a^4 + 5*a^3*b + a^2* \\
& b^2) * \cosh(dx + c) * \sinh(dx + c)^6 + 2*(4*a^4 + 5*a^3*b + a^2*b^2) * \sinh(dx \\
& + c)^7 + 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)^5 + 2*(4 \\
& *a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2) * \cosh \\
& (dx + c)^2) * \sinh(dx + c)^5 + 10*(7*(4*a^4 + 5*a^3*b + a^2*b^2) * \cosh(dx + \\
& c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)) * \sinh(dx + \\
& c)^4 - 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)^3 + 2*(35*(\\
& 4*a^4 + 5*a^3*b + a^2*b^2) * \cosh(dx + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2*b^2 + \\
& 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)^2) * sin \\
& h(dx + c)^3 + 2*(21*(4*a^4 + 5*a^3*b + a^2*b^2) * \cosh(dx + c)^5 + 10*(4*a^ \\
& 4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3) * \cosh(dx + c)^3 - 3*(4*a^4 + 13*a^3*b +
\end{aligned}$$

$$\begin{aligned}
& 5a^2b^2 - 4ab^3) \cosh(dx + c) \sinh(dx + c)^2 + ((4a^3 + a^2b) \cos \\
& h(dx + c)^8 + 8(4a^3 + a^2b) \cosh(dx + c) \sinh(dx + c)^7 + (4a^3 + a \\
& ^2b) \sinh(dx + c)^8 + 4(4a^3 + 9a^2b + 2ab^2) \cosh(dx + c)^6 + 4(\\
& 4a^3 + 9a^2b + 2ab^2 + 7(4a^3 + a^2b) \cosh(dx + c)^2) \sinh(dx + c \\
&)^6 + 8(7(4a^3 + a^2b) \cosh(dx + c)^3 + 3(4a^3 + 9a^2b + 2ab^2) * \\
& \cosh(dx + c)) \sinh(dx + c)^5 + 2(12a^3 + 35a^2b + 40ab^2 + 8b^3) * \\
& \cosh(dx + c)^4 + 2(35(4a^3 + a^2b) \cosh(dx + c)^4 + 12a^3 + 35a^2b \\
& + 40ab^2 + 8b^3 + 30(4a^3 + 9a^2b + 2ab^2) \cosh(dx + c)^2) \sinh(dx \\
& *x + c)^4 + 8(7(4a^3 + a^2b) \cosh(dx + c)^5 + 10(4a^3 + 9a^2b + 2 \\
& ab^2) \cosh(dx + c)^3 + (12a^3 + 35a^2b + 40ab^2 + 8b^3) \cosh(dx + \\
& c)) \sinh(dx + c)^3 + 4a^3 + a^2b + 4(4a^3 + 9a^2b + 2ab^2) \cosh(dx \\
& x + c)^2 + 4(7(4a^3 + a^2b) \cosh(dx + c)^6 + 15(4a^3 + 9a^2b + 2a \\
& *b^2) \cosh(dx + c)^4 + 4a^3 + 9a^2b + 2ab^2 + 3(12a^3 + 35a^2b + \\
& 40ab^2 + 8b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((4a^3 + a^2b) \cos \\
& h(dx + c)^7 + 3(4a^3 + 9a^2b + 2ab^2) \cosh(dx + c)^5 + (12a^3 + 35 \\
& *a^2b + 40ab^2 + 8b^3) \cosh(dx + c)^3 + (4a^3 + 9a^2b + 2ab^2) \co \\
& sh(dx + c)) \sinh(dx + c)) \sqrt{a^2 + ab} \arctan(1/2(a \cosh(dx + c)^3 + \\
& 3a \cosh(dx + c) \sinh(dx + c)^2 + a \sinh(dx + c)^3 + (3a + 4b) \cosh(dx \\
& *x + c) + (3a \cosh(dx + c)^2 + 3a + 4b) \sinh(dx + c)) / \sqrt{a^2 + ab}) \\
& + ((4a^3 + a^2b) \cosh(dx + c)^8 + 8(4a^3 + a^2b) \cosh(dx + c) \sinh(dx \\
& + c)^7 + (4a^3 + a^2b) \sinh(dx + c)^8 + 4(4a^3 + 9a^2b + 2ab^2) \\
&) \cosh(dx + c)^6 + 4(4a^3 + 9a^2b + 2ab^2 + 7(4a^3 + a^2b) \cosh(dx \\
& *x + c)^2) \sinh(dx + c)^6 + 8(7(4a^3 + a^2b) \cosh(dx + c)^3 + 3(4a^ \\
& 3 + 9a^2b + 2ab^2) \cosh(dx + c)) \sinh(dx + c)^5 + 2(12a^3 + 35a^2* \\
& b + 40ab^2 + 8b^3) \cosh(dx + c)^4 + 2(35(4a^3 + a^2b) \cosh(dx + c) \\
& ^4 + 12a^3 + 35a^2b + 40ab^2 + 8b^3 + 30(4a^3 + 9a^2b + 2ab^2) * \\
& \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(7(4a^3 + a^2b) \cosh(dx + c)^5 + 1 \\
& 0(4a^3 + 9a^2b + 2ab^2) \cosh(dx + c)^3 + (12a^3 + 35a^2b + 40ab \\
& ^2 + 8b^3) \cosh(dx + c)) \sinh(dx + c)^3 + 4a^3 + a^2b + 4(4a^3 + 9a \\
& ^2b + 2ab^2) \cosh(dx + c)^2 + 4(7(4a^3 + a^2b) \cosh(dx + c)^6 + 15 \\
& *(4a^3 + 9a^2b + 2ab^2) \cosh(dx + c)^4 + 4a^3 + 9a^2b + 2ab^2 + \\
& 3(12a^3 + 35a^2b + 40ab^2 + 8b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + \\
& 8((4a^3 + a^2b) \cosh(dx + c)^7 + 3(4a^3 + 9a^2b + 2ab^2) \cosh(dx \\
& x + c)^5 + (12a^3 + 35a^2b + 40ab^2 + 8b^3) \cosh(dx + c)^3 + (4a^3 \\
& + 9a^2b + 2ab^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{a^2 + ab} \arctan(1 \\
& /2 \sqrt{a^2 + ab} (\cosh(dx + c) + \sinh(dx + c)) / (a + b)) - 2(4a^4 + 5 \\
& a^3b + a^2b^2) \cosh(dx + c) + 2(7(4a^4 + 5a^3b + a^2b^2) \cosh(dx \\
& + c)^6 + 5(4a^4 + 13a^3b + 5a^2b^2 - 4ab^3) \cosh(dx + c)^4 - 4a^4 \\
& - 5a^3b - a^2b^2 - 3(4a^4 + 13a^3b + 5a^2b^2 - 4ab^3) \cosh(dx \\
& + c)^2) \sinh(dx + c)) / ((a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + \\
& c)^8 + 8(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c) \sinh(dx + c \\
&)^7 + (a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \sinh(dx + c)^8 + 4(a^7 + 5 \\
& a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)^6 + 4(7(a^7 + \\
& 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c)^2 + (a^7 + 5a^6b + 9a^5b \\
& ^2 + 7a^4b^3 + 2a^3b^4) d) \sinh(dx + c)^6 + 2(3a^7 + 17a^6b + 41a \\
& ^5b^2 + 51a^4b^3 + 32a^3b^4 + 8a^2b^5) d \cosh(dx + c)^4 + 8(7(a^7 \\
& + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c)^3 + 3(a^7 + 5a^6b + 9 \\
& a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(\\
& a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c)^4 + 30(a^7 + 5a^6b \\
& + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)^2 + (3a^7 + 17a^6b \\
& + 41a^5b^2 + 51a^4b^3 + 32a^3b^4 + 8a^2b^5) d) \sinh(dx + c)^4 + 4 \\
& (a^7 + 5a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)^2 + 8(\\
& 7(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c)^5 + 10(a^7 + 5a^6 \\
& *b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)^3 + (3a^7 + 17a^6 \\
& *b + 41a^5b^2 + 51a^4b^3 + 32a^3b^4 + 8a^2b^5) d \cosh(dx + c)) \sin \\
& h(dx + c)^3 + 4(7(a^7 + 3a^6b + 3a^5b^2 + a^4b^3) d \cosh(dx + c)^6 \\
& + 15(a^7 + 5a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d \cosh(dx + c)^4 \\
& + 3(3a^7 + 17a^6b + 41a^5b^2 + 51a^4b^3 + 32a^3b^4 + 8a^2b^5) * \\
& d \cosh(dx + c)^2 + (a^7 + 5a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4) d) *
\end{aligned}$$

$$\sinh(dx + c)^2 + (a^7 + 3a^6b + 3a^5b^2 + a^4b^3)d + 8*((a^7 + 3a^6b + 3a^5b^2 + a^4b^3)d * \cosh(dx + c)^7 + 3(a^7 + 5a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4)d * \cosh(dx + c)^5 + (3a^7 + 17a^6b + 41a^5b^2 + 51a^4b^3 + 32a^3b^4 + 8a^2b^5)d * \cosh(dx + c)^3 + (a^7 + 5a^6b + 9a^5b^2 + 7a^4b^3 + 2a^3b^4)d * \cosh(dx + c)) * \sinh(dx + c))]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^3/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-43,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered in limitEvaluation time: 1.54Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.34, size = 1038, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^3/(a+b*sech(dx+c)^2)^3,x)

[Out]
$$\begin{aligned} & -1/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)*\tanh(1/2*dx+1/2*c)^7+1/4/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2*b / (a+b) / a*\tanh(1/2*dx+1/2*c)^7-1/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)^2*\tanh(1/2*dx+1/2*c)^5+a+5/4/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2*b / (a+b)^2*\tanh(1/2*dx+1/2*c)^5-3/4/d/a*b^2 / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)^2*\tanh(1/2*dx+1/2*c)^5+1/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)^2*\tanh(1/2*dx+1/2*c)^3*a-5/4/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2*b / (a+b)^2*\tanh(1/2*dx+1/2*c)^3+3/4/d/a*b^2 / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)^2*\tanh(1/2*dx+1/2*c)^3+1/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2 / (a+b)*\tanh(1/2*dx+1/2*c)-1/4/d / (\tanh(1/2*dx+1/2*c)^4*a+b*\tanh(1/2*dx+1/2*c)^4+2*\tanh(1/2*dx+1/2*c)^2*a-2*\tanh(1/2*dx+1/2*c)^2*b+a+b)^2*b / (a+b) \end{aligned}$$

) / a * tanh(1/2*d*x+1/2*c) + 1/2/d / (a^2+2*a*b+b^2) / (a+b)^(1/2) / a^(1/2) * arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2)) / a^(1/2)) + 1/2/d / (a^2+2*a*b+b^2) / (a+b)^(1/2) / a^(1/2) * arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2)) / a^(1/2)) + 1/8/d / a^(3/2) * b / (a^2+2*a*b+b^2) / (a+b)^(1/2) * arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2)) / a^(1/2)) + 1/8/d / a^(3/2) * b / (a^2+2*a*b+b^2) / (a+b)^(1/2) * arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2)) / a^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(4a^2e^{7c} + abe^{7c})e^{7dx} + (4a^2e^{5c} + 9ab^2e^{5c} - 4b^2e^{5c})e^{5dx} - (4a^2e^{3c} + 9ab^2e^{3c} - 4b^2e^{3c})e^{3dx} - (4a^2e^c + abe^c)e^{dx}}{4(a^5d + 2a^4bd + a^3b^2d + (a^5de^{8c} + 2a^4bde^{8c} + a^3b^2de^{8c})e^{8dx} + 4(a^5de^{6c} + 4a^4bde^{6c} + 5a^3b^2de^{6c} + 2a^2b^3de^{6c} + 2ab^3de^{6c} + 5a^3b^2de^{6c} + 2a^2b^3de^{6c})e^{6dx} + 2(a^5de^{4c} + 4a^4bde^{4c} + 5a^3b^2de^{4c} + 2a^2b^3de^{4c} + 2ab^3de^{4c} + 5a^3b^2de^{4c} + 2a^2b^3de^{4c})e^{4dx} + 4(a^5de^{2c} + 4a^4bde^{2c} + 5a^3b^2de^{2c} + 2a^2b^3de^{2c} + 2ab^3de^{2c} + 5a^3b^2de^{2c} + 2a^2b^3de^{2c})e^{2dx} + 4(a^5de^c + 4a^4bde^c + 5a^3b^2de^c + 2a^2b^3de^c + 2ab^3de^c + 5a^3b^2de^c + 2a^2b^3de^c)e^c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*((4*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) + 9*a*b*e^(5*c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + a*b*e^c)*e^(d*x)) / (a^5*d + 2*a^4*b*d + a^3*b^2*d + (a^5*d*e^(8*c) + 2*a^4*b*d*e^(8*c) + a^3*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^5*d*e^(6*c) + 4*a^4*b*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*d*e^(6*c) + 2*a*b^3*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + 14*a^4*b*d*e^(4*c) + 27*a^3*b^2*d*e^(4*c) + 24*a^2*b^3*d*e^(4*c) + 8*a*b^4*d*e^(4*c)) * e^(4*d*x) + 4*(a^5*d*e^(2*c) + 4*a^4*b*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c) + 2*a*b^3*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c))*e^(2*d*x) + 8*integrate(1/32*((4*a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (4*a*e^c + b*e^c)*e^(d*x)) / (a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e^(2*c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^3 \left(a + \frac{b}{\cosh(c+dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)

$$3.98 \quad \int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=125

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}} + \frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2(a-b\tanh^2(c+dx)+b)} - \frac{a\tanh(c+dx)}{4bd(a+b)(a-b\tanh^2(c+dx)+b)^2}$$

[Out] 1/8*(a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/d-1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(a+4*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 385, 199, 208}

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}} + \frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2(a-b\tanh^2(c+dx)+b)} - \frac{a\tanh(c+dx)}{4bd(a+b)(a-b\tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(5/2)*d) - (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + ((a + 4*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+b-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{a \tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{(a+4b) \operatorname{Subst}\left(\int \frac{1}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{4b(a+b)d} \\
&= -\frac{a \tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{(a+4b) \tanh(c+dx)}{8b(a+b)^2 d(a+b-b\tanh^2(c+dx))} + \\
&= \frac{(a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d} - \frac{a \tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2} + \frac{1}{8b(a+b)}
\end{aligned}$$

Mathematica [A] time = 3.39, size = 250, normalized size = 2.00

$$\operatorname{sech}^6(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(-\frac{4(a+b)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a} + \frac{\operatorname{sech}(2c)((a+4b)\sinh(2c)-(a-2b)\sinh(2dx))}{b} \right)$$

$$64d(a+b)^2(a+b)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(((a + 4*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]])/(2* Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a + ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[2*c]*((a + 4*b)*Sinh[2*c] - (a - 2*b)*Sinh[2*d*x]))/b)/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.52, size = 5447, normalized size = 43.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 24*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*sinh(d*x + c)^6 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 4*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4)*cosh(d*x + c)^2 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4 + 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 + 4*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4

$$\begin{aligned}
& + 4a^3b) \sinh(dx + c)^8 + 4(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^6 \\
& + 4(a^4 + 6a^3b + 8a^2b^2 + 7(a^4 + 4a^3b) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(a^4 + 4a^3b) \cosh(dx + c)^3 + 3(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)) \sinh(dx + c)^5 + 2(3a^4 + 20a^3b + 40a^2b^2 + 32ab^3) \cosh(dx + c)^4 + 2(35(a^4 + 4a^3b) \cosh(dx + c)^4 + 3a^4 + 20a^3b + 40a^2b^2 + 32ab^3 + 30(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^4 + a^4 + 4a^3b + 8(7(a^4 + 4a^3b) \cosh(dx + c)^5 + 10(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^3 + (3a^4 + 20a^3b + 40a^2b^2 + 32ab^3) \cosh(dx + c)) \sinh(dx + c)^3 + 4(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^2 + 4(7(a^4 + 4a^3b) \cosh(dx + c)^6 + 15(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^4 + a^4 + 6a^3b + 8a^2b^2 + 3(3a^4 + 20a^3b + 40a^2b^2 + 32ab^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^4 + 4a^3b) \cosh(dx + c)^7 + 3(a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)^5 + (3a^4 + 20a^3b + 40a^2b^2 + 32ab^3) \cosh(dx + c)^3 + (a^4 + 6a^3b + 8a^2b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{ab + b^2} \log((a^2 \cosh(dx + c)^4 + 4a^2 \cosh(dx + c) \sinh(dx + c)^3 + a^2 \sinh(dx + c)^4 + 2(a^2 + 2ab) \cosh(dx + c)^2 + 2(3a^2 \cosh(dx + c)^2 + a^2 + 2ab) \sinh(dx + c)^2 + a^2 + 8ab + 8b^2 + 4(a^2 \cosh(dx + c)^3 + (a^2 + 2ab) \cosh(dx + c)) \sinh(dx + c) - 4(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{ab + b^2})) / (a \cosh(dx + c)^4 + 4a \cosh(dx + c) \sinh(dx + c)^3 + a \sinh(dx + c)^4 + 2(a + 2b) \cosh(dx + c)^2 + 2(3a \cosh(dx + c)^2 + a + 2b) \sinh(dx + c)^2 + 4(a \cosh(dx + c)^3 + (a + 2b) \cosh(dx + c)) \sinh(dx + c) + a)) + 8(3(a^4b + 5a^3b^2 + 4a^2b^3) \cosh(dx + c)^5 + 2(3a^4b + 5a^3b^2 - 6a^2b^3 - 24ab^4 - 16b^5) \cosh(dx + c)^3 + (3a^4b - a^3b^2 - 20a^2b^3 - 16ab^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^8 + 8(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \sinh(dx + c)^8 + 4(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^6 + 4(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d) \sinh(dx + c)^6 + 2(3a^6b^2 + 17a^5b^3 + 41a^4b^4 + 51a^3b^5 + 32a^2b^6 + 8ab^7) d \cosh(dx + c)^4 + 8(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + 3(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 30(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^2 + (3a^6b^2 + 17a^5b^3 + 41a^4b^4 + 51a^3b^5 + 32a^2b^6 + 8ab^7) d) \sinh(dx + c)^4 + 4(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^2 + 8(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + 10(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^3 + (3a^6b^2 + 17a^5b^3 + 41a^4b^4 + 51a^3b^5 + 32a^2b^6 + 8ab^7) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 15(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^4 + 3(3a^6b^2 + 17a^5b^3 + 41a^4b^4 + 51a^3b^5 + 32a^2b^6 + 8ab^7) d \cosh(dx + c)^2 + (a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d) \sinh(dx + c)^2 + (a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d + 8((a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)^5 + (3a^6b^2 + 17a^5b^3 + 41a^4b^4 + 51a^3b^5 + 32a^2b^6 + 8ab^7) d \cosh(dx + c)^3 + (a^6b^2 + 5a^5b^3 + 9a^4b^4 + 7a^3b^5 + 2a^2b^6) d \cosh(dx + c)) \sinh(dx + c)), 1/8(2(a^4b + 5a^3b^2 + 4a^2b^3) \cosh(dx + c)^6 + 12(a^4b + 5a^3b^2 + 4a^2b^3) \cosh(dx + c) \sinh(dx + c)^5 + 2(a^4b + 5a^3b^2 + 4a^2b^3) \sinh(dx + c)^6 + 2a^4b - 2a^3b^2 - 4a^2b^3 + 2(3a^4b + 5a^3b^2 - 6a^2b^3 - 24ab^4 - 16b^5) \cosh(dx + c)^4 + 2(3a^4b + 5a^3b^2 - 6a^2b^3 - 24ab^4 - 16b^5 + 15(a^4b + 5a^3b^2 + 4a^2b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(5(a^4b + 5a^3b^2 + 4a^2b^3) \cosh(dx + c)^3 + (3a^4b + 5a^3b^2 - 6a^2b^3 - 24ab^4 -
\end{aligned}$$

```

16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3
- 16*a*b^4)*cosh(d*x + c)^2 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
+ 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b
^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^
4 + 4*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c
)^7 + (a^4 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^6 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a
^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4
+ 3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*co
sh(d*x + c)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 2
0*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6
+ 15*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b
^2 + 3*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2
)*cosh(d*x + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
)^3 + (a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b -
b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(a^4*b + 5*a
^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 -
24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a
*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3
*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*
d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*
b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6
)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5
+ 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a
^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 5*a^5*b^
3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 17*
a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)*sinh(d*x + c)^
4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5
+ 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8
*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17*a^5*b^3 + 41*a
^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 +
5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^2 + (a^6*b^
2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^
4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3
*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4
+ 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^6*b^2 + 5*a^5*b
^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]

```

giac [B] time = 1.73, size = 274, normalized size = 2.19

$$\frac{(a+4b) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^2b+2ab^2+b^3)\sqrt{-ab-b^2}} + \frac{2(a^3e^{(6dx+6c)+4a^2be^{(6dx+6c)+3a^3e^{(4dx+4c)+2a^2be^{(4dx+4c)-8ab^2e^{(4dx+4c)-16b^3e^{(4dx+4c)+3a^3e^{(2dx+2c)-4a^2b}}}}}}}}{(a^3b+2a^2b^2+ab^3)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}}})^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8} * ((a + 4*b) * \arctan(1/2 * (a * e^{(2*d*x + 2*c)} + a + 2*b) / \sqrt{-a*b - b^2})) / ((a^2*b + 2*a*b^2 + b^3) * \sqrt{-a*b - b^2}) + 2 * (a^3 * e^{(6*d*x + 6*c)} + 4*a^2*b * e^{(6*d*x + 6*c)} + 3*a^3 * e^{(4*d*x + 4*c)} + 2*a^2*b * e^{(4*d*x + 4*c)} - 8*a*b^2 * e^{(4*d*x + 4*c)} - 16*b^3 * e^{(4*d*x + 4*c)} + 3*a^3 * e^{(2*d*x + 2*c)} - 4*a^2*b * e^{(2*d*x + 2*c)} - 16*a*b^2 * e^{(2*d*x + 2*c)} + a^3 - 2*a^2*b) / ((a^3*b + 2*a^2*b^2 + a*b^3) * (a * e^{(4*d*x + 4*c)} + 2*a * e^{(2*d*x + 2*c)} + 4*b * e^{(2*d*x + 2*c)} + a)^2) / d$

maple [B] time = 0.28, size = 1084, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)

[Out] $-\frac{1}{4} * \frac{d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b) * \tanh(1/2*d*x+1/2*c)^7} * \frac{a+1/d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b) * \tanh(1/2*d*x+1/2*c)^7} * \frac{3/4 * d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{5*a^2 + 5/4 * d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{5*a - 1/d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{5*a - 1/d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{3*a - 1/d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{5/4 * d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{3*a - 1/d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{1/4 * d}{(\tanh(1/2*d*x+1/2*c)^4 * a + b * \tanh(1/2*d*x+1/2*c)^4 + 2 * \tanh(1/2*d*x+1/2*c)^2 * a - 2 * \tanh(1/2*d*x+1/2*c)^2 * b + a + b)^2} * \frac{b}{(a+b)^2} * \frac{1/16 * d}{b^{(3/2)}} * \frac{1}{(a^2 + 2*a*b + b^2) * a} * \frac{1}{(a+b)^{(1/2)}} * \ln(-(a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) - (a+b)^{(1/2)}) + 1/16 * d / b^{(3/2)} / (a^2 + 2*a*b + b^2) * a / (a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)}) - 1/4 * d / (a^2 + 2*a*b + b^2) / b^{(1/2)} / (a+b)^{(1/2)} * \ln(-(a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) - (a+b)^{(1/2)}) + 1/4 * d / (a^2 + 2*a*b + b^2) / b^{(1/2)} / (a+b)^{(1/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2*d*x+1/2*c)^2 + 2*b^{(1/2)} * \tanh(1/2*d*x+1/2*c) + (a+b)^{(1/2)})$

maxima [B] time = 0.53, size = 369, normalized size = 2.95

$$\frac{(a + 4b) \log\left(\frac{ae^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{ae^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right)}{16(a^2b + 2ab^2 + b^3)\sqrt{(a+b)bd}} - \frac{a^3 - 2a^2b + (3a^3)}{4(a^5b + 2a^4b^2 + a^3b^3 + 4(a^5b + 4a^4b^2 + 5a^3b^3 + 2a^2b^4)e^{(-2dx-2c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-\frac{1}{16} * (a + 4*b) * \log((a * e^{(-2*d*x - 2*c)} + a + 2*b - 2 * \sqrt{(a + b) * b})) / (a * e^{(-2*d*x - 2*c)} + a + 2*b + 2 * \sqrt{(a + b) * b})) / ((a^2*b + 2*a*b^2 + b^3) * \sqrt{(a + b) * b} * d) - \frac{1}{4} * (a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2) * e^{(-2*d*x - 2*c)} + (3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3) * e^{(-4*d*x - 4*c)} + (a^3 + 4*a^2*b) * e^{(-6*d*x - 6*c)}) / ((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^4*b^2 + 5*a^3*b^3 + 2*a^2*b^4) * e^{(-2*d*x - 2*c)} + 2*(3*a^5*b + 14*a^4*b^2 + 27*a^3*b^3 + 24*a^2*b^4 + 8*a*b^5) * e^{(-4*d*x - 4*c)} + 4*(a^5*b + 4*a^4*b^2$

+ 5*a^3*b^3 + 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a^3*b^3)*
e^(-8*d*x - 8*c))*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^4 \left(a + \frac{b}{\cosh(c+dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**3, x)

[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)

$$3.99 \quad \int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=106

$$\frac{3 \sinh(c+dx)}{8d(a+b)^2(a \sinh^2(c+dx) + a+b)} + \frac{\sinh(c+dx)}{4d(a+b)(a \sinh^2(c+dx) + a+b)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

[Out] 1/4*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d/a^(1/2)

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4147, 199, 205}

$$\frac{3 \sinh(c+dx)}{8d(a+b)^2(a \sinh^2(c+dx) + a+b)} + \frac{\sinh(c+dx)}{4d(a+b)(a \sinh^2(c+dx) + a+b)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*d + Sinh[c + d*x]/(4*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 4147

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bax^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{4(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+bax^2)^2} dx, x, \sinh(c+dx)\right)}{4(a+b)d} \\
&= \frac{\sinh(c+dx)}{4(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{3\sinh(c+dx)}{8(a+b)^2d(a+b+a\sinh^2(c+dx))} + \frac{3S}{8(a+b)^2d(a+b)} \\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{4(a+b)d(a+b+a\sinh^2(c+dx))^2} + \frac{3\sinh}{8(a+b)^2d(a+b)}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 125, normalized size = 1.18

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2(a\sinh^2(c+dx)+a+b)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}\right)}{64d(a+b\operatorname{sech}^2(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2)))/(64*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.51, size = 5006, normalized size = 47.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(12*(a^3 + a^2*b)*cosh(d*x + c)^7 + 84*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 12*(a^3 + a^2*b)*sinh(d*x + c)^7 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2 + 63*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a^3 + a^2*b)*cosh(d*x + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 4*(105*(a^3 + a^2*b)*cosh(d*x + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*

$$\begin{aligned}
& \cosh(dx + c)^6 + 15*(a^2 + 2*a*b)*\cosh(dx + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*\cosh(dx + c)^2 + a^2 + 2*a*b*\sinh(dx + c)^2 + a^2 + 8*(a^2*\cosh(dx + c)^7 + 3*(a^2 + 2*a*b)*\cosh(dx + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*\cosh(dx + c)^3 + (a^2 + 2*a*b)*\cosh(dx + c))*\sinh(dx + c))*\sqrt{-a^2 - a*b}*\log((a*\cosh(dx + c)^4 + 4*a*\cosh(dx + c)*\sinh(dx + c)^3 + a*\sinh(dx + c)^4 - 2*(3*a + 2*b)*\cosh(dx + c)^2 + 2*(3*a*\cosh(dx + c)^2 - 3*a - 2*b)*\sinh(dx + c)^2 + 4*(a*\cosh(dx + c)^3 - (3*a + 2*b)*\cosh(dx + c))*\sinh(dx + c)) - 4*(\cosh(dx + c)^3 + 3*\cosh(dx + c)*\sinh(dx + c)^2 + \sinh(dx + c)^3 + (3*\cosh(dx + c)^2 - 1)*\sinh(dx + c) - \cosh(dx + c))*\sqrt{-a^2 - a*b} + a)/(a*\cosh(dx + c)^4 + 4*a*\cosh(dx + c)*\sinh(dx + c)^3 + a*\sinh(dx + c)^4 + 2*(a + 2*b)*\cosh(dx + c)^2 + 2*(3*a*\cosh(dx + c)^2 + a + 2*b)*\sinh(dx + c)^2 + 4*(a*\cosh(dx + c)^3 + (a + 2*b)*\cosh(dx + c))*\sinh(dx + c) + a)) - 12*(a^3 + a^2*b)*\cosh(dx + c) + 4*(21*(a^3 + a^2*b)*\cosh(dx + c)^6 + 5*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^4 - 3*a^3 - 3*a^2*b - 3*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)*\sinh(dx + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\sinh(dx + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*\sinh(dx + c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(dx + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d)*\sinh(dx + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(dx + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*\sinh(dx + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(dx + c)^7 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c)^5 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(dx + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(dx + c))*\sinh(dx + c)), 1/8*(6*(a^3 + a^2*b)*\cosh(dx + c)^7 + 42*(a^3 + a^2*b)*\cosh(dx + c)*\sinh(dx + c)^6 + 6*(a^3 + a^2*b)*\sinh(dx + c)^7 + 2*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^5 + 2*(11*a^3 + 31*a^2*b + 20*a*b^2 + 63*(a^3 + a^2*b)*\cosh(dx + c)^2)*\sinh(dx + c)^5 + 10*(21*(a^3 + a^2*b)*\cosh(dx + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^4 - 2*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^3 + 2*(105*(a^3 + a^2*b)*\cosh(dx + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^3 + 2*(63*(a^3 + a^2*b)*\cosh(dx + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c)^3 - 3*(11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^2 + 3*(a^2*\cosh(dx + c)^8 + 8*a^2*\cosh(dx + c)*\sinh(dx + c)^7 + a^2*\sinh(dx + c)^8 + 4*(a^2 + 2*a*b)*\cosh(dx + c)^6 + 4*(7*a^2*\cosh(dx + c)^2 + a^2 + 2*a*b)*\sinh(dx + c)^6 + 8*(7*a^2*\cosh(dx + c)^3 + 3*(a^2 + 2*a*b)*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*\cosh(dx + c)^4 + 2*(35*a^2*\cosh(dx + c)^4 + 30*(a^2 + 2*a*b)*\cosh(dx + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*\sinh(dx + c)^4 + 8*(7*a^2*\cosh(dx + c)^5 + 10*(a^2 + 2*a*b)*\cosh(dx + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(a^2 + 2*a*b)*\cosh(dx + c)^2 + 4*(7*a^2*\cosh(dx + c)^6 + 15*(a^2 + 2*a*b)*\cosh(dx + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*\cosh(dx + c)^2 + a^2 + 2*a*b)*\sinh(dx + c)^2 + a^2 + 8*(a^2*\cosh(dx + c)^7 + 3*(a^2 + 2*a*b)*\cosh(dx + c)^5 + (3*a^2 + 8*
\end{aligned}$$

$$\begin{aligned}
& a*b + 8*b^2)*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \sqrt{a^2 + a*b}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x \\
& + c)^2 + a*\sinh(d*x + c)^3 + (3*a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c) \\
& ^2 + 3*a + 4*b)*\sinh(d*x + c))/\sqrt{a^2 + a*b})) + 3*(a^2*\cosh(d*x + c)^8 + \\
& 8*a^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^2*\sinh(d*x + c)^8 + 4*(a^2 + 2*a*b) \\
& *\cosh(d*x + c)^6 + 4*(7*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^6 \\
& + 8*(7*a^2*\cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 \\
& + 2*(3*a^2 + 8*a*b + 8*b^2)*\cosh(d*x + c)^4 + 2*(35*a^2*\cosh(d*x + c)^4 + \\
& 30*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*\sinh(d*x + c)^4 + \\
& 8*(7*a^2*\cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*\cosh(d*x + c)^3 + (3*a^2 + 8*a \\
& *b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*\cosh(d*x + c)^ \\
& 2 + 4*(7*a^2*\cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*\cosh(d*x + c)^4 + 3*(3*a^2 \\
& + 8*a*b + 8*b^2)*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*(\\
& a^2*\cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*\cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8* \\
& b^2)*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 \\
& + a*b}*\arctan(1/2*\sqrt{a^2 + a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/(a + b)) \\
& - 6*(a^3 + a^2*b)*\cosh(d*x + c) + 2*(21*(a^3 + a^2*b)*\cosh(d*x + c)^6 + 5* \\
& (11*a^3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*(11*a^ \\
& 3 + 31*a^2*b + 20*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^6 + 3*a^5*b + \\
& 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3 \\
& *b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^ \\
& 3)*d*\sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4 \\
&)*d*\cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x \\
& + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*\sinh(d*x + \\
& c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^ \\
& 5)*d*\cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d* \\
& x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh \\
& (d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh \\
& (d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8* \\
& a*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^ \\
& 2*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cos \\
& h(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cos \\
& h(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8 \\
& *a*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 \\
& + a^3*b^3)*d*\cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + \\
& 2*a^2*b^4)*d*\cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^ \\
& 3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + \\
& 7*a^3*b^3 + 2*a^2*b^4)*d)*\sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^ \\
& 3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^7 + 3*(\\
& a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(d*x + c)^5 + (3*a \\
& ^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*\cosh(d*x \\
& + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*\cosh(d*x + c \\
&))*\sinh(d*x + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with

parameters. This might be wrong. The choice was done assuming [a,b]=[-28,94] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[-7,46] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[-35,-99] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[7,50] Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a,b]=[-63,-70] Undef/Unsigned Inf encountered in limitEvaluation time: 1.6 Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.32, size = 592, normalized size = 5.58

$$\frac{5 \left(\tanh^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + b \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a + b \right)^2 (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$-5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^7+3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5+a+15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5-3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3+a-15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(11ae^{5c} + 20be^{5c})e^{5dx} - (11ae^{3c} + 20be^{3c})e^{3dx} + 3ae^{7dx+7c} - 3ae^{dx+c}}{4(a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx}) + 4(a^4de^{6c} + 4a^3bde^{6c} + 5a^2b^2de^{6c} + 4a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx})e^{8dx} + 4(a^4de^{6c} + 4a^3bde^{6c} + 5a^2b^2de^{6c} + 2a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx})e^{8dx} + 4(a^4de^{6c} + 4a^3bde^{6c} + 5a^2b^2de^{6c} + 2a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx})e^{8dx} + 2(3a^4d + 14a^3bd + 27a^2b^2d + 24a^4d + 24a^3bd + 8b^4d)e^{4c} + 4(a^4de^{2c} + 4a^3bde^{2c} + 5a^2b^2de^{2c} + 2a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx})e^{8dx} + 2(a^4de^{2c} + 4a^3bde^{2c} + 5a^2b^2de^{2c} + 2a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx})e^{8dx} + 32 \int \frac{3}{128} (e^{3dx+3c} + e^{dx+c}) / (a^3 + 2a^2b + ab^2 + (a^3e^{4c} + 2a^2be^{4c} + ab^2e^{4c})e^{4dx} + 2(a^3e^{2c} + 4a^2be^{2c} + 5ab^2e^{2c} + 2b^3e^{2c}))e^{2dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$1/4*((11*a*e^{5*c} + 20*b*e^{5*c})*e^{5*d*x} - (11*a*e^{3*c} + 20*b*e^{3*c})*e^{3*d*x} + 3*a*e^{7*d*x + 7*c} - 3*a*e^{d*x + c})/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^{8*c} + 2*a^3*b*d*e^{8*c} + a^2*b^2*d*e^{8*c})*e^{8*d*x} + 4*(a^4*d*e^{6*c} + 4*a^3*b*d*e^{6*c} + 5*a^2*b^2*d*e^{6*c} + 2*a*b^3*d*e^{6*c})*e^{6*d*x} + 2*(3*a^4*d*e^{4*c} + 14*a^3*b*d*e^{4*c} + 27*a^2*b^2*d*e^{4*c} + 24*a*b^3*d*e^{4*c} + 8*b^4*d*e^{4*c})*e^{4*d*x} + 4*(a^4*d*e^{2*c} + 4*a^3*b*d*e^{2*c} + 5*a^2*b^2*d*e^{2*c} + 2*a*b^3*d*e^{2*c})*e^{2*d*x} + 32*integrate(3/128*(e^{3*d*x + 3*c} + e^{d*x + c})/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^{4*c} + 2*a^2*b*e^{4*c} + a*b^2*e^{4*c})*e^{4*d*x} + 2*(a^3*e^{2*c} + 4*a^2*b*e^{2*c} + 5*a*b^2*e^{2*c} + 2*b^3*e^{2*c})*e^{2*d*x}), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^5 \left(a + \frac{b}{\cosh(c+dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**3, x)

[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**3, x)

$$3.100 \quad \int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=144

$$\frac{(3a^2 + 8ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}} - \frac{3a(a+2b) \tanh(c+dx)}{8b^2d(a+b)^2(a-b \tanh^2(c+dx)+b)} - \frac{a \tanh(c+dx) \operatorname{sech}^2(c+dx)}{4bd(a+b)(a-b \tanh^2(c+dx))}$$

[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a+b)^(5/2)/d-1/4*a*sech(d*x+c)^2*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-3/8*a*(a+2*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4146, 413, 385, 208}

$$\frac{(3a^2 + 8ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}} - \frac{3a(a+2b) \tanh(c+dx)}{8b^2d(a+b)^2(a-b \tanh^2(c+dx)+b)} - \frac{a \tanh(c+dx) \operatorname{sech}^2(c+dx)}{4bd(a+b)(a-b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(5/2)*(a + b)^(5/2)*d) - (a*Sech[c + d*x]^2*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) - (3*a*(a + 2*b)*Tanh[c + d*x])/(8*b^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(a*b*n*(p+1)), x] - Dist[1/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^(q-2)*Simp[c*(a*d - c*b*(n*(p+1)+1)) + d*(a*d*(n*(q-1)+1) - b*c*(n*(p+q)+1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 4146

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-a-4b+(3a+4b)x^2}{(a+b-x^2)^2} dx, x, \tanh(c+dx)\right)}{4b(a+b)d} \\
&= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2} - \frac{3a(a+2b)\tanh(c+dx)}{8b^2(a+b)^2d(a+b-b\tanh^2(c+dx))} \\
&= \frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d} - \frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d(a+b-b\tanh^2(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 1.01, size = 125, normalized size = 0.87

$$\frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\sinh(2(c+dx))(3a^2+3a(a+2b)\cosh(2(c+dx))+16ab+16b^2)}{(a+b)^2(a\cosh(2(c+dx))+a+2b)^2}$$

$8b^{5/2}d$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(8*b^(5/2)*d)

fricas [B] time = 0.53, size = 5887, normalized size = 40.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 24*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 12*a^4*b + 36*a^3*b^2 + 24*a^2*b^3 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5) + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4 + 15*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a

$$\begin{aligned}
& ^3b + 8a^2b^2) * \cosh(dx + c)^2 * \sinh(dx + c)^6 + 8 * (7 * (3a^4 + 8a^3b \\
& + 8a^2b^2) * \cosh(dx + c)^3 + 3 * (3a^4 + 14a^3b + 24a^2b^2 + 16ab^3) \\
& * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (9a^4 + 48a^3b + 112a^2b^2 + 128a \\
& * b^3 + 64b^4) * \cosh(dx + c)^4 + 2 * (35 * (3a^4 + 8a^3b + 8a^2b^2) * \cosh(d \\
& * x + c)^4 + 9a^4 + 48a^3b + 112a^2b^2 + 128a * b^3 + 64b^4 + 30 * (3a^4 \\
& + 14a^3b + 24a^2b^2 + 16ab^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 3a \\
& ^4 + 8a^3b + 8a^2b^2 + 8 * (7 * (3a^4 + 8a^3b + 8a^2b^2) * \cosh(dx + c) \\
& ^5 + 10 * (3a^4 + 14a^3b + 24a^2b^2 + 16ab^3) * \cosh(dx + c)^3 + (9a^4 \\
& + 48a^3b + 112a^2b^2 + 128a * b^3 + 64b^4) * \cosh(dx + c)) * \sinh(dx + c \\
&)^3 + 4 * (3a^4 + 14a^3b + 24a^2b^2 + 16ab^3) * \cosh(dx + c)^2 + 4 * (7 * (\\
& 3a^4 + 8a^3b + 8a^2b^2) * \cosh(dx + c)^6 + 15 * (3a^4 + 14a^3b + 24a^2 \\
& * b^2 + 16ab^3) * \cosh(dx + c)^4 + 3a^4 + 14a^3b + 24a^2b^2 + 16ab^3 \\
& + 3 * (9a^4 + 48a^3b + 112a^2b^2 + 128a * b^3 + 64b^4) * \cosh(dx + c)^2 \\
&) * \sinh(dx + c)^2 + 8 * ((3a^4 + 8a^3b + 8a^2b^2) * \cosh(dx + c)^7 + 3 * (3 \\
& * a^4 + 14a^3b + 24a^2b^2 + 16ab^3) * \cosh(dx + c)^5 + (9a^4 + 48a^3 * \\
& b + 112a^2b^2 + 128a * b^3 + 64b^4) * \cosh(dx + c)^3 + (3a^4 + 14a^3b + \\
& 24a^2b^2 + 16ab^3) * \cosh(dx + c)) * \sinh(dx + c) * \sqrt{a * b + b^2} * \log((\\
& a^2 * \cosh(dx + c)^4 + 4a^2 * \cosh(dx + c) * \sinh(dx + c)^3 + a^2 * \sinh(dx + \\
& c)^4 + 2 * (a^2 + 2a * b) * \cosh(dx + c)^2 + 2 * (3a^2 * \cosh(dx + c)^2 + a^2 + 2 \\
& * a * b) * \sinh(dx + c)^2 + a^2 + 8a * b + 8b^2 + 4 * (a^2 * \cosh(dx + c)^3 + (a^2 \\
& + 2a * b) * \cosh(dx + c)) * \sinh(dx + c) - 4 * (a * \cosh(dx + c)^2 + 2a * \cosh(dx \\
& + c) * \sinh(dx + c) + a * \sinh(dx + c)^2 + a + 2b) * \sqrt{a * b + b^2})) / (a * \cos \\
& h(dx + c)^4 + 4a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + c)^4 + 2 * (a \\
& + 2b) * \cosh(dx + c)^2 + 2 * (3a * \cosh(dx + c)^2 + a + 2b) * \sinh(dx + c)^2 \\
& + 4 * (a * \cosh(dx + c)^3 + (a + 2b) * \cosh(dx + c)) * \sinh(dx + c) + a)) + 8 * \\
& (3 * (3a^4 * b + 11a^3 * b^2 + 16a^2 * b^3 + 8a * b^4) * \cosh(dx + c)^5 + 6 * (3a^4 \\
& * b + 17a^3 * b^2 + 38a^2 * b^3 + 40a * b^4 + 16b^5) * \cosh(dx + c)^3 + (9a^4 * \\
& b + 49a^3 * b^2 + 80a^2 * b^3 + 40a * b^4) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^5 \\
& * b^3 + 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c)^8 + 8 * (a^5 * b^3 + 3 * \\
& a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^5 * b^3 + \\
& 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \sinh(dx + c)^8 + 4 * (a^5 * b^3 + 5a^4 * b^4 \\
& + 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d * \cosh(dx + c)^6 + 4 * (7 * (a^5 * b^3 + 3 * \\
& a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c)^2 + (a^5 * b^3 + 5a^4 * b^4 + 9 \\
& * a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d) * \sinh(dx + c)^6 + 2 * (3a^5 * b^3 + 17a^4 * \\
& b^4 + 41a^3 * b^5 + 51a^2 * b^6 + 32a * b^7 + 8b^8) * d * \cosh(dx + c)^4 + 8 * (7 * \\
& (a^5 * b^3 + 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c)^3 + 3 * (a^5 * b^3 \\
& + 5a^4 * b^4 + 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d * \cosh(dx + c)) * \sinh(dx + \\
& c)^5 + 2 * (35 * (a^5 * b^3 + 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c)^4 \\
& + 30 * (a^5 * b^3 + 5a^4 * b^4 + 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d * \cosh(dx + c) \\
&)^2 + (3a^5 * b^3 + 17a^4 * b^4 + 41a^3 * b^5 + 51a^2 * b^6 + 32a * b^7 + 8b^8) \\
& * d) * \sinh(dx + c)^4 + 4 * (a^5 * b^3 + 5a^4 * b^4 + 9a^3 * b^5 + 7a^2 * b^6 + 2a * \\
& b^7) * d * \cosh(dx + c)^2 + 8 * (7 * (a^5 * b^3 + 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d \\
& * \cosh(dx + c)^5 + 10 * (a^5 * b^3 + 5a^4 * b^4 + 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7 \\
&) * d * \cosh(dx + c)^3 + (3a^5 * b^3 + 17a^4 * b^4 + 41a^3 * b^5 + 51a^2 * b^6 + \\
& 32a * b^7 + 8b^8) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (7 * (a^5 * b^3 + 3a^4 * \\
& b^4 + 3a^3 * b^5 + a^2 * b^6) * d * \cosh(dx + c)^6 + 15 * (a^5 * b^3 + 5a^4 * b^4 + 9 * \\
& a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d * \cosh(dx + c)^4 + 3 * (3a^5 * b^3 + 17a^4 * b^4 \\
& + 41a^3 * b^5 + 51a^2 * b^6 + 32a * b^7 + 8b^8) * d * \cosh(dx + c)^2 + (a^5 * b^3 \\
& + 5a^4 * b^4 + 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d) * \sinh(dx + c)^2 + (a^5 * \\
& b^3 + 3a^4 * b^4 + 3a^3 * b^5 + a^2 * b^6) * d + 8 * ((a^5 * b^3 + 3a^4 * b^4 + 3a^3 * \\
& b^5 + a^2 * b^6) * d * \cosh(dx + c)^7 + 3 * (a^5 * b^3 + 5a^4 * b^4 + 9a^3 * b^5 + 7a \\
& ^2 * b^6 + 2a * b^7) * d * \cosh(dx + c)^5 + (3a^5 * b^3 + 17a^4 * b^4 + 41a^3 * b^5 \\
& + 51a^2 * b^6 + 32a * b^7 + 8b^8) * d * \cosh(dx + c)^3 + (a^5 * b^3 + 5a^4 * b^4 + \\
& 9a^3 * b^5 + 7a^2 * b^6 + 2a * b^7) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/8 * (2 * (\\
& 3a^4 * b + 11a^3 * b^2 + 16a^2 * b^3 + 8a * b^4) * \cosh(dx + c)^6 + 12 * (3a^4 * b \\
& + 11a^3 * b^2 + 16a^2 * b^3 + 8a * b^4) * \cosh(dx + c) * \sinh(dx + c)^5 + 2 * (3a \\
& ^4 * b + 11a^3 * b^2 + 16a^2 * b^3 + 8a * b^4) * \sinh(dx + c)^6 + 6a^4 * b + 18a^3 \\
& * b^2 + 12a^2 * b^3 + 6 * (3a^4 * b + 17a^3 * b^2 + 38a^2 * b^3 + 40a * b^4 + 16b \\
& ^5) * \cosh(dx + c)^4 + 6 * (3a^4 * b + 17a^3 * b^2 + 38a^2 * b^3 + 40a * b^4 + 16b
\end{aligned}$$

$$\begin{aligned}
& b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*\cosh(d*x + c)^2*\sinh \\
& (d*x + c)^4 + 8*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*\cosh(d*x + \\
& c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^3 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*\co \\
& sh(d*x + c)^2 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4 + 15*(3*a^4 \\
& *b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*\cosh(d*x + c)^4 + 18*(3*a^4*b + 17* \\
& a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 \\
& + ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^8 + 8*(3*a^4 + 8*a^3*b + 8*a \\
& ^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*\sinh(\\
& d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^6 + \\
& 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a^3*b + 8*a^2*b \\
& ^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\c \\
& osh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4) \\
& *\cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^4 + 9* \\
& a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4 + 14*a^3*b + \\
& 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 3*a^4 + 8*a^3*b + \\
& 8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^5 + 10*(3*a^4 \\
& + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^3 + (9*a^4 + 48*a^3*b + \\
& 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a^4 \\
& + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(3*a^4 + 8*a^3* \\
& b + 8*a^2*b^2)*\cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b \\
& ^3)*\cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 + \\
& 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c \\
&)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^7 + 3*(3*a^4 + 14*a^3* \\
& b + 24*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^5 + (9*a^4 + 48*a^3*b + 112*a^2*b^ \\
& 2 + 128*a*b^3 + 64*b^4)*\cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 24*a^2*b^2 + \\
& 16*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b - b^2}*\arctan(1/2*(a*\cosh \\
& (d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b \\
&)*\sqrt{-a*b - b^2}/(a*b + b^2)) + 4*(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + \\
& 8*a*b^4)*\cosh(d*x + c)^5 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 \\
& + 16*b^5)*\cosh(d*x + c)^3 + (9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4) \\
& *\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6) \\
& *d*\cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\cosh(d \\
& *x + c)*\sinh(d*x + c)^7 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\sin \\
& h(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d* \\
& cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\cosh(d \\
& *x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*\sinh \\
& (d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^ \\
& 7 + 8*b^8)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2* \\
& b^6)*d*\cosh(d*x + c)^3 + 3*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2 \\
& *a*b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a \\
& ^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^4 + 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + \\
& 7*a^2*b^6 + 2*a*b^7)*d*\cosh(d*x + c)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3* \\
& b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d)*\sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^ \\
& 4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^3 \\
& + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4* \\
& b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*\cosh(d*x + c)^3 + (3*a^5*b^3 + 17* \\
& a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + \\
& c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*\cosh(d \\
& *x + c)^4 + 3*(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 \\
& + 8*b^8)*d*\cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + \\
& 2*a*b^7)*d)*\sinh(d*x + c)^2 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)* \\
& d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^7 + 3*(a \\
& ^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*\cosh(d*x + c)^5 + (\\
& 3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*\cosh \\
& (d*x + c)^3 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*\cos \\
& h(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 1.75, size = 302, normalized size = 2.10

$$\frac{(3a^2+8ab+8b^2) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^2b^2+2ab^3+b^4)\sqrt{-ab-b^2}} + \frac{2(3a^3e^{(6dx+6c)}+8a^2be^{(6dx+6c)}+8ab^2e^{(6dx+6c)}+9a^3e^{(4dx+4c)}+42a^2be^{(4dx+4c)}+72ab^2e^{(4dx+4c)}+4a^3e^{(2dx+2c)})}{(a^2b^2+2ab^3+b^4)(ae^{(4dx+4c)}+2ae^{(2dx+2c)})}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a*b - b^2)) + 2*(3*a^3*e^(6*d*x + 6*c) + 8*a^2*b*e^(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 9*a^3*e^(4*d*x + 4*c) + 42*a^2*b*e^(4*d*x + 4*c) + 72*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) + 40*a^2*b*e^(2*d*x + 2*c) + 40*a*b^2*e^(2*d*x + 2*c) + 3*a^3 + 6*a^2*b)/((a^2*b^2 + 2*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a^2))/d

maple [B] time = 0.34, size = 1245, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x)

[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7-2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5-13/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3-13/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)-2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a-3/16/d/b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))*a^2-1/2/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/16/d/b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))*a^2+1/2/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))

maxima [B] time = 1.37, size = 395, normalized size = 2.74

$$\frac{(3a^2 + 8ab + 8b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^2b^2 + 2ab^3 + b^4)\sqrt{(a+b)b}d} + \frac{3a^3 + 6a^2b + (3a^2 + 8ab + 8b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4(a^4b^2 + 2a^3b^3 + a^2b^4 + 4(a^4b^2 + 4a^3b^3 + 5a^2b^4 + 2ab^5))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/16*(3*a^2 + 8*a*b + 8*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^2*b^2 + 2*a*b^3 + b^4)*\sqrt{(a + b)*b}*d) - 1/4*(3*a^3 + 6*a^2*b + (9*a^3 + 40*a^2*b + 40*a*b^2)*e^{(-2*d*x - 2*c)} + 3*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3)*e^{(-4*d*x - 4*c)} + (3*a^3 + 8*a^2*b + 8*a*b^2)*e^{(-6*d*x - 6*c)})/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^{(-2*d*x - 2*c)} + 2*(3*a^4*b^2 + 14*a^3*b^3 + 27*a^2*b^4 + 24*a*b^5 + 8*b^6)*e^{(-4*d*x - 4*c)} + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^{(-6*d*x - 6*c)} + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^{(-8*d*x - 8*c)})*d$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^6 \left(a + \frac{b}{\cosh(c + dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)

$$3.101 \quad \int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=153

$$\frac{\sqrt{a} (8a^2 + 20ab + 15b^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}} - \frac{a(4a+7b) \sinh(c+dx)}{8b^2d(a+b)^2(a \sinh^2(c+dx) + a+b)} - \frac{a \sinh(c+dx)}{4bd(a+b)(a \sinh^2(c+dx) + a+b)}$$

[Out] arctan(sinh(d*x+c))/b^3/d-1/4*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2-1/8*a*(4*a+7*b)*sinh(d*x+c)/b^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)-1/8*(8*a^2+20*a*b+15*b^2)*arctan(sinh(d*x+c))*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^3/(a+b)^(5/2)/d

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4147, 414, 527, 522, 203, 205}

$$\frac{\sqrt{a} (8a^2 + 20ab + 15b^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}} - \frac{a(4a+7b) \sinh(c+dx)}{8b^2d(a+b)^2(a \sinh^2(c+dx) + a+b)} - \frac{a \sinh(c+dx)}{4bd(a+b)(a \sinh^2(c+dx) + a+b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ArcTan[Sinh[c + d*x]]/(b^3*d) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*b^3*(a + b)^(5/2)*d) - (a*Sinh[c + d*x])/(4*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (a*(4*a + 7*b)*Sinh[c + d*x])/(8*b^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4147

```
Int[sec[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)^3} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{a \sinh(c + dx)}{4b(a + b)d (a + b + a \sinh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{a+4b-3ax^2}{(1+x^2)(a+b+ax^2)^2} dx, x, \sinh(c + dx)\right)}{4b(a + b)d}$$

$$= -\frac{a \sinh(c + dx)}{4b(a + b)d (a + b + a \sinh^2(c + dx))^2} - \frac{a(4a + 7b) \sinh(c + dx)}{8b^2(a + b)^2d (a + b + a \sinh^2(c + dx))} + \dots$$

$$= -\frac{a \sinh(c + dx)}{4b(a + b)d (a + b + a \sinh^2(c + dx))^2} - \frac{a(4a + 7b) \sinh(c + dx)}{8b^2(a + b)^2d (a + b + a \sinh^2(c + dx))} + \dots$$

$$= \frac{\tan^{-1}(\sinh(c + dx))}{b^3d} - \frac{\sqrt{a} (8a^2 + 20ab + 15b^2) \tan^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a + b)^{5/2}d} - \frac{\dots}{4b(a + b)d}$$

Mathematica [A] time = 4.72, size = 247, normalized size = 1.61

$$\operatorname{sech}^5(c + dx)(a \cosh(2(c + dx)) + a + 2b) \frac{\sqrt{a} (8a^2 + 20ab + 15b^2) (\cosh(c) - \sinh(c)) \operatorname{sech}(c + dx) (a \cosh(2(c + dx)) + a + 2b)^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sqrt{a \cosh(2(c + dx)) + a + 2b}}{\sqrt{a+b}}\right)}{(a+b)^{5/2} \sqrt{(\cosh(c) - \sinh(c))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(16*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x] + (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c])]/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/(a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) - (8*a
```

$*b^2*\text{Tanh}[c + d*x])/(a + b) - (2*a*b*(4*a + 7*b)*(a + 2*b + a*\text{Cosh}[2*(c + d*x)])*\text{Tanh}[c + d*x])/(a + b)^2)/(64*b^3*d*(a + b*\text{Sech}[c + d*x]^2)^3)$

fricas [B] time = 0.62, size = 7993, normalized size = 52.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^7 + 28*(4*a^3*b + 7*a^2*b^2)* \\ & \cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(4*a^3*b + 7*a^2*b^2)*\sinh(d*x + c)^7 + 4 \\ & *(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*\cosh(d*x + c)^5 + 4*(4*a^3*b + 31*a^2*b^2 \\ & + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + \\ & 20*(7*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 + 36*a*b^3) \\ & *\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*\c \\ & \cosh(d*x + c)^3 + 4*(35*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^4 - 4*a^3*b - 31 \\ & *a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*\cosh(d*x + c)^2) \\ & *\sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^5 + 10*(4*a^3*b \\ & b + 31*a^2*b^2 + 36*a*b^3)*\cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a \\ & *b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cos \\ & h(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c \\ &)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*\sinh(d*x + c)^8 + 4*(8*a^4 + 36*a^3*b \\ & + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^2 \\ & + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x \\ & + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^3 + 3*(8*a^4 + \\ & 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(24*a^4 \\ & + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^4 + 2*(35* \\ & (8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b + 269* \\ & a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3) \\ & *\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 + 8*(7* \\ & (8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b + 55 \\ & *a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 + \\ & 280*a*b^3 + 120*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^3*b + \\ & 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2 \\ & ^2)*\cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d* \\ & x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b \\ & + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8* \\ & ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55 \\ & *a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 + \\ & 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30* \\ & a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a/(a + b)}*\log((a*\cosh(d*x + c))^4 \\ & + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 - 2*(3*a + 2*b)*\c \\ & \cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 - 3*a - 2*b)*\sinh(d*x + c)^2 + 4*(a \\ & *\cosh(d*x + c)^3 - (3*a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\co \\ & sh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x \\ & + c)^3 - (a + b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 - a - b)*\sinh(d \\ & *x + c))*\sqrt{-a/(a + b)} + a)/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(\\ & d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(\\ & d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\co \\ & sh(d*x + c))*\sinh(d*x + c) + a)) - 32*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + \\ & c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 + \\ & 2*a^3*b + a^2*b^2)*\sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 \\ &)*\cosh(d*x + c)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3 \\ & *b + a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2* \\ & b^2)*\cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c \\ &))*\sinh(d*x + c)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\c \\ & \cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^4 + 3*a^4 + \\ & 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + \\ & 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7* \end{aligned}$$

$$\begin{aligned}
& (a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^5 + 10(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) \cosh(dx + c)^3 + (3a^4 + 14a^3b + 27a^2b^2 + 24ab^3 + 8b^4) \cosh(dx + c) \sinh(dx + c)^3 + 4(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) \cosh(dx + c)^2 + 4(7(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^6 + 15(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) \cosh(dx + c)^4 + a^4 + 4a^3b + 5a^2b^2 + 2ab^3 + 3(3a^4 + 14a^3b + 27a^2b^2 + 24ab^3 + 8b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^7 + 3(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) \cosh(dx + c)^5 + (3a^4 + 14a^3b + 27a^2b^2 + 24ab^3 + 8b^4) \cosh(dx + c)^3 + (a^4 + 4a^3b + 5a^2b^2 + 2ab^3) \cosh(dx + c)) \sinh(dx + c) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 4(4a^3b + 7a^2b^2) \cosh(dx + c) + 4(7(4a^3b + 7a^2b^2) \cosh(dx + c)^6 + 5(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^4 - 4a^3b - 7a^2b^2 - 3(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^2) \sinh(dx + c)) / ((a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^8 + 8(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^4b^3 + 2a^3b^4 + a^2b^5) d \sinh(dx + c)^8 + 4(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^6 + 4(7(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^2 + (a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d) \sinh(dx + c)^6 + 2(3a^4b^3 + 14a^3b^4 + 27a^2b^5 + 24ab^6 + 8b^7) d \cosh(dx + c)^4 + 8(7(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^3 + 3(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^4 + 30(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^2 + (3a^4b^3 + 14a^3b^4 + 27a^2b^5 + 24ab^6 + 8b^7) d) \sinh(dx + c)^4 + 4(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^2 + 8(7(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^5 + 10(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^3 + (3a^4b^3 + 14a^3b^4 + 27a^2b^5 + 24ab^6 + 8b^7) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^6 + 15(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^4 + 3(3a^4b^3 + 14a^3b^4 + 27a^2b^5 + 24ab^6 + 8b^7) d \cosh(dx + c)^2 + (a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d) \sinh(dx + c)^2 + (a^4b^3 + 2a^3b^4 + a^2b^5) d + 8((a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^7 + 3(a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)^5 + (3a^4b^3 + 14a^3b^4 + 27a^2b^5 + 24ab^6 + 8b^7) d \cosh(dx + c)^3 + (a^4b^3 + 4a^3b^4 + 5a^2b^5 + 2ab^6) d \cosh(dx + c)) \sinh(dx + c)), -1/8(2(4a^3b + 7a^2b^2) \cosh(dx + c)^7 + 14(4a^3b + 7a^2b^2) \cosh(dx + c) \sinh(dx + c)^6 + 2(4a^3b + 7a^2b^2) \sinh(dx + c)^7 + 2(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^5 + 2(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^3 + 21(4a^3b + 7a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^5 + 10(7(4a^3b + 7a^2b^2) \cosh(dx + c)^3 + (4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)) \sinh(dx + c)^4 - 2(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^3 + 2(35(4a^3b + 7a^2b^2) \cosh(dx + c)^4 - 4a^3b - 31a^2b^2 - 36ab^3 + 10(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(21(4a^3b + 7a^2b^2) \cosh(dx + c)^5 + 10(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)^3 - 3(4a^3b + 31a^2b^2 + 36ab^3) \cosh(dx + c)) \sinh(dx + c)^2 + ((8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c)^8 + 8(8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c) \sinh(dx + c)^7 + (8a^4 + 20a^3b + 15a^2b^2) \sinh(dx + c)^8 + 4(8a^4 + 36a^3b + 55a^2b^2 + 30ab^3) \cosh(dx + c)^6 + 4(8a^4 + 36a^3b + 55a^2b^2 + 30ab^3 + 7(8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c)^3 + 3(8a^4 + 36a^3b + 55a^2b^2 + 30ab^3) \cosh(dx + c)) \sinh(dx + c)^5 + 2(24a^4 + 124a^3b + 269a^2b^2 + 280ab^3 + 120b^4) \cosh(dx + c)^4 + 2(35(8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c)^4 + 24a^4 + 124a^3b + 269a^2b^2 + 280ab^3 + 120b^4 + 30(8a^4 + 36a^3b + 55a^2b^2 + 30ab^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8a^4 + 20a^3b + 15a^2b^2 + 8(7(8a^4 + 20a^3b + 15a^2b^2) \cosh(dx + c)^5 + 10(8a^4 + 36a^3b + 55a^2b^2 + 30ab^3) \cosh(dx + c)^3 + (24a^4 + 124a^3b + 269a^2b^2 + 280ab^3 + 120b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(8a^4 + 36a^
\end{aligned}$$

$$\begin{aligned}
& 3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 \\
& + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a/(a + b)}*\arctan(1/2*\sqrt{a/(a + b)}*(\cosh(d*x + c) + \sinh(d*x + c))) + ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*\sinh(d*x + c)^8 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^3 + 3*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^4 + 2*(35*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*\cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*\cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a/(a + b)}*\arctan(1/2*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 + (3*a + 4*b)*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 + 3*a + 4*b)*\sinh(d*x + c)))*\sqrt{a/(a + b)})/a - 16*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 2*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c) + 2*(7*(4*a^3*b + 7*a^2*b^2)*\cosh(d*x + c)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*\cosh(d*x + c)^4 - 4*a^3*b - 7*a^2*b^2 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^8 + 8*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^2 + (a^4
\end{aligned}$$

$$\begin{aligned}
& *b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*\sinh(d*x + c)^6 + 2*(3*a^4*b^3 + \\
& 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*\cosh(d*x + c)^4 + 8*(7*(a^4* \\
& b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^3 + 3*(a^4*b^3 + 4*a^3*b^4 + 5*a \\
& ^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^4*b^3 + 2*a^3 \\
& *b^4 + a^2*b^5)*d*\cosh(d*x + c)^4 + 30*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2 \\
& *a*b^6)*d*\cosh(d*x + c)^2 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 \\
& + 8*b^7)*d)*\sinh(d*x + c)^4 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6 \\
&)*d*\cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^ \\
& 5 + 10*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c)^3 + (3*a \\
& ^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^6 + 15*(a \\
& ^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c)^4 + 3*(3*a^4*b^3 \\
& + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*\cosh(d*x + c)^2 + (a^4*b^3 \\
& + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*\sinh(d*x + c)^2 + (a^4*b^3 + 2*a^3*b^ \\
& 4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^7 + 3*(\\
& a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c)^5 + (3*a^4*b^3 + \\
& 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*\cosh(d*x + c)^3 + (a^4*b^3 + \\
& 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered i
n limitEvaluation time: 1.51Limit: Max order reached or unable to make seri
es expansion Error: Bad Argument Value

maple [B] time = 0.30, size = 1201, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x)

[Out] $1/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*\tanh(1/2*d*x+1/2*c)^7+9/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*\tanh(1/2*d*x+1/2*c)^7*a+1/d/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*\tanh(1/2*d*x+1/2*c)^5-11/4/d$

$$\frac{1}{(\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)^2/b} \tanh(1/2*dx+1/2*c)^{5*a-27/4} / d / (\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)^2} \tanh(1/2*dx+1/2*c)^{5*a-1} / d / (\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)^2/b} \tanh(1/2*dx+1/2*c)^{3*a-2+27/4} / d / (\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)^2} \tanh(1/2*dx+1/2*c)^{3*a-1} / d / (\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)/b^2} \tanh(1/2*dx+1/2*c)^{-9/4} / d / (\tanh(1/2*dx+1/2*c)^{4*a+b} \tanh(1/2*dx+1/2*c)^{4+2*\tanh(1/2*dx+1/2*c)^{2*a-2*\tanh(1/2*dx+1/2*c)^{2*b+a+b}})^2} \frac{1}{(a+b)*\tanh(1/2*dx+1/2*c)} \frac{1}{b^3} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(-2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) + \frac{5/2}{d} \frac{1}{b^2} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(-2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) + \frac{15/8}{d} \frac{1}{b} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(-2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) - \frac{1}{d} \frac{1}{b^3} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) - \frac{5/2}{d} \frac{1}{b^2} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) - \frac{15/8}{d} \frac{1}{b} \frac{1}{(a^2+2*a*b+b^2)} \frac{1}{(a+b)^{1/2}} \arctan\left(\frac{1/2*(2*(a+b)^{1/2}*\tanh(1/2*dx+1/2*c)+2*b^{1/2})}{a^{1/2}}\right) + \frac{2}{d} \frac{1}{b^3} \arctan(\tanh(1/2*dx+1/2*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(4a^3e^{7c} + 7a^2be^{7c})e^{7dx} + (4a^3e^{5c} + 31a^2*b*e^{5c} + 36*a*b^2*e^{5c})e^{5dx} - (4a^3e^{3c} + 31a^2*b*e^{3c} + 36*a*b^2*e^{3c})e^{3dx} - (4a^3e^c + 7a^2*b*e^c)e^{dx}}{4(a^4b^2d + 2a^3b^3d + a^2b^4d + (a^4b^2de^{8c} + 2a^3b^3de^{8c} + a^2b^4de^{8c})e^{8dx} + 4(a^4b^2de^{6c} + 4a^3b^3de^{6c} + 5a^2b^4de^{6c})e^{6dx} + 4(a^4b^2de^{4c} + 4a^3b^3de^{4c} + 5a^2b^4de^{4c})e^{4dx} + 4(a^4b^2de^{2c} + 4a^3b^3de^{2c} + 5a^2b^4de^{2c})e^{2dx} + 4(a^4b^2d + 2a^3b^3d + a^2b^4d))e^{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^7/(a+b*sech(dx+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/4*((4*a^3*e^{7*c} + 7*a^2*b*e^{7*c})*e^{7*d*x} + (4*a^3*e^{5*c} + 31*a^2*b*e^{5*c} + 36*a*b^2*e^{5*c})*e^{5*d*x} - (4*a^3*e^{3*c} + 31*a^2*b*e^{3*c} + 36*a*b^2*e^{3*c})*e^{3*d*x} - (4*a^3*e^c + 7*a^2*b*e^c)*e^{d*x}) / (a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^{8*c} + 2*a^3*b^3*d*e^{8*c} + a^2*b^4*d*e^{8*c})*e^{8*d*x} + 4*(a^4*b^2*d*e^{6*c} + 4*a^3*b^3*d*e^{6*c} + 5*a^2*b^4*d*e^{6*c} + 2*a*b^5*d*e^{6*c})*e^{6*d*x} + 2*(3*a^4*b^2*d*e^{4*c} + 14*a^3*b^3*d*e^{4*c} + 27*a^2*b^4*d*e^{4*c} + 24*a*b^5*d*e^{4*c} + 8*b^6*d*e^{4*c})*e^{4*d*x} + 4*(a^4*b^2*d*e^{2*c} + 4*a^3*b^3*d*e^{2*c} + 5*a^2*b^4*d*e^{2*c} + 2*a*b^5*d*e^{2*c})*e^{2*d*x} + 2*\arctan(e^{d*x + c}) / (b^3*d) - 128*\integrate(1/512*((8*a^3*e^{3*c} + 20*a^2*b*e^{3*c} + 15*a*b^2*e^{3*c})*e^{3*d*x} + (8*a^3*e^c + 20*a^2*b*e^c + 15*a*b^2*e^c)*e^{d*x}) / (a^3*b^3 + 2*a^2*b^4 + a*b^5 + (a^3*b^3*e^{4*c} + 2*a^2*b^4*e^{4*c} + a*b^5*e^{4*c})*e^{4*d*x} + 2*(a^3*b^3*e^{2*c} + 4*a^2*b^4*e^{2*c} + 5*a*b^5*e^{2*c} + 2*b^6*e^{2*c})*e^{2*d*x}), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^7 \left(a + \frac{b}{\cosh(c + dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + dx)^7*(a + b/cosh(c + dx)^2)^3), x)

[Out] int(1/(cosh(c + dx)^7*(a + b/cosh(c + dx)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.102 $\int (a + b \operatorname{sech}^2(c + dx)) \tanh^4(c + dx) dx$

Optimal. Leaf size=48

$$-\frac{a \tanh^3(c + dx)}{3d} - \frac{a \tanh(c + dx)}{d} + ax + \frac{b \tanh^5(c + dx)}{5d}$$

[Out] a*x-a*tanh(d*x+c)/d-1/3*a*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4141, 1802, 206}

$$-\frac{a \tanh^3(c + dx)}{3d} - \frac{a \tanh(c + dx)}{d} + ax + \frac{b \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]

[Out] a*x - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \tanh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^{4(a+b(1-x^2))}}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-a - ax^2 + bx^4 + \frac{a}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a \tanh(c + dx)}{d} - \frac{a \tanh^3(c + dx)}{3d} + \frac{b \tanh^5(c + dx)}{5d} + \frac{a \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= ax - \frac{a \tanh(c + dx)}{d} - \frac{a \tanh^3(c + dx)}{3d} + \frac{b \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.19

$$\frac{a \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a \tanh^3(c + dx)}{3d} - \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4, x]

[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)

fricas [B] time = 0.42, size = 327, normalized size = 6.81

$$(15 \, a \, d \, x + 20 \, a - 3 \, b) \cosh(dx + c)^5 + 5(15 \, a \, d \, x + 20 \, a - 3 \, b) \cosh(dx + c) \sinh(dx + c)^4 - (20 \, a - 3 \, b) \sinh(dx + c)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4, x, algorithm="fricas")

[Out] 1/15*((15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a - 3*b)*sinh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 - 5*(2*(20*a - 3*b)*cosh(d*x + c)^2 + 8*a + 3*b)*sinh(d*x + c)^3 + 5*(2*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 + 3*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c) - 5*((20*a - 3*b)*cosh(d*x + c)^4 + 3*(8*a + 3*b)*cosh(d*x + c)^2 + 4*a - 6*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.20, size = 105, normalized size = 2.19

$$\frac{15 \, a \, d \, x + \frac{2(30 \, a e^{(8 \, d \, x + 8 \, c)} - 15 \, b e^{(8 \, d \, x + 8 \, c)} + 90 \, a e^{(6 \, d \, x + 6 \, c)} + 110 \, a e^{(4 \, d \, x + 4 \, c)} - 30 \, b e^{(4 \, d \, x + 4 \, c)} + 70 \, a e^{(2 \, d \, x + 2 \, c)} + 20 \, a - 3 \, b)}{(e^{(2 \, d \, x + 2 \, c)} + 1)^5}}{15 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4, x, algorithm="giac")

[Out] 1/15*(15*a*d*x + 2*(30*a*e^(8*d*x + 8*c) - 15*b*e^(8*d*x + 8*c) + 90*a*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 20*a - 3*b)/(e^(2*d*x + 2*c) + 1)^5)/d

maple [B] time = 0.33, size = 98, normalized size = 2.04

$$\frac{a \left(dx + c - \tanh(dx + c) - \frac{(\tanh^3(dx + c))}{3} \right) + b \left(-\frac{\sinh^3(dx + c)}{2 \cosh(dx + c)^5} - \frac{3 \sinh(dx + c)}{8 \cosh(dx + c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx + c)^4}{5} + \frac{4 \operatorname{sech}(dx + c)^2}{15} \right) \tanh(dx + c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4, x)

[Out] 1/d*(a*(d*x+c-tanh(d*x+c))-1/3*tanh(d*x+c)^3)+b*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.61, size = 92, normalized size = 1.92

$$\frac{b \tanh(dx + c)^5}{5d} + \frac{1}{3} a \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="maxima")

[Out] 1/5*b*tanh(d*x + c)^5/d + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))

mupad [B] time = 1.54, size = 433, normalized size = 9.02

$$ax + \frac{2(2a-3b)}{15d} + \frac{4e^{2c+2dx}(a+b)}{5d} + \frac{2e^{4c+4dx}(2a-b)}{5d} + \frac{2(2a-b)}{5d} + \frac{8e^{2c+2dx}(a+b)}{5d} + \frac{8e^{6c+6dx}(a+b)}{5d} + \frac{4e^{4c+4dx}(2a-3b)}{5d} + \frac{2e^{8c+8dx}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)

[Out] a*x + ((2*(2*a - 3*b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d) + (2*exp(4*c + 4*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(2*a - b))/(5*d) + (8*exp(2*c + 2*d*x)*(a + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (4*exp(4*c + 4*d*x)*(2*a - 3*b))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(a + b))/(5*d) + (6*exp(4*c + 4*d*x)*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - 3*b))/(5*d) + (2*exp(6*c + 6*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + (2*(2*a - b))/(5*d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \tanh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**4,x)

[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**4, x)

3.103 $\int (a + b \operatorname{sech}^2(c + dx)) \tanh^3(c + dx) dx$

Optimal. Leaf size=49

$$\frac{(a-b)\operatorname{sech}^2(c+dx)}{2d} + \frac{a \log(\cosh(c+dx))}{d} + \frac{b\operatorname{sech}^4(c+dx)}{4d}$$

[Out] $a \ln(\cosh(dx+c))/d + 1/2*(a-b)*\operatorname{sech}(dx+c)^2/d + 1/4*b*\operatorname{sech}(dx+c)^4/d$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 76}

$$\frac{(a-b)\operatorname{sech}^2(c+dx)}{2d} + \frac{a \log(\cosh(c+dx))}{d} + \frac{b\operatorname{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sech}[c + d*x]^2)*\text{Tanh}[c + d*x]^3, x]$

[Out] $(a*\text{Log}[\text{Cosh}[c + d*x]])/d + ((a - b)*\text{Sech}[c + d*x]^2)/(2*d) + (b*\text{Sech}[c + d*x]^4)/(4*d)$

Rule 76

```
Int[((d_.)*(x_.))^(n_.)*((a_) + (b_.)*(x_.))*((e_) + (f_.)*(x_.))^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x]
&& IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol]
:> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \tanh^3(c + dx) dx &= - \frac{\operatorname{Subst} \left(\int \frac{(1-x^2)(b+ax^2)}{x^5} dx, x, \cosh(c + dx) \right)}{d} \\ &= - \frac{\operatorname{Subst} \left(\int \frac{(1-x)(b+ax)}{x^3} dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= - \frac{\operatorname{Subst} \left(\int \left(\frac{b}{x^3} + \frac{a-b}{x^2} - \frac{a}{x} \right) dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= \frac{a \log(\cosh(c + dx))}{d} + \frac{(a-b) \operatorname{sech}^2(c + dx)}{2d} + \frac{b \operatorname{sech}^4(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.92

$$-\frac{a \tanh^2(c + dx)}{2d} + \frac{a \log(\cosh(c + dx))}{d} + \frac{b \tanh^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]

[Out] (a*Log[Cosh[c + d*x]])/d - (a*Tanh[c + d*x]^2)/(2*d) + (b*Tanh[c + d*x]^4)/(4*d)

fricas [B] time = 0.43, size = 1072, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="fricas")

[Out] $-(a*d*x*\cosh(d*x + c)^8 + 8*a*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + a*d*x*\sinh(d*x + c)^8 + 2*(2*a*d*x - a + b)*\cosh(d*x + c)^6 + 2*(14*a*d*x*\cosh(d*x + c)^2 + 2*a*d*x - a + b)*\sinh(d*x + c)^6 + 4*(14*a*d*x*\cosh(d*x + c)^3 + 3*(2*a*d*x - a + b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*\cosh(d*x + c)^4 + 2*(35*a*d*x*\cosh(d*x + c)^4 + 3*a*d*x + 15*(2*a*d*x - a + b)*\cosh(d*x + c)^2 - 2*a)*\sinh(d*x + c)^4 + 8*(7*a*d*x*\cosh(d*x + c)^5 + 5*(2*a*d*x - a + b)*\cosh(d*x + c)^3 + (3*a*d*x - 2*a)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a*d*x + 2*(2*a*d*x - a + b)*\cosh(d*x + c)^2 + 2*(14*a*d*x*\cosh(d*x + c)^6 + 15*(2*a*d*x - a + b)*\cosh(d*x + c)^4 + 2*a*d*x + 6*(3*a*d*x - 2*a)*\cosh(d*x + c)^2 - a + b)*\sinh(d*x + c)^2 - (a*\cosh(d*x + c)^8 + 8*a*\cosh(d*x + c)*\sinh(d*x + c)^7 + a*\sinh(d*x + c)^8 + 4*a*\cosh(d*x + c)^6 + 4*(7*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c)^6 + 8*(7*a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*a*\cosh(d*x + c)^4 + 2*(35*a*\cosh(d*x + c)^4 + 30*a*\cosh(d*x + c)^2 + 3*a)*\sinh(d*x + c)^4 + 8*(7*a*\cosh(d*x + c)^5 + 10*a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a*\cosh(d*x + c)^2 + 4*(7*a*\cosh(d*x + c)^6 + 15*a*\cosh(d*x + c)^4 + 9*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c)^2 + 8*(a*\cosh(d*x + c)^7 + 3*a*\cosh(d*x + c)^5 + 3*a*\cosh(d*x + c)^3 + a*\cosh(d*x + c))*\sinh(d*x + c) + a*\log(2*\cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*\cosh(d*x + c)^7 + 3*(2*a*d*x - a + b)*\cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*\cosh(d*x + c)^3 + (2*a*d*x - a + b)*\cosh(d*x + c))*\sinh(d*x + c)/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 + 4*d*\cosh(d*x + c)^6 + 4*(7*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 8*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x +$

$$c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.18, size = 116, normalized size = 2.37

$$\frac{12\,adx - 12\,a \log\left(e^{(2dx+2c)} + 1\right) + \frac{25ae^{(8dx+8c)} + 76ae^{(6dx+6c)} + 24be^{(6dx+6c)} + 102ae^{(4dx+4c)} + 76ae^{(2dx+2c)} + 24be^{(2dx+2c)} + 25a}{(e^{(2dx+2c)} + 1)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="giac")

[Out] -1/12*(12*a*d*x - 12*a*log(e^(2*d*x + 2*c) + 1) + (25*a*e^(8*d*x + 8*c) + 76*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) + 76*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) + 1)^4)/d

maple [A] time = 0.20, size = 64, normalized size = 1.31

$$\frac{a \ln(\cosh(dx + c))}{d} - \frac{(\tanh^2(dx + c))a}{2d} - \frac{b(\sinh^2(dx + c))}{2d \cosh(dx + c)^4} - \frac{b}{4d \cosh(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x)

[Out] a*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a-1/2/d*b*sinh(d*x+c)^2/cosh(d*x+c)^4-1/4/d*b/cosh(d*x+c)^4

maxima [A] time = 0.75, size = 78, normalized size = 1.59

$$\frac{b \tanh(dx + c)^4}{4d} + a \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="maxima")

[Out] 1/4*b*tanh(d*x + c)^4/d + a*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))

mupad [B] time = 0.12, size = 173, normalized size = 3.53

$$\frac{2(a-b)}{d(e^{2c+2dx} + 1)} - a x - \frac{2(a-3b)}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{8b}{d(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} + \frac{b}{d(4e^{2c+2dx} + 6e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)

[Out] (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1)) - a*x - (2*(a - 3*b))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*b)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*b)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (a*log(exp(2*c + 2*d*x) + 1))/d

sympy [A] time = 1.66, size = 80, normalized size = 1.63

$$\begin{cases} ax - \frac{a \log(\tanh(c+dx)+1)}{d} - \frac{a \tanh^2(c+dx)}{2d} - \frac{b \tanh^2(c+dx) \operatorname{sech}^2(c+dx)}{4d} - \frac{b \operatorname{sech}^2(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a + b \operatorname{sech}^2(c)) \tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) - b*  
tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - b*sech(c + d*x)**2/(4*d), Ne(d, 0  
)), (x*(a + b*sech(c)**2)*tanh(c)**3, True))
```

3.104 $\int (a + b \operatorname{sech}^2(c + dx)) \tanh^2(c + dx) dx$

Optimal. Leaf size=32

$$-\frac{a \tanh(c + dx)}{d} + ax + \frac{b \tanh^3(c + dx)}{3d}$$

[Out] a*x-a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4141, 1802, 206}

$$-\frac{a \tanh(c + dx)}{d} + ax + \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]

[Out] a*x - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \tanh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b(1-x^2))}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-a + bx^2 + \frac{a}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d} + \frac{a \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= ax - \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$\frac{a \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]

[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)

fricas [B] time = 0.40, size = 155, normalized size = 4.84

$$\frac{(3 \operatorname{adx} + 3a - b) \cosh(dx + c)^3 + 3(3 \operatorname{adx} + 3a - b) \cosh(dx + c) \sinh(dx + c)^2 - (3a - b) \sinh(dx + c)^3 + 3}{3(d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/3*((3*a*d*x + 3*a - b)*cosh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a - b)*sinh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x + c) - 3*((3*a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.16, size = 69, normalized size = 2.16

$$\frac{3 \operatorname{adx} + \frac{2(3ae^{4dx+4c} - 3be^{4dx+4c} + 6ae^{2dx+2c} + 3a-b)}{(e^{2dx+2c} + 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="giac")

[Out] 1/3*(3*a*d*x + 2*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*a - b)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.33, size = 60, normalized size = 1.88

$$\frac{a(dx + c - \tanh(dx + c)) + b \left(-\frac{\sinh(dx + c)}{2 \cosh(dx + c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx + c)^2}{3} \right) \tanh(dx + c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x)

[Out] 1/d*(a*(d*x+c-tanh(d*x+c))+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [A] time = 0.40, size = 42, normalized size = 1.31

$$\frac{b \tanh(dx + c)^3}{3d} + a \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="maxima")

[Out] 1/3*b*tanh(d*x + c)^3/d + a*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))

mupad [B] time = 1.50, size = 163, normalized size = 5.09

$$\frac{\frac{2(a+b)}{3d} + \frac{2e^{2c+2dx}(a-b)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} + ax + \frac{\frac{2(a-b)}{3d} + \frac{4e^{2c+2dx}(a+b)}{3d} + \frac{2e^{4c+4dx}(a-b)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} + \frac{2(a-b)}{3d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)`

[Out] $((2*(a + b))/(3*d) + (2*\exp(2*c + 2*d*x)*(a - b))/(3*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) + a*x + ((2*(a - b))/(3*d) + (4*\exp(2*c + 2*d*x)*(a + b))/(3*d) + (2*\exp(4*c + 4*d*x)*(a - b))/(3*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) + (2*(a - b))/(3*d*(\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \tanh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**2, x)`

3.105 $\int (a + b \operatorname{sech}^2(c + dx)) \tanh(c + dx) dx$

Optimal. Leaf size=29

$$\frac{a \log(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}^2(c + dx)}{2d}$$

[Out] a*ln(cosh(d*x+c))/d-1/2*b*sech(d*x+c)^2/d

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4138, 14}

$$\frac{a \log(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]

[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)

Rule 14

Int[(u_)*((c_.)*(x_.))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) \tanh(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^3} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{b}{x^3} + \frac{a}{x}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{a \log(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{a \log(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]

[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)

fricas [B] time = 0.42, size = 359, normalized size = 12.38

$$adx \cosh(dx + c)^4 + 4 adx \cosh(dx + c) \sinh(dx + c)^3 + adx \sinh(dx + c)^4 + adx + 2(adx + b) \cosh(dx + c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="fricas")

[Out] $-(a*d*x*\cosh(d*x + c)^4 + 4*a*d*x*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*d*x*\sinh(d*x + c)^4 + a*d*x + 2*(a*d*x + b)*\cosh(d*x + c)^2 + 2*(3*a*d*x*\cosh(d*x + c)^2 + a*d*x + b)*\sinh(d*x + c)^2 - (a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*a*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + a*\cosh(d*x + c))*\sinh(d*x + c) + a)*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*(a*d*x*\cosh(d*x + c)^3 + (a*d*x + b)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 + 2*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$

giac [B] time = 0.13, size = 80, normalized size = 2.76

$$\frac{2 adx - 2 a \log(e^{(2 dx+2c)} + 1) + \frac{3 a e^{(4 dx+4c)} + 6 a e^{(2 dx+2c)} + 4 b e^{(2 dx+2c)} + 3 a}{(e^{(2 dx+2c)} + 1)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="giac")

[Out] $-1/2*(2*a*d*x - 2*a*\log(e^{(2*d*x + 2*c)} + 1) + (3*a*e^{(4*d*x + 4*c)} + 6*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + 3*a)/(e^{(2*d*x + 2*c)} + 1)^2)/d$

maple [A] time = 0.14, size = 29, normalized size = 1.00

$$-\frac{b \operatorname{sech}(dx + c)^2}{2d} - \frac{a \ln(\operatorname{sech}(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c),x)

[Out] $-1/2*b*\operatorname{sech}(d*x+c)^2/d - 1/d*a*\ln(\operatorname{sech}(d*x+c))$

maxima [A] time = 0.33, size = 27, normalized size = 0.93

$$\frac{b \tanh(dx + c)^2}{2d} + \frac{a \log(\cosh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="maxima")

[Out] $1/2*b*\tanh(d*x + c)^2/d + a*\log(\cosh(d*x + c))/d$

mupad [B] time = 1.47, size = 72, normalized size = 2.48

$$\frac{2b}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{2b}{d(e^{2c+2dx} + 1)} - ax + \frac{a \ln(e^{2c} e^{2dx} + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - a*x + (a*log(exp(2*c)*exp(2*d*x) + 1))/d
```

sympy [A] time = 0.49, size = 42, normalized size = 1.45

$$\begin{cases} ax - \frac{a \log(\tanh(c+dx)+1)}{d} - \frac{b \operatorname{sech}^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b \operatorname{sech}^2(c)) \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - b*sech(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sech(c)**2)*tanh(c), True))
```

3.106 $\int (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=15

$$ax + \frac{b \tanh(c + dx)}{d}$$

[Out] a*x+b*tanh(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3767, 8}

$$ax + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sech[c + d*x]^2,x]

[Out] a*x + (b*Tanh[c + d*x])/d

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx)) dx &= ax + b \int \operatorname{sech}^2(c + dx) dx \\ &= ax + \frac{(ib) \operatorname{Subst}(\int 1 dx, x, -i \tanh(c + dx))}{d} \\ &= ax + \frac{b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$ax + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sech[c + d*x]^2,x]

[Out] a*x + (b*Tanh[c + d*x])/d

fricas [B] time = 0.40, size = 36, normalized size = 2.40

$$\frac{(adx - b) \cosh(dx + c) + b \sinh(dx + c)}{d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="fricas")

[Out] $((a*d*x - b)*\cosh(d*x + c) + b*\sinh(d*x + c))/(d*\cosh(d*x + c))$

giac [A] time = 0.12, size = 23, normalized size = 1.53

$$ax - \frac{2b}{d(e^{2dx+2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="giac")

[Out] $a*x - 2*b/(d*(e^{(2*d*x + 2*c)} + 1))$

maple [A] time = 0.27, size = 16, normalized size = 1.07

$$ax + \frac{b \tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*sech(d*x+c)^2,x)

[Out] $a*x+b*\tanh(d*x+c)/d$

maxima [A] time = 0.37, size = 23, normalized size = 1.53

$$ax + \frac{2b}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="maxima")

[Out] $a*x + 2*b/(d*(e^{(-2*d*x - 2*c)} + 1))$

mupad [B] time = 1.39, size = 23, normalized size = 1.53

$$ax - \frac{2b}{d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b/cosh(c + d*x)^2,x)

[Out] $a*x - (2*b)/(d*(\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sech(d*x+c)**2,x)

[Out] Integral(a + b*sech(c + d*x)**2, x)

3.107 $\int \coth(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=28

$$\frac{(a + b) \log(\sinh(c + dx))}{d} - \frac{b \log(\cosh(c + dx))}{d}$$

[Out] $-b \cdot \ln(\cosh(d \cdot x + c)) / d + (a + b) \cdot \ln(\sinh(d \cdot x + c)) / d$

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4138, 446, 72}

$$\frac{(a + b) \log(\sinh(c + dx))}{d} - \frac{b \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[c + d \cdot x] \cdot (a + b \cdot \text{Sech}[c + d \cdot x]^2), x]$

[Out] $-((b \cdot \text{Log}[\text{Cosh}[c + d \cdot x]]) / d) + ((a + b) \cdot \text{Log}[\text{Sinh}[c + d \cdot x]]) / d$

Rule 72

$\text{Int}[(e_{.}) + (f_{.}) \cdot (x_{.})^{(p_{.})} / ((a_{.}) + (b_{.}) \cdot (x_{.})) \cdot ((c_{.}) + (d_{.}) \cdot (x_{.}))], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f \cdot x)^p / ((a + b \cdot x) \cdot (c + d \cdot x)), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[p]$

Rule 446

$\text{Int}[(x_{.})^{(m_{.})} \cdot ((a_{.}) + (b_{.}) \cdot (x_{.})^{(n_{.})})^{(p_{.})} \cdot ((c_{.}) + (d_{.}) \cdot (x_{.})^{(n_{.})})^{(q_{.})}], x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p, q, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 4138

$\text{Int}[(a_{.}) + (b_{.}) \cdot \sec[(e_{.}) + (f_{.}) \cdot (x_{.})]^{(n_{.})}]^{(p_{.})} \cdot \tan[(e_{.}) + (f_{.}) \cdot (x_{.})]^{(m_{.})}, x_Symbol] \rightarrow \text{Module}\{ff = \text{FreeFactors}[\text{Cos}[e + f \cdot x], x]\}, -\text{Dist}[(f \cdot ff^{(m + n \cdot p - 1)})^{(-1)}, \text{Subst}[\text{Int}[(1 - ff^2 \cdot x^2)^{(m - 1)/2} \cdot (b + a \cdot (ff \cdot x)^n)^p / x^{(m + n \cdot p)}], x], x, \text{Cos}[e + f \cdot x] / ff], x] /;$ $\text{FreeQ}\{a, b, e, f, n, x\} \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \coth(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x} + \frac{b}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{b \log(\cosh(c + dx))}{d} + \frac{(a + b) \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.57

$$\frac{a(\log(\tanh(c + dx)) + \log(\cosh(c + dx)))}{d} - \frac{b(\log(\cosh(c + dx)) - \log(\sinh(c + dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2), x]

[Out] -(b*(Log[Cosh[c + d*x]] - Log[Sinh[c + d*x]]))/d + (a*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d

fricas [B] time = 0.42, size = 69, normalized size = 2.46

$$-\frac{adx + b \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - (a+b) \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -(a*d*x + b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d

giac [A] time = 0.15, size = 56, normalized size = 2.00

$$-\frac{adx - (ae^{2c} + be^{2c})e^{-2c} \log(|e^{2dx+2c} - 1|) + b \log(e^{2dx+2c} + 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -(a*d*x - (a*e^(2*c) + b*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + b*log(e^(2*d*x + 2*c) + 1))/d

maple [A] time = 0.26, size = 26, normalized size = 0.93

$$\frac{b \ln(\tanh(dx + c))}{d} + \frac{a \ln(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2), x)

[Out] b*ln(tanh(d*x+c))/d+a*ln(sinh(d*x+c))/d

maxima [B] time = 0.53, size = 65, normalized size = 2.32

$$b\left(\frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} - \frac{\log(e^{-2dx-2c} + 1)}{d}\right) + \frac{a \log(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d) + a*log(sinh(d*x + c))/d

mupad [B] time = 0.17, size = 167, normalized size = 5.96

$$\frac{a \ln(4a^2 e^{4c} e^{4dx} - 4a^2 - 16b^2 - 16ab + 16b^2 e^{4c} e^{4dx} + 16abe^{4c} e^{4dx})}{2d} - ax - \frac{\operatorname{atan}\left(\frac{ae^{2c} e^{2dx} \sqrt{-d^2}}{d\sqrt{a^2+4ab+4b^2}} + \frac{2be^{2c}}{d\sqrt{a^2+4ab+4b^2}}\right)}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)*(a + b/cosh(c + d*x)^2),x)`

[Out] $(a \log(4a^2 \exp(4c) \exp(4dx) - 4a^2 - 16b^2 - 16ab + 16b^2 \exp(4c) \exp(4dx) + 16ab \exp(4c) \exp(4dx)) / (2d) - ax - (\operatorname{atan}((a \exp(2c) \exp(2dx) * (-d^2)^{1/2}) / (d(4ab + a^2 + 4b^2)^{1/2}) + (2b \exp(2c) \exp(2dx) * (-d^2)^{1/2}) / (d(4ab + a^2 + 4b^2)^{1/2})) * (4ab + a^2 + 4b^2)^{1/2}) / (-d^2)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{coth}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2),x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*coth(c + d*x), x)`

3.108 $\int \coth^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=18

$$ax - \frac{(a + b) \coth(c + dx)}{d}$$

[Out] a*x-(a+b)*coth(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4141, 1802, 207}

$$ax - \frac{(a + b) \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] a*x - ((a + b)*Coth[c + d*x])/d

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(n_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b(1-x^2)}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{x^2} - \frac{a}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a + b) \coth(c + dx)}{d} - \frac{a \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= ax - \frac{(a + b) \coth(c + dx)}{d} \end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 2.28

$$-\frac{a \coth(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d} - \frac{b \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2), x]

[Out] -((b*Coth[c + d*x])/d) - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d

fricas [B] time = 0.41, size = 39, normalized size = 2.17

$$\frac{(a + b) \cosh(dx + c) - (adx + a + b) \sinh(dx + c)}{d \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -((a + b)*cosh(d*x + c) - (a*d*x + a + b)*sinh(d*x + c))/(d*sinh(d*x + c))

giac [A] time = 0.15, size = 27, normalized size = 1.50

$$\frac{adx - \frac{2(a+b)}{e^{2dx+2c}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] (a*d*x - 2*(a + b)/(e^(2*d*x + 2*c) - 1))/d

maple [A] time = 0.33, size = 30, normalized size = 1.67

$$\frac{a(dx + c - \coth(dx + c)) - b \coth(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2), x)

[Out] 1/d*(a*(d*x+c-coth(d*x+c))-b*coth(d*x+c))

maxima [B] time = 0.34, size = 47, normalized size = 2.61

$$a \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)} \right) + \frac{2b}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))

mupad [B] time = 0.11, size = 25, normalized size = 1.39

$$ax - \frac{2(a + b)}{d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2), x)

[Out] a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \coth^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**2, x)
```

3.109 $\int \coth^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=31

$$\frac{a \log(\sinh(c + dx))}{d} - \frac{(a + b) \operatorname{csch}^2(c + dx)}{2d}$$

[Out] $-1/2*(a+b)*\operatorname{csch}(d*x+c)^2/d+a*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 444, 43}

$$\frac{a \log(\sinh(c + dx))}{d} - \frac{(a + b) \operatorname{csch}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[c + d*x]^3*(a + b*\text{Sech}[c + d*x]^2), x]$

[Out] $-((a + b)*\text{Csch}[c + d*x]^2)/(2*d) + (a*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 444

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)*((c_.) + (d_.)*(x_.)^{(n_.))^{(q_.)}, x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m - n + 1, 0]$

Rule 4138

$\text{Int}[(a_. + (b_.)*\sec[(e_.) + (f_.)*(x_.)]^{(n_.))^{(p_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] :> \text{Module}\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[(ff^{m + n*p - 1})^{-1}, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m - 1)/2}*(b + a*(ff*x)^n)^p/x^{m + n*p}, x], x, \text{Cos}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \coth^3(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^{b+ax^2}}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2} + \frac{a}{-1+x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{(a + b) \operatorname{csch}^2(c + dx)}{2d} + \frac{a \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.18, size = 52, normalized size = 1.68

$$\frac{a \left(\coth^2(c + dx) - 2 \log(\tanh(c + dx)) - 2 \log(\cosh(c + dx)) \right)}{2d} - \frac{b \operatorname{csch}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2), x]

[Out] -1/2*(b*Csch[c + d*x]^2)/d - (a*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2*Log[Tanh[c + d*x]]))/(2*d)

fricas [B] time = 0.42, size = 378, normalized size = 12.19

$$\frac{adx \cosh(dx + c)^4 + 4 adx \cosh(dx + c) \sinh(dx + c)^3 + adx \sinh(dx + c)^4 + adx - 2(adx - a - b) \cosh(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sinh(d*x + c)^4 + a*d*x - 2*(a*d*x - a - b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(d*x + c)^2 - a*d*x + a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(a*d*x*cosh(d*x + c)^3 - (a*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.21, size = 81, normalized size = 2.61

$$\frac{2 adx - 2 a \log \left(\left| e^{(2 dx + 2 c)} - 1 \right| \right) + \frac{3 a e^{(4 dx + 4 c)} - 2 a e^{(2 dx + 2 c)} + 4 b e^{(2 dx + 2 c)} + 3 a}{\left(e^{(2 dx + 2 c)} - 1 \right)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(2*a*d*x - 2*a*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a*e^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) - 1)^2)/d

maple [A] time = 0.33, size = 42, normalized size = 1.35

$$\frac{a \ln(\sinh(dx + c))}{d} - \frac{a \left(\coth^2(dx + c) \right)}{2d} - \frac{b}{2d \sinh(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2), x)

[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/2/d/sinh(d*x+c)^2*b

maxima [B] time = 0.33, size = 108, normalized size = 3.48

$$a \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2 e^{(-2dx-2c)}}{d(2 e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - \frac{2 b}{d(e^{(dx+c)} - e^{(-dx-c)})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b/(d*(e^(d*x + c) - e^(-d*x - c))^2)

mupad [B] time = 1.41, size = 76, normalized size = 2.45

$$\frac{a \ln(e^{2c} e^{2dx} - 1)}{d} - ax - \frac{2(a+b)}{d(e^{2c+2dx} - 1)} - \frac{2(a+b)}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)

[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x) - 1)) - (2*(a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{coth}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2),x)

[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**3, x)

3.110 $\int \coth^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=34

$$-\frac{(a+b)\coth^3(c+dx)}{3d} - \frac{a\coth(c+dx)}{d} + ax$$

[Out] a*x-a*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4141, 1802, 207}

$$-\frac{(a+b)\coth^3(c+dx)}{3d} - \frac{a\coth(c+dx)}{d} + ax$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] a*x - (a*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d)

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(n_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b(1-x^2)}{x^4(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{x^4} + \frac{a}{x^2} - \frac{a}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a\coth(c + dx)}{d} - \frac{(a+b)\coth^3(c + dx)}{3d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x\right)}{d} \\ &= ax - \frac{a\coth(c + dx)}{d} - \frac{(a+b)\coth^3(c + dx)}{3d} \end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 1.44

$$\frac{a \coth^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(c + dx)\right)}{3d} - \frac{b \coth^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2), x]

[Out] -1/3*(b*Coth[c + d*x]^3)/d - (a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/(3*d)

fricas [B] time = 0.41, size = 140, normalized size = 4.12

$$\frac{(4a + b) \cosh(dx + c)^3 + 3(4a + b) \cosh(dx + c) \sinh(dx + c)^2 - (3adx + 4a + b) \sinh(dx + c)^3 + 3b \cosh(dx + c)^3}{3(d \sinh(dx + c)^3 + 3(d \cosh(dx + c)^2 - d) \sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/3*((4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a*d*x + 4*a + b)*sinh(d*x + c)^3 + 3*b*cosh(d*x + c) + 3*(3*a*d*x - (3*a*d*x + 4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))

giac [B] time = 0.21, size = 67, normalized size = 1.97

$$\frac{3adx - \frac{2(6ae^{(4dx+4c)} + 3be^{(4dx+4c)} - 6ae^{(2dx+2c)} + 4a+b)}{(e^{(2dx+2c)} - 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] 1/3*(3*a*d*x - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 4*a + b)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [B] time = 0.39, size = 70, normalized size = 2.06

$$\frac{a \left(dx + c - \coth(dx + c) - \frac{\coth^3(dx + c)}{3} \right) + b \left(-\frac{\cosh(dx + c)}{2 \sinh(dx + c)^3} - \frac{\left(\frac{2}{3} - \frac{\operatorname{csch}(dx + c)^2}{3} \right) \coth(dx + c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2), x)

[Out] 1/d*(a*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b*(-1/2/sinh(d*x+c)^3*cosh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)))

maxima [B] time = 0.34, size = 170, normalized size = 5.00

$$\frac{1}{3} a \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + \frac{2}{3} b \left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $1/3*a*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} - 2)/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + 2/3*b*(3*e^{(-4*d*x - 4*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))$

mupad [B] time = 1.49, size = 161, normalized size = 4.74

$$ax - \frac{\frac{2b}{3d} + \frac{2e^{2c+2dx}(2a+b)}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{\frac{2(2a+b)}{3d} + \frac{4be^{2c+2dx}}{3d} + \frac{2e^{4c+4dx}(2a+b)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} - \frac{2(2a+b)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2), x)`

[Out] $ax - ((2*b)/(3*d) + (2*\exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) + (4*b*\exp(2*c + 2*d*x))/(3*d) + (2*\exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1) - (2*(2*a + b))/(3*d*(\exp(2*c + 2*d*x) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{coth}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2), x)`

[Out] `Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**4, x)`

3.111 $\int \coth^5(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d} - \frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d} + \frac{a \log(\sinh(c+dx))}{d}$$

[Out] $-1/2*(2*a+b)*\operatorname{csch}(d*x+c)^2/d-1/4*(a+b)*\operatorname{csch}(d*x+c)^4/d+a*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 77}

$$-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d} - \frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d} + \frac{a \log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]`

[Out] $-\frac{(2a+b)*\operatorname{Csch}[c+d*x]^2}{2d} - \frac{(a+b)*\operatorname{Csch}[c+d*x]^4}{4d} + (a*\operatorname{Log}[\operatorname{Sinh}[c+d*x]])/d$

Rule 77

```
Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \coth^5(c + dx) (a + b \operatorname{sech}^2(c + dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3} + \frac{-2a-b}{(-1+x)^2} - \frac{a}{-1+x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{(2a+b)\operatorname{csch}^2(c + dx)}{2d} - \frac{(a+b)\operatorname{csch}^4(c + dx)}{4d} + \frac{a \log(\sinh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 62, normalized size = 1.22

$$-\frac{a(\coth^4(c + dx) + 2\coth^2(c + dx) - 4\log(\tanh(c + dx)) - 4\log(\cosh(c + dx)))}{4d} - \frac{b\coth^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2), x]

[Out] -1/4*(b*Coth[c + d*x]^4)/d - (a*(2*Coth[c + d*x]^2 + Coth[c + d*x]^4 - 4*Log[Cosh[c + d*x]] - 4*Log[Tanh[c + d*x]]))/(4*d)

fricas [B] time = 0.43, size = 1099, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sinh(d*x + c)^8 - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x + c)^2 - 2*a*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3 - 3*(2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 - 5*(2*a*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sinh(d*x + c)^3 + a*d*x - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*a*d*x*cosh(d*x + c)^6 - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*a*d*x + 6*(3*a*d*x - 2*a)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 4*a*cosh(d*x + c)^6 + 4*(7*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 - 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*cosh(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 - 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - 3*a*cosh(d*x + c)^5 + 3*a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 - 3*(2*a*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 - (2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*

$$\cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^3 - 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c)^6 - 15d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 - d) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 - 3d \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 - d \cosh(dx + c)) \sinh(dx + c) + d$$

giac [B] time = 0.23, size = 117, normalized size = 2.29

$$\frac{12 dx - 12 a \log(|e^{(2dx+2c)} - 1|) + \frac{25 a e^{(8dx+8c)} - 52 a e^{(6dx+6c)} + 24 b e^{(6dx+6c)} + 102 a e^{(4dx+4c)} - 52 a e^{(2dx+2c)} + 24 b e^{(2dx+2c)} + 25 a}{(e^{(2dx+2c)} - 1)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^5*(a+b*sech(dx+c)^2),x, algorithm="giac")

[Out] -1/12*(12*a*d*x - 12*a*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a*e^(8*d*x + 8*c) - 52*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) - 52*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) - 1)^4)/d

maple [A] time = 0.31, size = 78, normalized size = 1.53

$$\frac{a \ln(\sinh(dx + c))}{d} - \frac{a (\coth^2(dx + c))}{2d} - \frac{a (\coth^4(dx + c))}{4d} - \frac{b (\cosh^2(dx + c))}{2d \sinh(dx + c)^4} + \frac{b}{4d \sinh(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^5*(a+b*sech(dx+c)^2),x)

[Out] a*ln(sinh(dx+c))/d-1/2*a*coth(dx+c)^2/d-1/4*a*coth(dx+c)^4/d-1/2/d*b/sinh(dx+c)^4*cosh(dx+c)^2+1/4/d*b/sinh(dx+c)^4

maxima [B] time = 0.51, size = 251, normalized size = 4.92

$$a \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{4(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^5*(a+b*sech(dx+c)^2),x, algorithm="maxima")

[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)))

mupad [B] time = 0.11, size = 179, normalized size = 3.51

$$\frac{a \ln(e^{2c} e^{2dx} - 1)}{d} - \frac{8(a + b)}{d(3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1)} - \frac{2(2a + b)}{d(e^{2c+2dx} - 1)} - \frac{4(a + b)}{d(6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2),x)

[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - (8*(a + b))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*(2*a + b))/(d*(exp(2*c + 2*d*x) - 1)) - (4*(a + b))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) - 1))

$p(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - a*x - (2*(4*a + 3*b))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx)) \operatorname{coth}^5(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2), x)

[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**5, x)

3.112 $\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^4(c + dx) dx$

Optimal. Leaf size=77

$$-\frac{a^2 \tanh^3(c + dx)}{3d} - \frac{a^2 \tanh(c + dx)}{d} + a^2 x + \frac{b(2a + b) \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

[Out] $a^2 x - a^2 \tanh(d x + c) / d - 1/3 a^2 \tanh(d x + c)^3 / d + 1/5 b (2 a + b) \tanh(d x + c)^5 / d - 1/7 b^2 \tanh(d x + c)^7 / d$

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 206}

$$-\frac{a^2 \tanh^3(c + dx)}{3d} - \frac{a^2 \tanh(c + dx)}{d} + a^2 x + \frac{b(2a + b) \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]

[Out] $a^2 x - (a^2 \operatorname{Tanh}[c + d x]) / d - (a^2 \operatorname{Tanh}[c + d x]^3) / (3 d) + (b (2 a + b) \operatorname{Tanh}[c + d x]^5) / (5 d) - (b^2 \operatorname{Tanh}[c + d x]^7) / (7 d)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4 (a + b(1 - x^2))^2}{1 - x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-a^2 - a^2 x^2 + b(2a + b)x^4 - b^2 x^6 + \frac{a^2}{1 - x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \tanh(c + dx)}{d} - \frac{a^2 \tanh^3(c + dx)}{3d} + \frac{b(2a + b) \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^7(c + dx)}{7d} \\ &= a^2 x - \frac{a^2 \tanh(c + dx)}{d} - \frac{a^2 \tanh^3(c + dx)}{3d} + \frac{b(2a + b) \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 1.18, size = 395, normalized size = 5.13

$$\frac{\operatorname{sech}(c)\operatorname{sech}^7(c+dx)\left(4480a^2\sinh(2c+dx)-3780a^2\sinh(2c+3dx)+2100a^2\sinh(4c+3dx)-1540a^2\sinh(6c+3dx)\right)}{13440d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]

[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^2*d*x*Cosh[d*x] + 3675*a^2*d*x*Cosh[2*c + d*x] + 2205*a^2*d*x*Cosh[2*c + 3*d*x] + 2205*a^2*d*x*Cosh[4*c + 3*d*x] + 735*a^2*d*x*Cosh[4*c + 5*d*x] + 735*a^2*d*x*Cosh[6*c + 5*d*x] + 105*a^2*d*x*Cosh[6*c + 7*d*x] + 105*a^2*d*x*Cosh[8*c + 7*d*x] - 5320*a^2*Sinh[d*x] + 1680*a*b*Sinh[d*x] + 840*b^2*Sinh[d*x] + 4480*a^2*Sinh[2*c + d*x] - 1260*a*b*Sinh[2*c + d*x] + 420*b^2*Sinh[2*c + d*x] - 3780*a^2*Sinh[2*c + 3*d*x] + 924*a*b*Sinh[2*c + 3*d*x] - 168*b^2*Sinh[2*c + 3*d*x] + 2100*a^2*Sinh[4*c + 3*d*x] - 840*a*b*Sinh[4*c + 3*d*x] - 420*b^2*Sinh[4*c + 3*d*x] - 1540*a^2*Sinh[4*c + 5*d*x] + 168*a*b*Sinh[4*c + 5*d*x] + 84*b^2*Sinh[4*c + 5*d*x] + 420*a^2*Sinh[6*c + 5*d*x] - 420*a*b*Sinh[6*c + 5*d*x] - 280*a^2*Sinh[6*c + 7*d*x] + 84*a*b*Sinh[6*c + 7*d*x] + 12*b^2*Sinh[6*c + 7*d*x]))/(13440*d)

fricas [B] time = 0.42, size = 721, normalized size = 9.36

$$\frac{(105a^2dx + 140a^2 - 42ab - 6b^2)\cosh(dx + c)^7 + 7(105a^2dx + 140a^2 - 42ab - 6b^2)\cosh(dx + c)\sinh(dx + c)^6}{13440d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="fricas")

[Out] 1/105*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 - 21*a*b - 3*b^2)*sinh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5 - 14*(3*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^2 + 40*a^2 + 9*a*b - 3*b^2)*sinh(d*x + c)^5 + 35*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + (105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^2 + 60*a^2 - 3*a*b + 21*b^2)*sinh(d*x + c)^3 + 7*(3*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5 + 10*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + 9*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c) - 14*((70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^6 + 5*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 - a*b + 7*b^2)*cosh(d*x + c)^2 + 30*a^2 - 15*a*b - 45*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))

giac [B] time = 0.25, size = 275, normalized size = 3.57

$$105a^2dx + \frac{4(105a^2e^{12dx+12c}-105abe^{12dx+12c}+525a^2e^{10dx+10c}-210abe^{10dx+10c}-105b^2e^{10dx+10c}+1120a^2e^{8dx+8c}-315abe^{8dx+8c})}{13440d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="giac")

[Out] 1/105*(105*a^2*d*x + 4*(105*a^2*e^(12*d*x + 12*c) - 105*a*b*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x + 10*c) - 210*a*b*e^(10*d*x + 10*c) - 105*b^2*e^(10*d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) - 315*a*b*e^(8*d*x + 8*c)))/(13440*d)

$$\frac{d^2x + 10c + 1120a^2e^{(8dx + 8c)} - 315ab e^{(8dx + 8c)} + 105b^2e^{(8dx + 8c)} + 1330a^2e^{(6dx + 6c)} - 420ab e^{(6dx + 6c)} - 210b^2e^{(6dx + 6c)} + 945a^2e^{(4dx + 4c)} - 231ab e^{(4dx + 4c)} + 42b^2e^{(4dx + 4c)} + 385a^2e^{(2dx + 2c)} - 42ab e^{(2dx + 2c)} - 21b^2e^{(2dx + 2c)} + 70a^2 - 21ab - 3b^2}{(e^{(2dx + 2c)} + 1)^7} \cdot d$$

maple [B] time = 0.44, size = 181, normalized size = 2.35

$$\frac{a^2 \left(dx + c - \tanh(dx + c) - \frac{(\tanh^3(dx+c))}{3} \right) + 2ab \left(-\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)^5} - \frac{3 \sinh(dx+c)}{8 \cosh(dx+c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4 \operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x)

[Out] 1/d*(a^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+2*a*b*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [B] time = 0.55, size = 649, normalized size = 8.43

$$\frac{2ab \tanh(dx + c)^5}{5d} + \frac{1}{3} a^2 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + \frac{4}{35} b^2 \left(\frac{1}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="maxima")

[Out] 2/5*a*b*tanh(d*x + c)^5/d + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 4/35*b^2*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) - 14*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 70*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) - 35*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-10*d*x - 10*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)))

mupad [B] time = 0.18, size = 1022, normalized size = 13.27

$$\frac{\frac{4(7a^2+ab+8b^2)}{105d} - \frac{4e^{8c+8dx}(ab-a^2)}{7d} - \frac{16e^{2c+2dx}(-2a^2+ab+3b^2)}{35d} + \frac{16e^{6c+6dx}(2a^2+ab-b^2)}{21d} + \frac{8e^{4c+4dx}(7a^2+ab+8b^2)}{35d} - \frac{4(-2a^2+ab+b^2)}{35d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)

```
[Out] ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(8*c + 8*d*x)*(a*b - a^2))/(7*d)
- (16*exp(2*c + 2*d*x)*(a*b - 2*a^2 + 3*b^2))/(35*d) + (16*exp(6*c + 6*d*x)
)*(a*b + 2*a^2 - b^2))/(21*d) + (8*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/
(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5
*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(a*b - 2*a^2 + 3*b^2))/(3
5*d) + (4*exp(6*c + 6*d*x)*(a*b - a^2))/(7*d) - (4*exp(4*c + 4*d*x)*(a*b +
2*a^2 - b^2))/(7*d) - (4*exp(2*c + 2*d*x)*(a*b + 7*a^2 + 8*b^2))/(35*d))/(4
*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + a^2*x + ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(4*c + 4*d*x)
*(a*b - a^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a*b + 2*a^2 - b^2))/(21*d))/(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((8*exp(2*c
+ 2*d*x)*(a*b + 2*a^2 - b^2))/(7*d) - (4*exp(12*c + 12*d*x)*(a*b - a^2))/(7
*d) - (4*(a*b - a^2))/(7*d) - (16*exp(6*c + 6*d*x)*(a*b - 2*a^2 + 3*b^2))/(
7*d) + (4*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/(7*d) + (8*exp(10*c + 10*
d*x)*(a*b + 2*a^2 - b^2))/(7*d) + (4*exp(8*c + 8*d*x)*(a*b + 7*a^2 + 8*b^2)
))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) +
35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14
*c + 14*d*x) + 1) + ((4*(a*b + 2*a^2 - b^2))/(21*d) - (4*exp(2*c + 2*d*x)*(
a*b - a^2))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*(a*b +
2*a^2 - b^2))/(21*d) - (4*exp(10*c + 10*d*x)*(a*b - a^2))/(7*d) - (8*exp(4
*c + 4*d*x)*(a*b - 2*a^2 + 3*b^2))/(7*d) + (4*exp(2*c + 2*d*x)*(a*b + 7*a^2
+ 8*b^2))/(21*d) + (20*exp(8*c + 8*d*x)*(a*b + 2*a^2 - b^2))/(21*d) + (8*exp
(6*c + 6*d*x)*(a*b + 7*a^2 + 8*b^2))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp
(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10
*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a*b - a^2))/(7*d*(exp(2*c + 2*d*x) +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**4, x)
```

3.113 $\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^3(c + dx) dx$

Optimal. Leaf size=77

$$\frac{a^2 \log(\cosh(c + dx))}{d} + \frac{b(2a - b) \operatorname{sech}^4(c + dx)}{4d} + \frac{a(a - 2b) \operatorname{sech}^2(c + dx)}{2d} + \frac{b^2 \operatorname{sech}^6(c + dx)}{6d}$$

[Out] $a^2 \ln(\cosh(d*x+c))/d + 1/2*a*(a-2*b)*\operatorname{sech}(d*x+c)^2/d + 1/4*(2*a-b)*b*\operatorname{sech}(d*x+c)^4/d + 1/6*b^2*\operatorname{sech}(d*x+c)^6/d$

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 76}

$$\frac{a^2 \log(\cosh(c + dx))}{d} + \frac{b(2a - b) \operatorname{sech}^4(c + dx)}{4d} + \frac{a(a - 2b) \operatorname{sech}^2(c + dx)}{2d} + \frac{b^2 \operatorname{sech}^6(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]

[Out] $(a^2*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d + (a*(a - 2*b)*\operatorname{Sech}[c + d*x]^2)/(2*d) + ((2*a - b)*b*\operatorname{Sech}[c + d*x]^4)/(4*d) + (b^2*\operatorname{Sech}[c + d*x]^6)/(6*d)$

Rule 76

Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^3(c + dx) dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^7} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^4} + \frac{(2a-b)b}{x^3} + \frac{a(a-2b)}{x^2} - \frac{a^2}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= \frac{a^2 \log(\cosh(c + dx))}{d} + \frac{a(a-2b) \operatorname{sech}^2(c + dx)}{2d} + \frac{(2a-b)b \operatorname{sech}^4(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 107, normalized size = 1.39

$$\frac{\cosh^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 (12a^2 \log(\cosh(c + dx)) + 3b(2a - b) \operatorname{sech}^4(c + dx) + 6a(a - 2b) \operatorname{sech}^2(c + dx))}{3d(a \cosh(2c + 2dx) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]

[Out] (Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(12*a^2*Log[Cosh[c + d*x]] + 6*a*(a - 2*b)*Sech[c + d*x]^2 + 3*(2*a - b)*b*Sech[c + d*x]^4 + 2*b^2*Sech[c + d*x]^6))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^2)

fricas [B] time = 0.45, size = 2591, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="fricas")

[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*a^2*d*x*sinh(d*x + c)^12 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 + 3*a^2*d*x - a^2 + 2*a*b)*sinh(d*x + c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x + 90*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*d*x*cosh(d*x + c)^5 + 30*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^3 + (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^6 + 315*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^4 + 15*a^2*d*x + 21*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - 9*a^2 + 6*a*b - 2*b^2)*sinh(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 + 63*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 + (15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 + 420*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 + 180*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^7 + 42*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^5 + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(33*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2*d*x*cosh(d*x + c)^10 + 45*(3*a^2*d*

```

*x - a^2 + 2*a*b)*cosh(d*x + c)^8 + 14*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)
*cosh(d*x + c)^6 + 10*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^4
+ 3*a^2*d*x + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^12 + 12*a^2*cosh(d*x + c)*
sinh(d*x + c)^11 + a^2*sinh(d*x + c)^12 + 6*a^2*cosh(d*x + c)^10 + 6*(11*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 + 15*a^2*cosh(d*x + c)^8 + 20*(11*
a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(33*a^2*cos
h(d*x + c)^4 + 18*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 20*a^2*cosh(
d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 + 30*a^2*cosh(d*x + c)^3 + 5*a^2*co
sh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d*x + c)^6 + 315*a^2*cosh(d*
x + c)^4 + 105*a^2*cosh(d*x + c)^2 + 5*a^2)*sinh(d*x + c)^6 + 15*a^2*cosh(d
*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 + 63*a^2*cosh(d*x + c)^5 + 35*a^2*co
sh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(33*a^2*cosh(d*x
+ c)^8 + 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x
+ c)^2 + a^2)*sinh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 20*(11*a^2*cosh(d*x
+ c)^9 + 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d*x + c)^5 + 20*a^2*cosh(d*x
+ c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^2*cosh(d*x + c)^10
+ 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)^6 + 50*a^2*cosh(d*x + c)^4
+ 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 12*(a^2*cosh(d*x +
c)^11 + 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x + c)^7 + 10*a^2*cosh(d*x +
c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^2*d*x*cosh(d*x + c)^1
1 + 5*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^9 + 2*(15*a^2*d*x - 8*a^2 + 8
*a*b + 4*b^2)*cosh(d*x + c)^7 + 2*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh
(d*x + c)^5 + (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 + (3*a^2
*d*x - a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*
d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^1
0 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^
3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*co
sh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d
*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*
d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*
cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5
+ 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 +
70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*
d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*
(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d
*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d
*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c
)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

```

giac [B] time = 0.20, size = 241, normalized size = 3.13

$$60 a^2 dx - 60 a^2 \log(e^{(2dx+2c)} + 1) + \frac{147 a^2 e^{(12dx+12c)} + 762 a^2 e^{(10dx+10c)} + 240 a b e^{(10dx+10c)} + 1725 a^2 e^{(8dx+8c)} + 480 a b e^{(8dx+8c)} + 240 b^2}{e^{(2dx+2c)} + 1} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="giac")

[Out] -1/60*(60*a^2*d*x - 60*a^2*log(e^(2*d*x + 2*c) + 1) + (147*a^2*e^(12*d*x + 12*c) + 762*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1725*a^2*e^(8*d*x + 8*c) + 480*a*b*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) + 2220*a^2*e^(6*d*x + 6*c) + 480*a*b*e^(6*d*x + 6*c) - 160*b^2*e^(6*d*x + 6*c) + 1725*a^2*e^(4*d*x + 4*c) + 480*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) + 762*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2*c) + 1)^6)/d

maple [A] time = 0.24, size = 110, normalized size = 1.43

$$\frac{a^2 \ln(\cosh(dx+c))}{d} - \frac{(\tanh^2(dx+c))a^2}{2d} - \frac{ab(\sinh^2(dx+c))}{d \cosh(dx+c)^4} - \frac{ab}{2d \cosh(dx+c)^4} - \frac{b^2(\sinh^2(dx+c))}{4d \cosh(dx+c)^6} - \frac{b^2}{12d \cosh(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x)

[Out] a^2*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^2-1/d*a*b*sinh(d*x+c)^2/cosh(d*x+c)^4-1/2/d*a*b/cosh(d*x+c)^4-1/4/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^6-1/12/d*b^2/cosh(d*x+c)^6

maxima [B] time = 0.44, size = 333, normalized size = 4.32

$$\frac{ab \tanh(dx+c)^4}{2d} + a^2 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) - \frac{4}{3} b^2 \left(\frac{1}{d(6e^{(-2dx-2c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="maxima")

[Out] 1/2*a*b*tanh(d*x+c)^4/d + a^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)))

mupad [B] time = 1.52, size = 349, normalized size = 4.53

$$\frac{4(2ab - 9b^2)}{d(4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1)} - \frac{32b^2}{3d(6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^3*(a+b/cosh(c+d*x)^2)^2,x)

[Out] (4*(2*a*b - 9*b^2))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (2*(a^2 - 6*a*b + 2*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*(2*a*b - a^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*(6*a*b - 7*b^2))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - a^2*x + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d + (32*b^2)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))

sympy [A] time = 5.09, size = 129, normalized size = 1.68

$$\left\{ \begin{array}{l} a^2 x - \frac{a^2 \log(\tanh(c+dx)+1)}{d} - \frac{a^2 \tanh^2(c+dx)}{2d} - \frac{ab \tanh^2(c+dx) \operatorname{sech}^2(c+dx)}{2d} - \frac{ab \operatorname{sech}^2(c+dx)}{2d} - \frac{b^2 \tanh^2(c+dx) \operatorname{sech}^4(c+dx)}{6d} - \frac{b^2}{12d} \\ x(a + b \operatorname{sech}^2(c))^2 \tanh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(2*d) - a*b*tanh(c + d*x)**2*sech(c + d*x)**2/(2*d) - a*b*sech(c + d*x)**2/(2*d) - b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(6*d) - b**2*sech(c + d*x)**4/(12*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c)**3, True))
```

3.114 $\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^2(c + dx) dx$

Optimal. Leaf size=59

$$-\frac{a^2 \tanh(c + dx)}{d} + a^2 x + \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d}$$

[Out] $a^2 x - a^2 \tanh(d x + c) / d + 1/3 b (2 a + b) \tanh(d x + c)^3 / d - 1/5 b^2 \tanh(d x + c)^5 / d$

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 206}

$$-\frac{a^2 \tanh(c + dx)}{d} + a^2 x + \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]

[Out] $a^2 x - (a^2 \operatorname{Tanh}[c + d x]) / d + (b(2 a + b) \operatorname{Tanh}[c + d x]^3) / (3 d) - (b^2 \operatorname{Tanh}[c + d x]^5) / (5 d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(n_)])^(m_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b(1-x^2))^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-a^2 + b(2a + b)x^2 - b^2x^4 + \frac{a^2}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \tanh(c + dx)}{d} + \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} \\ &= a^2 x - \frac{a^2 \tanh(c + dx)}{d} + \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 0.86, size = 281, normalized size = 4.76

$$\frac{\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(120a^2\sinh(2c+dx)-120a^2\sinh(2c+3dx)+30a^2\sinh(4c+3dx)-30a^2\sinh(4c+5dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]

[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] + 75*a^2*d*x*Cosh[4*c + 3*d*x] + 15*a^2*d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] - 180*a^2*Sinh[d*x] + 80*a*b*Sinh[d*x] - 20*b^2*Sinh[d*x] + 120*a^2*Sinh[2*c + d*x] - 120*a*b*Sinh[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 120*a^2*Sinh[2*c + 3*d*x] + 40*a*b*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] + 30*a^2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] - 30*a^2*Sinh[4*c + 5*d*x] + 20*a*b*Sinh[4*c + 5*d*x] + 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)

fricas [B] time = 0.42, size = 435, normalized size = 7.37

$$\frac{(15a^2dx + 15a^2 - 10ab - 2b^2)\cosh(dx+c)^5 + 5(15a^2dx + 15a^2 - 10ab - 2b^2)\cosh(dx+c)\sinh(dx+c)^4 - \dots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/15*((15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 - 10*a*b - 2*b^2)*sinh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^3 - 5*(2*(15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 10*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c) - 5*((15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.17, size = 193, normalized size = 3.27

$$\frac{15a^2dx + \frac{2(15a^2e^{8dx+8c}-30abe^{8dx+8c}+60a^2e^{6dx+6c}-60abe^{6dx+6c}-30b^2e^{6dx+6c}+90a^2e^{4dx+4c}-40abe^{4dx+4c}+10b^2e^{4dx+4c}+60a^2e^{2dx+2c})}{(e^{2dx+2c}+1)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="giac")

[Out] 1/15*(15*a^2*d*x + 2*(15*a^2*e^(8*d*x + 8*c) - 30*a*b*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) - 60*a*b*e^(6*d*x + 6*c) - 30*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) - 40*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) - 20*a*b*e^(2*d*x + 2*c) - 10*b^2*e^(2*d*x + 2*c) + 15*a^2 - 10*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^5/d

maple [B] time = 0.40, size = 115, normalized size = 1.95

$$\frac{a^2(dx+c - \tanh(dx+c)) + 2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right) + b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5} + \frac{\left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)}{1}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x)`

[Out] $1/d*(a^2*(d*x+c-\tanh(d*x+c))+2*a*b*(-1/2*\sinh(d*x+c)/\cosh(d*x+c)^3+1/2*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))+b^2*(-1/4*\sinh(d*x+c)/\cosh(d*x+c)^5+1/4*(8/15+1/5*\operatorname{sech}(d*x+c)^4+1/5*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.52, size = 325, normalized size = 5.51

$$\frac{2ab \tanh(dx+c)^3}{3d} + a^2 \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)}+1)} \right) + \frac{4}{15} b^2 \left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="maxima")`

[Out] $2/3*a*b*tanh(d*x+c)^3/d + a^2*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1))) + 4/15*b^2*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) - 5*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 15*e^{(-6*d*x - 6*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)))$

mupad [B] time = 0.14, size = 513, normalized size = 8.69

$$\frac{\frac{8e^{2c+2dx}(a^2-b^2)}{5d} - \frac{2(2ab-a^2)}{5d} + \frac{8e^{6c+6dx}(a^2-b^2)}{5d} - \frac{2e^{8c+8dx}(2ab-a^2)}{5d} + \frac{4e^{4c+4dx}(3a^2+2ab+4b^2)}{5d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1} + \frac{2(3a^2+2ab+4b^2)}{15d} + \frac{4e^{2c+2dx}}{3e^{2c+2dx} + 3e^{4c+4dx} + 3e^{6c+6dx} + 3e^{8c+8dx} + e^{10c+10dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c+d*x)^2*(a+b/cosh(c+d*x)^2)^2,x)`

[Out] $((8*\exp(2*c+2*d*x)*(a^2-b^2))/(5*d) - (2*(2*a*b-a^2))/(5*d) + (8*\exp(6*c+6*d*x)*(a^2-b^2))/(5*d) - (2*\exp(8*c+8*d*x)*(2*a*b-a^2))/(5*d) + (4*\exp(4*c+4*d*x)*(2*a*b+3*a^2+4*b^2))/(5*d))/(5*\exp(2*c+2*d*x) + 10*\exp(4*c+4*d*x) + 10*\exp(6*c+6*d*x) + 5*\exp(8*c+8*d*x) + \exp(10*c+10*d*x) + 1) + ((2*(2*a*b+3*a^2+4*b^2))/(15*d) + (4*\exp(2*c+2*d*x)*(a^2-b^2))/(5*d) - (2*\exp(4*c+4*d*x)*(2*a*b-a^2))/(5*d))/(3*\exp(2*c+2*d*x) + 3*\exp(4*c+4*d*x) + \exp(6*c+6*d*x) + 1) + ((2*(a^2-b^2))/(5*d) - (2*\exp(2*c+2*d*x)*(2*a*b-a^2))/(5*d))/(2*\exp(2*c+2*d*x) + \exp(4*c+4*d*x) + 1) + ((2*(a^2-b^2))/(5*d) + (6*\exp(4*c+4*d*x)*(a^2-b^2))/(5*d) - (2*\exp(6*c+6*d*x)*(2*a*b-a^2))/(5*d) + (2*\exp(2*c+2*d*x)*(2*a*b+3*a^2+4*b^2))/(5*d))/(4*\exp(2*c+2*d*x) + 6*\exp(4*c+4*d*x) + 4*\exp(6*c+6*d*x) + \exp(8*c+8*d*x) + 1) + a^2*x - (2*(2*a*b-a^2))/(5*d*(\exp(2*c+2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**2, x)`

3.115 $\int (a + b \operatorname{sech}^2(c + dx))^2 \tanh(c + dx) dx$

Optimal. Leaf size=48

$$\frac{a^2 \log(\cosh(c + dx))}{d} - \frac{ab \operatorname{sech}^2(c + dx)}{d} - \frac{b^2 \operatorname{sech}^4(c + dx)}{4d}$$

[Out] $a^2 \ln(\cosh(d*x+c))/d - a*b*\operatorname{sech}(d*x+c)^2/d - 1/4*b^2*\operatorname{sech}(d*x+c)^4/d$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 266, 43}

$$\frac{a^2 \log(\cosh(c + dx))}{d} - \frac{ab \operatorname{sech}^2(c + dx)}{d} - \frac{b^2 \operatorname{sech}^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x], x]

[Out] $(a^2*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d - (a*b*\operatorname{Sech}[c + d*x]^2)/d - (b^2*\operatorname{Sech}[c + d*x]^4)/(4*d)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 \tanh(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^5} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{x^3} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^3} + \frac{2ab}{x^2} + \frac{a^2}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{a^2 \log(\cosh(c + dx))}{d} - \frac{ab \operatorname{sech}^2(c + dx)}{d} - \frac{b^2 \operatorname{sech}^4(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.14, size = 81, normalized size = 1.69

$$\frac{\operatorname{sech}^4(c + dx) \left(a \cosh^2(c + dx) + b \right)^2 \left(4a^2 \cosh^4(c + dx) \log(\cosh(c + dx)) - 4ab \cosh^2(c + dx) - b^2 \right)}{d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x], x]

[Out] ((b + a*Cosh[c + d*x]^2)^2*(-b^2 - 4*a*b*Cosh[c + d*x]^2 + 4*a^2*Cosh[c + d*x]^4*Log[Cosh[c + d*x]])*Sech[c + d*x]^4)/(d*(a + 2*b + a*Cosh[2*(c + d*x)]))^2)

fricas [B] time = 0.42, size = 1180, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c), x, algorithm="fricas")

[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d*x*sinh(d*x + c)^8 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cosh(d*x + c)^2 + a^2*d*x + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^3 + 3*(a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x + 30*(a^2*d*x + a*b)*cosh(d*x + c)^2 + 4*a*b + 2*b^2)*sinh(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 + 10*(a^2*d*x + a*b)*cosh(d*x + c)^3 + (3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 + 15*(a^2*d*x + a*b)*cosh(d*x + c)^4 + a^2*d*x + 3*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 + 3*(a^2*d*x + a*b)*cosh(d*x + c)^5 + (3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^3 + (a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.14, size = 159, normalized size = 3.31

$$\frac{12 a^2 dx - 12 a^2 \log \left(e^{(2 dx+2c)} + 1 \right) + \frac{25 a^2 e^{(8 dx+8c)} + 100 a^2 e^{(6 dx+6c)} + 48 a b e^{(6 dx+6c)} + 150 a^2 e^{(4 dx+4c)} + 96 a b e^{(4 dx+4c)} + 48 b^2 e^{(4 dx+4c)}}{(e^{(2 dx+2c)} + 1)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c), x, algorithm="giac")

[Out] $-1/12*(12*a^2*d*x - 12*a^2*\log(e^{(2*d*x + 2*c)} + 1) + (25*a^2*e^{(8*d*x + 8*c)} + 100*a^2*e^{(6*d*x + 6*c)} + 48*a*b*e^{(6*d*x + 6*c)} + 150*a^2*e^{(4*d*x + 4*c)} + 96*a*b*e^{(4*d*x + 4*c)} + 48*b^2*e^{(4*d*x + 4*c)} + 100*a^2*e^{(2*d*x + 2*c)} + 48*a*b*e^{(2*d*x + 2*c)} + 25*a^2)/(e^{(2*d*x + 2*c)} + 1)^4)/d$

maple [A] time = 0.16, size = 48, normalized size = 1.00

$$\frac{b^2 \operatorname{sech}(dx+c)^4}{4d} - \frac{ab \operatorname{sech}(dx+c)^2}{d} - \frac{a^2 \ln(\operatorname{sech}(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x)`

[Out] $-1/4*b^2*\operatorname{sech}(d*x+c)^4/d - a*b*\operatorname{sech}(d*x+c)^2/d - 1/d*a^2*\ln(\operatorname{sech}(d*x+c))$

maxima [A] time = 0.48, size = 55, normalized size = 1.15

$$\frac{ab \tanh(dx+c)^2}{d} + \frac{a^2 \log(\cosh(dx+c))}{d} - \frac{4b^2}{d(e^{(dx+c)} + e^{(-dx-c)})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="maxima")`

[Out] $a*b*\tanh(d*x + c)^2/d + a^2*\log(\cosh(d*x + c))/d - 4*b^2/(d*(e^{(d*x + c)} + e^{(-d*x - c)})^4)$

mupad [B] time = 1.69, size = 182, normalized size = 3.79

$$\frac{4(ab - b^2)}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - a^2 x + \frac{8b^2}{d(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{4b^2}{d(4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)`

[Out] $(4*(a*b - b^2))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - a^2*x + (8*b^2)/(d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (4*b^2)/(d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (a^2*\log(\exp(2*c)*\exp(2*d*x) + 1))/d - (4*a*b)/(d*(\exp(2*c + 2*d*x) + 1))$

sympy [A] time = 1.67, size = 63, normalized size = 1.31

$$\begin{cases} a^2 x - \frac{a^2 \log(\tanh(c+dx)+1)}{d} - \frac{ab \operatorname{sech}^2(c+dx)}{d} - \frac{b^2 \operatorname{sech}^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a + b \operatorname{sech}^2(c))^2 \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c),x)`

[Out] `Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a*b*sech(c + d*x)**2/d - b**2*sech(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c), True))`

3.116 $\int (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=40

$$a^2x + \frac{b(2a + b) \tanh(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] $a^2x + b(2a + b) \tanh(dx + c)/d - 1/3 b^2 \tanh(dx + c)^3/d$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4128, 390, 206}

$$a^2x + \frac{b(2a + b) \tanh(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^2, x]

[Out] $a^2x + (b(2a + b) \tanh[c + d*x])/d - (b^2 \tanh[c + d*x]^3)/(3d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4128

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && NeQ[a + b, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(b(2a + b) - b^2x^2 + \frac{a^2}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b(2a + b) \tanh(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d} + \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= a^2x + \frac{b(2a + b) \tanh(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 0.38, size = 106, normalized size = 2.65

$$\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3a^2dx\cosh^3(c+dx)+2b(3a+b)\operatorname{sech}(c)\sinh(dx)\cosh^2(c+dx)+b^2\tanh(c)\right)}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2, x]

[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(3*a^2*d*x*Cosh[c + d*x]^3 + b^2*Sech[c]*Sinh[d*x] + 2*b*(3*a + b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)]^2)

fricas [B] time = 0.40, size = 176, normalized size = 4.40

$$\frac{\left(3a^2dx - 6ab - 2b^2\right)\cosh(dx+c)^3 + 3\left(3a^2dx - 6ab - 2b^2\right)\cosh(dx+c)\sinh(dx+c)^2 + 2\left(3ab + b^2\right)\sinh(dx+c)}{3\left(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/3*((3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c) + 6*((3*a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.14, size = 79, normalized size = 1.98

$$\frac{3(dx+c)a^2 - \frac{4(3abe^{4dx+4c}+6abe^{2dx+2c}+3b^2e^{2dx+2c}+3ab+b^2)}{(e^{2dx+2c}+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*a^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.36, size = 47, normalized size = 1.18

$$\frac{a^2(dx+c) + 2ab\tanh(dx+c) + b^2\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.46, size = 120, normalized size = 3.00

$$a^2x + \frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + \frac{4}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $a^2x + \frac{4}{3}b^2 \frac{(3e^{-2dx} - 2c)}{(d(3e^{-2dx} - 2c) + 3e^{-4dx} - 4c) + e^{-6dx} - 6c + 1)} + \frac{1}{(d(3e^{-2dx} - 2c) + 3e^{-4dx} - 4c) + e^{-6dx} - 6c + 1)} + \frac{4ab}{(d(e^{-2dx} - 2c) + 1)}$

mupad [B] time = 1.46, size = 163, normalized size = 4.08

$$a^2x - \frac{\frac{4(b^2+ab)}{3d} + \frac{4abe^{2c+2dx}}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{\frac{8e^{2c+2dx}(b^2+ab)}{3d} + \frac{4ab}{3d} + \frac{4abe^{4c+4dx}}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{4ab}{3d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2, x)`

[Out] $a^2x - \left(\frac{4(ab + b^2)}{3d} + \frac{4ab \exp(2c + 2dx)}{3d} \right) / (2 \exp(2c + 2dx) + \exp(4c + 4dx) + 1) - \left(\frac{8 \exp(2c + 2dx)(ab + b^2)}{3d} + \frac{4ab}{3d} + \frac{4ab \exp(4c + 4dx)}{3d} \right) / (3 \exp(2c + 2dx) + 3 \exp(4c + 4dx) + \exp(6c + 6dx) + 1) - \frac{4ab}{3d(\exp(2c + 2dx) + 1)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**2, x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2, x)`

3.117 $\int \coth(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2 \log(\sinh(c+dx))}{d} - \frac{b(2a+b) \log(\cosh(c+dx))}{d} + \frac{b^2 \operatorname{sech}^2(c+dx)}{2d}$$

[Out] $-b*(2*a+b)*\ln(\cosh(d*x+c))/d+(a+b)^2*\ln(\sinh(d*x+c))/d+1/2*b^2*\operatorname{sech}(d*x+c)^2/d$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2 \log(\sinh(c+dx))}{d} - \frac{b(2a+b) \log(\cosh(c+dx))}{d} + \frac{b^2 \operatorname{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]`

[Out] $-((b*(2*a + b)*\text{Log}[\text{Cosh}[c + d*x]])/d) + ((a + b)^2*\text{Log}[\text{Sinh}[c + d*x]])/d + (b^2*\text{Sech}[c + d*x]^2)/(2*d)$

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 4138

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned} \int \coth(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst} \left(\int \frac{(b+ax^2)^2}{x^3(1-x^2)} dx, x, \cosh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \frac{(b+ax)^2}{(1-x)x^2} dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= \frac{\operatorname{Subst} \left(\int \left(-\frac{(a+b)^2}{-1+x} + \frac{b^2}{x^2} + \frac{b(2a+b)}{x} \right) dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= -\frac{b(2a+b) \log(\cosh(c + dx))}{d} + \frac{(a+b)^2 \log(\sinh(c + dx))}{d} + \frac{b^2 \operatorname{sech}^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.25, size = 84, normalized size = 1.58

$$\frac{2(a \cosh(c + dx) + b \operatorname{sech}(c + dx))^2 \left(2 \cosh^2(c + dx) \left((a + b)^2 \log(\sinh(c + dx)) - b(2a + b) \log(\cosh(c + dx)) \right) \right)}{d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (2*(b^2 + 2*Cosh[c + d*x]^2*(-(b*(2*a + b)*Log[Cosh[c + d*x]])) + (a + b)^2*Log[Sinh[c + d*x]]))*(a*Cosh[c + d*x] + b*Sech[c + d*x])^2)/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.46, size = 665, normalized size = 12.55

$$\frac{a^2 dx \cosh(dx + c)^4 + 4 a^2 dx \cosh(dx + c) \sinh(dx + c)^3 + a^2 dx \sinh(dx + c)^4 + a^2 dx + 2(a^2 dx - b^2) \cosh(dx + c)}{d(a \cosh(2(dx + c)) + a + 2b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*x*sinh(d*x + c)^4 + a^2*d*x + 2*(a^2*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 + a^2*d*x - b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 + (a^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.17, size = 171, normalized size = 3.23

$$\frac{2 a^2 dx + 2 \left(2 a b e^{(2c)} + b^2 e^{(2c)} \right) e^{(-2c)} \log \left(e^{(2dx+2c)} + 1 \right) - 2 \left(a^2 e^{(2c)} + 2 a b e^{(2c)} + b^2 e^{(2c)} \right) e^{(-2c)} \log \left(\left| e^{(2dx+2c)} - 1 \right| \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*(2*a^2*d*x + 2*(2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2*c) + 1) - 2*(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) - (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) + 6*a*b + 3*b^2)/(e^(2*d*x + 2*c) + 1)^2)/d

maple [A] time = 0.26, size = 60, normalized size = 1.13

$$\frac{a^2 \ln(\sinh(dx + c))}{d} + \frac{2ab \ln(\tanh(dx + c))}{d} + \frac{b^2}{2d \cosh(dx + c)^2} + \frac{b^2 \ln(\tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)`

[Out] $a^2 \ln(\sinh(dx+c))/d + 2ab \ln(\tanh(dx+c))/d + 1/2 db^2 / \cosh(dx+c)^2 + 1/d b^2 \ln(\tanh(dx+c))$

maxima [B] time = 0.70, size = 161, normalized size = 3.04

$$b^2 \left(\frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} - \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right) + 2ab \left(\frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} - \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $b^2 * (\log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d - \log(e^{-2dx-2c} + 1)/d + 2e^{-2dx-2c}/(d(2e^{-2dx-2c} + e^{-4dx-4c} + 1))) + 2ab * (\log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d - \log(e^{-2dx-2c} + 1)/d) + a^2 * \log(\sinh(dx+c))/d$

mupad [B] time = 0.30, size = 308, normalized size = 5.81

$$\frac{2b^2}{d(e^{2c+2dx} + 1)} - a^2 x + \frac{a^2 \ln(e^{4c+4dx} - 1)}{2d} - \frac{2b^2}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \operatorname{atan} \left(\frac{e^{2c} e^{2dx} (a^4 \sqrt{-d^2}}{a^2 d \sqrt{a^4 + 8a^3 b + 20a^2 b^2 + 16ab^3 + 4b^4 + 2d^2}}{a^2 d \sqrt{a^4 + 8a^3 b + 20a^2 b^2 + 16ab^3 + 4b^4 + 2d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)`

[Out] $(2b^2)/(d(\exp(2c + 2dx) + 1)) - a^2 x + (a^2 \log(\exp(4c + 4dx) - 1))/(2d) - (2b^2)/(d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)) - (\operatorname{atan}((\exp(2c) \exp(2dx) * (a^4 (-d^2)^{1/2} + 4b^4 (-d^2)^{1/2} + 16ab^3 (-d^2)^{1/2} + 8a^3 b (-d^2)^{1/2} + 20a^2 b^2 (-d^2)^{1/2}))/ (a^2 d * (16ab^3 + 8a^3 b + a^4 + 4b^4 + 20a^2 b^2)^{1/2} + 2b^2 d * (16ab^3 + 8a^3 b + a^4 + 4b^4 + 20a^2 b^2)^{1/2} + 4ab * d * (16ab^3 + 8a^3 b + a^4 + 4b^4 + 20a^2 b^2)^{1/2}))/ (16ab^3 + 8a^3 b + a^4 + 4b^4 + 20a^2 b^2)^{1/2}))/ (-d^2)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{coth}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x), x)`

3.118 $\int \coth^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$

Optimal. Leaf size=36

$$a^2x - \frac{(a+b)^2 \coth(c+dx)}{d} - \frac{b^2 \tanh(c+dx)}{d}$$

[Out] $a^2x - (a+b)^2 \coth(dx+c)/d - b^2 \tanh(dx+c)/d$

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$a^2x - \frac{(a+b)^2 \coth(c+dx)}{d} - \frac{b^2 \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2, x]

[Out] $a^2x - ((a+b)^2 \operatorname{Coth}[c + d*x])/d - (b^2 \operatorname{Tanh}[c + d*x])/d$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(n_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+b(1-x^2))^2}{x^2(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(-b^2 + \frac{(a+b)^2}{x^2} - \frac{a^2}{-1+x^2} \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{(a+b)^2 \coth(c + dx)}{d} - \frac{b^2 \tanh(c + dx)}{d} - \frac{a^2 \operatorname{Subst} \left(\int \frac{1}{-1+x^2} dx \right)}{d} \\ &= a^2x - \frac{(a+b)^2 \coth(c + dx)}{d} - \frac{b^2 \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.74, size = 82, normalized size = 2.28

$$\frac{4\operatorname{sech}(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(a^2dx\cosh(c+dx)+\sinh(dx)\left((a+b)^2\operatorname{csch}(c)\coth(c+dx)-b^2\operatorname{sech}(c)\right)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]*(a^2*d*x*Cosh[c + d*x] + ((a + b)^2*Coth[c + d*x]*Csch[c] - b^2*Sech[c])*Sinh[d*x]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.44, size = 106, normalized size = 2.94

$$\frac{\left(a^2+2ab+2b^2\right)\cosh(dx+c)^2-2\left(a^2dx+a^2+2ab+2b^2\right)\cosh(dx+c)\sinh(dx+c)+\left(a^2+2ab+2b^2\right)\sinh(dx+c)}{2d\cosh(dx+c)\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*((a^2+2*a*b+2*b^2)*cosh(d*x+c)^2-2*(a^2*d*x+a^2+2*a*b+2*b^2)*cosh(d*x+c)*sinh(d*x+c)+(a^2+2*a*b+2*b^2)*sinh(d*x+c)^2+a^2+2*a*b)/(d*cosh(d*x+c)*sinh(d*x+c))

giac [A] time = 0.20, size = 65, normalized size = 1.81

$$\frac{a^2dx-\frac{2\left(a^2e^{2dx+2c}+2abe^{2dx+2c}+a^2+2ab+2b^2\right)}{e^{4dx+4c}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] (a^2*d*x-2*(a^2*e^(2*d*x+2*c)+2*a*b*e^(2*d*x+2*c)+a^2+2*a*b+2*b^2)/(e^(4*d*x+4*c)-1))/d

maple [A] time = 0.39, size = 64, normalized size = 1.78

$$\frac{a^2(dx+c-\coth(dx+c))-2ab\coth(dx+c)+b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))-2*a*b*coth(d*x+c)+b^2*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))

maxima [A] time = 0.55, size = 71, normalized size = 1.97

$$a^2\left(x+\frac{c}{d}+\frac{2}{d(e^{-2dx-2c}-1)}\right)+\frac{4ab}{d(e^{-2dx-2c}-1)}+\frac{4b^2}{d(e^{-4dx-4c}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b^2/(d*(e^(-4*d*x - 4*c) - 1))

mupad [B] time = 1.42, size = 60, normalized size = 1.67

$$a^2 x - \frac{\frac{2(a^2 + 2ab + 2b^2)}{d} + \frac{2ae^{2c+2dx}(a+2b)}{d}}{e^{4c+4dx} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2), x)

[Out] a^2*x - ((2*(2*a*b + a^2 + 2*b^2))/d + (2*a*exp(2*c + 2*d*x)*(a + 2*b))/d)/(exp(4*c + 4*d*x) - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{coth}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**2, x)

$$3.119 \quad \int \coth^3(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^2 dx$$

Optimal. Leaf size=55

$$\frac{(a^2 - b^2) \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^2(c + dx)}{2d} + \frac{b^2 \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*(a+b)^2*\operatorname{csch}(d*x+c)^2/d+b^2*\ln(\cosh(d*x+c))/d+(a^2-b^2)*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{(a^2 - b^2) \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^2(c + dx)}{2d} + \frac{b^2 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $-((a + b)^2*\operatorname{Csch}[c + d*x]^2)/(2*d) + (b^2*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d + ((a^2 - b^2)*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/d$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \coth^3(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2 x} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2} + \frac{a^2-b^2}{-1+x} + \frac{b^2}{x}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+b)^2 \operatorname{csch}^2(c+dx)}{2d} + \frac{b^2 \log(\cosh(c+dx))}{d} + \frac{(a^2-b^2) \log(\sinh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 82, normalized size = 1.49

$$\frac{2(a \cosh^2(c+dx) + b)^2 \left((a+b)^2 \operatorname{csch}^2(c+dx) - 2 \left((a^2-b^2) \log(\sinh(c+dx)) + b^2 \log(\cosh(c+dx)) \right) \right)}{d(a \cosh(2(c+dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (-2*(b + a*Cosh[c + d*x]^2)^2*((a + b)^2*Csch[c + d*x]^2 - 2*(b^2*Log[Cosh[c + d*x]] + (a^2 - b^2)*Log[Sinh[c + d*x]])))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.45, size = 637, normalized size = 11.58

$$\frac{a^2 dx \cosh(dx+c)^4 + 4a^2 dx \cosh(dx+c) \sinh(dx+c)^3 + a^2 dx \sinh(dx+c)^4 + a^2 dx - 2(a^2 dx - a^2 - 2ab)}{d(a \cosh(2(c+dx)) + a + 2b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*x*sinh(d*x + c)^4 + a^2*d*x - 2*(a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.25, size = 161, normalized size = 2.93

$$\frac{2a^2 dx - 2b^2 \log(e^{(2dx+2c)} + 1) - 2(a^2 e^{(2c)} - b^2 e^{(2c)})e^{(-2c)} \log(|e^{(2dx+2c)} - 1|) + \frac{3a^2 e^{(4dx+4c)} - 3b^2 e^{(4dx+4c)} - 2a^2 e^{(2c)}}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2*(2*a^2*d*x - 2*b^2*\log(e^{(2*d*x + 2*c)} + 1) - 2*(a^2*e^{(2*c)} - b^2*e^{(2*c)})*e^{(-2*c)}*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1))) + (3*a^2*e^{(4*d*x + 4*c)} - 3*b^2*e^{(4*d*x + 4*c)} - 2*a^2*e^{(2*d*x + 2*c)} + 8*a*b*e^{(2*d*x + 2*c)} + 10*b^2*e^{(2*d*x + 2*c)} + 3*a^2 - 3*b^2)/(e^{(2*d*x + 2*c)} - 1)^2/d$

maple [A] time = 0.31, size = 78, normalized size = 1.42

$$\frac{a^2 \ln(\sinh(dx+c))}{d} - \frac{a^2 (\coth^2(dx+c))}{2d} - \frac{ab}{d \sinh(dx+c)^2} - \frac{b^2}{2d \sinh(dx+c)^2} - \frac{b^2 \ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)

[Out] $a^2*\ln(\sinh(d*x+c))/d - 1/2*a^2*\coth(d*x+c)^2/d - 1/d/\sinh(d*x+c)^2*a*b - 1/2/d*b^2/\sinh(d*x+c)^2 - 1/d*b^2*\ln(\tanh(d*x+c))$

maxima [B] time = 0.48, size = 206, normalized size = 3.75

$$a^2 \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - b^2 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $a^2*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - b^2*(\log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d - \log(e^{(-2*d*x - 2*c)} + 1)/d - 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - 4*a*b/(d*(e^{(d*x + c)} - e^{(-d*x - c)})^2)$

mupad [B] time = 0.22, size = 240, normalized size = 4.36

$$\frac{\ln(e^{4c+4dx} - 1) (d(a^2 - b^2) + b^2 d)}{2d^2} - a^2 x - \frac{2(a^2 + 2ab + b^2)}{d(e^{2c+2dx} - 1)} - \frac{\operatorname{atan}\left(\frac{e^{2c} e^{2dx} (a^4 \sqrt{-d^2} + 4b^4 \sqrt{-d^2} - 4a^2 b^2 \sqrt{-d^2})}{a^2 d \sqrt{a^4 - 4a^2 b^2 + 4b^4} - 2b^2 d \sqrt{a^4 - 4a^2 b^2 + 4b^4}}\right)}{\sqrt{-d^2}} \sqrt{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $(\log(\exp(4*c + 4*d*x) - 1)*(d*(a^2 - b^2) + b^2*d))/(2*d^2) - a^2*x - (2*(2*a*b + a^2 + b^2))/(d*(\exp(2*c + 2*d*x) - 1)) - (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*(a^4*(-d^2)^{(1/2)} + 4*b^4*(-d^2)^{(1/2)} - 4*a^2*b^2*(-d^2)^{(1/2)})))/(a^2*d*(a^4 + 4*b^4 - 4*a^2*b^2)^{(1/2)} - 2*b^2*d*(a^4 + 4*b^4 - 4*a^2*b^2)^{(1/2)}))*(a^4 + 4*b^4 - 4*a^2*b^2)^{(1/2)})/(-d^2)^{(1/2)} - (2*(2*a*b + a^2 + b^2))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \coth^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**3, x)

3.120 $\int \coth^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=46

$$-\frac{(a^2 - b^2) \coth(c + dx)}{d} + a^2 x - \frac{(a + b)^2 \coth^3(c + dx)}{3d}$$

[Out] $a^2 x - (a^2 - b^2) \coth(d x + c) / d - 1/3 (a + b)^2 \coth(d x + c)^3 / d$

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$-\frac{(a^2 - b^2) \coth(c + dx)}{d} + a^2 x - \frac{(a + b)^2 \coth^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $a^2 x - ((a^2 - b^2) \operatorname{Coth}[c + d x]) / d - ((a + b)^2 \operatorname{Coth}[c + d x]^3) / (3 d)$

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f_)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b(1-x^2))^2}{x^4(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{x^4} + \frac{a^2-b^2}{x^2} - \frac{a^2}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a^2 - b^2) \coth(c + dx)}{d} - \frac{(a + b)^2 \coth^3(c + dx)}{3d} - \frac{a^2 \operatorname{Subst}\left(\int \right)}{d} \\ &= a^2 x - \frac{(a^2 - b^2) \coth(c + dx)}{d} - \frac{(a + b)^2 \coth^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 0.81, size = 160, normalized size = 3.48

$$\frac{\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\left(-12a^2\sinh(2c+dx)+8a^2\sinh(2c+3dx)-9a^2dx\cosh(2c+dx)-3a^2dx\cosh(2c+3dx)\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (Csch[c]*Csch[c + d*x]^3*(9*a^2*d*x*Cosh[d*x] - 9*a^2*d*x*Cosh[2*c + d*x] - 3*a^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh[4*c + 3*d*x] - 12*a^2*Sinh[d*x] + 12*b^2*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 12*a*b*Sinh[2*c + d*x] + 8*a^2*Sinh[2*c + 3*d*x] + 4*a*b*Sinh[2*c + 3*d*x] - 4*b^2*Sinh[2*c + 3*d*x])/(24*d)

fricas [B] time = 0.43, size = 201, normalized size = 4.37

$$\frac{2(2a^2 + ab - b^2)\cosh(dx + c)^3 + 6(2a^2 + ab - b^2)\cosh(dx + c)\sinh(dx + c)^2 - (3a^2dx + 4a^2 + 2ab - 2b^2)}{3(d\sinh(dx + c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + 6*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^3 + 6*(a*b + b^2)*cosh(d*x + c) + 3*(3*a^2*d*x - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))

giac [B] time = 0.27, size = 97, normalized size = 2.11

$$\frac{3a^2dx - \frac{4(3a^2e^{4dx+4c} + 3abe^{4dx+4c} - 3a^2e^{2dx+2c} + 3b^2e^{2dx+2c} + 2a^2 + ab - b^2)}{(e^{2dx+2c} - 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/3*(3*a^2*d*x - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 2*a^2 + a*b - b^2)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [B] time = 0.36, size = 96, normalized size = 2.09

$$\frac{a^2\left(dx + c - \coth(dx + c) - \frac{\coth^3(dx + c)}{3}\right) + 2ab\left(\frac{\cosh(dx + c)}{2\sinh(dx + c)^3} - \frac{\left(\frac{2}{3} - \frac{\operatorname{csch}(dx + c)^2}{3}\right)\coth(dx + c)}{2}\right) + b^2\left(\frac{2}{3} - \frac{\operatorname{csch}(dx + c)^2}{3}\right)\coth(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+2*a*b*(-1/2/sinh(d*x+c)^3*cosh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+b^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))

maxima [B] time = 0.43, size = 268, normalized size = 5.83

$$\frac{1}{3}a^2\left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + \frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{3}a^2(3x + 3c/d - 4(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} - 2)/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1))) + \frac{4}{3}b^2(3e^{(-2dx - 2c)}/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1)) - 1/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1))) + \frac{4}{3}ab(3e^{(-4dx - 4c)}/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1)) + 1/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1)))$

mupad [B] time = 1.42, size = 183, normalized size = 3.98

$$a^2 x - \frac{4(a^2+ba)}{3d} + \frac{4e^{4c+4dx}(a^2+ba)}{3d} + \frac{8e^{2c+2dx}(b^2+ab)}{3d} - \frac{4(b^2+ab)}{3d} + \frac{4e^{2c+2dx}(a^2+ba)}{3d} - \frac{4(a^2+ba)}{3d(e^{2c+2dx}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $a^2x - ((4(ab + a^2))/(3d) + (4\exp(4c + 4dx)(ab + a^2))/(3d) + (8\exp(2c + 2dx)(ab + b^2))/(3d))/(3\exp(2c + 2dx) - 3\exp(4c + 4dx) + \exp(6c + 6dx) - 1) - ((4(ab + b^2))/(3d) + (4\exp(2c + 2dx)(ab + a^2))/(3d))/(\exp(4c + 4dx) - 2\exp(2c + 2dx) + 1) - (4(ab + a^2))/(3d(\exp(2c + 2dx) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^2 \operatorname{coth}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**4, x)

3.121 $\int \coth^5(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=52

$$\frac{a^2 \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} - \frac{a(a + b) \operatorname{csch}^2(c + dx)}{d}$$

[Out] $-a*(a+b)*\operatorname{csch}(d*x+c)^2/d-1/4*(a+b)^2*\operatorname{csch}(d*x+c)^4/d+a^2*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 444, 43}

$$\frac{a^2 \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} - \frac{a(a + b) \operatorname{csch}^2(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[c + d*x]^5*(a + b*\operatorname{Sech}[c + d*x]^2)^2, x]$

[Out] $-((a*(a + b)*\operatorname{Csch}[c + d*x]^2)/d) - ((a + b)^2*\operatorname{Csch}[c + d*x]^4)/(4*d) + (a^2*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/d$

Rule 43

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) || \operatorname{LtQ}[9*m + 5*(n + 1), 0] || \operatorname{GtQ}[m + n + 2, 0])$

Rule 444

$\operatorname{Int}[(x_.)^{(m_.)*((a_. + (b_.)*(x_.))^{(n_.))^{(p_.)*((c_. + (d_.)*(x_.))^{(q_.)}, x_Symbol] :> \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{EqQ}[m - n + 1, 0]$

Rule 4138

$\operatorname{Int}[(a_. + (b_.)*\operatorname{sec}[(e_. + (f_.)*(x_.))^{(n_.))^{(p_.)*\tan[(e_. + (f_.)*(x_.))^{(m_.)}, x_Symbol] :> \operatorname{Module}\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[(ff^{m + n*p - 1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m - 1)/2}*(b + a*(ff*x)^n)^p/x^{m + n*p}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, n\}, x\} \&\& \operatorname{IntegerQ}[(m - 1)/2] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \coth^5(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{x(b+ax^2)^2}{(1-x^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(a+b)}{(-1+x)^2} - \frac{a^2}{-1+x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{a(a + b) \operatorname{csch}^2(c + dx)}{d} - \frac{(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} + \frac{a^2 \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.23, size = 77, normalized size = 1.48

$$\frac{(a \cosh^2(c + dx) + b)^2 (-4a^2 \log(\sinh(c + dx)) + (a + b)^2 \operatorname{csch}^4(c + dx) + 4a(a + b) \operatorname{csch}^2(c + dx))}{d(a \cosh(2(c + dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]

[Out] -(((b + a*Cosh[c + d*x]^2)^2*(4*a*(a + b)*Csch[c + d*x]^2 + (a + b)^2*Csch[c + d*x]^4 - 4*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)]))^2))

fricas [B] time = 0.44, size = 1252, normalized size = 24.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d*x*sinh(d*x + c)^8 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^3 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 30*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 2*b^2)*sinh(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 - 10*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^3 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 - 15*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^4 - a^2*d*x + 3*(3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 - 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 - 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^5 - 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^6 - 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 - 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^5 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.34, size = 147, normalized size = 2.83

$$\frac{12 a^2 dx - 12 a^2 \log\left(\left|e^{2 dx+2 c}-1\right|\right) + \frac{25 a^2 e^{(8 dx+8 c)}-52 a^2 e^{(6 dx+6 c)}+48 a b e^{(6 dx+6 c)}+102 a^2 e^{(4 dx+4 c)}+48 b^2 e^{(4 dx+4 c)}-52 a^2 e^{(2 dx+2 c)}+b^2}{\left(e^{2 dx+2 c}-1\right)^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out]
$$-1/12*(12*a^2*d*x - 12*a^2*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1))) + (25*a^2*e^{(8*d*x + 8*c)} - 52*a^2*e^{(6*d*x + 6*c)} + 48*a*b*e^{(6*d*x + 6*c)} + 102*a^2*e^{(4*d*x + 4*c)} + 48*b^2*e^{(4*d*x + 4*c)} - 52*a^2*e^{(2*d*x + 2*c)} + 48*a*b*e^{(2*d*x + 2*c)} + 25*a^2)/(e^{(2*d*x + 2*c)} - 1)^4/d$$

maple [B] time = 0.32, size = 102, normalized size = 1.96

$$\frac{a^2 \ln(\sinh(dx+c))}{d} - \frac{a^2 (\coth^2(dx+c))}{2d} - \frac{a^2 (\coth^4(dx+c))}{4d} - \frac{ab (\cosh^2(dx+c))}{d \sinh(dx+c)^4} + \frac{ab}{2d \sinh(dx+c)^4} - \frac{b^2}{4d \sinh(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$a^2*\ln(\sinh(d*x+c))/d - 1/2*a^2*\coth(d*x+c)^2/d - 1/4*a^2*\coth(d*x+c)^4/d - 1/d*a*b/\sinh(d*x+c)^4*\cosh(d*x+c)^2 + 1/2/d*a*b/\sinh(d*x+c)^4 - 1/4/d/\sinh(d*x+c)^4*b^2$$

maxima [B] time = 0.45, size = 282, normalized size = 5.42

$$a^2 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{4(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$a^2*(x + c/d + \log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d + 4*(e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) + 4*a*b*(e^{(-2*d*x - 2*c)})/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1)) + e^{(-6*d*x - 6*c)}/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) - 4*b^2/(d*(e^{(d*x + c)} - e^{(-d*x - c)}))^4$$

mupad [B] time = 1.47, size = 207, normalized size = 3.98

$$\frac{a^2 \ln(e^{2c} e^{2dx} - 1)}{d} - \frac{4(a^2 + 2ab + b^2)}{d(6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1)} - \frac{4(2a^2 + 3ab + b^2)}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - a^2 x - \frac{b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2,x)

[Out]
$$(a^2*\log(\exp(2*c)*\exp(2*d*x) - 1))/d - (4*(2*a*b + a^2 + b^2))/(d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - (4*(3*a*b + 2*a^2 + b^2))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - a^2*x - (4*(a*b + a^2))/(d*(\exp(2*c + 2*d*x) - 1)) - (8*(2*a*b + a^2 + b^2))/(d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

3.122 $\int \coth^6(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=64

$$-\frac{(a^2 - b^2) \coth^3(c + dx)}{3d} - \frac{a^2 \coth(c + dx)}{d} + a^2 x - \frac{(a + b)^2 \coth^5(c + dx)}{5d}$$

[Out] $a^2 x - a^2 \coth(d x + c) / d - 1/3 (a^2 - b^2) \coth(d x + c)^3 / d - 1/5 (a + b)^2 \coth(d x + c)^5 / d$

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$-\frac{(a^2 - b^2) \coth^3(c + dx)}{3d} - \frac{a^2 \coth(c + dx)}{d} + a^2 x - \frac{(a + b)^2 \coth^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]

[Out] $a^2 x - (a^2 \operatorname{Coth}[c + d x]) / d - ((a^2 - b^2) \operatorname{Coth}[c + d x]^3) / (3 d) - ((a + b)^2 \operatorname{Coth}[c + d x]^5) / (5 d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^6(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b(1-x^2))^2}{x^6(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{x^6} + \frac{a^2-b^2}{x^4} + \frac{a^2}{x^2} - \frac{a^2}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \coth(c + dx)}{d} - \frac{(a^2 - b^2) \coth^3(c + dx)}{3d} - \frac{(a + b)^2 \coth^5(c + dx)}{5d} \\ &= a^2 x - \frac{a^2 \coth(c + dx)}{d} - \frac{(a^2 - b^2) \coth^3(c + dx)}{3d} - \frac{(a + b)^2 \coth^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 1.10, size = 256, normalized size = 4.00

$$\operatorname{csch}(c)\operatorname{csch}^5(c+dx)\left(180a^2\sinh(2c+dx)-140a^2\sinh(2c+3dx)-90a^2\sinh(4c+3dx)+46a^2\sinh(4c+5dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]

[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] - 75*a^2*d*x*Cosh[4*c + 3*d*x] - 15*a^2*d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] + 280*a^2*Sinh[d*x] + 120*a*b*Sinh[d*x] + 20*b^2*Sinh[d*x] + 180*a^2*Sinh[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 140*a^2*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] - 90*a^2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] + 46*a^2*Sinh[4*c + 5*d*x] + 12*a*b*Sinh[4*c + 5*d*x] - 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)

fricas [B] time = 0.42, size = 425, normalized size = 6.64

$$\frac{(23a^2 + 6ab - 2b^2)\cosh(dx+c)^5 + 5(23a^2 + 6ab - 2b^2)\cosh(dx+c)\sinh(dx+c)^4 - (15a^2dx + 23a^2 + 6ab - 2b^2)\sinh(dx+c)^5}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 5*(15*a^2*d*x - 2*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 23*a^2 + 6*a*b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 6*a*b + 4*b^2)*cosh(d*x + c) - 5*((15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^4 + 30*a^2*d*x - 3*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 46*a^2 + 12*a*b - 4*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))

giac [B] time = 0.37, size = 167, normalized size = 2.61

$$15a^2dx - \frac{2(45a^2e^{(8dx+8c)}+30abe^{(8dx+8c)}-90a^2e^{(6dx+6c)}+30b^2e^{(6dx+6c)}+140a^2e^{(4dx+4c)}+60abe^{(4dx+4c)}+10b^2e^{(4dx+4c)}-70a^2e^{(2dx+2c)}+10b^2e^{(2dx+2c)})}{(e^{(2dx+2c)}-1)^5}$$

$$15d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/15*(15*a^2*d*x - 2*(45*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) + 30*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 60*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) + 23*a^2 + 6*a*b - 2*b^2)/(e^(2*d*x + 2*c) - 1)^5/d

maple [B] time = 0.44, size = 163, normalized size = 2.55

$$a^2\left(dx+c-\operatorname{coth}(dx+c)-\frac{\operatorname{coth}^3(dx+c)}{3}-\frac{\operatorname{coth}^5(dx+c)}{5}\right)+2ab\left(-\frac{\cosh^3(dx+c)}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)}{15}\right)}{8}\right)$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x)

[Out] $\frac{1}{d} \left(a^2 \left(d*x+c - \coth(d*x+c) - \frac{1}{3} \coth(d*x+c)^3 - \frac{1}{5} \coth(d*x+c)^5 \right) + 2*a*b \left(-\frac{1}{2} \frac{1}{\sinh(d*x+c)^5} \cosh(d*x+c)^3 + \frac{3}{8} \frac{1}{\sinh(d*x+c)^5} \cosh(d*x+c) + \frac{3}{8} \left(-\frac{8}{15} - \frac{1}{5} \operatorname{csch}(d*x+c)^4 + \frac{4}{15} \operatorname{csch}(d*x+c)^2 \right) \coth(d*x+c) \right) + b^2 \left(-\frac{1}{4} \frac{1}{\sinh(d*x+c)^5} \cosh(d*x+c) - \frac{1}{4} \left(-\frac{8}{15} - \frac{1}{5} \operatorname{csch}(d*x+c)^4 + \frac{4}{15} \operatorname{csch}(d*x+c)^2 \right) \coth(d*x+c) \right) \right)$

maxima [B] time = 0.54, size = 613, normalized size = 9.58

$$\frac{1}{15} a^2 \left(15x + \frac{15c}{d} - \frac{2 \left(70e^{(-2dx-2c)} - 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} - 45e^{(-8dx-8c)} - 23 \right)}{d \left(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1 \right)} \right) + \frac{4}{15} b^2 \left(\frac{1}{d \left(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{15} a^2 \left(15*x + \frac{15*c}{d} - \frac{2 \left(70*e^{(-2*d*x - 2*c)} - 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} - 45*e^{(-8*d*x - 8*c)} - 23 \right)}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} \right) + \frac{4}{15} b^2 \left(\frac{1}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} \right) + \frac{5*e^{(-4*d*x - 4*c)}}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} + \frac{10*e^{(-6*d*x - 6*c)}}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} + \frac{15*e^{(-6*d*x - 6*c)}}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} - \frac{1}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} + \frac{4}{5} a*b \left(\frac{10*e^{(-4*d*x - 4*c)}}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} + \frac{5*e^{(-8*d*x - 8*c)}}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} + \frac{1}{d \left(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1 \right)} \right)$

mupad [B] time = 1.44, size = 511, normalized size = 7.98

$$a^2 x - \frac{2(5a^2+6ab+4b^2)}{15d} + \frac{4e^{2c+2dx}(b^2+2ab)}{5d} + \frac{2e^{4c+4dx}(3a^2+2ba)}{5d} - \frac{2(b^2+2ab)}{5d} + \frac{2e^{2c+2dx}(3a^2+2ba)}{5d} - \frac{2(b^2+2ab)}{5d} + \frac{6e^{4c+4dx}}{6e^{4c+4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2,x)

[Out] $a^2*x - \left(\frac{2 \left(6*a*b + 5*a^2 + 4*b^2 \right)}{15*d} + \frac{4*\exp(2*c + 2*d*x) \left(2*a*b + b^2 \right)}{5*d} + \frac{2*\exp(4*c + 4*d*x) \left(2*a*b + 3*a^2 \right)}{5*d} \right) / \left(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1 \right) - \left(\frac{2 \left(2*a*b + b^2 \right)}{5*d} + \frac{2*\exp(2*c + 2*d*x) \left(2*a*b + 3*a^2 \right)}{5*d} \right) / \left(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1 \right) - \left(\frac{2 \left(2*a*b + b^2 \right)}{5*d} + \frac{6*\exp(4*c + 4*d*x) \left(2*a*b + b^2 \right)}{5*d} + \frac{2*\exp(6*c + 6*d*x) \left(2*a*b + 3*a^2 \right)}{5*d} + \frac{2*\exp(2*c + 2*d*x) \left(6*a*b + 5*a^2 + 4*b^2 \right)}{5*d} \right) / \left(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1 \right) - \left(\frac{2 \left(2*a*b + 3*a^2 \right)}{5*d} + \frac{8*\exp(2*c + 2*d*x) \left(2*a*b + b^2 \right)}{5*d} + \frac{8*\exp(6*c + 6*d*x) \left(2*a*b + b^2 \right)}{5*d} + \frac{2*\exp(8*c + 8*d*x) \left(2*a*b + 3*a^2 \right)}{5*d} + \frac{4*\exp(4*c + 4*d*x) \left(6*a*b + 5*a^2 + 4*b^2 \right)}{5*d} \right) / \left(5*\exp(2*c + 2*d*x) - 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) - 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) - 1 \right) - \frac{2 \left(2*a*b + 3*a^2 \right)}{5*d \left(\exp(2*c + 2*d*x) - 1 \right)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.123 $\int \coth^7(c + dx) (a + b \operatorname{sech}^2(c + dx))^2 dx$

Optimal. Leaf size=86

$$\frac{a^2 \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} - \frac{a(a + b) \operatorname{csch}^2(c + dx)}{d} - \frac{\operatorname{csch}^6(c + dx) (a \cosh^2(c + dx) + b)^3}{6d(a + b)}$$

[Out] $-a*(a+b)*\operatorname{csch}(d*x+c)^2/d-1/4*(a+b)^2*\operatorname{csch}(d*x+c)^4/d-1/6*(b+a*\cosh(d*x+c)^2)^3*\operatorname{csch}(d*x+c)^6/(a+b)/d+a^2*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4138, 446, 78, 43}

$$\frac{a^2 \log(\sinh(c + dx))}{d} - \frac{(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} - \frac{a(a + b) \operatorname{csch}^2(c + dx)}{d} - \frac{\operatorname{csch}^6(c + dx) (a \cosh^2(c + dx) + b)^3}{6d(a + b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[c + d*x]^7*(a + b*\operatorname{Sech}[c + d*x]^2)^2, x]$

[Out] $-((a*(a + b)*\operatorname{Csch}[c + d*x]^2)/d) - ((a + b)^2*\operatorname{Csch}[c + d*x]^4)/(4*d) - ((b + a*\operatorname{Cosh}[c + d*x]^2)^3*\operatorname{Csch}[c + d*x]^6)/(6*(a + b)*d) + (a^2*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/d$

Rule 43

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 78

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow -\operatorname{Simp}[(b*e - a*f)*(c + d*x)^{n+1}*(e + f*x)^{p+1}]/(f*(p+1)*(c*f - d*e)), x] - \operatorname{Dist}[(a*d*f*(n+1) - b*(d*e*(n+1) + c*f*(p+1))]/(f*(p+1)*(c*f - d*e)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^{p+1}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 446

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n) - 1})*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m+1)/n]]

Rule 4138

$\operatorname{Int}[(a + b*\sec(e + f*x))^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Module}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[(ff^{m+n*p-1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(b + a*(ff*x)^n)^p/x^{m+n*p}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m-1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \coth^7(c+dx) (a+b\operatorname{sech}^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3(b+ax^2)^2}{(1-x^2)^4} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x(b+ax)^2}{(1-x)^4} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(b+a\cosh^2(c+dx))^3 \operatorname{csch}^6(c+dx)}{6(a+b)d} - \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(b+a\cosh^2(c+dx))^3 \operatorname{csch}^6(c+dx)}{6(a+b)d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(a+b)}{(-1+x)^2}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d} - \frac{(a+b)^2 \operatorname{csch}^4(c+dx)}{4d} - \frac{(b+a\cosh^2(c+dx))^3 \operatorname{csch}^6(c+dx)}{6(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 107, normalized size = 1.24

$$\frac{(a \cosh^2(c+dx) + b)^2 (3(3a^2 + 4ab + b^2) \operatorname{csch}^4(c+dx) - 12a^2 \log(\sinh(c+dx)) + 2(a+b)^2 \operatorname{csch}^6(c+dx) + 6(a+b) \operatorname{csch}^4(c+dx))}{3d(a \cosh(2(c+dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]

[Out] -1/3*((b + a*Cosh[c + d*x]^2)^2*(6*a*(3*a + 2*b)*Csch[c + d*x]^2 + 3*(3*a^2 + 4*a*b + b^2)*Csch[c + d*x]^4 + 2*(a + b)^2*Csch[c + d*x]^6 - 12*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.48, size = 2548, normalized size = 29.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*a^2*d*x*sinh(d*x + c)^12 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x + 3*a^2 + 2*a*b)*sinh(d*x + c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x - 90*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*d*x*cosh(d*x + c)^5 - 30*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^3 + (15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^6 - 315*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 15*a^2*d*x + 21*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^2 + 17*a^2 + 10*a*b + 2*b^2)*sinh(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 - 63*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 - 420*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x - 20*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 - 180*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^7 + 42*(15*a

$$\begin{aligned} &^2d*x - 12*a^2 + 4*b^2)*\cosh(d*x + c)^5 - 20*(15*a^2*d*x - 17*a^2 - 10*a*b \\ &- 2*b^2)*\cosh(d*x + c)^3 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*\cosh(d*x + c))* \\ &\sinh(d*x + c)^3 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*\cosh(d*x + c)^2 + 6*(33*a^2 \\ &*d*x*\cosh(d*x + c)^10 - 45*(3*a^2*d*x - 3*a^2 - 2*a*b)*\cosh(d*x + c)^8 + 14 \\ &*(15*a^2*d*x - 12*a^2 + 4*b^2)*\cosh(d*x + c)^6 - 10*(15*a^2*d*x - 17*a^2 - \\ &10*a*b - 2*b^2)*\cosh(d*x + c)^4 - 3*a^2*d*x + 3*(15*a^2*d*x - 12*a^2 + 4*b^ \\ &2)*\cosh(d*x + c)^2 + 3*a^2 + 2*a*b)*\sinh(d*x + c)^2 - 3*(a^2*\cosh(d*x + c)^ \\ &12 + 12*a^2*\cosh(d*x + c)*\sinh(d*x + c)^11 + a^2*\sinh(d*x + c)^12 - 6*a^2*c \\ &osh(d*x + c)^10 + 6*(11*a^2*\cosh(d*x + c)^2 - a^2)*\sinh(d*x + c)^10 + 15*a^ \\ &2*\cosh(d*x + c)^8 + 20*(11*a^2*\cosh(d*x + c)^3 - 3*a^2*\cosh(d*x + c))*\sinh(\\ &d*x + c)^9 + 15*(33*a^2*\cosh(d*x + c)^4 - 18*a^2*\cosh(d*x + c)^2 + a^2)*\sin \\ &h(d*x + c)^8 - 20*a^2*\cosh(d*x + c)^6 + 24*(33*a^2*\cosh(d*x + c)^5 - 30*a^2 \\ &*\cosh(d*x + c)^3 + 5*a^2*\cosh(d*x + c))*\sinh(d*x + c)^7 + 4*(231*a^2*\cosh(d \\ &*x + c)^6 - 315*a^2*\cosh(d*x + c)^4 + 105*a^2*\cosh(d*x + c)^2 - 5*a^2)*\sinh \\ &(d*x + c)^6 + 15*a^2*\cosh(d*x + c)^4 + 24*(33*a^2*\cosh(d*x + c)^7 - 63*a^2* \\ &cosh(d*x + c)^5 + 35*a^2*\cosh(d*x + c)^3 - 5*a^2*\cosh(d*x + c))*\sinh(d*x + \\ &c)^5 + 15*(33*a^2*\cosh(d*x + c)^8 - 84*a^2*\cosh(d*x + c)^6 + 70*a^2*\cosh(d* \\ &x + c)^4 - 20*a^2*\cosh(d*x + c)^2 + a^2)*\sinh(d*x + c)^4 - 6*a^2*\cosh(d*x + \\ &c)^2 + 20*(11*a^2*\cosh(d*x + c)^9 - 36*a^2*\cosh(d*x + c)^7 + 42*a^2*\cosh(d \\ &*x + c)^5 - 20*a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\ &6*(11*a^2*\cosh(d*x + c)^10 - 45*a^2*\cosh(d*x + c)^8 + 70*a^2*\cosh(d*x + c) \\ &^6 - 50*a^2*\cosh(d*x + c)^4 + 15*a^2*\cosh(d*x + c)^2 - a^2)*\sinh(d*x + c)^2 \\ &+ a^2 + 12*(a^2*\cosh(d*x + c)^11 - 5*a^2*\cosh(d*x + c)^9 + 10*a^2*\cosh(d*x \\ &+ c)^7 - 10*a^2*\cosh(d*x + c)^5 + 5*a^2*\cosh(d*x + c)^3 - a^2*\cosh(d*x + c \\ &))*\sinh(d*x + c))*\log(2*\sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12 \\ &*(3*a^2*d*x*\cosh(d*x + c)^11 - 5*(3*a^2*d*x - 3*a^2 - 2*a*b)*\cosh(d*x + c)^ \\ &9 + 2*(15*a^2*d*x - 12*a^2 + 4*b^2)*\cosh(d*x + c)^7 - 2*(15*a^2*d*x - 17*a^ \\ &2 - 10*a*b - 2*b^2)*\cosh(d*x + c)^5 + (15*a^2*d*x - 12*a^2 + 4*b^2)*\cosh(d* \\ &x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c))/(d*cos \\ &h(d*x + c)^12 + 12*d*\cosh(d*x + c)*\sinh(d*x + c)^11 + d*\sinh(d*x + c)^12 - \\ &6*d*\cosh(d*x + c)^10 + 6*(11*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^10 + 20*(\\ &11*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*d*\cosh(d*x + \\ &c)^8 + 15*(33*d*\cosh(d*x + c)^4 - 18*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^ \\ &8 + 24*(33*d*\cosh(d*x + c)^5 - 30*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\si \\ &nh(d*x + c)^7 - 20*d*\cosh(d*x + c)^6 + 4*(231*d*\cosh(d*x + c)^6 - 315*d*cos \\ &h(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 - 5*d)*\sinh(d*x + c)^6 + 24*(33*d*\cosh \\ &(d*x + c)^7 - 63*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 - 5*d*\cosh(d*x + \\ &c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c)^4 + 15*(33*d*\cosh(d*x + c)^8 - 84* \\ &d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^4 - 20*d*\cosh(d*x + c)^2 + d)*\sinh(d \\ &*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 - 36*d*\cosh(d*x + c)^7 + 42*d*\cosh(d*x \\ &+ c)^5 - 20*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 - 6*d*c \\ &osh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^10 - 45*d*\cosh(d*x + c)^8 + 70*d*cos \\ &h(d*x + c)^6 - 50*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + \\ &c)^2 + 12*(d*\cosh(d*x + c)^11 - 5*d*\cosh(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 \\ &- 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + \\ &c) + d) \end{aligned}$$

giac [B] time = 0.46, size = 216, normalized size = 2.51

$$60 a^2 dx - 60 a^2 \log \left(\left| e^{(2dx+2c)} - 1 \right| \right) + \frac{147 a^2 e^{(12dx+12c)} - 522 a^2 e^{(10dx+10c)} + 240 a b e^{(10dx+10c)} + 1485 a^2 e^{(8dx+8c)} + 240 b^2 e^{(8dx+8c)} - 1580 a^2 e^{(6dx+6c)} + 160 a b e^{(6dx+6c)} + 1485 a^2 e^{(4dx+4c)}}{60 d}$$

60 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/60*(60*a^2*d*x - 60*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^2*e^(12*d*x + 12*c) - 522*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1485*a^2*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) - 1580*a^2*e^(6*d*x + 6*c) + 800*a*b*e^(6*d*x + 6*c) + 160*b^2*e^(6*d*x + 6*c) + 1485*a^2*e^(4*d*x + 4*c)

$$+ 240*b^2*e^(4*d*x + 4*c) - 522*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2*c) - 1)^6)/d$$

maple [A] time = 0.34, size = 164, normalized size = 1.91

$$\frac{a^2 \ln(\sinh(dx + c))}{d} - \frac{a^2 (\coth^2(dx + c))}{2d} - \frac{a^2 (\coth^4(dx + c))}{4d} - \frac{a^2 (\coth^6(dx + c))}{6d} - \frac{ab (\cosh^4(dx + c))}{d \sinh(dx + c)^6} + \frac{ab (\cos}{d \sinh$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x)

[Out] a^2*ln(sinh(d*x+c))/d-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/6*a^2*coth(d*x+c)^6/d-1/d*a*b/sinh(d*x+c)^6*cosh(d*x+c)^4+1/d*a*b/sinh(d*x+c)^6*cosh(d*x+c)^2-1/3/d*a*b/sinh(d*x+c)^6-1/4/d*b^2/sinh(d*x+c)^6*cosh(d*x+c)^2+1/12/d*b^2/sinh(d*x+c)^6

maxima [B] time = 0.41, size = 696, normalized size = 8.09

$$\frac{1}{3} a^2 \left(3x + \frac{3c}{d} + \frac{3 \log(e^{-dx-c} + 1)}{d} + \frac{3 \log(e^{-dx-c} - 1)}{d} + \frac{2(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 9e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)} \right) + \frac{4/3 a b (3e^{(-2dx-2c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)) + 10e^{(-6dx-6c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)) + 3e^{(-10dx-10c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)))}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1))} + \frac{4/3 b^2 (3e^{(-4dx-4c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)) + 2e^{(-6dx-6c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)) + 3e^{(-8dx-8c)} / (d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)))}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 4/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 10*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3*e^(-10*d*x - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)))

mupad [B] time = 1.53, size = 377, normalized size = 4.38

$$\frac{a^2 \ln(e^{2c} e^{2dx} - 1)}{d} - \frac{32(a^2 + 2ab + b^2)}{d(5e^{2c+2dx} - 10e^{4c+4dx} + 10e^{6c+6dx} - 5e^{8c+8dx} + e^{10c+10dx} - 1)} - \frac{2(3a^2 + 2ba)}{d(e^{2c+2dx} - 1)} - \frac{2(2ab + a^2 + b^2)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2,x)

[Out] (a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(2*a*b + a^2 + b^2))/(d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (2*(2*a*b + 3*a^2))/(d*(exp(2*c + 2*d*x) - 1)) - (32*(2*a*b + a^2 + b^2))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (2*(10*a*b + 9*a^2 + 2*b^2))/(d*(exp(4*c + 4*d*x) - 1))

$$2*\exp(2*c + 2*d*x) + 1)) - (8*(20*a*b + 13*a^2 + 7*b^2))/(3*d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (4*(20*a*b + 11*a^2 + 9*b^2))/(d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - a^2*x$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

3.124 $\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^4(c + dx) dx$

Optimal. Leaf size=110

$$\frac{a^3 \tanh^3(c + dx)}{3d} - \frac{a^3 \tanh(c + dx)}{d} + a^3 x + \frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} - \frac{b^2(3a + 2b) \tanh^7(c + dx)}{7d} + \frac{b^3 \tanh^9(c + dx)}{9d}$$

[Out] $a^3 x - a^3 \tanh(d x + c) / d - 1/3 a^3 \tanh(d x + c)^3 / d + 1/5 b (3 a^2 + 3 a b + b^2) \tanh^5(d x + c) / d - 1/7 b^2 (3 a + 2 b) \tanh^7(d x + c) / d + 1/9 b^3 \tanh^9(d x + c) / d$

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 206}

$$\frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} - \frac{a^3 \tanh^3(c + dx)}{3d} - \frac{a^3 \tanh(c + dx)}{d} + a^3 x - \frac{b^2(3a + 2b) \tanh^7(c + dx)}{7d} + \frac{b^3 \tanh^9(c + dx)}{9d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]

[Out] $a^3 x - (a^3 \tanh[c + d x]) / d - (a^3 \tanh[c + d x]^3) / (3 d) + (b (3 a^2 + 3 a b + b^2) \tanh[c + d x]^5) / (5 d) - (b^2 (3 a + 2 b) \tanh[c + d x]^7) / (7 d) + (b^3 \tanh[c + d x]^9) / (9 d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^4(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4(a+b(1-x^2))^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (-a^3 - a^3 x^2 + b(3a^2 + 3ab + b^2)x^4 - b^2(3a + 2b)x^6 + b^3 x^8) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^3 \tanh(c + dx)}{d} - \frac{a^3 \tanh^3(c + dx)}{3d} + \frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} \\ &= a^3 x - \frac{a^3 \tanh(c + dx)}{d} - \frac{a^3 \tanh^3(c + dx)}{3d} + \frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 5.94, size = 301, normalized size = 2.74

$$8\operatorname{sech}^9(c + dx) \left(a \cosh^2(c + dx) + b \right)^3 \left(315a^3 dx \cosh^9(c + dx) + 3b \left(63a^2 - 72ab + b^2 \right) \tanh(c) \cosh^5(c + dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]

[Out] (8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^9*(315*a^3*d*x*Cosh[c + d*x]^9 + 35*b^3*Sech[c]*Sinh[d*x] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + (105*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*Cosh[c + d*x]^8*Sech[c]*Sinh[d*x] + 35*b^3*Cosh[c + d*x]*Tanh[c] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^3*Tanh[c] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^5*Tanh[c] + (105*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(315*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.43, size = 1323, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="fricas")

[Out] 1/315*((315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*sinh(d*x + c)^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^7 - 9*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3 + 4*(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 - 9*(14*(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 700*a^3 + 84*a^2*b + 204*a*b^2 - 32*b^3 + 21*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 35*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 20*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 3*(28*(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 2660*a^3 - 252*a^2*b - 252*a*b^2 + 896*b^3 + 120*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^7 + 21*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 40*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 28*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 126*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c) - 9*((420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^8 + 7*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 20*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 420*a^3 - 126*a^2*b - 336*a*b^2 - 672*b^3 + 28*(95*a^3 - 9*a^2*b - 9*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(d*x + c))

giac [B] time = 0.30, size = 472, normalized size = 4.29

$$315 a^3 dx + \frac{2(630 a^3 e^{(16 dx+16 c)} - 945 a^2 b e^{(16 dx+16 c)} + 4410 a^3 e^{(14 dx+14 c)} - 3780 a^2 b e^{(14 dx+14 c)} - 1890 a b^2 e^{(14 dx+14 c)} + 13650 a^3 e^{(12 dx+12 c)} - 7560 a^2 b e^{(12 dx+12 c)} - 1890 a b^2 e^{(12 dx+12 c)} - 1680 b^3 e^{(12 dx+12 c)} + 24570 a^3 e^{(10 dx+10 c)} - 11340 a^2 b e^{(10 dx+10 c)} - 1890 a b^2 e^{(10 dx+10 c)} + 2520 b^3 e^{(10 dx+10 c)} + 28350 a^3 e^{(8 dx+8 c)} - 12474 a^2 b e^{(8 dx+8 c)} - 4914 a b^2 e^{(8 dx+8 c)} - 3528 b^3 e^{(8 dx+8 c)} + 21630 a^3 e^{(6 dx+6 c)} - 8316 a^2 b e^{(6 dx+6 c)} - 2646 a b^2 e^{(6 dx+6 c)} + 1008 b^3 e^{(6 dx+6 c)} + 10710 a^3 e^{(4 dx+4 c)} - 3024 a^2 b e^{(4 dx+4 c)} - 54 a b^2 e^{(4 dx+4 c)} - 288 b^3 e^{(4 dx+4 c)} + 3150 a^3 e^{(2 dx+2 c)} - 756 a^2 b e^{(2 dx+2 c)} - 486 a b^2 e^{(2 dx+2 c)} - 72 b^3 e^{(2 dx+2 c)} + 420 a^3 - 189 a^2 b - 54 a b^2 - 8 b^3)}{(e^{(2 dx+2 c)} + 1)^9}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="giac")

[Out] 1/315*(315*a^3*d*x + 2*(630*a^3*e^(16*d*x + 16*c) - 945*a^2*b*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) - 3780*a^2*b*e^(14*d*x + 14*c) - 1890*a*b^2*e^(14*d*x + 14*c) + 13650*a^3*e^(12*d*x + 12*c) - 7560*a^2*b*e^(12*d*x + 12*c) - 1890*a*b^2*e^(12*d*x + 12*c) - 1680*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x + 10*c) - 11340*a^2*b*e^(10*d*x + 10*c) - 1890*a*b^2*e^(10*d*x + 10*c) + 2520*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) - 12474*a^2*b*e^(8*d*x + 8*c) - 4914*a*b^2*e^(8*d*x + 8*c) - 3528*b^3*e^(8*d*x + 8*c) + 21630*a^3*e^(6*d*x + 6*c) - 8316*a^2*b*e^(6*d*x + 6*c) - 2646*a*b^2*e^(6*d*x + 6*c) + 1008*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) - 3024*a^2*b*e^(4*d*x + 4*c) - 54*a*b^2*e^(4*d*x + 4*c) - 288*b^3*e^(4*d*x + 4*c) + 3150*a^3*e^(2*d*x + 2*c) - 756*a^2*b*e^(2*d*x + 2*c) - 486*a*b^2*e^(2*d*x + 2*c) - 72*b^3*e^(2*d*x + 2*c) + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)/(e^(2*d*x + 2*c) + 1)^9)/d

maple [B] time = 0.55, size = 274, normalized size = 2.49

$$a^3 \left(dx + c - \tanh(dx + c) - \frac{(\tanh^3(dx + c))}{3} \right) + 3a^2b \left(-\frac{\sinh^3(dx + c)}{2 \cosh(dx + c)^5} - \frac{3 \sinh(dx + c)}{8 \cosh(dx + c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx + c)^4}{5} + \frac{4 \operatorname{sech}(dx + c)^2}{15} \right) \tanh(dx + c)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x)

[Out] 1/d*(a^3*(d*x+c-tanh(d*x+c))-1/3*tanh(d*x+c)^3)+3*a^2*b*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/6*sinh(d*x+c)^3/cosh(d*x+c)^9-1/16*sinh(d*x+c)/cosh(d*x+c)^9+1/16*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.39, size = 1453, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="maxima")

[Out] 3/5*a^2*b*tanh(d*x + c)^5/d + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 16/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 126*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))

$$\begin{aligned}
& (-8dx - 8c) + 126e^{(-10dx - 10c)} + 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} + 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} + 1)) + 441e^{(-8dx - 8c)} / (d(9e^{(-2dx - 2c)} + 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} + 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} + 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} + 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} + 1)) - \\
& 315e^{(-10dx - 10c)} / (d(9e^{(-2dx - 2c)} + 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} + 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} + 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} + 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} + 1)) + 210e^{(-12dx - 12c)} / (d(9e^{(-2dx - 2c)} + 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} + 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} + 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} + 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} + 1)) + 1 / (d(9e^{(-2dx - 2c)} + 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} + 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} + 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} + 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} + 1))) + 12/35ab^2(7e^{(-2dx - 2c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) - 14e^{(-4dx - 4c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) + 70e^{(-6dx - 6c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) - 35e^{(-8dx - 8c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) + 35e^{(-10dx - 10c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) + 1 / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)))
\end{aligned}$$

mupad [B] time = 1.62, size = 1834, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\tanh(c + dx))^4 (a + b/\cosh(c + dx))^2 dx, x$

[Out]
$$\begin{aligned}
& ((3ab^2 + 13a^3 + 16b^3)/(63d) + (10\exp(4c + 4dx))(3ab^2 + 13a^3 + 16b^3)/(63d) + (20\exp(6c + 6dx))(6ab^2 + 3a^2b + 8a^3 - 4b^3)/(63d) - (2\exp(2c + 2dx))(8ab^2 + 3a^2b - 10a^3 + 16b^3)/(21d) - (5\exp(8c + 8dx))(ab^2 - a^3)/(3d) - (2\exp(10c + 10dx))(3a^2b - 2a^3)/(9d)) / (6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1) - ((2\exp(2c + 2dx))(ab^2 - a^3)/(3d) - (2(6ab^2 + 3a^2b + 8a^3 - 4b^3))/(63d) + (2\exp(4c + 4dx))(3a^2b - 2a^3)/(9d)) / (3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1) + ((3ab^2 + 13a^3 + 16b^3)/(63d) + (2\exp(2c + 2dx))(6ab^2 + 3a^2b + 8a^3 - 4b^3)/(21d) - (\exp(4c + 4dx))(ab^2 - a^3)/d - (2\exp(6c + 6dx))(3a^2b - 2a^3)/(9d)) / (4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1) - ((ab^2 - a^3)/(3d) + (2\exp(2c + 2dx))(3a^2b - 2a^3)/(9d)) / (2\exp(2c + 2dx) + \exp(4c + 4dx) + 1) + a^3x + ((2(6ab^2 + 3a^2b + 8a^3 - 4b^3))/(63d) + (2\exp(2c + 2dx))(3ab^2 + 13a^3 + 16b^3))/(21d) + (20\exp(6c + 6dx))(3ab^2 + 13a^3 + 16b^3)/(63d) + (10\exp(8c + 8dx))(6ab^2 + 3a^2b + 8a^3 - 4b^3)/(21d) - (2\exp(4c + 4dx))(8ab^2 + 3a^2b - 10a^3 + 16b^3)/(7d) - (2\exp(10c + 10dx))(ab^2 - a^3)/d - (2\exp(12c + 12dx))(3a^2b - 2a^3)/(9d)) / (7\exp(2c + 2dx) + 21\exp(4c + 4dx) + 35\exp(6c + 6dx) + 35\exp(8c + 8dx) + 21\exp(10c + 10dx) + 7\exp(12c + 12dx) + \exp(14c + 14dx) + 1) - ((ab^2 - a^3)/(3d) - (\exp(4c + 4dx))(3ab^2 + 13a^3 + 16b^3)/(3d) - (5\exp(8c + 8dx))(3ab^2 + 1
\end{aligned}$$

```

3*a^3 + 16*b^3))/(9*d) - (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4
*b^3))/(9*d) - (2*exp(10*c + 10*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(
3*d) + (2*exp(6*c + 6*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(3*d) + (
7*exp(12*c + 12*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(14*c + 14*d*x)*(3*a^2*b
- 2*a^3))/(9*d))/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6
*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x)
+ 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((2*(3*a^2*b - 2*a^3))/
(9*d) - (8*exp(6*c + 6*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(10*
c + 10*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(4*c + 4*d*x)*(6*a*b
^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(9*d) - (8*exp(12*c + 12*d*x)*(6*a*b^2 + 3*a
^2*b + 8*a^3 - 4*b^3))/(9*d) + (4*exp(8*c + 8*d*x)*(8*a*b^2 + 3*a^2*b - 10*
a^3 + 16*b^3))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) + (8*exp(14
*c + 14*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(16*c + 16*d*x)*(3*a^2*b - 2*a^3)
))/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) +
126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) + 36*
exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - ((2*(
8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(105*d) - (4*exp(2*c + 2*d*x)*(3*a*b^
2 + 13*a^3 + 16*b^3))/(63*d) - (4*exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 8*a
^3 - 4*b^3))/(21*d) + (4*exp(6*c + 6*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(8*c
+ 8*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*
x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
2*(3*a^2*b - 2*a^3))/(9*d*(exp(2*c + 2*d*x) + 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**4,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**4, x)

3.125 $\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^3(c + dx) dx$

Optimal. Leaf size=103

$$\frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} + \frac{\operatorname{sech}^8(c + dx) (a \cosh^2(c + dx) + b)^4}{8bd} - \frac{b^3 \operatorname{sech}^6(c + dx)}{6d}$$

[Out] $a^3 \ln(\cosh(dx+c))/d - 3/2 a^2 b \operatorname{sech}(dx+c)^2/d - 3/4 a^2 b^2 \operatorname{sech}(dx+c)^4/d - 1/6 b^3 \operatorname{sech}(dx+c)^6/d + 1/8 (b+a \cosh(dx+c)^2)^4 \operatorname{sech}(dx+c)^8/b/d$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4138, 446, 78, 43}

$$-\frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} + \frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} + \frac{\operatorname{sech}^8(c + dx) (a \cosh^2(c + dx) + b)^4}{8bd} - \frac{b^3 \operatorname{sech}^6(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]

[Out] $(a^3 \operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d - (3a^2 b \operatorname{Sech}[c + d*x]^2)/(2*d) - (3a^2 b^2 \operatorname{Sech}[c + d*x]^4)/(4*d) - (b^3 \operatorname{Sech}[c + d*x]^6)/(6*d) + ((b + a \operatorname{Cosh}[c + d*x]^2)^4 \operatorname{Sech}[c + d*x]^8)/(8*b*d)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n]))))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^3(c + dx) dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^9} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)(b+ax)^3}{x^5} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= \frac{(b + a \cosh^2(c + dx))^4 \operatorname{sech}^8(c + dx)}{8bd} + \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{x^4} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= \frac{(b + a \cosh^2(c + dx))^4 \operatorname{sech}^8(c + dx)}{8bd} + \frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4} + \frac{3ab^2}{x^3} + \frac{3a^2b}{x^2} + \frac{a^3}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= \frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} - \frac{b^3 \operatorname{sech}^6(c + dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 0.75, size = 128, normalized size = 1.24

$$\frac{\cosh^6(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 (24a^3 \log(\cosh(c + dx)) + 12a^2(a - 3b) \operatorname{sech}^2(c + dx) + 4b^2(3a - b) \operatorname{sech}^6(c + dx) + 3b^3 \operatorname{sech}^8(c + dx))}{3d(a \cosh(2c + 2dx) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]

[Out] (Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(24*a^3*Log[Cosh[c + d*x]] + 12*a^2*(a - 3*b)*Sech[c + d*x]^2 + 18*a*(a - b)*b*Sech[c + d*x]^4 + 4*(3*a - b)*b^2*Sech[c + d*x]^6 + 3*b^3*Sech[c + d*x]^8))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)

fricas [B] time = 0.51, size = 4658, normalized size = 45.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="fricas")

[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^16 + 48*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^15 + 3*a^3*d*x*sinh(d*x + c)^16 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^14 + 6*(60*a^3*d*x*cosh(d*x + c)^2 + 4*a^3*d*x - a^3 + 3*a^2*b)*sinh(d*x + c)^14 + 84*(20*a^3*d*x*cosh(d*x + c)^3 + (4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^13 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^12 + 6*(910*a^3*d*x*cosh(d*x + c)^4 + 14*a^3*d*x - 6*a^3 + 12*a^2*b + 6*a*b^2 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24*(546*a^3*d*x*cosh(d*x + c)^5 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^10 + 2*(12012*a^3*d*x*cosh(d*x + c)^6 + 84*a^3*d*x + 3003*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3 + 396*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(8580*a^3*d*x*cosh(d*x + c)^7 + 3003*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^5 + 660*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^8 + 2*(19305*a^3*d*x*cosh(d*x + c)^8 + 9009*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^6 + 105*a^3*d*x + 2970*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3 + 45

$$\begin{aligned}
&*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + c)^2*\sinh \\
&(d*x + c)^8 + 16*(2145*a^3*d*x*\cosh(d*x + c)^9 + 1287*(4*a^3*d*x - a^3 + 3* \\
&a^2*b)*\cosh(d*x + c)^7 + 594*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cosh(d \\
&*x + c)^5 + 15*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d* \\
&x + c)^3 + (105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*\cosh(d*x + \\
&c))*\sinh(d*x + c)^7 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^ \\
&3)*\cosh(d*x + c)^6 + 2*(12012*a^3*d*x*\cosh(d*x + c)^10 + 9009*(4*a^3*d*x - \\
&a^3 + 3*a^2*b)*\cosh(d*x + c)^8 + 5544*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^ \\
&2)*\cosh(d*x + c)^6 + 84*a^3*d*x + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24* \\
&a*b^2 + 16*b^3)*\cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3 + 2 \\
&8*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*\cosh(d*x + c)^2)*\si \\
&nh(d*x + c)^6 + 4*(3276*a^3*d*x*\cosh(d*x + c)^11 + 3003*(4*a^3*d*x - a^3 + \\
&3*a^2*b)*\cosh(d*x + c)^9 + 2376*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cos \\
&h(d*x + c)^7 + 126*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cos \\
&h(d*x + c)^5 + 28*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*\cos \\
&h(d*x + c)^3 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(\\
&d*x + c))*\sinh(d*x + c)^5 + 3*a^3*d*x + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3 \\
&*a*b^2)*\cosh(d*x + c)^4 + 2*(2730*a^3*d*x*\cosh(d*x + c)^12 + 3003*(4*a^3*d* \\
&x - a^3 + 3*a^2*b)*\cosh(d*x + c)^10 + 2970*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3 \\
&*a*b^2)*\cosh(d*x + c)^8 + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + \\
&16*b^3)*\cosh(d*x + c)^6 + 42*a^3*d*x + 70*(105*a^3*d*x - 60*a^3 + 72*a^2*b \\
&+ 12*a*b^2 - 16*b^3)*\cosh(d*x + c)^4 - 18*a^3 + 36*a^2*b + 18*a*b^2 + 15*(8 \\
&4*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + c)^2)*\sinh(d* \\
&x + c)^4 + 8*(210*a^3*d*x*\cosh(d*x + c)^13 + 273*(4*a^3*d*x - a^3 + 3*a^2*b \\
&)*\cosh(d*x + c)^11 + 330*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cosh(d*x + \\
&c)^9 + 30*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + \\
&c)^7 + 14*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*\cosh(d*x + \\
&c)^5 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + c) \\
&^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c) \\
&^3 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*\cosh(d*x + c)^2 + 2*(180*a^3*d*x*\cosh(d* \\
&x + c)^14 + 273*(4*a^3*d*x - a^3 + 3*a^2*b)*\cosh(d*x + c)^12 + 396*(7*a^3*d \\
&*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cosh(d*x + c)^10 + 45*(84*a^3*d*x - 45*a^3 \\
&+ 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + c)^8 + 28*(105*a^3*d*x - 60*a^3 \\
&+ 72*a^2*b + 12*a*b^2 - 16*b^3)*\cosh(d*x + c)^6 + 12*a^3*d*x + 15*(84*a^3*d \\
&*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*\cosh(d*x + c)^4 - 3*a^3 + 9*a^2 \\
&*b + 36*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + \\
&c)^2 - 3*(a^3*\cosh(d*x + c)^16 + 16*a^3*\cosh(d*x + c)*\sinh(d*x + c)^15 + a \\
&^3*\sinh(d*x + c)^16 + 8*a^3*\cosh(d*x + c)^14 + 28*a^3*\cosh(d*x + c)^12 + 8* \\
&(15*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^14 + 112*(5*a^3*\cosh(d*x + c)^ \\
&3 + a^3*\cosh(d*x + c))*\sinh(d*x + c)^13 + 56*a^3*\cosh(d*x + c)^10 + 28*(65* \\
&a^3*\cosh(d*x + c)^4 + 26*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^12 + 112* \\
&(39*a^3*\cosh(d*x + c)^5 + 26*a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(d*x + c))*\sin \\
&h(d*x + c)^11 + 70*a^3*\cosh(d*x + c)^8 + 56*(143*a^3*\cosh(d*x + c)^6 + 143* \\
&a^3*\cosh(d*x + c)^4 + 33*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^10 + 16*(\\
&715*a^3*\cosh(d*x + c)^7 + 1001*a^3*\cosh(d*x + c)^5 + 385*a^3*\cosh(d*x + c)^ \\
&3 + 35*a^3*\cosh(d*x + c))*\sinh(d*x + c)^9 + 56*a^3*\cosh(d*x + c)^6 + 2*(643 \\
&5*a^3*\cosh(d*x + c)^8 + 12012*a^3*\cosh(d*x + c)^6 + 6930*a^3*\cosh(d*x + c)^ \\
&4 + 1260*a^3*\cosh(d*x + c)^2 + 35*a^3)*\sinh(d*x + c)^8 + 16*(715*a^3*\cosh(d \\
&*x + c)^9 + 1716*a^3*\cosh(d*x + c)^7 + 1386*a^3*\cosh(d*x + c)^5 + 420*a^3*\c \\
&osh(d*x + c)^3 + 35*a^3*\cosh(d*x + c))*\sinh(d*x + c)^7 + 28*a^3*\cosh(d*x + \\
&c)^4 + 56*(143*a^3*\cosh(d*x + c)^10 + 429*a^3*\cosh(d*x + c)^8 + 462*a^3*\cos \\
&h(d*x + c)^6 + 210*a^3*\cosh(d*x + c)^4 + 35*a^3*\cosh(d*x + c)^2 + a^3)*\sinh \\
&(d*x + c)^6 + 112*(39*a^3*\cosh(d*x + c)^11 + 143*a^3*\cosh(d*x + c)^9 + 198* \\
&a^3*\cosh(d*x + c)^7 + 126*a^3*\cosh(d*x + c)^5 + 35*a^3*\cosh(d*x + c)^3 + 3* \\
&a^3*\cosh(d*x + c))*\sinh(d*x + c)^5 + 8*a^3*\cosh(d*x + c)^2 + 28*(65*a^3*\cos \\
&h(d*x + c)^12 + 286*a^3*\cosh(d*x + c)^10 + 495*a^3*\cosh(d*x + c)^8 + 420*a^ \\
&3*\cosh(d*x + c)^6 + 175*a^3*\cosh(d*x + c)^4 + 30*a^3*\cosh(d*x + c)^2 + a^3) \\
&*\sinh(d*x + c)^4 + 112*(5*a^3*\cosh(d*x + c)^13 + 26*a^3*\cosh(d*x + c)^11 + \\
&55*a^3*\cosh(d*x + c)^9 + 60*a^3*\cosh(d*x + c)^7 + 35*a^3*\cosh(d*x + c)^5 +
\end{aligned}$$

$$\begin{aligned}
& 10a^3 \cosh(dx + c)^3 + a^3 \cosh(dx + c) \sinh(dx + c)^3 + a^3 + 8(15a^3 \cosh(dx + c)^{14} + 91a^3 \cosh(dx + c)^{12} + 231a^3 \cosh(dx + c)^{10} + \\
& 315a^3 \cosh(dx + c)^8 + 245a^3 \cosh(dx + c)^6 + 105a^3 \cosh(dx + c)^4 + 21a^3 \cosh(dx + c)^2 + a^3) \sinh(dx + c)^2 + 16(a^3 \cosh(dx + c)^{15} \\
& + 7a^3 \cosh(dx + c)^{13} + 21a^3 \cosh(dx + c)^{11} + 35a^3 \cosh(dx + c)^9 + 35a^3 \cosh(dx + c)^7 + 21a^3 \cosh(dx + c)^5 + 7a^3 \cosh(dx + c)^3 \\
& + a^3 \cosh(dx + c)) \sinh(dx + c) \log(2 \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 4(12a^3 dx \cosh(dx + c)^{15} + 21(4a^3 dx - a^3 + 3a^2 b) \cosh(dx + c)^{13} + \\
& 36(7a^3 dx - 3a^3 + 6a^2 b + 3ab^2) \cosh(dx + c)^{11} + 5(84a^3 dx - 45a^3 + 63a^2 b + 24ab^2 + 16b^3) \cosh(dx + c)^9 + 4(105a^3 dx - 60a^3 + 72a^2 b + 12ab^2 - 16b^3) \cosh(dx + c)^7 + \\
& 3(84a^3 dx - 45a^3 + 63a^2 b + 24ab^2 + 16b^3) \cosh(dx + c)^5 + 12(7a^3 dx - 3a^3 + 6a^2 b + 3ab^2) \cosh(dx + c)^3 + 3(4a^3 dx - a^3 + 3a^2 b) \cosh(dx + c) \sinh(dx + c)) / (d \cosh(dx + c)^{16} + \\
& 16d \cosh(dx + c) \sinh(dx + c)^{15} + d \sinh(dx + c)^{16} + 8d \cosh(dx + c)^{14} + 8(15d \cosh(dx + c)^2 + d) \sinh(dx + c)^{14} + 112(5d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^{13} + \\
& 28d \cosh(dx + c)^{12} + 28(65d \cosh(dx + c)^4 + 26d \cosh(dx + c)^2 + d) \sinh(dx + c)^{12} + 112(39d \cosh(dx + c)^5 + 26d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^{11} + \\
& 56d \cosh(dx + c)^{10} + 56(143d \cosh(dx + c)^6 + 143d \cosh(dx + c)^4 + 33d \cosh(dx + c)^2 + d) \sinh(dx + c)^{10} + 16(715d \cosh(dx + c)^7 + 1001d \cosh(dx + c)^5 + 385d \cosh(dx + c)^3 + 35d \cosh(dx + c)) \sinh(dx + c)^9 + \\
& 70d \cosh(dx + c)^8 + 2(6435d \cosh(dx + c)^8 + 12012d \cosh(dx + c)^6 + 6930d \cosh(dx + c)^4 + 1260d \cosh(dx + c)^2 + 35d) \sinh(dx + c)^8 + 16(715d \cosh(dx + c)^9 + 1716d \cosh(dx + c)^7 + 1386d \cosh(dx + c)^5 + 420d \cosh(dx + c)^3 + 35d \cosh(dx + c)) \sinh(dx + c)^7 + \\
& 56d \cosh(dx + c)^6 + 56(143d \cosh(dx + c)^{10} + 429d \cosh(dx + c)^8 + 462d \cosh(dx + c)^6 + 210d \cosh(dx + c)^4 + 35d \cosh(dx + c)^2 + d) \sinh(dx + c)^6 + 112(39d \cosh(dx + c)^{11} + 143d \cosh(dx + c)^9 + 198d \cosh(dx + c)^7 + 126d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^5 + 28d \cosh(dx + c)^4 + 28(65d \cosh(dx + c)^{12} + 286d \cosh(dx + c)^{10} + 495d \cosh(dx + c)^8 + 420d \cosh(dx + c)^6 + 175d \cosh(dx + c)^4 + 30d \cosh(dx + c)^2 + d) \sinh(dx + c)^4 + 112(5d \cosh(dx + c)^{13} + 26d \cosh(dx + c)^{11} + 55d \cosh(dx + c)^9 + 60d \cosh(dx + c)^7 + 35d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^3 + 8d \cosh(dx + c)^2 + 8(15d \cosh(dx + c)^{14} + 91d \cosh(dx + c)^{12} + 231d \cosh(dx + c)^{10} + 315d \cosh(dx + c)^8 + 245d \cosh(dx + c)^6 + 105d \cosh(dx + c)^4 + 21d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 16(d \cosh(dx + c)^{15} + 7d \cosh(dx + c)^{13} + 21d \cosh(dx + c)^{11} + 35d \cosh(dx + c)^9 + 35d \cosh(dx + c)^7 + 21d \cosh(dx + c)^5 + 7d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)
\end{aligned}$$

giac [B] time = 0.27, size = 384, normalized size = 3.73

$$840 a^3 dx - 840 a^3 \log(e^{(2dx+2c)} + 1) + \frac{2283 a^3 e^{(16dx+16c)} + 16584 a^3 e^{(14dx+14c)} + 5040 a^2 b e^{(14dx+14c)} + 53844 a^3 e^{(12dx+12c)} + 20160 a^2 b e^{(12dx+12c)}}{e^{(2dx+2c)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(dx+c)^2)^3*tanh(dx+c)^3,x, algorithm="giac")

[Out] -1/840*(840*a^3*dx - 840*a^3*log(e^(2*dx + 2*c) + 1) + (2283*a^3*e^(16*d*x + 16*c) + 16584*a^3*e^(14*d*x + 14*c) + 5040*a^2*b*e^(14*d*x + 14*c) + 53844*a^3*e^(12*d*x + 12*c) + 20160*a^2*b*e^(12*d*x + 12*c) + 10080*a*b^2*e^(12*d*x + 12*c) + 102648*a^3*e^(10*d*x + 10*c) + 35280*a^2*b*e^(10*d*x + 10*c) + 13440*a*b^2*e^(10*d*x + 10*c) + 8960*b^3*e^(10*d*x + 10*c) + 126210*a^3*e^(8*d*x + 8*c) + 40320*a^2*b*e^(8*d*x + 8*c) + 6720*a*b^2*e^(8*d*x + 8*c) - 8960*b^3*e^(8*d*x + 8*c) + 102648*a^3*e^(6*d*x + 6*c) + 35280*a^2*b*e^(6*d*x + 6*c) + 13440*a*b^2*e^(6*d*x + 6*c) + 8960*b^3*e^(6*d*x + 6*c) + 53844*a^3*e^(4*d*x + 4*c) + 20160*a^2*b*e^(4*d*x + 4*c) + 10080*a*b^2*e^(4*d*x

$$+ 4*c) + 16584*a^3*e^{(2*d*x + 2*c)} + 5040*a^2*b*e^{(2*d*x + 2*c)} + 2283*a^3)/(e^{(2*d*x + 2*c)} + 1)^8)/d$$

maple [A] time = 0.30, size = 156, normalized size = 1.51

$$\frac{a^3 \ln(\cosh(dx + c))}{d} - \frac{(\tanh^2(dx + c))a^3}{2d} - \frac{3a^2b(\sinh^2(dx + c))}{2d \cosh(dx + c)^4} - \frac{3a^2b}{4d \cosh(dx + c)^4} - \frac{3ab^2(\sinh^2(dx + c))}{4d \cosh(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x)

[Out] a^3*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^3-3/2/d*a^2*b*sinh(d*x+c)^2/cosh(d*x+c)^4-3/4/d*a^2*b/cosh(d*x+c)^4-3/4/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^6-1/4/d*a*b^2/cosh(d*x+c)^6-1/6/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/d*b^3/cosh(d*x+c)^8

maxima [B] time = 0.46, size = 652, normalized size = 6.33

$$\frac{3a^2b \tanh(dx + c)^4}{4d} + a^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) - 4ab^2 \left(\frac{1}{d(6e^{(-2dx-2c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="maxima")

[Out] 3/4*a^2*b*tanh(d*x + c)^4/d + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 32/3*b^3*(e^(-6*d*x - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))

mupad [B] time = 1.64, size = 573, normalized size = 5.56

$$\frac{32(3ab^2 - 5b^3)}{d(5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1)} - a^3 x - \frac{1}{d(7e^{2c+2dx} + 21e^{4c+4dx} + 35e^{6c+6dx} + 35e^{8c+8dx} + 21e^{10c+10dx} + 7e^{12c+12dx} + \exp(14c + 14dx) + 1)} - \frac{32(3ab^2 - 19b^3)}{3d(6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1)} + \frac{32b^3}{d(8\exp(2c + 2dx) + 28\exp(4c + 4dx) + 56\exp(6c + 6dx) + 70\exp(8c + 8dx) + 56\exp(10c + 10dx) + 28\exp(12c + 12dx) + 8\exp(14c + 14dx) + \exp(16c + 16dx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (32*(3*a*b^2 - 5*b^3))/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - a^3*x - (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (32*(3*a*b^2 - 19*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))

$8\exp(4c + 4dx) + 56\exp(6c + 6dx) + 70\exp(8c + 8dx) + 56\exp(10c + 10dx) + 28\exp(12c + 12dx) + 8\exp(14c + 14dx) + \exp(16c + 16dx) + 1) - (8(9a^2b - 21ab^2 + 4b^3))/(3d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)) + (4(3a^2b - 27ab^2 + 16b^3))/(d(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1)) - (2(3a^2b - a^3))/(d(\exp(2c + 2dx) + 1)) + (a^3 \log(\exp(2c)\exp(2dx) + 1))/d - (2(6a^2b^2 - 9a^2b + a^3))/(d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1))$

sympy [A] time = 13.79, size = 178, normalized size = 1.73

$$\left\{ \begin{array}{l} a^3 x - \frac{a^3 \log(\tanh(c+dx)+1)}{d} - \frac{a^3 \tanh^2(c+dx)}{2d} - \frac{3a^2 b \tanh^2(c+dx) \operatorname{sech}^2(c+dx)}{4d} - \frac{3a^2 b \operatorname{sech}^2(c+dx)}{4d} - \frac{ab^2 \tanh^2(c+dx) \operatorname{sech}^4(c+dx)}{2d} - \dots \\ x(a + b \operatorname{sech}^2(c))^3 \tanh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(dx+c)**2)**3*tanh(dx+c)**3,x)

[Out] Piecewise((a**3*x - a**3*log(tanh(c + dx) + 1)/d - a**3*tanh(c + dx)**2/(2*d) - 3*a**2*b*tanh(c + dx)**2*sech(c + dx)**2/(4*d) - 3*a**2*b*sech(c + dx)**2/(4*d) - a*b**2*tanh(c + dx)**2*sech(c + dx)**4/(2*d) - a*b**2*sech(c + dx)**4/(4*d) - b**3*tanh(c + dx)**2*sech(c + dx)**6/(8*d) - b**3*sech(c + dx)**6/(24*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c)**3, True))

3.126 $\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^2(c + dx) dx$

Optimal. Leaf size=92

$$\frac{a^3 \tanh(c + dx)}{d} + a^3 x - \frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a + 2b) \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d}$$

[Out] $a^3 x - a^3 \tanh(d x + c) / d + 1/3 * b * (3 * a^2 + 3 * a * b + b^2) * \tanh(d x + c)^3 / d - 1/5 * b^2 * (3 * a + 2 * b) * \tanh(d x + c)^5 / d + 1/7 * b^3 * \tanh(d x + c)^7 / d$

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 206}

$$\frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{a^3 \tanh(c + dx)}{d} + a^3 x - \frac{b^2(3a + 2b) \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]

[Out] $a^3 x - (a^3 \operatorname{Tanh}[c + d x]) / d + (b * (3 a^2 + 3 a b + b^2) * \operatorname{Tanh}[c + d x]^3) / (3 d) - (b^2 * (3 a + 2 b) * \operatorname{Tanh}[c + d x]^5) / (5 d) + (b^3 * \operatorname{Tanh}[c + d x]^7) / (7 d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(n_)])^(m_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^2(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b(1-x^2))^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (-a^3 + b(3a^2 + 3ab + b^2)x^2 - b^2(3a + 2b)x^4 + b^3x^6 + \dots) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^3 \tanh(c + dx)}{d} + \frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a + 2b) \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d} \\ &= a^3 x - \frac{a^3 \tanh(c + dx)}{d} + \frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a + 2b) \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d} \end{aligned}$$

Mathematica [B] time = 1.79, size = 479, normalized size = 5.21

$$\operatorname{sech}(c)\operatorname{sech}^7(c+dx)\left(3150a^3\sinh(2c+dx)-3150a^3\sinh(2c+3dx)+1260a^3\sinh(4c+3dx)-1260a^3\sinh(4c+5dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]

[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^3*d*x*Cosh[d*x] + 3675*a^3*d*x*Cosh[2*c + d*x] + 2205*a^3*d*x*Cosh[2*c + 3*d*x] + 2205*a^3*d*x*Cosh[4*c + 3*d*x] + 735*a^3*d*x*Cosh[4*c + 5*d*x] + 735*a^3*d*x*Cosh[6*c + 5*d*x] + 105*a^3*d*x*Cosh[6*c + 7*d*x] + 105*a^3*d*x*Cosh[8*c + 7*d*x] - 4200*a^3*Sinh[d*x] + 3360*a^2*b*Sinh[d*x] + 840*a*b^2*Sinh[d*x] - 560*b^3*Sinh[d*x] + 3150*a^3*Sinh[2*c + d*x] - 3990*a^2*b*Sinh[2*c + d*x] - 2100*a*b^2*Sinh[2*c + d*x] - 1120*b^3*Sinh[2*c + d*x] - 3150*a^3*Sinh[2*c + 3*d*x] + 1890*a^2*b*Sinh[2*c + 3*d*x] + 504*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] + 1260*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] - 1260*a*b^2*Sinh[4*c + 3*d*x] - 1260*a^3*Sinh[4*c + 5*d*x] + 840*a^2*b*Sinh[4*c + 5*d*x] + 588*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] + 210*a^3*Sinh[6*c + 5*d*x] - 630*a^2*b*Sinh[6*c + 5*d*x] - 210*a^3*Sinh[6*c + 7*d*x] + 210*a^2*b*Sinh[6*c + 7*d*x] + 84*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(13440*d)

fricas [B] time = 0.42, size = 881, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/105*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 - (105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*sinh(d*x + c)^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^5 - 7*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3 + 3*(105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + (105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 7*(5*(105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 135*a^3 + 45*a^2*b + 54*a*b^2 - 24*b^3 + 10*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 10*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c) - 7*((105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 75*a^3 + 45*a^2*b + 90*a*b^2 + 120*b^3 + 9*(45*a^3 + 15*a^2*b + 18*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))

giac [B] time = 0.20, size = 356, normalized size = 3.87

$$105a^3dx + \frac{2(105a^3e^{(12dx+12c)} - 315a^2be^{(12dx+12c)} + 630a^3e^{(10dx+10c)} - 1260a^2be^{(10dx+10c)} - 630ab^2e^{(10dx+10c)} + 1575a^3e^{(8dx+8c)} - 1995a^2be^{(8dx+8c)})}{13440d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="giac")

[Out] $\frac{1}{105}*(105*a^3*d*x + 2*(105*a^3*e^{(12*d*x + 12*c)} - 315*a^2*b*e^{(12*d*x + 12*c)} + 630*a^3*e^{(10*d*x + 10*c)} - 1260*a^2*b*e^{(10*d*x + 10*c)} - 630*a*b^2*e^{(10*d*x + 10*c)} + 1575*a^3*e^{(8*d*x + 8*c)} - 1995*a^2*b*e^{(8*d*x + 8*c)} - 1050*a*b^2*e^{(8*d*x + 8*c)} - 560*b^3*e^{(8*d*x + 8*c)} + 2100*a^3*e^{(6*d*x + 6*c)} - 1680*a^2*b*e^{(6*d*x + 6*c)} - 420*a*b^2*e^{(6*d*x + 6*c)} + 280*b^3*e^{(6*d*x + 6*c)} + 1575*a^3*e^{(4*d*x + 4*c)} - 945*a^2*b*e^{(4*d*x + 4*c)} - 252*a*b^2*e^{(4*d*x + 4*c)} - 168*b^3*e^{(4*d*x + 4*c)} + 630*a^3*e^{(2*d*x + 2*c)} - 420*a^2*b*e^{(2*d*x + 2*c)} - 294*a*b^2*e^{(2*d*x + 2*c)} - 56*b^3*e^{(2*d*x + 2*c)} + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)/(e^{(2*d*x + 2*c)} + 1)^7/d$

maple [B] time = 0.52, size = 180, normalized size = 1.96

$$a^3(dx+c-\tanh(dx+c))+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3ab^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)}{5}\right)\tanh(dx+c)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x)

[Out] $\frac{1}{d}*(a^3*(d*x+c-\tanh(d*x+c))+3*a^2*b*(-1/2*\sinh(d*x+c)/\cosh(d*x+c)^3+1/2*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))+3*a*b^2*(-1/4*\sinh(d*x+c)/\cosh(d*x+c)^5+1/4*(8/15+1/5*\operatorname{sech}(d*x+c)^4+4/15*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))+b^3*(-1/6*\sinh(d*x+c)/\cosh(d*x+c)^7+1/6*(16/35+1/7*\operatorname{sech}(d*x+c)^6+6/35*\operatorname{sech}(d*x+c)^4+8/35*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [B] time = 0.44, size = 788, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="maxima")

[Out] $a^2*b*\tanh(d*x+c)^3/d+a^3*(x+c/d-2/(d*(e^{(-2*d*x-2*c)}+1))) + 16/105*b^3*(7*e^{(-2*d*x-2*c)}/(d*(7*e^{(-2*d*x-2*c)}+21*e^{(-4*d*x-4*c)}+35*e^{(-6*d*x-6*c)}+35*e^{(-8*d*x-8*c)}+21*e^{(-10*d*x-10*c)}+7*e^{(-12*d*x-12*c)}+e^{(-14*d*x-14*c)}+1))+21*e^{(-4*d*x-4*c)}/(d*(7*e^{(-2*d*x-2*c)}+21*e^{(-4*d*x-4*c)}+35*e^{(-6*d*x-6*c)}+35*e^{(-8*d*x-8*c)}+21*e^{(-10*d*x-10*c)}+7*e^{(-12*d*x-12*c)}+e^{(-14*d*x-14*c)}+1))-35*e^{(-6*d*x-6*c)}/(d*(7*e^{(-2*d*x-2*c)}+21*e^{(-4*d*x-4*c)}+35*e^{(-6*d*x-6*c)}+35*e^{(-8*d*x-8*c)}+21*e^{(-10*d*x-10*c)}+7*e^{(-12*d*x-12*c)}+e^{(-14*d*x-14*c)}+1))+70*e^{(-8*d*x-8*c)}/(d*(7*e^{(-2*d*x-2*c)}+21*e^{(-4*d*x-4*c)}+35*e^{(-6*d*x-6*c)}+35*e^{(-8*d*x-8*c)}+21*e^{(-10*d*x-10*c)}+7*e^{(-12*d*x-12*c)}+e^{(-14*d*x-14*c)}+1))+1/(d*(7*e^{(-2*d*x-2*c)}+21*e^{(-4*d*x-4*c)}+35*e^{(-6*d*x-6*c)}+35*e^{(-8*d*x-8*c)}+21*e^{(-10*d*x-10*c)}+7*e^{(-12*d*x-12*c)}+e^{(-14*d*x-14*c)}+1)))+4/5*a*b^2*(5*e^{(-2*d*x-2*c)}/(d*(5*e^{(-2*d*x-2*c)}+10*e^{(-4*d*x-4*c)}+10*e^{(-6*d*x-6*c)}+5*e^{(-8*d*x-8*c)}+e^{(-10*d*x-10*c)}+1))-5*e^{(-4*d*x-4*c)}/(d*(5*e^{(-2*d*x-2*c)}+10*e^{(-4*d*x-4*c)}+10*e^{(-6*d*x-6*c)}+5*e^{(-8*d*x-8*c)}+e^{(-10*d*x-10*c)}+1))+15*e^{(-6*d*x-6*c)}/(d*(5*e^{(-2*d*x-2*c)}+10*e^{(-4*d*x-4*c)}+10*e^{(-6*d*x-6*c)}+5*e^{(-8*d*x-8*c)}+e^{(-10*d*x-10*c)}+1))+1/(d*(5*e^{(-2*d*x-2*c)}+10*e^{(-4*d*x-4*c)}+10*e^{(-6*d*x-6*c)}+5*e^{(-8*d*x-8*c)}+e^{(-10*d*x-10*c)}+1)))$

mupad [B] time = 0.21, size = 1133, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)`

[Out] $a^3x - \left(\frac{(2(2ab^2 + a^2b - a^3))}{(7d)} - \frac{(2\exp(2c + 2dx)(3a^2b + 15a^3 - 16b^3))}{(21d)} - \frac{(4\exp(6c + 6dx)(3a^2b + 15a^3 - 16b^3))}{(21d)} + \frac{(10\exp(8c + 8dx)(2ab^2 + a^2b - a^3))}{(7d)} - \frac{(4\exp(4c + 4dx)(6ab^2 + 3a^2b + 5a^3 + 8b^3))}{(7d)} + \frac{(2\exp(10c + 10dx)(3a^2b - a^3))}{(7d)} \right) / (6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1) - \left(\frac{(2(3a^2b - a^3))}{(7d)} - \frac{(2\exp(4c + 4dx)(3a^2b + 15a^3 - 16b^3))}{(7d)} - \frac{(2\exp(8c + 8dx)(3a^2b + 15a^3 - 16b^3))}{(7d)} + \frac{(12\exp(2c + 2dx)(2ab^2 + a^2b - a^3))}{(7d)} + \frac{(12\exp(10c + 10dx)(2ab^2 + a^2b - a^3))}{(7d)} - \frac{(8\exp(6c + 6dx)(6ab^2 + 3a^2b + 5a^3 + 8b^3))}{(7d)} + \frac{(2\exp(12c + 12dx)(3a^2b - a^3))}{(7d)} \right) / (7\exp(2c + 2dx) + 21\exp(4c + 4dx) + 35\exp(6c + 6dx) + 35\exp(8c + 8dx) + 21\exp(10c + 10dx) + 7\exp(12c + 12dx) + \exp(14c + 14dx) + 1) - \left(\frac{(4\exp(2c + 2dx)(2ab^2 + a^2b - a^3))}{(7d)} - \frac{(2(3a^2b + 15a^3 - 16b^3))}{(105d)} + \frac{(2\exp(4c + 4dx)(3a^2b - a^3))}{(7d)} \right) / (3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1) + \left(\frac{(2(6ab^2 + 3a^2b + 5a^3 + 8b^3))}{(35d)} + \frac{(2\exp(2c + 2dx)(3a^2b + 15a^3 - 16b^3))}{(35d)} - \frac{(6\exp(4c + 4dx)(2ab^2 + a^2b - a^3))}{(7d)} - \frac{(2\exp(6c + 6dx)(3a^2b - a^3))}{(7d)} \right) / (4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1) + \left(\frac{(2(3a^2b + 15a^3 - 16b^3))}{(105d)} + \frac{(4\exp(4c + 4dx)(3a^2b + 15a^3 - 16b^3))}{(35d)} - \frac{(8\exp(6c + 6dx)(2ab^2 + a^2b - a^3))}{(7d)} + \frac{(8\exp(2c + 2dx)(6ab^2 + 3a^2b + 5a^3 + 8b^3))}{(35d)} - \frac{(2\exp(8c + 8dx)(3a^2b - a^3))}{(7d)} \right) / (5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1) - \left(\frac{(2(2ab^2 + a^2b - a^3))}{(7d)} + \frac{(2\exp(2c + 2dx)(3a^2b - a^3))}{(7d)} \right) / (2\exp(2c + 2dx) + \exp(4c + 4dx) + 1) - \frac{(2(3a^2b - a^3))}{(7d(\exp(2c + 2dx) + 1))}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**2,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**2, x)`

3.127 $\int (a + b \operatorname{sech}^2(c + dx))^3 \tanh(c + dx) dx$

Optimal. Leaf size=71

$$\frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} - \frac{b^3 \operatorname{sech}^6(c + dx)}{6d}$$

[Out] $a^3 \ln(\cosh(d*x+c))/d - 3/2*a^2*b*\operatorname{sech}(d*x+c)^2/d - 3/4*a*b^2*\operatorname{sech}(d*x+c)^4/d - 1/6*b^3*\operatorname{sech}(d*x+c)^6/d$

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 266, 43}

$$-\frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} + \frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} - \frac{b^3 \operatorname{sech}^6(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x], x]

[Out] $(a^3 \operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d - (3*a^2*b*\operatorname{Sech}[c + d*x]^2)/(2*d) - (3*a*b^2*\operatorname{Sech}[c + d*x]^4)/(4*d) - (b^3*\operatorname{Sech}[c + d*x]^6)/(6*d)$

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] := Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 \tanh(c + dx) dx &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x^7} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{x^4} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4} + \frac{3ab^2}{x^3} + \frac{3a^2b}{x^2} + \frac{a^3}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{a^3 \log(\cosh(c + dx))}{d} - \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} - \frac{3ab^2 \operatorname{sech}^4(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.30, size = 100, normalized size = 1.41

$$\frac{2\operatorname{sech}^6(c + dx) \left(a \cosh^2(c + dx) + b \right)^3 \left(12a^3 \cosh^6(c + dx) \log(\cosh(c + dx)) - 18a^2b \cosh^4(c + dx) - 9ab^2 \cosh^2(c + dx) \right)}{3d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x], x]

[Out] (2*(b + a*Cosh[c + d*x]^2)^3*(-2*b^3 - 9*a*b^2*Cosh[c + d*x]^2 - 18*a^2*b*Cosh[c + d*x]^4 + 12*a^3*Cosh[c + d*x]^6*Log[Cosh[c + d*x]])*Sech[c + d*x]^6)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.44, size = 2519, normalized size = 35.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c), x, algorithm="fricas")

[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*a^3*d*x*sinh(d*x + c)^12 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^10 + 18*(11*a^3*d*x*cosh(d*x + c)^2 + a^3*d*x + a^2*b)*sinh(d*x + c)^10 + 60*(11*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x + 8*a^2*b + 4*a*b^2 + 90*(a^3*d*x + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 + 30*(a^3*d*x + a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 + 15*a^3*d*x + 945*(a^3*d*x + a^2*b)*cosh(d*x + c)^4 + 27*a^2*b + 18*a*b^2 + 8*b^3 + 63*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 + 189*(a^3*d*x + a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^3 + (15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 3*(495*a^3*d*x*cosh(d*x + c)^8 + 1260*(a^3*d*x + a^2*b)*cosh(d*x + c)^6 + 15*a^3*d*x + 210*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 24*a^2*b + 12*a*b^2 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 + 540*(a^3*d*x + a^2*b)*cosh(d*x + c)^7 + 126*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 + 135*(a^3*d*x + a^2*b)*cosh(d*x + c)^8 + 42*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 3*a^3*d*x + 10*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 3*a^2*b + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(d*x + c)^12 + 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d*x + c)^8 + 6*(11*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 20*a^3*cosh(d*x + c)^6 + 15*(33*a^3*cosh(d*x + c)^4 + 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 + 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 + 315*a^3*cosh(d*x + c)^4 + 105*a^3*cosh(d*x + c)^2 + 5*a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh(d*x + c)^7 + 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a^3*cosh(d*x + c)^2 + 15*(33*a^3*cosh(d*x + c)^8 + 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^4 + 20*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c)^9 + 36*a^3*cosh(d*x + c)^7 + 42*a^3*cosh(d*x + c)^5 + 20*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^10 + 45*a^3*cosh(d*x + c)^8 + 36*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4 + 3*a^3)*sinh(d*x + c)^2 + 6*(33*a^3*cosh(d*x + c)^11 + 135*a^3*cosh(d*x + c)^9 + 42*a^3*cosh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^5 + 3*a^3)*sinh(d*x + c) + 3*a^3*cosh(d*x + c)^2 + a^3

$$c)^8 + 70a^3 \cosh(dx + c)^6 + 50a^3 \cosh(dx + c)^4 + 15a^3 \cosh(dx + c)^2 + a^3 \sinh(dx + c)^2 + 12(a^3 \cosh(dx + c)^{11} + 5a^3 \cosh(dx + c)^9 + 10a^3 \cosh(dx + c)^7 + 10a^3 \cosh(dx + c)^5 + 5a^3 \cosh(dx + c)^3 + a^3 \cosh(dx + c)) \sinh(dx + c) \log(2 \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 12(3a^3 dx \cosh(dx + c)^{11} + 15(a^3 dx + a^2 b) \cosh(dx + c)^9 + 6(5a^3 dx + 8a^2 b + 4ab^2) \cosh(dx + c)^7 + 2(15a^3 dx + 27a^2 b + 18ab^2 + 8b^3) \cosh(dx + c)^5 + 3(5a^3 dx + 8a^2 b + 4ab^2) \cosh(dx + c)^3 + 3(a^3 dx + a^2 b) \cosh(dx + c) \sinh(dx + c)) / (d \cosh(dx + c)^{12} + 12d \cosh(dx + c) \sinh(dx + c)^{11} + d \sinh(dx + c)^{12} + 6d \cosh(dx + c)^{10} + 6(11d \cosh(dx + c)^2 + d) \sinh(dx + c)^{10} + 20(11d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^9 + 15d \cosh(dx + c)^8 + 15(33d \cosh(dx + c)^4 + 18d \cosh(dx + c)^2 + d) \sinh(dx + c)^8 + 24(33d \cosh(dx + c)^5 + 30d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^7 + 20d \cosh(dx + c)^6 + 4(231d \cosh(dx + c)^6 + 315d \cosh(dx + c)^4 + 105d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^6 + 24(33d \cosh(dx + c)^7 + 63d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^5 + 15d \cosh(dx + c)^4 + 15(33d \cosh(dx + c)^8 + 84d \cosh(dx + c)^6 + 70d \cosh(dx + c)^4 + 20d \cosh(dx + c)^2 + d) \sinh(dx + c)^4 + 20(11d \cosh(dx + c)^9 + 36d \cosh(dx + c)^7 + 42d \cosh(dx + c)^5 + 20d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 + 6d \cosh(dx + c)^2 + 6(11d \cosh(dx + c)^{10} + 45d \cosh(dx + c)^8 + 70d \cosh(dx + c)^6 + 50d \cosh(dx + c)^4 + 15d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 12(d \cosh(dx + c)^{11} + 5d \cosh(dx + c)^9 + 10d \cosh(dx + c)^7 + 10d \cosh(dx + c)^5 + 5d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d$$

giac [B] time = 0.19, size = 268, normalized size = 3.77

$$60a^3 dx - 60a^3 \log(e^{2dx+2c} + 1) + \frac{147a^3 e^{(12dx+12c)} + 882a^3 e^{(10dx+10c)} + 360a^2 b e^{(10dx+10c)} + 2205a^3 e^{(8dx+8c)} + 1440a^2 b e^{(8dx+8c)}}{e^{(2dx+2c)} + 1} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(dx+c)^2)^3*tanh(dx+c),x, algorithm="giac")

[Out] $-1/60*(60a^3 dx - 60a^3 \log(e^{2dx+2c} + 1) + (147a^3 e^{(12dx+12c)} + 882a^3 e^{(10dx+10c)} + 360a^2 b e^{(10dx+10c)} + 2205a^3 e^{(8dx+8c)} + 1440a^2 b e^{(8dx+8c)} + 720a^2 b^2 e^{(8dx+8c)} + 2940a^3 e^{(6dx+6c)} + 2160a^2 b e^{(6dx+6c)} + 1440a^2 b^2 e^{(6dx+6c)} + 640b^3 e^{(6dx+6c)} + 2205a^3 e^{(4dx+4c)} + 1440a^2 b e^{(4dx+4c)} + 720a^2 b^2 e^{(4dx+4c)} + 882a^3 e^{(2dx+2c)} + 360a^2 b e^{(2dx+2c)} + 147a^3) / (e^{(2dx+2c)} + 1)^6) / d$

maple [A] time = 0.16, size = 67, normalized size = 0.94

$$\frac{b^3 \operatorname{sech}(dx+c)^6}{6d} - \frac{3ab^2 \operatorname{sech}(dx+c)^4}{4d} - \frac{3a^2 b \operatorname{sech}(dx+c)^2}{2d} - \frac{a^3 \ln(\operatorname{sech}(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(dx+c)^2)^3*tanh(dx+c),x)

[Out] $-1/6*b^3*\operatorname{sech}(dx+c)^6/d - 3/4*a*b^2*\operatorname{sech}(dx+c)^4/d - 3/2*a^2*b*\operatorname{sech}(dx+c)^2/d - 1/d*a^3*\ln(\operatorname{sech}(dx+c))$

maxima [A] time = 0.43, size = 85, normalized size = 1.20

$$\frac{3a^2 b \tanh(dx+c)^2}{2d} + \frac{a^3 \log(\cosh(dx+c))}{d} - \frac{12ab^2}{d(e^{(dx+c)} + e^{(-dx-c)})^4} - \frac{32b^3}{3d(e^{(dx+c)} + e^{(-dx-c)})^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="maxima")

[Out] $\frac{3}{2}a^2b \tanh(dx+c)^2/d + a^3 \log(\cosh(dx+c))/d - 12ab^2/(d(e^{dx+c} + e^{-dx-c}))^4 - 32/3b^3/(d(e^{dx+c} + e^{-dx-c}))^6$

mupad [B] time = 0.21, size = 347, normalized size = 4.89

$$\frac{32b^3}{3d(6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx} + 6e^{10c+10dx} + e^{12c+12dx} + 1)} - \frac{4(3a^2b \tanh(dx+c)^2/d + a^3 \log(\cosh(dx+c))/d - 12ab^2/(d(e^{dx+c} + e^{-dx-c}))^4 - 32/3b^3/(d(e^{dx+c} + e^{-dx-c}))^6)}{d(4e^{2c+2dx} + 6e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx} + 6e^{10c+10dx} + e^{12c+12dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)

[Out] $(32b^3)/(3d(6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1)) - (4(3a^2b^2 - 8b^3))/(d(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1)) - a^3x - (6(2ab^2 - a^2b))/(d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)) + (a^3 \log(\exp(2c) \exp(2dx) + 1))/d + (8(9ab^2 - 4b^3))/(3d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)) - (32b^3)/(d(5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1)) - (6a^2b)/(d(\exp(2c + 2dx) + 1))$

sympy [A] time = 4.91, size = 87, normalized size = 1.23

$$\begin{cases} a^3x - \frac{a^3 \log(\tanh(c+dx)+1)}{d} - \frac{3a^2b \operatorname{sech}^2(c+dx)}{2d} - \frac{3ab^2 \operatorname{sech}^4(c+dx)}{4d} - \frac{b^3 \operatorname{sech}^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x(a + b \operatorname{sech}^2(c))^3 \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c),x)

[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - 3*a**2*b*sech(c + d*x)**2/(2*d) - 3*a*b**2*sech(c + d*x)**4/(4*d) - b**3*sech(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c), True))

3.128 $\int (a + b \operatorname{sech}^2(c + dx))^3 dx$

Optimal. Leaf size=73

$$a^3x + \frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + 2b) \tanh^3(c + dx)}{3d} + \frac{b^3 \tanh^5(c + dx)}{5d}$$

[Out] $a^3x + b(3a^2 + 3ab + b^2) \tanh(dx + c)/d - 1/3 b^2 (3a + 2b) \tanh(dx + c)^3/d + 1/5 b^3 \tanh(dx + c)^5/d$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4128, 390, 206}

$$\frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} + a^3x - \frac{b^2(3a + 2b) \tanh^3(c + dx)}{3d} + \frac{b^3 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^3, x]

[Out] $a^3x + (b(3a^2 + 3ab + b^2) \operatorname{Tanh}[c + dx])/d - (b^2(3a + 2b) \operatorname{Tanh}[c + dx]^3)/(3d) + (b^3 \operatorname{Tanh}[c + dx]^5)/(5d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4128

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && NeQ[a + b, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(b(3a^2 + 3ab + b^2) - b^2(3a + 2b)x^2 + b^3x^4 + \frac{a^3}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + 2b) \tanh^3(c + dx)}{3d} + \frac{b^3 \tanh^5(c + dx)}{5d} \\ &= a^3x + \frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + 2b) \tanh^3(c + dx)}{3d} + \frac{b^3 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 0.93, size = 268, normalized size = 3.67

$$\frac{\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(150a^3dx\cosh(2c+dx)+75a^3dx\cosh(2c+3dx)+75a^3dx\cosh(4c+3dx)+15a^3dx\cosh(4c+5dx)+15a^3d^2x^2\cosh(6c+5dx)+540a^2b\sinh(dx)+420a^2b^2\sinh(dx)+160b^3\sinh(dx)-360a^2b\sinh(2c+dx)-180a^2b^2\sinh(2c+dx)+360a^2b\sinh(2c+3dx)+300ab^2\sinh(2c+3dx)+80b^3\sinh(2c+3dx)-90a^2b\sinh(4c+3dx)+90a^2b\sinh(4c+5dx)+60ab^2\sinh(4c+5dx)+16b^3\sinh(4c+5dx)\right)}{(480d)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^3,x]

[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] + 75*a^3*d*x*Cosh[4*c + 3*d*x] + 15*a^3*d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 540*a^2*b*Sinh[d*x] + 420*a^2*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] - 360*a^2*b*Sinh[2*c + d*x] - 180*a^2*b^2*Sinh[2*c + d*x] + 360*a^2*b*Sinh[2*c + 3*d*x] + 300*a*b^2*Sinh[2*c + 3*d*x] + 80*b^3*Sinh[2*c + 3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 90*a^2*b*Sinh[4*c + 5*d*x] + 60*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/(480*d)

fricas [B] time = 0.42, size = 470, normalized size = 6.44

$$\frac{(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)^5 + 5(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)\sinh(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/15*((15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (45*a^2*b + 30*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 5*(27*a^2*b + 30*a*b^2 + 8*b^3 + 2*(45*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c) + 5*((45*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 24*a*b^2 + 16*b^3 + 3*(27*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.15, size = 182, normalized size = 2.49

$$\frac{15(dx+c)a^3 - \frac{2(45a^2be^{8dx+8c}+180a^2be^{6dx+6c}+90ab^2e^{6dx+6c}+270a^2be^{4dx+4c}+210ab^2e^{4dx+4c}+80b^3e^{4dx+4c}+180a^2be^{2dx+2c}+150ab^2e^{2dx+2c})}{(e^{2dx+2c}+1)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/15*(15*(d*x + c)*a^3 - 2*(45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 210*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 150*a*b^2*e^(2*d*x + 2*c) + 40*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 30*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d

maple [A] time = 0.42, size = 83, normalized size = 1.14

$$\frac{a^3(dx+c) + 3a^2b \tanh(dx+c) + 3ab^2 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + b^3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^3,x)

[Out] $1/d*(a^3*(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))$

maxima [B] time = 0.41, size = 332, normalized size = 4.55

$$a^3x + \frac{16}{15}b^3 \left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{1}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $a^3x + \frac{16}{15}b^3 \left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{10e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{5e^{(-8dx-8c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{1}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + 4a^2b^2 \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + 6a^2b \left(\frac{e^{(-2dx-2c)}}{d(e^{(-2dx-2c)} + 1)} \right)$

mupad [B] time = 1.44, size = 502, normalized size = 6.88

$$a^3x - \frac{2(9a^2b+12ab^2+8b^3)}{15d} + \frac{12e^{2c+2dx}(a^2b+ab^2)}{5d} + \frac{6a^2be^{4c+4dx}}{5d} - \frac{6a^2b}{5d} + \frac{24e^{2c+2dx}(a^2b+ab^2)}{5d} + \frac{24e^{6c+6dx}(a^2b+ab^2)}{5d} + \frac{4e^{10c+10dx}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^3,x)

[Out] $a^3x - \left(\frac{(2*(12*a*b^2 + 9*a^2*b + 8*b^3))/(15*d) + (12*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*a^2*b*\exp(4*c + 4*d*x))/(5*d)}{(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)} - \frac{(6*a^2*b)/(5*d) + (24*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (24*\exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (4*\exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b*\exp(8*c + 8*d*x))/(5*d)}{(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)} - \frac{(6*(a*b^2 + a^2*b))/(5*d) + (18*\exp(4*c + 4*d*x)*(a*b^2 + a^2*b))/(5*d) + (2*\exp(2*c + 2*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b*\exp(6*c + 6*d*x))/(5*d)}{(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)} - \frac{(6*(a*b^2 + a^2*b))/(5*d) + (6*a^2*b*\exp(2*c + 2*d*x))/(5*d)}{(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)} - \frac{(6*a^2*b)/(5*d*(\exp(2*c + 2*d*x) + 1))}{(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)} \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3, x)

3.129 $\int \coth(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=84

$$\frac{b(3a^2 + 3ab + b^2) \log(\cosh(c + dx))}{d} + \frac{b^2(3a + b) \operatorname{sech}^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\sinh(c + dx))}{d} + \frac{b^3 \operatorname{sech}^4(c + dx)}{4d}$$

[Out] $-b(3a^2+3a*b+b^2)*\ln(\cosh(d*x+c))/d+(a+b)^3*\ln(\sinh(d*x+c))/d+1/2*b^2*(3*a+b)*\operatorname{sech}(d*x+c)^2/d+1/4*b^3*\operatorname{sech}(d*x+c)^4/d$

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 88}

$$\frac{b(3a^2 + 3ab + b^2) \log(\cosh(c + dx))}{d} + \frac{b^2(3a + b) \operatorname{sech}^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\sinh(c + dx))}{d} + \frac{b^3 \operatorname{sech}^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-((b*(3a^2 + 3a*b + b^2)*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/d) + ((a + b)^3*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/d + (b^2*(3a + b)*\operatorname{Sech}[c + d*x]^2)/(2*d) + (b^3*\operatorname{Sech}[c + d*x]^4)/(4*d)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \coth(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x^5(1-x^2)} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{(1-x)x^3} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x} + \frac{b^3}{x^3} + \frac{b^2(3a+b)}{x^2} + \frac{b(3a^2+3ab+b^2)}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{b(3a^2 + 3ab + b^2) \log(\cosh(c + dx))}{d} + \frac{(a + b)^3 \log(\sinh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 114, normalized size = 1.36

$$\frac{2 \cosh^6(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 (4b(3a^2 + 3ab + b^2) \log(\cosh(c + dx)) - 2b^2(3a + b) \operatorname{sech}^2(c + dx) - \dots)}{d(a \cosh(2c + 2dx) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(-2*\operatorname{Cosh}[c + d*x]^6*(a + b*\operatorname{Sech}[c + d*x]^2)^3*(4*b*(3*a^2 + 3*a*b + b^2)*\operatorname{Log}[\operatorname{Cosh}[c + d*x]] - 4*(a + b)^3*\operatorname{Log}[\operatorname{Sinh}[c + d*x]] - 2*b^2*(3*a + b)*\operatorname{Sech}[c + d*x]^2 - b^3*\operatorname{Sech}[c + d*x]^4))/(d*(a + 2*b + a*\operatorname{Cosh}[2*c + 2*d*x])^3)$

fricas [B] time = 0.46, size = 2376, normalized size = 28.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $-(a^3*d*x*\cosh(d*x + c)^8 + 8*a^3*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*d*x*\sinh(d*x + c)^8 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^6 + 2*(14*a^3*d*x*\cosh(d*x + c)^2 + 2*a^3*d*x - 3*a*b^2 - b^3)*\sinh(d*x + c)^6 + 4*(14*a^3*d*x*\cosh(d*x + c)^3 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*\cosh(d*x + c)^4 + 2*(35*a^3*d*x*\cosh(d*x + c)^4 + 3*a^3*d*x - 6*a*b^2 - 4*b^3 + 15*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*a^3*d*x*\cosh(d*x + c)^5 + 5*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^3 + (3*a^3*d*x - 6*a*b^2 - 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^2 + 2*(14*a^3*d*x*\cosh(d*x + c)^6 + 2*a^3*d*x + 15*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^4 - 3*a*b^2 - b^3 + 6*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^2 +$

$$\begin{aligned}
& b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 8*((3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^8 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*(2*a^3*d*x*\cosh(d*x + c)^7 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c)^5 + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*\cosh(d*x + c)^3 + (2*a^3*d*x - 3*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 + 4*d*\cosh(d*x + c)^6 + 4*(7*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 8*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)
\end{aligned}$$

giac [B] time = 0.18, size = 325, normalized size = 3.87

$$12 a^3 dx + 12 \left(3 a^2 b e^{(2c)} + 3 a b^2 e^{(2c)} + b^3 e^{(2c)} \right) e^{(-2c)} \log \left(e^{(2dx+2c)} + 1 \right) - 12 \left(a^3 e^{(2c)} + 3 a^2 b e^{(2c)} + 3 a b^2 e^{(2c)} + b^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/12*(12*a^3*d*x + 12*(3*a^2*b*e^{(2*c)} + 3*a*b^2*e^{(2*c)} + b^3*e^{(2*c)})*e^{(-2*c)}*\log(e^{(2*d*x + 2*c)} + 1) - 12*(a^3*e^{(2*c)} + 3*a^2*b*e^{(2*c)} + 3*a*b^2*e^{(2*c)} + b^3*e^{(2*c)})*e^{(-2*c)}*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) - (75*a^2*b*e^{(8*d*x + 8*c)} + 75*a*b^2*e^{(8*d*x + 8*c)} + 25*b^3*e^{(8*d*x + 8*c)} + 300*a^2*b*e^{(6*d*x + 6*c)} + 372*a*b^2*e^{(6*d*x + 6*c)} + 124*b^3*e^{(6*d*x + 6*c)} + 450*a^2*b*e^{(4*d*x + 4*c)} + 594*a*b^2*e^{(4*d*x + 4*c)} + 246*b^3*e^{(4*d*x + 4*c)} + 300*a^2*b*e^{(2*d*x + 2*c)} + 372*a*b^2*e^{(2*d*x + 2*c)} + 124*b^3*e^{(2*d*x + 2*c)} + 75*a^2*b + 75*a*b^2 + 25*b^3)/(e^{(2*d*x + 2*c)} + 1)^4)/d
\end{aligned}$$

maple [A] time = 0.31, size = 111, normalized size = 1.32

$$\frac{a^3 \ln(\sinh(dx+c))}{d} + \frac{3a^2b \ln(\tanh(dx+c))}{d} + \frac{3ab^2}{2d \cosh(dx+c)^2} + \frac{3ab^2 \ln(\tanh(dx+c))}{d} + \frac{b^3}{4d \cosh(dx+c)^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)`

[Out] $a^3 \ln(\sinh(dx+c))/d + 3a^2 b \ln(\tanh(dx+c))/d + 3/2 d a b^2 / \cosh(dx+c)^2 + 3/d a b^2 \ln(\tanh(dx+c)) + 1/4 d b^3 / \cosh(dx+c)^4 + 1/2 d b^3 / \cosh(dx+c)^2 + 1/d b^3 \ln(\tanh(dx+c))$

maxima [B] time = 0.47, size = 300, normalized size = 3.57

$$b^3 \left(\frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} - \frac{\log(e^{-2dx-2c} + 1)}{d} \right) + \frac{2(e^{-2dx-2c} + 4e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $b^3(\log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d - \log(e^{-2dx-2c} + 1)/d + 2(e^{-2dx-2c} + 4e^{-4dx-4c} + e^{-6dx-6c})/(d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c}))) + 3ab^2(\log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d - \log(e^{-2dx-2c} + 1)/d + 2e^{-2dx-2c}/(d(2e^{-2dx-2c} + e^{-4dx-4c} + 1))) + 3a^2 b(\log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d - \log(e^{-2dx-2c} + 1)/d) + a^3 \log(\sinh(dx+c))/d$

mupad [B] time = 1.64, size = 360, normalized size = 4.29

$$\frac{2(b^3 + 3ab^2)}{d(e^{2c+2dx} + 1)} - a^3 x - \frac{\operatorname{atan}\left(\frac{e^{2c} e^{2dx} (a^3 \sqrt{-d^2} + 2b^3 \sqrt{-d^2} + 6ab^2 \sqrt{-d^2} + 6a^2 b \sqrt{-d^2})}{d \sqrt{a^6 + 12a^5 b + 48a^4 b^2 + 76a^3 b^3 + 60a^2 b^4 + 24ab^5 + 4b^6}}\right)}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)`

[Out] $(2(3ab^2 + b^3))/(d(\exp(2c + 2dx) + 1)) - a^3 x - (\operatorname{atan}(\exp(2c) \exp(2dx) * (a^3(-d^2)^{1/2} + 2b^3(-d^2)^{1/2} + 6ab^2(-d^2)^{1/2} + 6a^2 b(-d^2)^{1/2}))/ (d(24a^5 b + 12a^6 + 4b^6 + 60a^2 b^4 + 76a^3 b^3 + 48a^4 b^2)^{1/2})) * (24a^5 b + 12a^6 + 4b^6 + 60a^2 b^4 + 76a^3 b^3 + 48a^4 b^2)^{1/2} / (-d^2)^{1/2} + (a^3 \log(\exp(4c + 4dx) - 1)) / (2d) - (8b^3) / (d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)) - (2(3ab^2 - b^3)) / (d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)) + (4b^3) / (d(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{coth}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**3*coth(c + d*x), x)`

3.130 $\int \coth^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=61

$$a^3x - \frac{b^2(3a + 2b) \tanh(c + dx)}{d} - \frac{(a + b)^3 \coth(c + dx)}{d} + \frac{b^3 \tanh^3(c + dx)}{3d}$$

[Out] $a^3x - (a+b)^3 \coth(dx+c)/d - b^2(3a+2b) \tanh(dx+c)/d + 1/3 b^3 \tanh(dx+c)^3/d$

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$a^3x - \frac{b^2(3a + 2b) \tanh(c + dx)}{d} - \frac{(a + b)^3 \coth(c + dx)}{d} + \frac{b^3 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $a^3x - ((a + b)^3 \coth[c + d*x])/d - (b^2(3a + 2b) \tanh[c + d*x])/d + (b^3 \tanh[c + d*x]^3)/(3*d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+b(1-x^2))^3}{x^2(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(-b^2(3a + 2b) + \frac{(a+b)^3}{x^2} + b^3x^2 - \frac{a^3}{-1+x^2} \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{(a + b)^3 \coth(c + dx)}{d} - \frac{b^2(3a + 2b) \tanh(c + dx)}{d} + \frac{b^3 \tanh^3(c + dx)}{3d} \\ &= a^3x - \frac{(a + b)^3 \coth(c + dx)}{d} - \frac{b^2(3a + 2b) \tanh(c + dx)}{d} + \frac{b^3 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 1.80, size = 126, normalized size = 2.07

$$\frac{8(a \cosh(c + dx) + b \operatorname{sech}(c + dx))^3 (3a^3 dx \cosh^3(c + dx) + \sinh(dx) \cosh^2(c + dx) (3(a + b)^3 \operatorname{csch}(c) \operatorname{coth}(c + dx) + 3d(a \cosh(2(c + dx)) + a + 2b)^3))}{3d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (8*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(3*a^3*d*x*Cosh[c + d*x]^3 - b^3*Sech[c]*Sinh[d*x] + Cosh[c + d*x]^2*(3*(a + b)^3*Coth[c + d*x]*Csch[c] - b^2*(9*a + 5*b)*Sech[c])*Sinh[d*x] - b^3*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.41, size = 359, normalized size = 5.89

$$\frac{(3a^3 + 9a^2b + 18ab^2 + 8b^3) \cosh(dx + c)^4 - 4(3a^3dx + 3a^3 + 9a^2b + 18ab^2 + 8b^3) \cosh(dx + c) \sinh(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*sinh(d*x + c)^4 + 9*a^3 + 27*a^2*b + 18*a*b^2 + 4*(3*a^3 + 9*a^2*b + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(6*a^3 + 18*a^2*b + 18*a*b^2 + 8*b^3 + 3*(3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c))^3 + d*cosh(d*x + c)*sinh(d*x + c))

giac [B] time = 0.21, size = 132, normalized size = 2.16

$$\frac{3a^3dx - \frac{6(a^3+3a^2b+3ab^2+b^3)}{e^{2dx+2c}-1} + \frac{2(9ab^2e^{4dx+4c}+3b^3e^{4dx+4c}+18ab^2e^{2dx+2c}+12b^3e^{2dx+2c}+9ab^2+5b^3)}{(e^{2dx+2c}+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/3*(3*a^3*d*x - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 5*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.57, size = 111, normalized size = 1.82

$$\frac{a^3(dx + c - \operatorname{coth}(dx + c)) - 3a^2b \operatorname{coth}(dx + c) + 3ab^2 \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2 \tanh(dx + c) \right) + b^3 \left(-\frac{1}{\sinh(dx+c)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))-3*a^2*b*coth(d*x+c)+3*a*b^2*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [B] time = 0.41, size = 172, normalized size = 2.82

$$a^3 \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)} \right) - \frac{16}{3} b^3 \left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} - e^{(-8dx-8c)} + 1)} + \frac{1}{d(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] a^3*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) - 16/3*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1)) + 12*a*b^2/(d*(e^(-4*d*x - 4*c) - 1))

mupad [B] time = 0.14, size = 234, normalized size = 3.84

$$\frac{2(b^3+3ab^2)}{3d} + \frac{4e^{2c+2dx}(b^3+ab^2)}{d} + \frac{2e^{4c+4dx}(b^3+3ab^2)}{3d} + a^3x + \frac{2(b^3+ab^2)}{d} + \frac{2e^{2c+2dx}(b^3+3ab^2)}{3d} + \frac{2(b^3+3ab^2)}{3d(e^{2c+2dx}+1)} - \frac{2(a^3+ab^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)

[Out] ((2*(3*a*b^2 + b^3))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b^2 + b^3))/d + (2*exp(4*c + 4*d*x)*(3*a*b^2 + b^3))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + a^3*x + ((2*(a*b^2 + b^3))/d + (2*exp(2*c + 2*d*x)*(3*a*b^2 + b^3))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + (2*(3*a*b^2 + b^3))/(3*d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^3 \operatorname{coth}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*coth(c + d*x)**2, x)

3.131 $\int \coth^3(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx$

Optimal. Leaf size=81

$$\frac{b^2(3a + 2b) \log(\cosh(c + dx))}{d} - \frac{(a + b)^3 \operatorname{csch}^2(c + dx)}{2d} + \frac{(a - 2b)(a + b)^2 \log(\sinh(c + dx))}{d} - \frac{b^3 \operatorname{sech}^2(c + dx)}{2d}$$

[Out] $-1/2*(a+b)^3*\operatorname{csch}(d*x+c)^2/d+b^2*(3*a+2*b)*\ln(\cosh(d*x+c))/d+(a-2*b)*(a+b)^2*\ln(\sinh(d*x+c))/d-1/2*b^3*\operatorname{sech}(d*x+c)^2/d$

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{b^2(3a + 2b) \log(\cosh(c + dx))}{d} - \frac{(a + b)^3 \operatorname{csch}^2(c + dx)}{2d} + \frac{(a - 2b)(a + b)^2 \log(\sinh(c + dx))}{d} - \frac{b^3 \operatorname{sech}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[c + d*x]^3*(a + b*\text{Sech}[c + d*x]^2)^3, x]$

[Out] $-((a + b)^3*\text{Csch}[c + d*x]^2)/(2*d) + (b^2*(3*a + 2*b)*\text{Log}[\text{Cosh}[c + d*x]])/d + ((a - 2*b)*(a + b)^2*\text{Log}[\text{Sinh}[c + d*x]])/d - (b^3*\text{Sech}[c + d*x]^2)/(2*d)$

Rule 88

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.))^{(n_.))^{(p_.)*((c_.) + (d_.)*(x_.))^{(q_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]

Rule 4138

$\text{Int}[(a_. + (b_.)*\sec[(e_.) + (f_.)*(x_.)]^{(n_.))^{(p_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] := \text{Module}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[(f*ff^{(m + n*p - 1)})^{-1}, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m - 1)/2}*(b + a*(ff*x)^n)^p/x^{(m + n*p)}, x], x, \text{Cos}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \coth^3(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\text{Subst} \left(\int \frac{(b+ax^2)^3}{x^3(1-x^2)^2} dx, x, \cosh(c + dx) \right)}{d} \\ &= \frac{\text{Subst} \left(\int \frac{(b+ax)^3}{(1-x)^2 x^2} dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= \frac{\text{Subst} \left(\int \left(\frac{(a+b)^3}{(-1+x)^2} + \frac{(a-2b)(a+b)^2}{-1+x} + \frac{b^3}{x^2} + \frac{b^2(3a+2b)}{x} \right) dx, x, \cosh^2(c + dx) \right)}{2d} \\ &= -\frac{(a + b)^3 \operatorname{csch}^2(c + dx)}{2d} + \frac{b^2(3a + 2b) \log(\cosh(c + dx))}{d} + \frac{(a - 2b)(a + b)^2 \log(\sinh(c + dx))}{d} - \frac{b^3 \operatorname{sech}^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 1.24, size = 110, normalized size = 1.36

$$\frac{4 \cosh^6(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 (-2b^2(3a + 2b) \log(\cosh(c + dx)) + (a + b)^3 \operatorname{csch}^2(c + dx) - 2(a - 2b)(a + b))}{d(a \cosh(2c + 2dx) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $(-4*\operatorname{Cosh}[c + d*x]^6*(a + b*\operatorname{Sech}[c + d*x]^2)^3*((a + b)^3*\operatorname{Csch}[c + d*x]^2 - 2*b^2*(3*a + 2*b)*\operatorname{Log}[\operatorname{Cosh}[c + d*x]] - 2*(a - 2*b)*(a + b)^2*\operatorname{Log}[\operatorname{Sinh}[c + d*x]]) + b^3*\operatorname{Sech}[c + d*x]^2))/(d*(a + 2*b + a*\operatorname{Cosh}[2*c + 2*d*x])^3)$

fricas [B] time = 0.44, size = 1701, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $-(a^3*d*x*\cosh(d*x + c)^8 + 8*a^3*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*d*x*\sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^6 + 2*(14*a^3*d*x*\cosh(d*x + c)^2 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\sinh(d*x + c)^6 + 4*(14*a^3*d*x*\cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + a^3*d*x - 2*(a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*\cosh(d*x + c)^4 + 2*(35*a^3*d*x*\cosh(d*x + c)^4 - a^3*d*x + 2*a^3 + 6*a^2*b + 6*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*a^3*d*x*\cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^3 - (a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^2 + 2*(14*a^3*d*x*\cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3 - 6*(a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((3*a*b^2 + 2*b^3)*\cosh(d*x + c)^8 + 56*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a*b^2 + 2*b^3)*\sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^4 - 3*a*b^2 - 2*b^3)*\sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^5 - (3*a*b^2 + 2*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*a*b^2 + 2*b^3 + 4*(7*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^6 - 3*(3*a*b^2 + 2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a*b^2 + 2*b^3)*\cosh(d*x + c)^7 - (3*a*b^2 + 2*b^3)*\cosh(d*x + c)^3)*\sinh(d*x + c)*\log(2*\cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^8 + 56*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3 - 3*a*b^2 - 2*b^3)*\sinh(d*x + c)^8 - 2*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^4 + 2*(35*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^4 - a^3 + 3*a*b^2 + 2*b^3)*\sinh(d*x + c)^4 + 8*(7*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^5 - (a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^3 - 3*a*b^2 - 2*b^3 + 4*(7*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^6 - 3*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^7 - (a^3 - 3*a*b^2 - 2*b^3)*\cosh(d*x + c)^3)*\sinh(d*x + c)*\log(2*\sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a^3*d*x*\cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c)^5 - 2*(a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^8 + 56*d*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*d*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 - 2*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 - d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 - d*\cosh(d*x + c))*\sinh(d*x + c)$

$)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)$

giac [B] time = 0.29, size = 292, normalized size = 3.60

$$4a^3 dx - 4(3ab^2 e^{2c} + 2b^3 e^{2c})e^{(-2c)} \log(e^{(2dx+2c)} + 1) - 4(a^3 e^{2c} - 3ab^2 e^{2c} - 2b^3 e^{2c})e^{(-2c)} \log(e^{(2dx+2c)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] $-1/4*(4*a^3*d*x - 4*(3*a*b^2*e^{(2*c)} + 2*b^3*e^{(2*c)})*e^{(-2*c)}*\log(e^{(2*d*x + 2*c)} + 1) - 4*(a^3*e^{(2*c)} - 3*a*b^2*e^{(2*c)} - 2*b^3*e^{(2*c)})*e^{(-2*c)}*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1))) + (3*a^3*e^{(8*d*x + 8*c)} + 8*a^3*e^{(6*d*x + 6*c)} + 24*a^2*b*e^{(6*d*x + 6*c)} + 24*a*b^2*e^{(6*d*x + 6*c)} + 16*b^3*e^{(6*d*x + 6*c)} + 10*a^3*e^{(4*d*x + 4*c)} + 48*a^2*b*e^{(4*d*x + 4*c)} + 48*a*b^2*e^{(4*d*x + 4*c)} + 8*a^3*e^{(2*d*x + 2*c)} + 24*a^2*b*e^{(2*d*x + 2*c)} + 24*a*b^2*e^{(2*d*x + 2*c)} + 16*b^3*e^{(2*d*x + 2*c)} + 3*a^3)/(e^{(4*d*x + 4*c)} - 1)^2/d$

maple [A] time = 0.36, size = 137, normalized size = 1.69

$$\frac{a^3 \ln(\sinh(dx+c))}{d} - \frac{a^3 (\coth^2(dx+c))}{2d} - \frac{3a^2 b}{2d \sinh(dx+c)^2} - \frac{3ab^2}{2d \sinh(dx+c)^2} - \frac{3ab^2 \ln(\tanh(dx+c))}{d} - \frac{2d \sinh(dx+c)}{2d \sinh(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)

[Out] $a^3*\ln(\sinh(d*x+c))/d - 1/2*a^3*\coth(d*x+c)^2/d - 3/2/d/\sinh(d*x+c)^2*a^2*b - 3/2/d*a*b^2/\sinh(d*x+c)^2 - 3/d*a*b^2*\ln(\tanh(d*x+c)) - 1/2/d*b^3/\sinh(d*x+c)^2/\cosh(d*x+c)^2 - 1/d*b^3/\cosh(d*x+c)^2 - 2/d*b^3*\ln(\tanh(d*x+c))$

maxima [B] time = 0.47, size = 314, normalized size = 3.88

$$a^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - 2b^3 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $a^3*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - 2*b^3*(\log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d - \log(e^{(-2*d*x - 2*c)} + 1)/d - 2*(e^{(-2*d*x - 2*c)} + e^{(-6*d*x - 6*c)})/(d*(2*e^{(-4*d*x - 4*c)} - e^{(-8*d*x - 8*c)} - 1))) - 3*a*b^2*(\log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d - \log(e^{(-2*d*x - 2*c)} + 1)/d - 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - 6*a^2*b/(d*(e^{(d*x + c)} - e^{(-d*x - c)})^2)$

mupad [B] time = 1.72, size = 324, normalized size = 4.00

$$\frac{\text{atan} \left(\frac{e^{2c} e^{2dx} (4b^3 \sqrt{-d^2} - a^3 \sqrt{-d^2} + 6ab^2 \sqrt{-d^2})}{d \sqrt{a^6 - 12a^4 b^2 - 8a^3 b^3 + 36a^2 b^4 + 48ab^5 + 16b^6}} \right)}{\sqrt{-d^2}} - \frac{4(a^3 + 3a^2 b + 3ab^2 + b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)

```
[Out] (atan((exp(2*c)*exp(2*d*x)*(4*b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 6*a*b^2
*(-d^2)^(1/2)))/(d*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a
^4*b^2)^(1/2)))*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a^4*
b^2)^(1/2))/(-d^2)^(1/2) - ((4*(3*a*b^2 + 3*a^2*b + a^3))/d + (2*exp(2*c +
2*d*x)*(3*a*b^2 + 3*a^2*b + a^3 + 2*b^3))/d)/(exp(4*c + 4*d*x) - 1) - ((4*(
3*a*b^2 + 3*a^2*b + a^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + a^3
+ 2*b^3))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - a^3*x + (a^3*log
(exp(4*c + 4*d*x) - 1))/(2*d)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.132 $\int \coth^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=60

$$a^3x - \frac{(a+b)^3 \coth^3(c+dx)}{3d} - \frac{(a-2b)(a+b)^2 \coth(c+dx)}{d} + \frac{b^3 \tanh(c+dx)}{d}$$

[Out] $a^3x - (a-2b)(a+b)^2 \coth(dx+c)/d - 1/3(a+b)^3 \coth(dx+c)^3/d + b^3 \tanh(dx+c)/d$

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$a^3x - \frac{(a+b)^3 \coth^3(c+dx)}{3d} - \frac{(a-2b)(a+b)^2 \coth(c+dx)}{d} + \frac{b^3 \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $a^3x - ((a-2b)(a+b)^2 \coth[c+d*x])/d - ((a+b)^3 \coth[c+d*x]^3)/(3*d) + (b^3 \tanh[c+d*x])/d$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)])^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+b(1-x^2))^3}{x^4(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(b^3 + \frac{(a+b)^3}{x^4} + \frac{(a-2b)(a+b)^2}{x^2} - \frac{a^3}{-1+x^2} \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{(a-2b)(a+b)^2 \coth(c+dx)}{d} - \frac{(a+b)^3 \coth^3(c+dx)}{3d} + \frac{b^3 \tanh(c+dx)}{d} \\ &= a^3x - \frac{(a-2b)(a+b)^2 \coth(c+dx)}{d} - \frac{(a+b)^3 \coth^3(c+dx)}{3d} + \frac{b^3 \tanh(c+dx)}{d} \end{aligned}$$

Mathematica [B] time = 1.73, size = 343, normalized size = 5.72

$$\frac{\operatorname{csch}(c)\operatorname{sech}(c)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)\left(-16a^3\sinh(2(c+dx))+8a^3\sinh(4(c+dx))+8a^3\sinh(2(c+2dx))-\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (Csch[c]*Csch[c + d*x]^3*Sech[c]*Sech[c + d*x]*(6*a^3*d*x*Cosh[2*d*x] - 3*a^3*d*x*Cosh[2*(c + 2*d*x)] - 6*a^3*d*x*Cosh[4*c + 2*d*x] + 3*a^3*d*x*Cosh[6*c + 4*d*x] - 18*a^2*b*Sinh[2*c] - 36*a*b^2*Sinh[2*c] - 4*a^3*Sinh[2*d*x] + 6*a^2*b*Sinh[2*d*x] + 24*a*b^2*Sinh[2*d*x] + 32*b^3*Sinh[2*d*x] - 16*a^3*Sinh[2*(c + d*x)] - 12*a^2*b*Sinh[2*(c + d*x)] + 24*a*b^2*Sinh[2*(c + d*x)] + 8*b^3*Sinh[2*(c + d*x)] + 8*a^3*Sinh[4*(c + d*x)] + 6*a^2*b*Sinh[4*(c + d*x)] - 12*a*b^2*Sinh[4*(c + d*x)] - 4*b^3*Sinh[4*(c + d*x)] + 8*a^3*Sinh[2*(c + 2*d*x)] + 6*a^2*b*Sinh[2*(c + 2*d*x)] - 12*a*b^2*Sinh[2*(c + 2*d*x)] - 16*b^3*Sinh[2*(c + 2*d*x)] - 12*a^3*Sinh[4*c + 2*d*x] - 18*a^2*b*Sinh[4*c + 2*d*x]))/(96*d)

fricas [B] time = 0.40, size = 354, normalized size = 5.90

$$\frac{\left(4a^3 + 3a^2b - 6ab^2 - 8b^3\right)\cosh(dx+c)^4 - 4\left(3a^3dx + 4a^3 + 3a^2b - 6ab^2 - 8b^3\right)\cosh(dx+c)\sinh(dx+c)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*sinh(d*x + c)^4 + 9*a^2*b + 18*a*b^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 + 6*a^2*b + 6*a*b^2 + 8*b^3 + 3*(4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - (3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.34, size = 155, normalized size = 2.58

$$\frac{3a^3dx - \frac{6b^3}{e^{(2dx+2c)+1}} - \frac{2(6a^3e^{(4dx+4c)} + 9a^2be^{(4dx+4c)} - 3b^3e^{(4dx+4c)} - 6a^3e^{(2dx+2c)} + 18ab^2e^{(2dx+2c)} + 12b^3e^{(2dx+2c)} + 4a^3 + 3a^2b - 6ab^2 - 5b^3)}{(e^{(2dx+2c)} - 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/3*(3*a^3*d*x - 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2*c) + 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 4*a^3 + 3*a^2*b - 6*a*b^2 - 5*b^3)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [B] time = 0.48, size = 149, normalized size = 2.48

$$\frac{a^3\left(dx+c-\operatorname{coth}(dx+c)-\frac{\operatorname{coth}^3(dx+c)}{3}\right)+3a^2b\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\operatorname{coth}(dx+c)}{2}\right)+3ab^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)`

[Out] $\frac{1}{d} \left(a^3 (d*x+c - \coth(d*x+c) - \frac{1}{3} \coth(d*x+c)^3) + 3a^2 b \left(-\frac{1}{2} \sinh(d*x+c)^3 \cosh(d*x+c) - \frac{1}{2} \left(\frac{2}{3} - \frac{1}{3} \operatorname{csch}(d*x+c)^2 \right) \coth(d*x+c) \right) + 3a b^2 \left(\frac{2}{3} - \frac{1}{3} \operatorname{csch}(d*x+c)^2 \right) \coth(d*x+c) + b^3 \left(-\frac{1}{3} \sinh(d*x+c)^3 / \cosh(d*x+c) + \frac{4}{3} \sinh(d*x+c) / \cosh(d*x+c) + 8/3 \tanh(d*x+c) \right) \right)$

maxima [B] time = 0.52, size = 366, normalized size = 6.10

$$\frac{1}{3} a^3 \left(3x + \frac{3c}{d} - \frac{4 \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2 \right)}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right) + 4ab^2 \left(\frac{3e^{(-2dx-2c)}}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{1}{3} a^3 \left(3x + \frac{3c}{d} - \frac{4 \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2 \right)}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right) + 4a^2 b^2 \left(\frac{3e^{(-2dx-2c)}}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} - \frac{1}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right) + \frac{16}{3} b^3 \left(\frac{2e^{(-2dx-2c)}}{d \left(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} - 1 \right)} - \frac{1}{d \left(2e^{(-2dx-2c)} - 2e^{(-6dx-6c)} + e^{(-8dx-8c)} - 1 \right)} \right) + 2a^2 b \left(\frac{3e^{(-4dx-4c)}}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} + \frac{1}{d \left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right)$

mupad [B] time = 1.48, size = 260, normalized size = 4.33

$$a^3 x - \frac{\frac{2(a^2 b + 2 a b^2 + b^3)}{d} + \frac{2e^{2c+2dx}(2a^3 + 3a^2 b - b^3)}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{\frac{2(2a^3 + 3a^2 b - b^3)}{3d} + \frac{2e^{4c+4dx}(2a^3 + 3a^2 b - b^3)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} + \frac{4e^{2c+2dx}(a^2 b + 2 a b^2 + b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)`

[Out] $a^3 x - \left(\frac{2(2a^2 b + a^2 b + b^3)}{d} + \frac{2 \exp(2c + 2d*x) (3a^2 b + 2a^3 - b^3)}{3d} \right) / (\exp(4c + 4d*x) - 2 \exp(2c + 2d*x) + 1) - \left(\frac{2(3a^2 b + 2a^3 - b^3)}{3d} + \frac{2 \exp(4c + 4d*x) (3a^2 b + 2a^3 - b^3)}{3d} \right) / (3 \exp(2c + 2d*x) - 3 \exp(4c + 4d*x) + \exp(6c + 6d*x) - 1) - \frac{2b^3}{d (\exp(2c + 2d*x) + 1)} - \frac{2(3a^2 b + 2a^3 - b^3)}{3d (\exp(2c + 2d*x) - 1)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)`

[Out] Timed out

3.133 $\int \coth^5(c + dx) \left(a + b \operatorname{sech}^2(c + dx)\right)^3 dx$

Optimal. Leaf size=81

$$\frac{(a^3 + b^3) \log(\sinh(c + dx))}{d} - \frac{(a + b)^3 \operatorname{csch}^4(c + dx)}{4d} - \frac{(2a - b)(a + b)^2 \operatorname{csch}^2(c + dx)}{2d} - \frac{b^3 \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*(2*a-b)*(a+b)^2*\operatorname{csch}(d*x+c)^2/d-1/4*(a+b)^3*\operatorname{csch}(d*x+c)^4/d-b^3*\ln(\cosh(d*x+c))/d+(a^3+b^3)*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{(a^3 + b^3) \log(\sinh(c + dx))}{d} - \frac{(a + b)^3 \operatorname{csch}^4(c + dx)}{4d} - \frac{(2a - b)(a + b)^2 \operatorname{csch}^2(c + dx)}{2d} - \frac{b^3 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]`

[Out] $-\frac{(2a - b)(a + b)^2 \operatorname{Csch}[c + d*x]^2}{2d} - \frac{(a + b)^3 \operatorname{Csch}[c + d*x]^4}{4d} - \frac{b^3 \operatorname{Log}[\operatorname{Cosh}[c + d*x]]}{d} + \frac{(a^3 + b^3) \operatorname{Log}[\operatorname{Sinh}[c + d*x]]}{d}$

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 4138

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned}
\int \coth^5(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x(1-x^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^3 x} dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^3}{(-1+x)^3} - \frac{(2a-b)(a+b)^2}{(-1+x)^2} + \frac{-a^3-b^3}{-1+x} + \frac{b^3}{x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\
&= -\frac{(2a-b)(a+b)^2 \operatorname{csch}^2(c + dx)}{2d} - \frac{(a+b)^3 \operatorname{csch}^4(c + dx)}{4d} - \frac{b^3 \log(\sinh(c + dx))}{4d}
\end{aligned}$$

Mathematica [A] time = 0.91, size = 101, normalized size = 1.25

$$\frac{2(a \cosh^2(c + dx) + b)^3 (-4(a^3 + b^3) \log(\sinh(c + dx)) + (a + b)^3 \operatorname{csch}^4(c + dx) + 2(2a - b)(a + b)^2 \operatorname{csch}^2(c + dx))}{d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(2*(2*a - b)*(a + b)^2*Csch[c + d*x]^2 + (a + b)^3*Csch[c + d*x]^4 + 4*b^3*Log[Cosh[c + d*x]]) - 4*(a^3 + b^3)*Log[Sinh[c + d*x]])/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.45, size = 1830, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d*x*sinh(d*x + c)^8 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^6 + 2*(14*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x + 2*a^3 + 3*a^2*b - b^3)*sinh(d*x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 - 3*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3 - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 - 5*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 - 2*a^3*d*x - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - b^3 + 6*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (b^3*cosh(d*x + c)^8 + 8*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4*b^3*cosh(d*x + c)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^6 + 8*(7*b^3*cosh(d*x + c) - 3*b^3*cosh(d*x + c))*sinh(d*x + c)^5 - 4*b^3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cosh(d*x + c)^2 + 3*b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh(d*x + c)^6 - 15*b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 8*(b^3*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 + b^3)*cosh(d*x + c)^8 + 8*(a^3 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + b^3)*sinh(d*x + c)^8 - 4*(a^3 + b^3)*cosh(d

```

*x + c)^6 - 4*(a^3 + b^3 - 7*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
8*(7*(a^3 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + b^3)*cosh(d*x + c))*sinh(d*x +
c)^5 + 6*(a^3 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^3 + b^3)*cosh(d*x + c)^4 +
3*a^3 + 3*b^3 - 30*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^
3 + b^3)*cosh(d*x + c)^5 - 10*(a^3 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a^3 + b^3 - 4*(a^3 + b^3)*cosh(d*x + c)^2 +
4*(7*(a^3 + b^3)*cosh(d*x + c)^6 - 15*(a^3 + b^3)*cosh(d*x + c)^4 - a^3 -
b^3 + 9*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + b^3)*cosh(
d*x + c)^7 - 3*(a^3 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + b^3)*cosh(d*x + c)^3
- (a^3 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 - 3*(2*a^3*d*x - 2*a^3
- 3*a^2*b + b^3)*cosh(d*x + c)^5 + 2*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d
*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)
^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh
(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh
(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + d)

```

giac [B] time = 0.41, size = 248, normalized size = 3.06

$$12a^3dx + 12b^3 \log(e^{(2dx+2c)} + 1) - 12(a^3e^{(2c)} + b^3e^{(2c)})e^{(-2c)} \log(|e^{(2dx+2c)} - 1|) + \frac{25a^3e^{(8dx+8c)} + 25b^3e^{(8dx+8c)} - 52}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

```

[Out] -1/12*(12*a^3*d*x + 12*b^3*log(e^(2*d*x + 2*c) + 1) - 12*(a^3*e^(2*c) + b^3
*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^3*e^(8*d*x + 8*c)
+ 25*b^3*e^(8*d*x + 8*c) - 52*a^3*e^(6*d*x + 6*c) + 72*a^2*b*e^(6*d*x + 6*c)
- 124*b^3*e^(6*d*x + 6*c) + 102*a^3*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x
+ 4*c) + 246*b^3*e^(4*d*x + 4*c) - 52*a^3*e^(2*d*x + 2*c) + 72*a^2*b*e^(2*d
*x + 2*c) - 124*b^3*e^(2*d*x + 2*c) + 25*a^3 + 25*b^3)/(e^(2*d*x + 2*c) - 1
)^4)/d

```

maple [A] time = 0.32, size = 153, normalized size = 1.89

$$\frac{a^3 \ln(\sinh(dx+c))}{d} - \frac{a^3 (\coth^2(dx+c))}{2d} - \frac{a^3 (\coth^4(dx+c))}{4d} - \frac{3a^2b (\cosh^2(dx+c))}{2d \sinh(dx+c)^4} + \frac{3a^2b}{4d \sinh(dx+c)^4} - \frac{3b^3}{4d \sinh(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x)

```

[Out] a^3*ln(sinh(d*x+c))/d-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-3/2/d
*a^2*b/sinh(d*x+c)^4*cosh(d*x+c)^2+3/4/d*a^2*b/sinh(d*x+c)^4-3/4/d/sinh(d*x
+c)^4*a*b^2-1/4/d*b^3/sinh(d*x+c)^4+1/2/d*b^3/sinh(d*x+c)^2+1/d*b^3*ln(tanh
(d*x+c))

```

maxima [B] time = 0.50, size = 422, normalized size = 5.21

$$a^3 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{4(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $a^3(x + c/d + \log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d + 4*(e^{-2*d*x - 2*c} - e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c}))/((d*(4*e^{-2*d*x - 2*c} - 6*e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} - e^{-8*d*x - 8*c} - 1))) + b^3(\log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d - \log(e^{-2*d*x - 2*c} + 1)/d - 2*(e^{-2*d*x - 2*c} - 4*e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c}))/((d*(4*e^{-2*d*x - 2*c} - 6*e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} - e^{-8*d*x - 8*c} - 1))) + 6*a^2*b*(e^{-2*d*x - 2*c}))/((d*(4*e^{-2*d*x - 2*c} - 6*e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} - e^{-8*d*x - 8*c} - 1))) + e^{-6*d*x - 6*c}/((d*(4*e^{-2*d*x - 2*c} - 6*e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} - e^{-8*d*x - 8*c} - 1))) - 12*a*b^2/((d*(e^{d*x + c} - e^{-d*x - c}))^4)$

mupad [B] time = 1.64, size = 384, normalized size = 4.74

$$-a^3 x - \frac{2(4a^3 + 9a^2b + 6ab^2 + b^3)}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{\ln(e^{4c+4dx} - 1)(b^3d - d(a^3 + b^3))}{2d^2} - \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}(a^6\sqrt{-d^2+4}b^6\sqrt{-d^2+4}}{a^3d\sqrt{a^6+4a^3b^3+4b^6}+2b^3d\sqrt{a^6-4a^3b^3+4b^6}}\right)}{\sqrt{-d^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3,x)

[Out] $-a^3x - (2*(6*a*b^2 + 9*a^2*b + 4*a^3 + b^3))/((d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (\log(\exp(4*c + 4*d*x) - 1)*(b^3*d - d*(a^3 + b^3))))/(2*d^2) - (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*(a^6*(-d^2)^{(1/2)} + 4*b^6*(-d^2)^{(1/2)} + 4*a^3*b^3*(-d^2)^{(1/2)}))/((a^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^{(1/2)} + 2*b^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^{(1/2)})))*(a^6 + 4*b^6 + 4*a^3*b^3)^{(1/2)})/(-d^2)^{(1/2)} - (2*(3*a^2*b + 2*a^3 - b^3))/((d*(\exp(2*c + 2*d*x) - 1)) - (8*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/((d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (4*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/((d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.134 $\int \coth^6(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx$

Optimal. Leaf size=69

$$-\frac{(a^3 + b^3) \coth(c + dx)}{d} + a^3 x - \frac{(a + b)^3 \coth^5(c + dx)}{5d} - \frac{(a - 2b)(a + b)^2 \coth^3(c + dx)}{3d}$$

[Out] $a^3 x - (a^3 + b^3) \coth(d x + c) / d - 1/3 (a - 2 b) (a + b)^2 \coth(d x + c)^3 / d - 1/5 (a + b)^3 \coth(d x + c)^5 / d$

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4141, 1802, 207}

$$-\frac{(a^3 + b^3) \coth(c + dx)}{d} + a^3 x - \frac{(a + b)^3 \coth^5(c + dx)}{5d} - \frac{(a - 2b)(a + b)^2 \coth^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]

[Out] $a^3 x - ((a^3 + b^3) \operatorname{Coth}[c + d x]) / d - ((a - 2 b) (a + b)^2 \operatorname{Coth}[c + d x]^3) / (3 d) - ((a + b)^3 \operatorname{Coth}[c + d x]^5) / (5 d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \coth^6(c + dx) \left(a + b \operatorname{sech}^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+b(1-x^2))^3}{x^6(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(\frac{(a+b)^3}{x^6} + \frac{(a-2b)(a+b)^2}{x^4} + \frac{a^3+b^3}{x^2} - \frac{a^3}{-1+x^2} \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{(a^3 + b^3) \coth(c + dx)}{d} - \frac{(a - 2b)(a + b)^2 \coth^3(c + dx)}{3d} - \frac{(a + b)^3}{-1 + x^2} \\ &= a^3 x - \frac{(a^3 + b^3) \coth(c + dx)}{d} - \frac{(a - 2b)(a + b)^2 \coth^3(c + dx)}{3d} - \frac{(a + b)^3}{-1 + x^2} \end{aligned}$$

Mathematica [B] time = 1.09, size = 303, normalized size = 4.39

$$\operatorname{csch}(c)\operatorname{csch}^5(c+dx)\left(180a^3\sinh(2c+dx)-140a^3\sinh(2c+3dx)-90a^3\sinh(4c+3dx)+46a^3\sinh(4c+5dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] - 75*a^3*d*x*Cosh[4*c + 3*d*x] - 15*a^3*d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 280*a^3*Sinh[d*x] + 180*a^2*b*Sinh[d*x] + 60*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] + 180*a^3*Sinh[2*c + d*x] - 180*a*b^2*Sinh[2*c + d*x] - 140*a^3*Sinh[2*c + 3*d*x] + 60*a*b^2*Sinh[2*c + 3*d*x] - 80*b^3*Sinh[2*c + 3*d*x] - 90*a^3*Sinh[4*c + 3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 46*a^3*Sinh[4*c + 5*d*x] + 18*a^2*b*Sinh[4*c + 5*d*x] - 12*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/(480*d)

fricas [B] time = 0.41, size = 521, normalized size = 7.55

$$\frac{(23a^3 + 9a^2b - 6ab^2 + 8b^3)\cosh(dx+c)^5 + 5(23a^3 + 9a^2b - 6ab^2 + 8b^3)\cosh(dx+c)\sinh(dx+c)^4 - \dots}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/15*((23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*sinh(d*x + c)^5 - 5*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 5*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3 - 2*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^3 - 3*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 9*a^2*b + 12*a*b^2 + 8*b^3)*cosh(d*x + c) - 5*(30*a^3*d*x + (15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 46*a^3 + 18*a^2*b - 12*a*b^2 + 16*b^3 - 3*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))

giac [B] time = 0.43, size = 210, normalized size = 3.04

$$15a^3dx - \frac{2(45a^3e^{(8dx+8c)}+45a^2be^{(8dx+8c)}-90a^3e^{(6dx+6c)}+90ab^2e^{(6dx+6c)}+140a^3e^{(4dx+4c)}+90a^2be^{(4dx+4c)}+30ab^2e^{(4dx+4c)}+80b^3e^{(4dx+4c)})}{(e^{(2dx+2c)}-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/15*(15*a^3*d*x - 2*(45*a^3*e^(8*d*x + 8*c) + 45*a^2*b*e^(8*d*x + 8*c) - 90*a^3*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*d*x + 4*c) + 90*a^2*b*e^(4*d*x + 4*c) + 30*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) - 70*a^3*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) - 40*b^3*e^(2*d*x + 2*c) + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) - 1)^5/d

maple [B] time = 0.42, size = 199, normalized size = 2.88

$$a^3\left(dx+c-\operatorname{coth}(dx+c)-\frac{(\operatorname{coth}^3(dx+c))}{3}-\frac{(\operatorname{coth}^5(dx+c))}{5}\right)+3a^2b\left(-\frac{\cosh^3(dx+c)}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4c}{8}\right)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\coth(dx+c)^6*(a+b*\text{sech}(dx+c)^2)^3,x)$

[Out] $\frac{1}{d}(a^3*(dx+c-\coth(dx+c)-\frac{1}{3}\coth(dx+c)^3-\frac{1}{5}\coth(dx+c)^5)+3a^2*b*(-\frac{1}{2}\sinh(dx+c)^5*\cosh(dx+c)^3+\frac{3}{8}\sinh(dx+c)^5*\cosh(dx+c)+\frac{3}{8}*(-\frac{8}{15}-\frac{1}{5}\text{csch}(dx+c)^4+\frac{4}{15}\text{csch}(dx+c)^2)*\coth(dx+c))+3a*b^2*(-\frac{1}{4}\sinh(dx+c)^5*\cosh(dx+c)-\frac{1}{4}*(-\frac{8}{15}-\frac{1}{5}\text{csch}(dx+c)^4+\frac{4}{15}\text{csch}(dx+c)^2)*\coth(dx+c))+b^3*(-\frac{8}{15}-\frac{1}{5}\text{csch}(dx+c)^4+\frac{4}{15}\text{csch}(dx+c)^2)*\coth(dx+c))$

maxima [B] time = 0.36, size = 826, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\coth(dx+c)^6*(a+b*\text{sech}(dx+c)^2)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] $\frac{1}{15}a^3*(15*x + 15*c/d - 2*(70*e^{(-2*d*x - 2*c)} - 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} - 45*e^{(-8*d*x - 8*c)} - 23)/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) + \frac{4}{5}a*b^2*(5*e^{(-2*d*x - 2*c)})/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) + \frac{5*e^{(-4*d*x - 4*c)}}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) + \frac{15*e^{(-6*d*x - 6*c)}}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - \frac{1}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) - \frac{16}{15}b^3*(5*e^{(-2*d*x - 2*c)})/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - \frac{10*e^{(-4*d*x - 4*c)}}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - \frac{1}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) + \frac{6}{5}a^2*b*(10*e^{(-4*d*x - 4*c)})/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) + \frac{5*e^{(-8*d*x - 8*c)}}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) + \frac{1}{d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)))$

mupad [B] time = 1.60, size = 547, normalized size = 7.93

$$a^3 x - \frac{\frac{6(a^3+b a^2)}{5d} + \frac{24e^{2c+2dx}(a^2 b+a b^2)}{5d} + \frac{24e^{6c+6dx}(a^2 b+a b^2)}{5d} + \frac{6e^{8c+8dx}(a^3+b a^2)}{5d} + \frac{4e^{4c+4dx}(5a^3+9a^2 b+12 a b^2+8 b^3)}{5d}}{5e^{2c+2dx} - 10e^{4c+4dx} + 10e^{6c+6dx} - 5e^{8c+8dx} + e^{10c+10dx} - 1} - \frac{6(a^2 b+a b^2)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\coth(c + dx)^6*(a + b/\cosh(c + dx)^2)^3,x)$

[Out] $a^3*x - ((6*(a^2*b + a^3))/(5*d) + (24*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (24*\exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*\exp(8*c + 8*d*x)*(a^2*b + a^3))/(5*d) + (4*\exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3)))/(5*d)))/(5*\exp(2*c + 2*d*x) - 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) - 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) - 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (6*\exp(2*c + 2*d*x)*(a^2*b + a^3))/(5*d))/(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (18*\exp(4*c + 4*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*\exp(6*c + 6*d*x)*(a^2*b + a^3))/(5*d) + (2*\exp(2*c + 2*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3)))/(5*d))/(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - ((2*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3))/(15*d) + (12*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b)))/$

$$(5*d) + (6*\exp(4*c + 4*d*x)*(a^2*b + a^3))/(5*d)/(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1) - (6*(a^2*b + a^3))/(5*d*(\exp(2*c + 2*d*x) - 1))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.135 $\int \coth^7(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx$

Optimal. Leaf size=77

$$\frac{a^3 \log(\sinh(c + dx))}{d} - \frac{3a^2(a + b) \operatorname{csch}^2(c + dx)}{2d} - \frac{(a + b)^3 \operatorname{csch}^6(c + dx)}{6d} - \frac{3a(a + b)^2 \operatorname{csch}^4(c + dx)}{4d}$$

[Out] $-3/2*a^2*(a+b)*\operatorname{csch}(d*x+c)^2/d-3/4*a*(a+b)^2*\operatorname{csch}(d*x+c)^4/d-1/6*(a+b)^3*\operatorname{csch}(d*x+c)^6/d+a^3*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 444, 43}

$$-\frac{3a^2(a + b) \operatorname{csch}^2(c + dx)}{2d} + \frac{a^3 \log(\sinh(c + dx))}{d} - \frac{(a + b)^3 \operatorname{csch}^6(c + dx)}{6d} - \frac{3a(a + b)^2 \operatorname{csch}^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[c + d*x]^7*(a + b*\text{Sech}[c + d*x]^2)^3, x]$

[Out] $(-3*a^2*(a + b)*\text{Csch}[c + d*x]^2)/(2*d) - (3*a*(a + b)^2*\text{Csch}[c + d*x]^4)/(4*d) - ((a + b)^3*\text{Csch}[c + d*x]^6)/(6*d) + (a^3*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 444

$\text{Int}[(x + a)^m*(b*x + c)^n*(d*x + e)^p, x] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m - n + 1, 0]$

Rule 4138

$\text{Int}[(a + b*\sec(e + f*x))^m*(c + d*x)^n, x] \rightarrow \text{Module}\{\text{ff} = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[(\text{ff}^m*(m + n*p - 1))^{-1}, \text{Subst}[\text{Int}[(1 - \text{ff}^2*x^2)^{(m-1)/2}*(b + a*(\text{ff}*x)^n)^p/x^{m+n*p}, x], x, \text{Cos}[e + f*x]/\text{ff}], x] /; \text{FreeQ}\{a, b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[(m-1)/2] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \coth^7(c + dx) (a + b \operatorname{sech}^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x(b+ax^2)^3}{(1-x^2)^4} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^4} dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^4} + \frac{3a(a+b)^2}{(-1+x)^3} + \frac{3a^2(a+b)}{(-1+x)^2} + \frac{a^3}{-1+x}\right) dx, x, \cosh^2(c + dx)\right)}{2d} \\ &= -\frac{3a^2(a + b) \operatorname{csch}^2(c + dx)}{2d} - \frac{3a(a + b)^2 \operatorname{csch}^4(c + dx)}{4d} - \frac{(a + b)^3 \operatorname{csch}^6(c + dx)}{6d} \end{aligned}$$

Mathematica [A] time = 0.80, size = 98, normalized size = 1.27

$$\frac{2(a \cosh^2(c + dx) + b)^3 (-12a^3 \log(\sinh(c + dx)) + 18a^2(a + b) \operatorname{csch}^2(c + dx) + 2(a + b)^3 \operatorname{csch}^6(c + dx) + 9a^2 \operatorname{csch}^4(c + dx) + 2(a + b)^3 \operatorname{csch}^2(c + dx) - 12a^3 \operatorname{Log}[\operatorname{Sinh}[c + dx]])}{3d(a \cosh(2(c + dx)) + a + 2b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(18*a^2*(a + b)*Csch[c + d*x]^2 + 9*a*(a + b)^2*Csch[c + d*x]^4 + 2*(a + b)^3*Csch[c + d*x]^6 - 12*a^3*Log[Sinh[c + d*x]])/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)

fricas [B] time = 0.46, size = 2632, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*a^3*d*x*sinh(d*x + c)^12 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^10 + 18*(11*a^3*d*x*cosh(d*x + c)^2 - a^3*d*x + a^3 + a^2*b)*sinh(d*x + c)^10 + 60*(11*a^3*d*x*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x - 4*a^3 + 4*a*b^2 - 90*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 - 30*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 - 15*a^3*d*x - 945*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^4 + 17*a^3 + 15*a^2*b + 6*a*b^2 + 8*b^3 + 63*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 - 189*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - (15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 + 3*(495*a^3*d*x*cosh(d*x + c)^8 - 1260*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^6 + 15*a^3*d*x + 210*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 - 12*a^3 + 12*a*b^2 - 20*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 - 540*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^7 + 126*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^5 - 20*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 - 135*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^8 + 42*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^6 - 3*a^3*d*x - 10*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 3*a^3 + 3*a^2*b + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(d*x + c)^12 - 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d*x + c)^8 + 6*(11*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 3*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 20*a^3*cosh(d*x + c)^6 + 15*(33*a^3*cosh(d*x + c)^4 - 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 - 315*a^3*cosh(d*x + c)^4 + 105*a^3*cosh(d*x + c)^2 - 5*a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 - 5*a^3*cosh(d*x + c))*sinh(d*x + c)^5 - 6*a^3*cosh(d*x + c)^2 + 15*(33*a^3*cosh(d*x + c)^8 - 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^4 - 20*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c)^9 - 36*a^3*cosh(d*x + c)^7 + 42*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)

)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^10 - 45*a^3*cosh(d*x + c)^8 + 70*a^3*cosh(d*x + c)^6 - 50*a^3*cosh(d*x + c)^4 + 15*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^11 - 5*a^3*cosh(d*x + c)^9 + 10*a^3*cosh(d*x + c)^7 - 10*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 - 15*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^9 + 6*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^7 - 2*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 3*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.59, size = 239, normalized size = 3.10

$$60 a^3 dx - 60 a^3 \log \left(\left| e^{(2 dx+2 c)} - 1 \right| \right) + \frac{147 a^3 e^{(12 dx+12 c)} - 522 a^3 e^{(10 dx+10 c)} + 360 a^2 b e^{(10 dx+10 c)} + 1485 a^3 e^{(8 dx+8 c)} + 720 a b^2 e^{(8 dx+8 c)} - 1580 a^3 e^{(6 dx+6 c)} + 1200 a^2 b e^{(6 dx+6 c)} + 480 a b^2 e^{(6 dx+6 c)} + 640 b^3 e^{(6 dx+6 c)} + 1485 a^3 e^{(4 dx+4 c)} + 720 a b^2 e^{(4 dx+4 c)} - 522 a^3 e^{(2 dx+2 c)} + 360 a^2 b e^{(2 dx+2 c)} + 147 a^3}{(e^{(2 dx+2 c)} - 1)^6} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/60*(60*a^3*d*x - 60*a^3*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^3*e^(12*d*x + 12*c) - 522*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 1485*a^3*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) - 1580*a^3*e^(6*d*x + 6*c) + 1200*a^2*b*e^(6*d*x + 6*c) + 480*a*b^2*e^(6*d*x + 6*c) + 640*b^3*e^(6*d*x + 6*c) + 1485*a^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) - 522*a^3*e^(2*d*x + 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) - 1)^6)/d

maple [B] time = 0.36, size = 189, normalized size = 2.45

$$\frac{a^3 \ln(\sinh(dx + c))}{d} - \frac{a^3 (\coth^2(dx + c))}{2d} - \frac{a^3 (\coth^4(dx + c))}{4d} - \frac{a^3 (\coth^6(dx + c))}{6d} - \frac{3a^2b (\cosh^4(dx + c))}{2d \sinh(dx + c)^6} + \frac{3a^2b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x)

[Out] a^3*ln(sinh(d*x+c))/d-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-1/6*a^3*coth(d*x+c)^6/d-3/2/d*a^2*b/sinh(d*x+c)^6*cosh(d*x+c)^4+3/2/d*a^2*b/sinh(d*x+c)^6*cosh(d*x+c)^2-1/2/d*a^2*b/sinh(d*x+c)^6-3/4/d*a*b^2/sinh(d*x+c)^6*cosh(d*x+c)^2+1/4/d*a*b^2/sinh(d*x+c)^6-1/6/d/sinh(d*x+c)^6*b^3

maxima [B] time = 0.42, size = 727, normalized size = 9.44

$$\frac{1}{3}a^3\left(3x + \frac{3c}{d} + \frac{3\log(e^{-dx-c} + 1)}{d} + \frac{3\log(e^{-dx-c} - 1)}{d} + \frac{2(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 18e^{(-8dx-8c)} + 9e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}\right) + \frac{2(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 18e^{(-8dx-8c)} + 9e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 2*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - 32/3*b^3/(d*(e^(d*x + c) - e^(-d*x - c))^6)

mupad [B] time = 1.61, size = 411, normalized size = 5.34

$$\frac{a^3 \ln(e^{2c} e^{2dx} - 1)}{d} - \frac{32(a^3 + 3a^2b + 3ab^2 + b^3)}{d(5e^{2c+2dx} - 10e^{4c+4dx} + 10e^{6c+6dx} - 5e^{8c+8dx} + e^{10c+10dx} - 1)} - \frac{3d(15e^{4c+4dx} - 10e^{6c+6dx} + 5e^{8c+8dx} - e^{10c+10dx} - 1)}{3d(15e^{4c+4dx} - 10e^{6c+6dx} + 5e^{8c+8dx} - e^{10c+10dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (a^3*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (6*(a^2*b + a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (6*(2*a*b^2 + 5*a^2*b + 3*a^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(21*a*b^2 + 30*a^2*b + 13*a^3 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(27*a*b^2 + 30*a^2*b + 11*a^3 + 8*b^3))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a^3*x

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

3.136 $\int (a + b \operatorname{sech}^2(c + dx))^4 dx$

Optimal. Leaf size=111

$$a^4x - \frac{b^2(6a^2 + 8ab + 3b^2) \tanh^3(c + dx)}{3d} + \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} + \frac{b^3(4a + 3b) \tanh^5(c + dx)}{5d} - \frac{b^4 \tanh^7(c + dx)}{7d}$$

[Out] $a^4x + b(2a + b)(2a^2 + 2ab + b^2) \operatorname{Tanh}(d*x + c)/d - 1/3*b^2*(6*a^2 + 8*a*b + 3*b^2) * \operatorname{Tanh}(d*x + c)^3/d + 1/5*b^3*(4*a + 3*b) * \operatorname{Tanh}(d*x + c)^5/d - 1/7*b^4 * \operatorname{Tanh}(d*x + c)^7/d$

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4128, 390, 206}

$$-\frac{b^2(6a^2 + 8ab + 3b^2) \tanh^3(c + dx)}{3d} + \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} + a^4x + \frac{b^3(4a + 3b) \tanh^5(c + dx)}{5d} - \frac{b^4 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^4, x]

[Out] $a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*\operatorname{Tanh}[c + d*x])/d - (b^2*(6*a^2 + 8*a*b + 3*b^2)*\operatorname{Tanh}[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*\operatorname{Tanh}[c + d*x]^5)/(5*d) - (b^4*\operatorname{Tanh}[c + d*x]^7)/(7*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4128

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && NeQ[a + b, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(c + dx))^4 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^4}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (b(2a + b)(2a^2 + 2ab + b^2) - b^2(6a^2 + 8ab + 3b^2)x^2 + b^3(4a + 3b)x^4 - b^4x^6) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(6a^2 + 8ab + 3b^2) \tanh^3(c + dx)}{3d} + \frac{b^3(4a + 3b) \tanh^5(c + dx)}{5d} - \frac{b^4 \tanh^7(c + dx)}{7d} \\ &= a^4x + \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(6a^2 + 8ab + 3b^2) \tanh^3(c + dx)}{3d} + \frac{b^3(4a + 3b) \tanh^5(c + dx)}{5d} - \frac{b^4 \tanh^7(c + dx)}{7d} \end{aligned}$$

Mathematica [B] time = 1.67, size = 455, normalized size = 4.10

$$\operatorname{sech}(c)\operatorname{sech}^7(c+dx)\left(3675a^4dx\cosh(2c+dx)+2205a^4dx\cosh(2c+3dx)+2205a^4dx\cosh(4c+3dx)+735a^4dx\cosh(4c+5dx)+735a^4dx\cosh(6c+5dx)+105a^4dx\cosh(6c+7dx)+105a^4dx\cosh(8c+7dx)+16800a^3b\sinh(dx)+18480a^2b^2\sinh(dx)+11200a^3b\sinh(dx)+3360b^4\sinh(dx)-12600a^3b\sinh(2c+dx)-10920a^2b^2\sinh(2c+dx)-4480ab^3\sinh(2c+dx)+12600a^3b\sinh(2c+3dx)+15120a^2b^2\sinh(2c+3dx)+9408ab^3\sinh(2c+3dx)+2016b^4\sinh(2c+3dx)-5040a^3b\sinh(4c+3dx)-2520a^2b^2\sinh(4c+3dx)+5040a^3b\sinh(4c+5dx)+5880a^2b^2\sinh(4c+5dx)+3136ab^3\sinh(4c+5dx)+672b^4\sinh(4c+5dx)-840a^3b\sinh(6c+5dx)+840a^3b\sinh(6c+7dx)+840a^2b^2\sinh(6c+7dx)+448ab^3\sinh(6c+7dx)+96b^4\sinh(6c+7dx)\right)/(13440d)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^4, x]

[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^4*d*x*Cosh[d*x] + 3675*a^4*d*x*Cosh[2*c + d*x] + 2205*a^4*d*x*Cosh[2*c + 3*d*x] + 2205*a^4*d*x*Cosh[4*c + 3*d*x] + 735*a^4*d*x*Cosh[4*c + 5*d*x] + 735*a^4*d*x*Cosh[6*c + 5*d*x] + 105*a^4*d*x*Cosh[6*c + 7*d*x] + 105*a^4*d*x*Cosh[8*c + 7*d*x] + 16800*a^3*b*Sinh[d*x] + 18480*a^2*b^2*Sinh[d*x] + 11200*a^3*b*Sinh[d*x] + 3360*b^4*Sinh[d*x] - 12600*a^3*b*Sinh[2*c + d*x] - 10920*a^2*b^2*Sinh[2*c + d*x] - 4480*a*b^3*Sinh[2*c + d*x] + 12600*a^3*b*Sinh[2*c + 3*d*x] + 15120*a^2*b^2*Sinh[2*c + 3*d*x] + 9408*a*b^3*Sinh[2*c + 3*d*x] + 2016*b^4*Sinh[2*c + 3*d*x] - 5040*a^3*b*Sinh[4*c + 3*d*x] - 2520*a^2*b^2*Sinh[4*c + 3*d*x] + 5040*a^3*b*Sinh[4*c + 5*d*x] + 5880*a^2*b^2*Sinh[4*c + 5*d*x] + 3136*a*b^3*Sinh[4*c + 5*d*x] + 672*b^4*Sinh[4*c + 5*d*x] - 840*a^3*b*Sinh[6*c + 5*d*x] + 840*a^3*b*Sinh[6*c + 7*d*x] + 840*a^2*b^2*Sinh[6*c + 7*d*x] + 448*a*b^3*Sinh[6*c + 7*d*x] + 96*b^4*Sinh[6*c + 7*d*x]))/(13440*d)

fricas [B] time = 0.42, size = 941, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")

[Out] 1/105*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*sinh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^5 + 28*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4 + 3*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + (105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)*sinh(d*x + c)^4 + 21*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 28*(5*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 135*a^3*b + 225*a^2*b^2 + 168*a*b^3 + 36*b^4 + 10*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^5 + 10*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 9*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c) + 28*((105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 5*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 75*a^3*b + 135*a^2*b^2 + 120*a*b^3 + 60*b^4 + 9*(45*a^3*b + 75*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))

giac [B] time = 0.15, size = 334, normalized size = 3.01

$$105(dx+c)a^4 - \frac{8(105a^3be^{12dx+12c})+630a^3be^{10dx+10c}+315a^2b^2e^{10dx+10c}+1575a^3be^{8dx+8c}+1365a^2b^2e^{8dx+8c}+560ab^3e^{8dx+8c}+224a^4be^{6dx+6c}+1680a^3b^2e^{6dx+6c}+1008a^2b^3e^{6dx+6c}+288a^4b^3e^{6dx+6c}+105a^4be^{4dx+4c}+756a^3b^2e^{4dx+4c}+252a^2b^3e^{4dx+4c}+36a^4b^3e^{4dx+4c}+105a^4be^{2dx+2c}+756a^3b^2e^{2dx+2c}+252a^2b^3e^{2dx+2c}+36a^4b^3e^{2dx+2c}+105a^4be^c+756a^3b^2e^c+252a^2b^3e^c+36a^4b^3e^c}{13440d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="giac")

[Out] $\frac{1}{105} \cdot (105 \cdot (d \cdot x + c) \cdot a^4 - 8 \cdot (105 \cdot a^3 \cdot b \cdot e^{(12 \cdot d \cdot x + 12 \cdot c)} + 630 \cdot a^3 \cdot b \cdot e^{(10 \cdot d \cdot x + 10 \cdot c)} + 315 \cdot a^2 \cdot b^2 \cdot e^{(10 \cdot d \cdot x + 10 \cdot c)} + 1575 \cdot a^3 \cdot b \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} + 1365 \cdot a^2 \cdot b^2 \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} + 560 \cdot a \cdot b^3 \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} + 2100 \cdot a^3 \cdot b \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 2310 \cdot a^2 \cdot b^2 \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 1400 \cdot a \cdot b^3 \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 420 \cdot b^4 \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 1575 \cdot a^3 \cdot b \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 1890 \cdot a^2 \cdot b^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 1176 \cdot a \cdot b^3 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 252 \cdot b^4 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 630 \cdot a^3 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 735 \cdot a^2 \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 392 \cdot a \cdot b^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 84 \cdot b^4 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 105 \cdot a^3 \cdot b + 105 \cdot a^2 \cdot b^2 + 56 \cdot a \cdot b^3 + 12 \cdot b^4) / (e^{(2 \cdot d \cdot x + 2 \cdot c)} + 1)^7) / d$

maple [A] time = 0.52, size = 129, normalized size = 1.16

$$\frac{a^4(dx+c) + 4a^3b \tanh(dx+c) + 6a^2b^2 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 4ab^3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^4,x)

[Out] $\frac{1}{d} \cdot (a^4 \cdot (d \cdot x + c) + 4 \cdot a^3 \cdot b \cdot \tanh(d \cdot x + c) + 6 \cdot a^2 \cdot b^2 \cdot (2/3 + 1/3 \cdot \operatorname{sech}(d \cdot x + c)^2) \cdot \tanh(d \cdot x + c) + 4 \cdot a \cdot b^3 \cdot (8/15 + 1/5 \cdot \operatorname{sech}(d \cdot x + c)^4 + 4/15 \cdot \operatorname{sech}(d \cdot x + c)^2) \cdot \tanh(d \cdot x + c) + b^4 \cdot (16/35 + 1/7 \cdot \operatorname{sech}(d \cdot x + c)^6 + 6/35 \cdot \operatorname{sech}(d \cdot x + c)^4 + 8/35 \cdot \operatorname{sech}(d \cdot x + c)^2) \cdot \tanh(d \cdot x + c))$

maxima [B] time = 0.57, size = 703, normalized size = 6.33

$$a^4x + \frac{32}{35} b^4 \left(\frac{7e^{(-2dx-2c)}}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")

[Out] $a^4x + \frac{32}{35} \cdot b^4 \cdot (7 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} / (d \cdot (7 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 21 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 35 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 35 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + 21 \cdot e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 7 \cdot e^{(-12 \cdot d \cdot x - 12 \cdot c)} + e^{(-14 \cdot d \cdot x - 14 \cdot c)} + 1)) + 21 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} / (d \cdot (7 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 21 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 35 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 35 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + 21 \cdot e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 7 \cdot e^{(-12 \cdot d \cdot x - 12 \cdot c)} + e^{(-14 \cdot d \cdot x - 14 \cdot c)} + 1)) + 35 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} / (d \cdot (7 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 21 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 35 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 35 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + 21 \cdot e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 7 \cdot e^{(-12 \cdot d \cdot x - 12 \cdot c)} + e^{(-14 \cdot d \cdot x - 14 \cdot c)} + 1)) + 1 / (d \cdot (7 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 21 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 35 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 35 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + 21 \cdot e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 7 \cdot e^{(-12 \cdot d \cdot x - 12 \cdot c)} + e^{(-14 \cdot d \cdot x - 14 \cdot c)} + 1))) + 64/15 \cdot a \cdot b^3 \cdot (5 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} / (d \cdot (5 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 10 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 10 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 5 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 1)) + 10 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} / (d \cdot (5 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 10 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 10 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 5 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 1)) + 1 / (d \cdot (5 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 10 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 10 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 5 \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} + e^{(-10 \cdot d \cdot x - 10 \cdot c)} + 1))) + 8 \cdot a^2 \cdot b^2 \cdot (3 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} / (d \cdot (3 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 3 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 1)) + 1 / (d \cdot (3 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 3 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 1))) + 8 \cdot a^3 \cdot b / (d \cdot (e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 1)))$

mupad [B] time = 0.20, size = 1083, normalized size = 9.76

$$a^4x - \frac{8(a^3b+a^2b^2)}{7d} + \frac{8e^{2c+2dx}(15a^3b+24a^2b^2+16ab^3)}{21d} + \frac{16e^{6c+6dx}(15a^3b+24a^2b^2+16ab^3)}{21d} + \frac{16e^{4c+4dx}(5a^3b+9a^2b^2+8ab^3+4b^4)}{7d} - \frac{6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx} + 6e^{10c+10dx} + e^{12c+12dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2)^4, x)`

[Out] $a^4x - \frac{(8(a^3b + a^2b^2))}{(7d)} + \frac{(8\exp(2c + 2dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(21d)} + \frac{(16\exp(6c + 6dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(21d)} + \frac{(16\exp(4c + 4dx))(8ab^3 + 5a^3b + 4b^4 + 9a^2b^2)}{(7d)} + \frac{(40\exp(8c + 8dx))(a^3b + a^2b^2)}{(7d)} + \frac{(8a^3b\exp(10c + 10dx))}{(7d)} + \frac{(6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx)x + 1)}{(6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx)x + 1)} - \frac{(8\exp(4c + 4dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(7d)} + \frac{(8\exp(8c + 8dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(7d)} + \frac{(8a^3b)}{(7d)} + \frac{(32\exp(6c + 6dx))(8ab^3 + 5a^3b + 4b^4 + 9a^2b^2)}{(7d)} + \frac{(48\exp(2c + 2dx))(a^3b + a^2b^2)}{(7d)} + \frac{(48\exp(10c + 10dx))(a^3b + a^2b^2)}{(7d)} + \frac{(8a^3b\exp(12c + 12dx))}{(7d)} + \frac{(7\exp(2c + 2dx) + 21\exp(4c + 4dx) + 35\exp(6c + 6dx) + 35\exp(8c + 8dx) + 21\exp(10c + 10dx) + 7\exp(12c + 12dx) + \exp(14c + 14dx)x + 1)}{(7\exp(2c + 2dx) + 21\exp(4c + 4dx) + 35\exp(6c + 6dx) + 35\exp(8c + 8dx) + 21\exp(10c + 10dx) + 7\exp(12c + 12dx) + \exp(14c + 14dx)x + 1)} - \frac{(8(a^3b + a^2b^2))}{(7d)} + \frac{(8a^3b\exp(2c + 2dx))}{(7d)} + \frac{(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)}{(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)} - \frac{(8(16ab^3 + 15a^3b + 24a^2b^2))}{(105d)} + \frac{(16\exp(4c + 4dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(35d)} + \frac{(32\exp(2c + 2dx))(8ab^3 + 5a^3b + 4b^4 + 9a^2b^2)}{(35d)} + \frac{(32\exp(6c + 6dx))(a^3b + a^2b^2)}{(7d)} + \frac{(8a^3b\exp(8c + 8dx))}{(7d)} + \frac{(5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1)}{(5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1)} - \frac{(8(8ab^3 + 5a^3b + 4b^4 + 9a^2b^2))}{(35d)} + \frac{(8\exp(2c + 2dx))(16ab^3 + 15a^3b + 24a^2b^2)}{(35d)} + \frac{(24\exp(4c + 4dx))(a^3b + a^2b^2)}{(7d)} + \frac{(8a^3b\exp(6c + 6dx))}{(7d)} + \frac{(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx)x + \exp(8c + 8dx) + 1)}{(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx)x + \exp(8c + 8dx) + 1)} - \frac{(8(16ab^3 + 15a^3b + 24a^2b^2))}{(105d)} + \frac{(16\exp(2c + 2dx))(a^3b + a^2b^2)}{(7d)} + \frac{(8a^3b\exp(4c + 4dx))}{(7d)} + \frac{(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)}{(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)} - \frac{(8a^3b)}{(7d(\exp(2c + 2dx) + 1))}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**4, x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**4, x)`

3.137 $\int (a + b \operatorname{sech}^2(c + dx))^5 dx$

Optimal. Leaf size=163

$$a^5 x + \frac{b^3 (10a^2 + 15ab + 6b^2) \tanh^5(c + dx)}{5d} - \frac{b^2 (10a^3 + 20a^2b + 15ab^2 + 4b^3) \tanh^3(c + dx)}{3d} + \frac{b (5a^4 + 10a^3b + 10a^2b^2 + 5a^2b^2 + 4b^3) \tanh(c + dx)}{d}$$

[Out] $a^5 x + b(5a^4 + 10a^3b + 10a^2b^2 + 5a^2b^2 + 4b^3) \tanh(d*x+c)/d - 1/3 b^2 (10a^3 + 20a^2b + 15a^2b^2 + 4b^3) \tanh(d*x+c)^3/d + 1/5 b^3 (10a^2 + 15a^2b + 6b^2) \tanh(d*x+c)^5/d - 1/7 b^4 (5a + 4b) \tanh(d*x+c)^7/d + 1/9 b^5 \tanh(d*x+c)^9/d$

Rubi [A] time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4128, 390, 206}

$$\frac{b^3 (10a^2 + 15ab + 6b^2) \tanh^5(c + dx)}{5d} - \frac{b^2 (20a^2b + 10a^3 + 15ab^2 + 4b^3) \tanh^3(c + dx)}{3d} + \frac{b (10a^2b^2 + 10a^3b + 5a^4) \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^5, x]

[Out] $a^5 x + (b(5a^4 + 10a^3b + 10a^2b^2 + 5a^2b^2 + 4b^3) \operatorname{Tanh}[c + d*x])/d - (b^2(10a^3 + 20a^2b + 15a^2b^2 + 4b^3) \operatorname{Tanh}[c + d*x]^3)/(3*d) + (b^3(10a^2 + 15a^2b + 6b^2) \operatorname{Tanh}[c + d*x]^5)/(5*d) - (b^4(5a + 4b) \operatorname{Tanh}[c + d*x]^7)/(7*d) + (b^5 \operatorname{Tanh}[c + d*x]^9)/(9*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 4128

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && NeQ[a + b, 0] && NeQ[p, -1]

Rubi steps

$$\int (a + b \operatorname{sech}^2(c + dx))^5 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^5}{1-x^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b(5a^4 + 10a^3b + 10a^2b^2 + 5ab^3 + b^4) - b^2(10a^3 + 20a^2b + 15ab^2 + 5b^3)\right) dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{b(5a^4 + 10a^3b + 10a^2b^2 + 5ab^3 + b^4) \tanh(c + dx)}{d} - \frac{b^2(10a^3 + 20a^2b + 15ab^2 + 5b^3)}{3d}$$

$$= a^5x + \frac{b(5a^4 + 10a^3b + 10a^2b^2 + 5ab^3 + b^4) \tanh(c + dx)}{d} - \frac{b^2(10a^3 + 20a^2b + 15ab^2 + 5b^3)}{3d}$$

Mathematica [B] time = 6.57, size = 724, normalized size = 4.44

$$\frac{32a^5x \cosh^{10}(c + dx) (a + b \operatorname{sech}^2(c + dx))^5}{(a \cosh(2c + 2dx) + a + 2b)^5} + \frac{64 \operatorname{sech}(c) (105a^2b^3 \sinh(c) + 45ab^4 \sinh(c) + 8b^5 \sinh(c)) \cosh(c)}{105d(a \cosh(2c + 2dx) + a + 2b)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^5, x]

[Out] (32*a^5*x*Cosh[c + d*x]^10*(a + b*Sech[c + d*x]^2)^5)/(a + 2*b + a*Cosh[2*c + 2*d*x])^5 + (32*Cosh[c + d*x]^4*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*b^4*Sinh[c] + 8*b^5*Sinh[c]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^6*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[c] + 45*a*b^4*Sinh[c] + 8*b^5*Sinh[c]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^8*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh[c] + 420*a^2*b^3*Sinh[c] + 180*a*b^4*Sinh[c] + 32*b^5*Sinh[c]))/(315*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]*Sech[c]*(a + b*Sech[c + d*x]^2)^5*Sinh[d*x])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*Cosh[c + d*x]^3*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*b^4*Sinh[d*x] + 8*b^5*Sinh[d*x]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^5*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[d*x] + 45*a*b^4*Sinh[d*x] + 8*b^5*Sinh[d*x]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^7*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh[d*x] + 420*a^2*b^3*Sinh[d*x] + 180*a*b^4*Sinh[d*x] + 32*b^5*Sinh[d*x]))/(315*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*Cosh[c + d*x]^9*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(1575*a^4*b*Sinh[d*x] + 2100*a^3*b^2*Sinh[d*x] + 1680*a^2*b^3*Sinh[d*x] + 720*a*b^4*Sinh[d*x] + 128*b^5*Sinh[d*x]))/(315*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^5*Tanh[c])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5)

fricas [B] time = 0.43, size = 1652, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="fricas")

[Out] 1/315*((315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*sinh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^7 + 9*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5 + 4*(1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 +

$$\begin{aligned}
& 128*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 21*(4*(315*a^5*d*x - 1575*a^4*b \\
& - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^3 + 3*(\\
& 315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^ \\
& 5)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 36*(315*a^5*d*x - 1575*a^4*b - 2100*a^3 \\
& *b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^5 + 9*(3500*a^4*b \\
& + 7000*a^3*b^2 + 6720*a^2*b^3 + 2880*a*b^4 + 512*b^5 + 14*(1575*a^4*b + 210 \\
& 0*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^4 + 21*(1225* \\
& a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^2) \\
& *\sinh(d*x + c)^5 + 9*(14*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^ \\
& 2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^5 + 35*(315*a^5*d*x - 1575*a^4*b \\
& - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^3 + 20* \\
& (315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b \\
& ^5)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 84*(315*a^5*d*x - 1575*a^4*b - 2100*a^ \\
& 3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^3 + 3*(28*(1575*a \\
& ^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^6 + \\
& 14700*a^4*b + 32200*a^3*b^2 + 35840*a^2*b^3 + 20160*a*b^4 + 3584*b^5 + 105 \\
& *(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x \\
& + c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 9*(4*(315*a^5*d*x - 1575*a^4*b - 2100* \\
& a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^7 + 21*(315*a^5 \\
& *d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh \\
& (d*x + c)^5 + 40*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - \\
& 720*a*b^4 - 128*b^5)*\cosh(d*x + c)^3 + 28*(315*a^5*d*x - 1575*a^4*b - 2100* \\
& a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^ \\
& 2 + 126*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 \\
& - 128*b^5)*\cosh(d*x + c) + 9*((1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + \\
& 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^8 + 7*(1225*a^4*b + 2100*a^3*b^2 + 1680* \\
& a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^6 + 2450*a^4*b + 5600*a^3*b^2 \\
& + 6720*a^2*b^3 + 4480*a*b^4 + 1792*b^5 + 20*(875*a^4*b + 1750*a^3*b^2 + 168 \\
& 0*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^4 + 28*(525*a^4*b + 1150*a^3 \\
& *b^2 + 1280*a^2*b^3 + 720*a*b^4 + 128*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c))/ \\
& (d*\cosh(d*x + c)^9 + 9*d*\cosh(d*x + c)*\sinh(d*x + c)^8 + 9*d*\cosh(d*x + c)^ \\
& 7 + 21*(4*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 36*d*\cos \\
& h(d*x + c)^5 + 9*(14*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 20*d*\cosh(d \\
& *x + c))*\sinh(d*x + c)^4 + 84*d*\cosh(d*x + c)^3 + 9*(4*d*\cosh(d*x + c)^7 + \\
& 21*d*\cosh(d*x + c)^5 + 40*d*\cosh(d*x + c)^3 + 28*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^2 + 126*d*\cosh(d*x + c))
\end{aligned}$$

giac [B] time = 0.17, size = 537, normalized size = 3.29

$$315(dx + c)a^5 - \frac{2(1575a^4be^{16dx+16c} + 12600a^4be^{14dx+14c} + 6300a^3b^2e^{14dx+14c} + 44100a^4be^{12dx+12c} + 39900a^3b^2e^{12dx+12c} + 16800a^2b^3e^{12dx+12c})}{(d*\cosh(d*x + c)^9 + 9*d*\cosh(d*x + c)*\sinh(d*x + c)^8 + 9*d*\cosh(d*x + c)^7 + 21*(4*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 36*d*\cosh(d*x + c)^5 + 9*(14*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 20*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 84*d*\cosh(d*x + c)^3 + 9*(4*d*\cosh(d*x + c)^7 + 21*d*\cosh(d*x + c)^5 + 40*d*\cosh(d*x + c)^3 + 28*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 126*d*\cosh(d*x + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c))^5,x, algorithm="giac")

[Out] 1/315*(315*(d*x + c)*a^5 - 2*(1575*a^4*b*e^(16*d*x + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 6300*a^3*b^2*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 39900*a^3*b^2*e^(12*d*x + 12*c) + 16800*a^2*b^3*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10*c) + 107100*a^3*b^2*e^(10*d*x + 10*c) + 75600*a^2*b^3*e^(10*d*x + 10*c) + 25200*a*b^4*e^(10*d*x + 10*c) + 110250*a^4*b*e^(8*d*x + 8*c) + 157500*a^3*b^2*e^(8*d*x + 8*c) + 136080*a^2*b^3*e^(8*d*x + 8*c) + 65520*a*b^4*e^(8*d*x + 8*c) + 16128*b^5*e^(8*d*x + 8*c) + 88200*a^4*b*e^(6*d*x + 6*c) + 136500*a^3*b^2*e^(6*d*x + 6*c) + 124320*a^2*b^3*e^(6*d*x + 6*c) + 60480*a*b^4*e^(6*d*x + 6*c) + 10752*b^5*e^(6*d*x + 6*c) + 44100*a^4*b*e^(4*d*x + 4*c) + 69300*a^3*b^2*e^(4*d*x + 4*c) + 60480*a^2*b^3*e^(4*d*x + 4*c) + 25920*a*b^4*e^(4*d*x + 4*c) + 4608*b^5*e^(4*d*x + 4*c) + 12600*a^4*b*e^(2*d*x + 2*c) + 18900*a^3*b^2*e^(2*d*x + 2*c) + 15120*a^2*b^3*e^(2*d*x + 2*c) + 4608*b^5*e^(2*d*x + 2*c))

$$d*x + 2*c) + 6480*a*b^4*e^{(2*d*x + 2*c)} + 1152*b^5*e^{(2*d*x + 2*c)} + 1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)/(e^{(2*d*x + 2*c)} + 1)^9)/d$$

maple [A] time = 0.61, size = 185, normalized size = 1.13

$$a^5(dx + c) + 5a^4b \tanh(dx + c) + 10a^3b^2 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx + c) + 10a^2b^3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)}{15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^5,x)

[Out] 1/d*(a^5*(d*x+c)+5*a^4*b*tanh(d*x+c)+10*a^3*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+10*a^2*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+5*a*b^4*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)+b^5*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))

maxima [B] time = 0.35, size = 1277, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="maxima")

[Out] a^5*x + 256/315*b^5*(9*e^{(-2*d*x - 2*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 36*e^{(-4*d*x - 4*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 84*e^{(-6*d*x - 6*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 126*e^{(-8*d*x - 8*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 126*e^{(-10*d*x - 10*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 32/7*a*b^4*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 32/3*a^2*b^3*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 40/3*a^3*b^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 10*a^4*b/(d*(e^{(-2*d*x - 2*c)} + 1))

mupad [B] time = 0.32, size = 1952, normalized size = 11.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x))^2)^5,x`

[Out] $a^5x - \left(\frac{10(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{63d} + \frac{10 \exp(2c + 2dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{21d} + \frac{10 \exp(4c + 4dx)(a^4b + a^3b^2)}{3d} + \frac{10a^4b \exp(6c + 6dx)}{9d} \right) / (4 \exp(2c + 2dx) + 6 \exp(4c + 4dx) + 4 \exp(6c + 6dx) + \exp(8c + 8dx) + 1) - \left(\frac{80 \exp(6c + 6dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{9d} + \frac{80 \exp(10c + 10dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{9d} + \frac{4 \exp(8c + 8dx)(320ab^4 + 175a^4b + 128b^5 + 480a^2b^3 + 400a^3b^2)}{9d} + \frac{10a^4b}{9d} + \frac{40 \exp(4c + 4dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{9d} + \frac{40 \exp(12c + 12dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{9d} + \frac{80 \exp(2c + 2dx)(a^4b + a^3b^2)}{9d} + \frac{80 \exp(14c + 14dx)(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(16c + 16dx)}{9d} \right) / (9 \exp(2c + 2dx) + 36 \exp(4c + 4dx) + 84 \exp(6c + 6dx) + 126 \exp(8c + 8dx) + 126 \exp(10c + 10dx) + 84 \exp(12c + 12dx) + 36 \exp(14c + 14dx) + 9 \exp(16c + 16dx) + \exp(18c + 18dx) + 1) - \left(\frac{10(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(2c + 2dx)}{9d} \right) / (2 \exp(2c + 2dx) + \exp(4c + 4dx) + 1) - \left(\frac{10(7a^4b + 8a^2b^3 + 12a^3b^2)}{63d} + \frac{20 \exp(2c + 2dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{21d} + \frac{200 \exp(6c + 6dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{63d} + \frac{2 \exp(4c + 4dx)(320ab^4 + 175a^4b + 128b^5 + 480a^2b^3 + 400a^3b^2)}{21d} + \frac{50 \exp(8c + 8dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{21d} + \frac{20 \exp(10c + 10dx)(a^4b + a^3b^2)}{3d} + \frac{10a^4b \exp(12c + 12dx)}{9d} \right) / (7 \exp(2c + 2dx) + 21 \exp(4c + 4dx) + 35 \exp(6c + 6dx) + 35 \exp(8c + 8dx) + 21 \exp(10c + 10dx) + 7 \exp(12c + 12dx) + \exp(14c + 14dx) + 1) - \left(\frac{2(320ab^4 + 175a^4b + 128b^5 + 480a^2b^3 + 400a^3b^2)}{315d} + \frac{40 \exp(2c + 2dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{63d} + \frac{20 \exp(4c + 4dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{21d} + \frac{40 \exp(6c + 6dx)(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(8c + 8dx)}{9d} \right) / (5 \exp(2c + 2dx) + 10 \exp(4c + 4dx) + 10 \exp(6c + 6dx) + 5 \exp(8c + 8dx) + \exp(10c + 10dx) + 1) - \left(\frac{10(a^4b + a^3b^2)}{9d} + \frac{10 \exp(4c + 4dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{3d} + \frac{50 \exp(8c + 8dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{9d} + \frac{2 \exp(6c + 6dx)(320ab^4 + 175a^4b + 128b^5 + 480a^2b^3 + 400a^3b^2)}{9d} + \frac{10 \exp(2c + 2dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{9d} + \frac{10 \exp(10c + 10dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{3d} + \frac{70 \exp(12c + 12dx)(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(14c + 14dx)}{9d} \right) / (8 \exp(2c + 2dx) + 28 \exp(4c + 4dx) + 56 \exp(6c + 6dx) + 70 \exp(8c + 8dx) + 56 \exp(10c + 10dx) + 28 \exp(12c + 12dx) + 8 \exp(14c + 14dx) + \exp(16c + 16dx) + 1) - \left(\frac{10(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{63d} + \frac{100 \exp(4c + 4dx)(8ab^4 + 7a^4b + 16a^2b^3 + 15a^3b^2)}{63d} + \frac{2 \exp(2c + 2dx)(320ab^4 + 175a^4b + 128b^5 + 480a^2b^3 + 400a^3b^2)}{63d} + \frac{100 \exp(6c + 6dx)(7a^4b + 8a^2b^3 + 12a^3b^2)}{63d} + \frac{50 \exp(8c + 8dx)(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(10c + 10dx)}{9d} \right) / (6 \exp(2c + 2dx) + 15 \exp(4c + 4dx) + 20 \exp(6c + 6dx) + 15 \exp(8c + 8dx) + 6 \exp(10c + 10dx) + \exp(12c + 12dx) + 1) - \left(\frac{10(7a^4b + 8a^2b^3 + 12a^3b^2)}{63d} + \frac{20 \exp(2c + 2dx)(a^4b + a^3b^2)}{9d} + \frac{10a^4b \exp(4c + 4dx)}{9d} \right) / (3 \exp(2c + 2dx) + 3 \exp(4c + 4dx) + \exp(6c + 6dx) + 1) - \frac{10a^4b}{9d \cdot (\exp(2c + 2dx) + 1)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**5,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**5, x)
```

$$3.138 \quad \int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=70

$$\frac{(a+b)^2 \log(a \cosh^2(c+dx) + b)}{2ab^2d} - \frac{(a+2b) \log(\cosh(c+dx))}{b^2d} - \frac{\operatorname{sech}^2(c+dx)}{2bd}$$

[Out] $-(a+2*b)*\ln(\cosh(d*x+c))/b^2/d+1/2*(a+b)^2*\ln(b+a*\cosh(d*x+c)^2)/a/b^2/d-1/2*\operatorname{sech}(d*x+c)^2/b/d$

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2 \log(a \cosh^2(c+dx) + b)}{2ab^2d} - \frac{(a+2b) \log(\cosh(c+dx))}{b^2d} - \frac{\operatorname{sech}^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2), x]

[Out] $-(((a + 2*b)*\text{Log}[\text{Cosh}[c + d*x]])/(b^2*d)) + ((a + b)^2*\text{Log}[b + a*\text{Cosh}[c + d*x]^2])/(2*a*b^2*d) - \text{Sech}[c + d*x]^2/(2*b*d)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^3(b+ax^2)} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{bx^2} + \frac{-a-2b}{b^2x} + \frac{(a+b)^2}{b^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+2b)\log(\cosh(c+dx))}{b^2d} + \frac{(a+b)^2\log(b+a\cosh^2(c+dx))}{2ab^2d} - \frac{\operatorname{sech}^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 98, normalized size = 1.40

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx)) + a + 2b)(ab\operatorname{sech}^2(c+dx) + (a+b)^2(-\log(a\sinh^2(c+dx) + a + b)) + 2a)}{4ab^2d(a+b\operatorname{sech}^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2), x]

[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(2*a*(a + 2*b)*Log[Cosh[c + d*x]] - (a + b)^2*Log[a + b + a*Sinh[c + d*x]^2] + a*b*Sech[c + d*x]^2))/(a*b^2*d*(a + b*Sech[c + d*x]^2))

fricas [B] time = 0.48, size = 736, normalized size = 10.51

$$\frac{2b^2dx \cosh(dx+c)^4 + 8b^2dx \cosh(dx+c) \sinh(dx+c)^3 + 2b^2dx \sinh(dx+c)^4 + 2b^2dx + 4(b^2dx + ab)}{4ab^2d(a+b\operatorname{sech}^2(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x + a*b)*cosh(d*x + c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x + a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c)^4 + 4*a*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*d*sinh(d*x + c)^4 + 2*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d + 2*(3*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d)*sinh(d*x + c)^2 + 4*(a*b^2*d*cosh(d*x + c)^3 + a*b^2*d*cosh(d*x + c))*sinh(d*x + c))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueEvaluation time: 0.46Done

maple [B] time = 0.32, size = 331, normalized size = 4.73

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{a \ln\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2}\right)}{2db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x)

[Out] $-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/2/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/b*\ln(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a*\ln(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+2/d/b/(\tanh(1/2*d*x+1/2*c)^2+1)-1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a-2/d/b*\ln(\tanh(1/2*d*x+1/2*c)^2+1)-2/d/b/(\tanh(1/2*d*x+1/2*c)^2+1)^2$

maxima [A] time = 0.46, size = 131, normalized size = 1.87

$$\frac{dx+c}{ad} - \frac{2e^{(-2dx-2c)}}{(2be^{(-2dx-2c)} + be^{(-4dx-4c)} + b)d} - \frac{(a+2b)\log(e^{(-2dx-2c)} + 1)}{b^2d} + \frac{(a^2 + 2ab + b^2)\log(2(a+2b)e^{(-2dx-2c)})}{2ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $(d*x + c)/(a*d) - 2*e^{(-2*d*x - 2*c)} / ((2*b*e^{(-2*d*x - 2*c)} + b*e^{(-4*d*x - 4*c)} + b)*d) - (a + 2*b)*\log(e^{(-2*d*x - 2*c)} + 1) / (b^2*d) + 1/2*(a^2 + 2*a*b + b^2)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a) / (a*b^2*d)$

mupad [B] time = 1.81, size = 421, normalized size = 6.01

$$\frac{2}{bd(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{2}{bd(e^{2c+2dx} + 1)} - \frac{x}{a} \ln(39ab^7 + 243a^7b + 27a^8 + 2b^8 + 289a^2b^6 + 1017a^3b^5 + 1791a^4b^4 + 1701a^5b^3 + 891a^6b^2 + 27a^8*\exp(2*c)*\exp(2*d*x) + 2*b^8*\exp(2*c)*\exp(2*d*x) + 39*a*b^7*\exp(2*c)*\exp(2*d*x) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2),x)

[Out] $2/(b*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - 2/(b*d*(\exp(2*c + 2*d*x) + 1)) - x/a - (\log(39*a*b^7 + 243*a^7*b + 27*a^8 + 2*b^8 + 289*a^2*b^6 + 1017*a^3*b^5 + 1791*a^4*b^4 + 1701*a^5*b^3 + 891*a^6*b^2 + 27*a^8*\exp(2*c)*\exp(2*d*x) + 2*b^8*\exp(2*c)*\exp(2*d*x) + 39*a*b^7*\exp(2*c)*\exp(2*d*x) + 2$

$43a^7b \exp(2c) \exp(2dx) + 289a^2b^6 \exp(2c) \exp(2dx) + 1017a^3b^5 \exp(2c) \exp(2dx) + 1791a^4b^4 \exp(2c) \exp(2dx) + 1701a^5b^3 \exp(2c) \exp(2dx) + 891a^6b^2 \exp(2c) \exp(2dx) \cdot (a + 2b) / (b^2d) + (\log(ab^2 + 6a^2b + 3a^3 + 6a^3 \exp(2c) \exp(2dx) + 3a^3 \exp(4c) \exp(4dx) + 4b^3 \exp(2c) \exp(2dx) + 26ab^2 \exp(2c) \exp(2dx) + 24a^2b \exp(2c) \exp(2dx) + ab^2 \exp(4c) \exp(4dx) + 6a^2b \exp(4c) \exp(4dx)) \cdot (2ab + a^2 + b^2) / (2ab^2d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2), x)

[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2), x)

$$3.139 \quad \int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=59

$$-\frac{(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{x}{a} + \frac{\tanh(c+dx)}{bd}$$

[Out] x/a-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/b^(3/2)/d+tanh(d*x+c)/b/d

Rubi [A] time = 0.18, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4141, 1975, 479, 522, 206, 208}

$$-\frac{(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{x}{a} + \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] x/a - ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*b^(3/2)*d) + Tanh[c + d*x]/(b*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 479

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975

Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\tanh(c + dx)}{bd} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(-a-2b)x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{bd} \\ &= \frac{\tanh(c + dx)}{bd} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{ad} - \frac{(a+b)^2 \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c + dx)\right)}{abd} \\ &= \frac{x}{a} - \frac{(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{\tanh(c + dx)}{bd} \end{aligned}$$

Mathematica [B] time = 1.09, size = 196, normalized size = 3.32

$$\frac{\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))} \right)^4 (a \operatorname{sech}(c) \sinh(dx) \operatorname{sech}(c + dx) + b \operatorname{sech}^2(c + dx))}{2abd \sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))}^4}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*((a + b)^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(b*d*x + a*Sech[c]*Sech[c + d*x]*Sinh[d*x])))/(2*a*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.45, size = 683, normalized size = 11.58

$$\left[\frac{2 b d x \cosh (d x + c)^2 + 4 b d x \cosh (d x + c) \sinh (d x + c) + 2 b d x \sinh (d x + c)^2 + 2 b d x + ((a + b) \cosh (d x + c) + b \operatorname{sech}^2(d x + c))}{2 a b d \sqrt{a + b} \sqrt{b(\cosh(c) - \sinh(c))}^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d*x*sinh(d*x + c)^2 + 2*b*d*x + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c) + b*sech^2(d*x + c)))]

$$\begin{aligned} & x + c) * \sinh(dx + c) + (a + b) * \sinh(dx + c)^2 + a + b) * \sqrt{(a + b)/b} * \log \\ & ((a^2 * \cosh(dx + c)^4 + 4 * a^2 * \cosh(dx + c) * \sinh(dx + c)^3 + a^2 * \sinh(dx \\ & + c)^4 + 2 * (a^2 + 2 * a * b) * \cosh(dx + c)^2 + 2 * (3 * a^2 * \cosh(dx + c)^2 + a^2 + \\ & 2 * a * b) * \sinh(dx + c)^2 + a^2 + 8 * a * b + 8 * b^2 + 4 * (a^2 * \cosh(dx + c)^3 + (a \\ & ^2 + 2 * a * b) * \cosh(dx + c)) * \sinh(dx + c) + 4 * (a * b * \cosh(dx + c)^2 + 2 * a * b * c \\ & \cosh(dx + c) * \sinh(dx + c) + a * b * \sinh(dx + c)^2 + a * b + 2 * b^2) * \sqrt{(a + b \\ &)/b)) / (a * \cosh(dx + c)^4 + 4 * a * \cosh(dx + c) * \sinh(dx + c)^3 + a * \sinh(dx + \\ & c)^4 + 2 * (a + 2 * b) * \cosh(dx + c)^2 + 2 * (3 * a * \cosh(dx + c)^2 + a + 2 * b) * \sin \\ & h(dx + c)^2 + 4 * (a * \cosh(dx + c)^3 + (a + 2 * b) * \cosh(dx + c)) * \sinh(dx + c \\ &) + a)) - 4 * a) / (a * b * d * \cosh(dx + c)^2 + 2 * a * b * d * \cosh(dx + c) * \sinh(dx + c) \\ & + a * b * d * \sinh(dx + c)^2 + a * b * d), (b * d * x * \cosh(dx + c)^2 + 2 * b * d * x * \cosh(dx \\ & + c) * \sinh(dx + c) + b * d * x * \sinh(dx + c)^2 + b * d * x - ((a + b) * \cosh(dx + \\ & c)^2 + 2 * (a + b) * \cosh(dx + c) * \sinh(dx + c) + (a + b) * \sinh(dx + c)^2 + a \\ & + b) * \sqrt{-(a + b)/b} * \arctan(1/2 * (a * \cosh(dx + c)^2 + 2 * a * \cosh(dx + c) * \sin \\ & h(dx + c) + a * \sinh(dx + c)^2 + a + 2 * b) * \sqrt{-(a + b)/b} / (a + b)) - 2 * a) / \\ & (a * b * d * \cosh(dx + c)^2 + 2 * a * b * d * \cosh(dx + c) * \sinh(dx + c) + a * b * d * \sinh(dx \\ & + c)^2 + a * b * d) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4/(a+b*sech(dx+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueDone

maple [B] time = 0.35, size = 386, normalized size = 6.54

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{a \ln\left(\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{a+b}\right)}{2db^2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^4/(a+b*sech(dx+c)^2),x)

[Out]
$$\begin{aligned} & -1/d/a * \ln(\tanh(1/2 * dx + 1/2 * c) - 1) + 1/d/a * \ln(\tanh(1/2 * dx + 1/2 * c) + 1) + 1/2/d * a/b^{\frac{3}{2}} \\ & / (a+b)^{\frac{1}{2}} * \ln((a+b)^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c)^2 - 2 * b^{\frac{1}{2}} * \tanh(1/2 * dx \\ & + 1/2 * c) + (a+b)^{\frac{1}{2}}) + 1/d/b^{\frac{1}{2}} / (a+b)^{\frac{1}{2}} * \ln((a+b)^{\frac{1}{2}} * \tanh(1/2 * dx + \\ & 1/2 * c)^2 - 2 * b^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c) + (a+b)^{\frac{1}{2}}) + 1/2/d/a * b^{\frac{1}{2}} / (a+b)^{\frac{1}{2}} \\ & * \ln((a+b)^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c)^2 - 2 * b^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c) + (a \\ & + b)^{\frac{1}{2}}) - 1/2/d * a/b^{\frac{3}{2}} / (a+b)^{\frac{1}{2}} * \ln((a+b)^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c)^2 + \\ & 2 * b^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c) + (a+b)^{\frac{1}{2}}) - 1/d/b^{\frac{1}{2}} / (a+b)^{\frac{1}{2}} * \ln((a+b) \\ & ^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c)^2 + 2 * b^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c) + (a+b)^{\frac{1}{2}}) - 1/2 \\ & /d/a * b^{\frac{1}{2}} / (a+b)^{\frac{1}{2}} * \ln((a+b)^{\frac{1}{2}} * \tanh(1/2 * dx + 1/2 * c)^2 + 2 * b^{\frac{1}{2}} * \tan \\ & h(1/2 * dx + 1/2 * c) + (a+b)^{\frac{1}{2}}) + 2/d/b * \tanh(1/2 * dx + 1/2 * c) / (\tanh(1/2 * dx + 1/2 * c) \\ &)^2 + 1) \end{aligned}$$

maxima [B] time = 0.64, size = 637, normalized size = 10.80

$$\frac{(a + 2b) \log\left(\frac{ae^{(2dx+2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(2dx+2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{8\sqrt{(a+b)b}bd} + \frac{(a + 2b) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{8\sqrt{(a+b)b}bd} + \frac{3a \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16\sqrt{(a+b)b}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out]
$$-1/8*(a + 2*b)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*b*d) + 1/8*(a + 2*b)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*b*d) + 3/16*a*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*b*d) + 1/8*(a + 2*b)*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/(a*b*d) + 1/4*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/(b*d) - 1/8*(a + 2*b)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a*b*d) - 1/4*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(b*d) - 3/4*\log(e^{(2*d*x + 2*c)} + 1)/(b*d) + 3/4*\log(e^{(-2*d*x - 2*c)} + 1)/(b*d) - 1/32*(a^2 + 8*a*b + 8*b^2)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*a*b*d) + 1/32*(a^2 + 8*a*b + 8*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*a*b*d) - 5/8/((b*e^{(2*d*x + 2*c)} + b)*d) + 11/8/((b*e^{(-2*d*x - 2*c)} + b)*d)$$

mupad [B] time = 1.87, size = 183, normalized size = 3.10

$$\frac{x}{a} - \frac{2}{b d (e^{2c+2dx} + 1)} + \frac{\ln\left(\frac{4e^{2c+2dx}(a+b)^2}{a^2 b} - \frac{2(a+b)^{3/2}(a+ae^{2c+2dx}+2be^{2c+2dx})}{a^2 b^{3/2}}\right)}{2 a b^{3/2} d} (a+b)^{3/2} - \frac{\ln\left(\frac{4e^{2c+2dx}(a+b)^2}{a^2 b} + \frac{2(a+b)^{3/2}(a+ae^{2c+2dx}+2be^{2c+2dx})}{a^2 b^{3/2}}\right)}{2 a b^{3/2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)

[Out]
$$x/a - 2/(b*d*(\exp(2*c + 2*d*x) + 1)) + (\log((4*\exp(2*c + 2*d*x)*(a + b)^2)/(a^2*b) - (2*(a + b)^{(3/2)}*(a + a*\exp(2*c + 2*d*x) + 2*b*\exp(2*c + 2*d*x)))/(a^2*b^{(3/2)}))*(a + b)^{(3/2)})/(2*a*b^{(3/2)}*d) - (\log((4*\exp(2*c + 2*d*x)*(a + b)^2)/(a^2*b) + (2*(a + b)^{(3/2)}*(a + a*\exp(2*c + 2*d*x) + 2*b*\exp(2*c + 2*d*x)))/(a^2*b^{(3/2)}))*(a + b)^{(3/2)})/(2*a*b^{(3/2)}*d)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)

[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)

$$3.140 \quad \int \frac{\tanh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=45

$$\frac{(a+b)\log(a\cosh^2(c+dx)+b)}{2abd} - \frac{\log(\cosh(c+dx))}{bd}$$

[Out] $-\ln(\cosh(d*x+c))/b/d+1/2*(a+b)*\ln(b+a*\cosh(d*x+c)^2)/a/b/d$

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log(a\cosh^2(c+dx)+b)}{2abd} - \frac{\log(\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]`

[Out] $-(\text{Log}[\text{Cosh}[c + d*x]]/(b*d)) + ((a + b)*\text{Log}[b + a*\text{Cosh}[c + d*x]^2])/(2*a*b*d)$

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 4138

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]`

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \frac{1-x}{x(b+ax)} dx, x, \cosh^2(c+dx)\right)}{2d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{bx} + \frac{-a-b}{b(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\ &= -\frac{\log(\cosh(c+dx))}{bd} + \frac{(a+b)\log(b+a\cosh^2(c+dx))}{2abd} \end{aligned}$$

Mathematica [A] time = 0.12, size = 41, normalized size = 0.91

$$\frac{(a+b)\log(a\cosh^2(c+dx)+b)-2a\log(\cosh(c+dx))}{2abd}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] (-2*a*Log[Cosh[c + d*x]] + (a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d)

fricas [B] time = 0.46, size = 112, normalized size = 2.49

$$\frac{2bdx - (a+b)\log\left(\frac{2(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2b)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right) + 2a\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*b*d*x - (a + b)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*a*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))/(a*b*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueDone

maple [B] time = 0.33, size = 196, normalized size = 4.36

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{\ln\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{2db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2), x)

[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b*ln((tanh(1/2*d*x+1/2*c)^4+a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a*ln(tanh(1/2*d*x+1/2*c)^4+a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d/b*ln(tanh(1/2*d*x+1/2*c)^2+1)

maxima [A] time = 0.47, size = 77, normalized size = 1.71

$$\frac{dx+c}{ad} + \frac{(a+b)\log(2(a+2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2abd} - \frac{\log(e^{(-2dx-2c)} + 1)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $(d*x + c)/(a*d) + 1/2*(a + b)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a*b*d) - \log(e^{(-2*d*x - 2*c)} + 1)/(b*d)$

mupad [B] time = 1.65, size = 238, normalized size = 5.29

$$\frac{\ln(ab + 3a^2 + 6a^2 e^{2c} e^{2dx} + 3a^2 e^{4c} e^{4dx} + 4b^2 e^{2c} e^{2dx} + 14ab e^{2c} e^{2dx} + ab e^{4c} e^{4dx})(a + b)}{2abd} - \frac{\ln(21ab^4 + \dots)}{2abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2), x)`

[Out] $(\log(a*b + 3*a^2 + 6*a^2*\exp(2*c)*\exp(2*d*x) + 3*a^2*\exp(4*c)*\exp(4*d*x) + 4*b^2*\exp(2*c)*\exp(2*d*x) + 14*a*b*\exp(2*c)*\exp(2*d*x) + a*b*\exp(4*c)*\exp(4*d*x))*(a + b)/(2*a*b*d) - \log(21*a*b^4 + 108*a^4*b + 27*a^5 + 2*b^5 + 82*a^2*b^3 + 144*a^3*b^2 + 27*a^5*\exp(2*c)*\exp(2*d*x) + 2*b^5*\exp(2*c)*\exp(2*d*x) + 21*a*b^4*\exp(2*c)*\exp(2*d*x) + 108*a^4*b*\exp(2*c)*\exp(2*d*x) + 82*a^2*b^3*\exp(2*c)*\exp(2*d*x) + 144*a^3*b^2*\exp(2*c)*\exp(2*d*x))/(b*d) - x/a$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2), x)`

[Out] `Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)`

$$3.141 \quad \int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=46

$$\frac{x}{a} - \frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

[Out] x/a-arc tanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a/d/b^(1/2)

Rubi [A] time = 0.14, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4141, 1975, 481, 206, 208}

$$\frac{x}{a} - \frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] x/a - (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 481

Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 1975

Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] :> Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)^(m_.)]), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c+dx)\right)}{ad} \\
&= \frac{x}{a} - \frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}
\end{aligned}$$

Mathematica [B] time = 0.30, size = 174, normalized size = 3.78

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(dx\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))^4} + (a+b)(\sinh(2c) - \cosh(2c)) \tanh(c+dx)\right)}{2ad\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))^4} (a + b\operatorname{sech}^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(Cosh[c] - Sinh[c])^4] + (a + b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.45, size = 419, normalized size = 9.11

$$\left[2dx + \sqrt{\frac{a+b}{b}} \log\left(\frac{a^2 \cosh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c)^3 + a^2 \sinh(dx+c)^4 + 2(a^2 + 2ab) \cosh(dx+c)^2 + 2(3a^2 \cosh(dx+c)^2 + a^2 + 2ab) \sinh(dx+c) \cosh(dx+c) + a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a+2b) \cosh(dx+c) \sinh(dx+c)}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2(a+2b) \cosh(dx+c) \sinh(dx+c)}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*d*x + sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a))] / (a*d), (d*x - sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)))/(a*d)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
 x_gcd Error: Bad Argument ValueDone

maple [B] time = 0.36, size = 251, normalized size = 5.46

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\sqrt{b}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{a+b}\right)}{2d\sqrt{b}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)

[Out] $-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/a*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-1/2/d/b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})+1/2/d/a*b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-1/2/d/a*b^{(1/2)}/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})$

maxima [B] time = 0.50, size = 291, normalized size = 6.33

$$-\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(2dx+2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{8\sqrt{(a+b)b}ad} + \frac{\log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4\sqrt{(a+b)b}d} + \frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{8\sqrt{(a+b)b}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/8*(a+2*b)*\log((a*e^{(2*d*x+2*c)}+a+2*b-2*\sqrt{(a+b)*b})/(a*e^{(2*d*x+2*c)}+a+2*b+2*\sqrt{(a+b)*b}))/(\sqrt{(a+b)*b}*a*d)+1/4*\log((a*e^{(-2*d*x-2*c)}+a+2*b-2*\sqrt{(a+b)*b})/(a*e^{(-2*d*x-2*c)}+a+2*b+2*\sqrt{(a+b)*b}))/(\sqrt{(a+b)*b}*d)+1/8*(a+2*b)*\log((a*e^{(-2*d*x-2*c)}+a+2*b-2*\sqrt{(a+b)*b})/(a*e^{(-2*d*x-2*c)}+a+2*b+2*\sqrt{(a+b)*b}))/(\sqrt{(a+b)*b}*a*d)+1/4*\log(a*e^{(4*d*x+4*c)}+2*(a+2*b)*e^{(2*d*x+2*c)}+a)/(a*d)-1/4*\log(2*(a+2*b)*e^{(-2*d*x-2*c)}+a*e^{(-4*d*x-4*c)}+a)/(a*d)$

mupad [B] time = 0.43, size = 105, normalized size = 2.28

$$\frac{x}{a} + \frac{\operatorname{atan}\left(\frac{\sqrt{-a^2 b d^2}}{a d \sqrt{a+b}} + \frac{\sqrt{-a^2 b d^2}}{2 b d \sqrt{a+b}} + \frac{e^{2c} e^{2dx} \sqrt{-a^2 b d^2}}{2 b d \sqrt{a+b}}\right) \sqrt{a+b}}{\sqrt{-a^2 b d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^2/(a+b/cosh(c+d*x)^2),x)

[Out] $x/a + (\operatorname{atan}((-a^2*b*d^2)^{(1/2)}/(a*d*(a+b)^{(1/2)})) + (-a^2*b*d^2)^{(1/2)}/(2*b*d*(a+b)^{(1/2)})) + (\exp(2*c)*\exp(2*d*x)*(-a^2*b*d^2)^{(1/2)}/(2*b*d*(a+b)^{(1/2)}))*(a+b)^{(1/2)}/(-a^2*b*d^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

$$3.142 \quad \int \frac{\tanh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=23

$$\frac{\log(a \cosh^2(c+dx) + b)}{2ad}$$

[Out] 1/2*ln(b+a*cosh(d*x+c)^2)/a/d

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4138, 260}

$$\frac{\log(a \cosh^2(c+dx) + b)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] Log[b + a*Cosh[c + d*x]^2]/(2*a*d)

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 4138

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*tan[(e_) + (f_)*(x_)^(n_)], x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\tanh(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{b+ax^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\log(b+a \cosh^2(c+dx))}{2ad} \end{aligned}$$

Mathematica [A] time = 0.18, size = 26, normalized size = 1.13

$$\frac{\log(a \cosh(2(c+dx)) + a + 2b)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] Log[a + 2*b + a*Cosh[2*(c + d*x)]]/(2*a*d)

fricas [B] time = 0.42, size = 76, normalized size = 3.30

$$\frac{2 dx - \log\left(\frac{2(a \cosh(dx+c)^2 + a \sinh(dx+c)^2 + a + 2b)}{\cosh(dx+c)^2 - 2 \cosh(dx+c) \sinh(dx+c) + \sinh(dx+c)^2}\right)}{2 ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] $-1/2*(2*d*x - \log(2*(a*\cosh(d*x + c)^2 + a*\sinh(d*x + c)^2 + a + 2*b)/(\cosh(d*x + c)^2 - 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2)))/(a*d)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error index.cc index_gcd Error: Bad Argument
ValueError index.cc index_gcd Error: Bad Argument ValueDone

maple [A] time = 0.15, size = 38, normalized size = 1.65

$$\frac{\ln(a + b\operatorname{sech}(dx + c)^2)}{2da} - \frac{\ln(\operatorname{sech}(dx + c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x)

[Out] $1/2/d/a*\ln(a+b*\operatorname{sech}(d*x+c)^2)-1/d/a*\ln(\operatorname{sech}(d*x+c))$

maxima [B] time = 0.38, size = 51, normalized size = 2.22

$$\frac{dx + c}{ad} + \frac{\log(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $(d*x + c)/(a*d) + 1/2*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a*d)$

mupad [B] time = 0.35, size = 51, normalized size = 2.22

$$\frac{\ln(a + 2ae^{2c}e^{2dx} + ae^{4c}e^{4dx} + 4be^{2c}e^{2dx}) - 2dx}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2),x)

[Out] $(\log(a + 2*a*\exp(2*c)*\exp(2*d*x) + a*\exp(4*c)*\exp(4*d*x) + 4*b*\exp(2*c)*\exp(2*d*x)) - 2*d*x)/(2*a*d)$

sympy [A] time = 7.04, size = 124, normalized size = 5.39

$$\left\{ \begin{array}{ll} \frac{\infty x \tanh(c)}{\operatorname{sech}^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{1}{2bd \operatorname{sech}^2(c+dx)} & \text{for } a = 0 \\ \frac{x \tanh(c)}{a+b \operatorname{sech}^2(c)} & \text{for } d = 0 \\ x \frac{\log(\tanh(c+dx)+1)}{d} & \text{for } b = 0 \\ \frac{x}{a} + \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \operatorname{sech}(c+dx)\right)}{2ad} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \operatorname{sech}(c+dx)\right)}{2ad} - \frac{\log(\tanh(c+dx)+1)}{ad} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2), x)

[Out] Piecewise((zoo*x*tanh(c)/sech(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2*b*d*sech(c + d*x)**2), Eq(a, 0)), (x*tanh(c)/(a + b*sech(c)**2), Eq(d, 0)), ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (x/a + log(-I*sqrt(a)*sqrt(1/b) + sech(c + d*x))/(2*a*d) + log(I*sqrt(a)*sqrt(1/b) + sech(c + d*x))/(2*a*d) - log(tanh(c + d*x) + 1)/(a*d), True))

$$3.143 \quad \int \frac{1}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=46

$$\frac{x}{a} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

[Out] x/a-arc tanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a/d/(a+b)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4127, 3181, 208}

$$\frac{x}{a} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-1), x]

[Out] x/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]*d)

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 4127

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := Simp[x/a, x] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{x}{a} - \frac{b \int \frac{1}{b+a \cosh^2(c+dx)} dx}{a} \\ &= \frac{x}{a} - \frac{b \operatorname{Subst}\left(\int \frac{1}{b-(a+b)x^2} dx, x, \operatorname{coth}(c+dx)\right)}{ad} \\ &= \frac{x}{a} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d} \end{aligned}$$

Mathematica [B] time = 0.25, size = 172, normalized size = 3.74

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(dx\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))^4} + b(\sinh(2c) - \cosh(2c)) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)\right)}{2ad\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))^4} (a + b\operatorname{sech}^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^(-1), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(Cosh[c] - Sinh[c])^4] + b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.44, size = 436, normalized size = 9.48

$$\left[2dx + \sqrt{\frac{b}{a+b}} \log \left(\frac{a^2 \cosh(dx+c)^4 + 4a^2 \cosh(dx+c) \sinh(dx+c)^3 + a^2 \sinh(dx+c)^4 + 2(a^2 + 2ab) \cosh(dx+c)^2 + 2(3a^2 \cosh(dx+c)^2 + a^2 + 2ab) \cosh(dx+c) \sinh(dx+c) + a \sinh(dx+c)^3}{a \cosh(dx+c)^4 + 4a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^3} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*d*x + sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c)*sinh(d*x + c) + a)))/(a*d), (d*x - sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b))/(a*d)]

giac [A] time = 0.39, size = 64, normalized size = 1.39

$$-\frac{b \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right) - \frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a) - (d*x + c)/a/d

maple [B] time = 0.28, size = 149, normalized size = 3.24

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{\sqrt{b} \ln\left(\sqrt{a+b} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\sqrt{b} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2da\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c)^2), x)

[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))

maxima [B] time = 0.47, size = 83, normalized size = 1.80

$$\frac{b \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{2\sqrt{(a+b)b}ad} + \frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + (d*x + c)/(a*d)

mupad [B] time = 2.16, size = 470, normalized size = 10.22

$$\frac{\sqrt{b} \operatorname{atan}\left(\frac{\left(a^5 \sqrt{-a^3 d^2 - b a^2 d^2} + a^4 b \sqrt{-a^3 d^2 - b a^2 d^2}\right) \left(e^{2c} e^{2dx} \left(\frac{2(a^2 + 8ab + 8b^2) \left(8b^{5/2} \sqrt{-a^3 d^2 - b a^2 d^2} + 8ab^{3/2} \sqrt{-a^3 d^2 - b a^2 d^2} + a^2 \sqrt{b} \sqrt{-a^3 d^2 - b a^2 d^2}\right)}{a^8 d(a+b)^2 \sqrt{-a^3 d^2 - b a^2 d^2}}\right)}{\right)}{\frac{x}{a} + \sqrt{-a^3 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cosh(c + d*x)^2),x)

[Out] x/a + (b^(1/2)*atan(((a^5*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^4*b*(- a^3*d^2 - a^2*b*d^2)^(1/2))*(exp(2*c)*exp(2*d*x))*((2*(8*a*b + a^2 + 8*b^2)*(8*b^(5/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + 8*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^2*b^(1/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2))))/(a^8*d*(a + b)^2*(- a^3*d^2 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(12*a^2*b^2*d + 8*a*b^3*d + 4*a^3*b*d))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2))) + (2*(2*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^2*b^(1/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2))/(a^8*d*(a + b)^2*(- a^3*d^2 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a^2*b^2*d + 2*a^3*b*d)*(2*a + 4*b))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2))))/(4*b)))/(- a^3*d^2 - a^2*b*d^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)**2),x)

[Out] Integral(1/(a + b*sech(c + d*x)**2), x)

$$3.144 \quad \int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=46

$$\frac{\log(\sinh(c+dx))}{d(a+b)} + \frac{b \log(a \cosh^2(c+dx) + b)}{2ad(a+b)}$$

[Out] 1/2*b*ln(b+a*cosh(d*x+c)^2)/a/d/(a+b)+ln(sinh(d*x+c))/(a+b)/d

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 72}

$$\frac{\log(\sinh(c+dx))}{d(a+b)} + \frac{b \log(a \cosh^2(c+dx) + b)}{2ad(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] (b*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a + b)*d) + Log[Sinh[c + d*x]]/((a + b)*d)

Rule 72

Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^(m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \frac{x}{(1-x)(b+ax)} dx, x, \cosh^2(c+dx)\right)}{2d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\ &= \frac{b \log(b + a \cosh^2(c+dx))}{2a(a+b)d} + \frac{\log(\sinh(c+dx))}{(a+b)d} \end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.91

$$\frac{b \log(a \sinh^2(c + dx) + a + b) + 2a \log(\sinh(c + dx))}{2a^2d + 2abd}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] (2*a*Log[Sinh[c + d*x]] + b*Log[a + b + a*Sinh[c + d*x]^2])/(2*a^2*d + 2*a*b*d)

fricas [B] time = 0.46, size = 115, normalized size = 2.50

$$\frac{2(a+b)dx - b \log\left(\frac{2(a \cosh(dx+c)^2 + a \sinh(dx+c)^2 + a + 2b)}{\cosh(dx+c)^2 - 2 \cosh(dx+c) \sinh(dx+c) + \sinh(dx+c)^2}\right) - 2a \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)}{2(a^2 + ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x - b*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*a*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))/((a^2 + a*b)*d)

giac [B] time = 0.33, size = 97, normalized size = 2.11

$$\frac{\frac{2dx}{a} - \frac{b \log(ae^{4dx+4c} + 2ae^{2dx+2c} + 4be^{2dx+2c} + a)}{a^2+ab}}{2d} - \frac{2e^{2c} \log(|e^{2dx+2c}-1|)}{ae^{2c}+be^{2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(2*d*x/a - b*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/(a^2 + a*b) - 2*e^(2*c)*log(abs(e^(2*d*x + 2*c) - 1))/(a*e^(2*c) + b*e^(2*c)))/d

maple [B] time = 0.38, size = 133, normalized size = 2.89

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{b \ln\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2da(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2), x)

[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b/a/(a+b)*ln(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))

maxima [B] time = 0.42, size = 100, normalized size = 2.17

$$\frac{b \log(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2(a^2 + ab)d} + \frac{dx + c}{ad} + \frac{\log(e^{(-dx-c)} + 1)}{(a + b)d} + \frac{\log(e^{(-dx-c)} - 1)}{(a + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="maxima")

[Out] $\frac{1}{2}b \log(2(a + 2b)e^{-2dx - 2c} + ae^{-4dx - 4c} + a) / ((a^2 + ab)d + (dx + c)/(ad) + \log(e^{-dx - c} + 1) / ((a + b)d) + \log(e^{-dx - c} - 1) / ((a + b)d))$

mupad [B] time = 1.82, size = 228, normalized size = 4.96

$$\frac{\ln(8ab^5 - b^6 - 24a^2b^4 + 32a^3b^3 - 16a^4b^2 + b^6e^{2c}e^{2dx} - 8ab^5e^{2c}e^{2dx} + 24a^2b^4e^{2c}e^{2dx} - 32a^3b^3e^{2c}e^{2dx})}{ad + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)/(a + b/cosh(c + d*x)^2), x)`

[Out] $\frac{\log(8a^2b^5 - b^6 - 24a^2b^4 + 32a^3b^3 - 16a^4b^2 + b^6\exp(2c)\exp(2dx) - 8a^2b^5\exp(2c)\exp(2dx) + 24a^2b^4\exp(2c)\exp(2dx) - 32a^3b^3\exp(2c)\exp(2dx) + 16a^4b^2\exp(2c)\exp(2dx)) / (ad + bd) - x/a + (b \log(2a^2 - ab + 4a^2\exp(2c)\exp(2dx) + 2a^2\exp(4c)\exp(4dx) - 4b^2\exp(2c)\exp(2dx) + 6ab\exp(2c)\exp(2dx) - ab\exp(4c)\exp(4dx))) / (2a^2d + 2ab*d)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2), x)`

[Out] `Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2), x)`

$$3.145 \quad \int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=62

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}} - \frac{\coth(c+dx)}{d(a+b)} + \frac{x}{a}$$

[Out] x/a-b^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(3/2)/d-coth(d*x+c)/(a+b)/d

Rubi [A] time = 0.18, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4141, 1975, 480, 522, 206, 208}

$$-\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}} - \frac{\coth(c+dx)}{d(a+b)} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] x/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(3/2)*d) - Coth[c + d*x]/((a + b)*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975

Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !

BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{\coth(c + dx)}{(a + b)d} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-bx^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{(a + b)d} \\ &= -\frac{\coth(c + dx)}{(a + b)d} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{ad} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \tanh(c + dx)\right)}{a(a + b)d} \\ &= \frac{x}{a} - \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a + b)^{3/2}d} - \frac{\coth(c + dx)}{(a + b)d} \end{aligned}$$

Mathematica [B] time = 1.16, size = 193, normalized size = 3.11

$$\frac{\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(b^2(\sinh(2c) - \cosh(2c)) \tanh^{-1}\left(\frac{(\cosh(2c) - \sinh(2c)) \operatorname{sech}(dx)(a+2b) \sinh(dx)}{2\sqrt{a+b} \sqrt{b(\cosh(c) - \sinh(c))}}\right) \right)}{2ad(a + b)^{3/2} \sqrt{b(\cosh(c) - \sinh(c))}^4}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*((a + b)*d*x + a*Csch[c]*Csch[c + d*x]*Sinh[d*x]))/(2*a*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])

fricas [B] time = 0.46, size = 749, normalized size = 12.08

$$\left[\frac{2(a + b)dx \cosh(dx + c)^2 + 4(a + b)dx \cosh(dx + c) \sinh(dx + c) + 2(a + b)dx \sinh(dx + c)^2 - 2(a + b)dx}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

```
[Out] [1/2*(2*(a + b)*d*x*cosh(d*x + c)^2 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*(a + b)*d*x*sinh(d*x + c)^2 - 2*(a + b)*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) - 4*a)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d), ((a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*x*sinh(d*x + c)^2 - (a + b)*d*x - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 2*a)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
ValueDone
```

maple [B] time = 0.46, size = 189, normalized size = 3.05

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d(a+b)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} - \frac{1}{2d(a+b)\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{b^{\frac{3}{2}} \ln\left(\sqrt{a+b}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)\right)}{2d(a+b)\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/(a+b)*tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(t
anh(1/2*d*x+1/2*c)+1)-1/2/d/(a+b)/tanh(1/2*d*x+1/2*c)+1/2/d*b^(3/2)/a/(a+b)
^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(
a+b)^(1/2))-1/2/d*b^(3/2)/a/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.48, size = 429, normalized size = 6.92

$$\frac{b \log\left(ae^{(4dx+4c)} + 2(a+2b)e^{(2dx+2c)} + a\right)}{4(a^2+ab)d} - \frac{b \log\left(2(a+2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a\right)}{4(a^2+ab)d} - \frac{(ab+2b^2) \log\left(\frac{ae^{2dx}}{ae^{2dx}}\right)}{8(a^2+ab)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```



```
[Out] 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^2 + a*b)*d) - 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^2 + a*b)*d) - 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b)))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*sqrt((a + b)*b)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b)))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*sqrt((a + b)*b)*d) - 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b)))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*sqrt((a + b)*b)*d) - 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b)))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*sqrt((a + b)*b)*d) - 1/2*log(e^(2*d*x + 2*c) - 1)/((a + b)*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/((a + b)*d) - 1/2/(((a + b)*e^(2*d*x + 2*c) - a - b)*d) + 3/2/(((a + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

mupad [B] time = 3.39, size = 977, normalized size = 15.76

$$\frac{x}{a} - \frac{2}{(e^{2c+2dx} - 1)(ad + bd)} + \operatorname{atan} \left(\frac{e^{2c} e^{2dx} \left(\frac{8(a+2b)(4da^4b^2 + 16da^3b^3 + 20da^2b^4 + 8dab^5)}{a^6(a+b)(a^3+2a^2b+ab^2)} \sqrt{-a^2d^2(a+b)^3} \sqrt{-a^5d^2-3a^4bd^2-3a^3b^2d^2-a^2b^3d^2} \right) + \frac{2\sqrt{b^3}(a^2+8ab+b^3)}}{a^6(a+b)(a^3+2a^2b+ab^2)} \sqrt{-a^2d^2(a+b)^3} \sqrt{-a^5d^2-3a^4bd^2-3a^3b^2d^2-a^2b^3d^2} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2), x)
```

```
[Out] x/a - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) + (atan(((exp(2*c)*exp(2*d*x))*((8*(a + 2*b)*(20*a^2*b^4*d + 16*a^3*b^3*d + 4*a^4*b^2*d + 8*a*b^5*d))/(a^6*(a + b)*(a*b^2 + 2*a^2*b + a^3))*(-a^2*d^2*(a + b)^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(b^3)^(1/2)*(8*a*b + a^2 + 8*b^2)*(a^2*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 8*b^2*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 8*a*b*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (8*(a + 2*b)*(2*a^2*b^4*d + 4*a^3*b^3*d + 2*a^4*b^2*d))/(a^6*(a + b)*(a*b^2 + 2*a^2*b + a^3))*(-a^2*d^2*(a + b)^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(a^2*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 2*a*b*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(4*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + a^4*b^3*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^5*b^2*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^6*b*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(4*(b^3)^(1/2))*(-a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2), x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

$$3.146 \quad \int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=73

$$\frac{b^2 \log(a \cosh^2(c+dx) + b)}{2ad(a+b)^2} - \frac{\operatorname{csch}^2(c+dx)}{2d(a+b)} + \frac{(a+2b) \log(\sinh(c+dx))}{d(a+b)^2}$$

[Out] $-1/2*\operatorname{csch}(d*x+c)^2/(a+b)/d+1/2*b^2*\ln(b+a*\cosh(d*x+c)^2)/a/(a+b)^2/d+(a+2*b)*\ln(\sinh(d*x+c))/(a+b)^2/d$

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{b^2 \log(a \cosh^2(c+dx) + b)}{2ad(a+b)^2} - \frac{\operatorname{csch}^2(c+dx)}{2d(a+b)} + \frac{(a+2b) \log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] $-\operatorname{Csch}[c + d*x]^2/(2*(a + b)*d) + (b^2*\operatorname{Log}[b + a*\operatorname{Cosh}[c + d*x]^2])/(2*a*(a + b)^2*d) + ((a + 2*b)*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/((a + b)^2*d)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2} + \frac{a+2b}{(a+b)^2(-1+x)} + \frac{b^2}{(a+b)^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\operatorname{csch}^2(c+dx)}{2(a+b)d} + \frac{b^2 \log(b+a \cosh^2(c+dx))}{2a(a+b)^2d} + \frac{(a+2b) \log(\sinh(c+dx))}{(a+b)^2d}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 100, normalized size = 1.37

$$\frac{\operatorname{sech}^2(c+dx)(a \cosh(2(c+dx)) + a + 2b) \left(b^2 \left(-\log(a \sinh^2(c+dx) + a + b) \right) + a(a+b) \operatorname{csch}^2(c+dx) - 2a \right)}{4ad(a+b)^2(a+b\operatorname{sech}^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2), x]

[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*(a*(a + b)*Csch[c + d*x]^2 - 2*a*(a + 2*b)*Log[Sinh[c + d*x]] - b^2*Log[a + b + a*Sinh[c + d*x]^2])*Sech[c + d*x]^2)/(a*(a + b)^2*d*(a + b*Sech[c + d*x]^2))

fricas [B] time = 0.52, size = 862, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^4 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d + 4*((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 - (a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error index.cc index_gcd Error: Bad Argument
ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
ValueEvaluation time: 0.58Done

maple [B] time = 0.48, size = 199, normalized size = 2.73

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d(a+b)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da} + \frac{b^2 \ln\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\right) + \left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x)

[Out] $-1/8/d*\tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a*\ln$
 $(\tanh(1/2*d*x+1/2*c)+1)+1/2/d*b^2/a/(a+b)^2*\ln(\tanh(1/2*d*x+1/2*c)^4+a+b*\tanh$
 $(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b$
 $)-1/8/d/(a+b)/\tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^2*\ln(\tanh(1/2*d*x+1/2*c))*a+2$
 $/d/(a+b)^2*\ln(\tanh(1/2*d*x+1/2*c))*b$

maxima [B] time = 0.44, size = 187, normalized size = 2.56

$$\frac{b^2 \log\left(2(a+2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a\right)}{2(a^3 + 2a^2b + ab^2)d} + \frac{(a+2b) \log\left(e^{(-dx-c)} + 1\right)}{(a^2 + 2ab + b^2)d} + \frac{(a+2b) \log\left(e^{(-dx-c)} - 1\right)}{(a^2 + 2ab + b^2)d} + \frac{dx+c}{ad} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $1/2*b^2*\log(2*(a+2*b)*e^{(-2*d*x-2*c)} + a*e^{(-4*d*x-4*c)} + a)/((a^3 +$
 $2*a^2*b + a*b^2)*d) + (a+2*b)*\log(e^{(-d*x-c)} + 1)/((a^2 + 2*a*b + b^2)*$
 $d) + (a+2*b)*\log(e^{(-d*x-c)} - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x+c)/(a$
 $*d) + 2*e^{(-2*d*x-2*c)}/((2*(a+b)*e^{(-2*d*x-2*c)} - (a+b)*e^{(-4*d*x-4*c)} - a - b)*d)$

mupad [B] time = 2.07, size = 523, normalized size = 7.16

$$\frac{\ln\left(23ab^7 + 8a^7b - 2b^8 - 72a^2b^6 - 10a^3b^5 + 184a^4b^4 + 180a^5b^3 + 64a^6b^2 + 2b^8e^{2c}e^{2dx} - 23ab^7e^{2c}e^{2dx}\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c+d*x)^3/(a+b/cosh(c+d*x)^2),x)

[Out] $(\log(23*a*b^7 + 8*a^7*b - 2*b^8 - 72*a^2*b^6 - 10*a^3*b^5 + 184*a^4*b^4 + 1$
 $80*a^5*b^3 + 64*a^6*b^2 + 2*b^8*\exp(2*c)*\exp(2*d*x) - 23*a*b^7*\exp(2*c)*\exp$
 $(2*d*x) - 8*a^7*b*\exp(2*c)*\exp(2*d*x) + 72*a^2*b^6*\exp(2*c)*\exp(2*d*x) + 10$
 $*a^3*b^5*\exp(2*c)*\exp(2*d*x) - 184*a^4*b^4*\exp(2*c)*\exp(2*d*x) - 180*a^5*b^3$
 $*\exp(2*c)*\exp(2*d*x) - 64*a^6*b^2*\exp(2*c)*\exp(2*d*x))*(a+2*b))/(a^2*d +$
 $b^2*d + 2*a*b*d) - x/a - 2/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2$

$d*x) + 1)) + (b^2*\log(a*b^4 + 16*a^4*b + 4*a^5 - 8*a^2*b^3 + 12*a^3*b^2 + 8$
 $*a^5*\exp(2*c)*\exp(2*d*x) + 4*a^5*\exp(4*c)*\exp(4*d*x) + 4*b^5*\exp(2*c)*\exp(2$
 $*d*x) - 30*a*b^4*\exp(2*c)*\exp(2*d*x) + 48*a^4*b*\exp(2*c)*\exp(2*d*x) + a*b^4$
 $*\exp(4*c)*\exp(4*d*x) + 16*a^4*b*\exp(4*c)*\exp(4*d*x) + 32*a^2*b^3*\exp(2*c)*e$
 $xp(2*d*x) + 88*a^3*b^2*\exp(2*c)*\exp(2*d*x) - 8*a^2*b^3*\exp(4*c)*\exp(4*d*x)$
 $+ 12*a^3*b^2*\exp(4*c)*\exp(4*d*x)))/(2*a^3*d + 2*a*b^2*d + 4*a^2*b*d) - (2*($
 $a*b + a^2))/(a*(\exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2), x)

[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2), x)

$$3.147 \quad \int \frac{\coth^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)} dx$$

Optimal. Leaf size=87

$$-\frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}} - \frac{\coth^3(c+dx)}{3d(a+b)} - \frac{(a+2b)\coth(c+dx)}{d(a+b)^2} + \frac{x}{a}$$

[Out] x/a-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(5/2)/d-(a+2*b)*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d

Rubi [A] time = 0.29, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4141, 1975, 480, 583, 522, 206, 208}

$$-\frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}} - \frac{\coth^3(c+dx)}{3d(a+b)} - \frac{(a+2b)\coth(c+dx)}{d(a+b)^2} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] x/a - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)*d) - ((a + 2*b)*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b)*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g^(m + 1)), x] + Dist[1/(a*c*g^n*(

$m + 1$), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{\coth^3(c + dx)}{3(a + b)d} + \frac{\operatorname{Subst}\left(\int \frac{3(a+2b)-3bx^2}{x^2(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{3(a + b)d} \\ &= -\frac{(a + 2b) \coth(c + dx)}{(a + b)^2 d} - \frac{\coth^3(c + dx)}{3(a + b)d} - \frac{\operatorname{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)+3b(a+2b)x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{3(a + b)^2 d} \\ &= -\frac{(a + 2b) \coth(c + dx)}{(a + b)^2 d} - \frac{\coth^3(c + dx)}{3(a + b)d} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{ad} - \frac{b}{a} \\ &= \frac{x}{a} - \frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d} - \frac{(a+2b) \coth(c+dx)}{(a+b)^2 d} - \frac{\coth^3(c+dx)}{3(a+b)d} \end{aligned}$$

Mathematica [B] time = 3.32, size = 380, normalized size = 4.37

$$\operatorname{sech}^2(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{1}{8} \sqrt{a + b} \operatorname{csch}(c) \sqrt{b(\cosh(c) - \sinh(c))^4} \operatorname{csch}^3(c + dx) (-12a^2 \sinh(c) \cosh(c) + 8a^2 \cosh^2(c) - 8a^2 \sinh^2(c) + 8ab \cosh(c) \sinh(c) - 8ab \cosh^2(c) - 8ab \sinh^2(c) + 8b^2 \cosh(c) \sinh(c) - 8b^2 \cosh^2(c) - 8b^2 \sinh^2(c))\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*b^3*ArcTanh[(Sech[d*x]*Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*sqrt[a + b]*sqrt[b*(Cosh[c] - Sinh[c])^4]))*(-Cosh[2*c] + Sinh[2*c]) + (sqrt[a + b]*Csch[c]*Csch[c + d*x]^3*sqrt[b*(Cosh[c] - Sinh[c])^4]*(9*(a + b)^2*d*x*

$$\frac{\text{Cosh}[d*x] - 9*(a + b)^2*d*x*\text{Cosh}[2*c + d*x] - 3*a^2*d*x*\text{Cosh}[2*c + 3*d*x] - 6*a*b*d*x*\text{Cosh}[2*c + 3*d*x] - 3*b^2*d*x*\text{Cosh}[2*c + 3*d*x] + 3*a^2*d*x*\text{Cosh}[4*c + 3*d*x] + 6*a*b*d*x*\text{Cosh}[4*c + 3*d*x] + 3*b^2*d*x*\text{Cosh}[4*c + 3*d*x] - 12*a^2*\text{Sinh}[d*x] - 24*a*b*\text{Sinh}[d*x] - 12*a^2*\text{Sinh}[2*c + d*x] - 18*a*b*\text{Sinh}[2*c + d*x] + 8*a^2*\text{Sinh}[2*c + 3*d*x] + 14*a*b*\text{Sinh}[2*c + 3*d*x])}{8} / (6*a*(a + b)^{(5/2)}*d*(a + b*\text{Sech}[c + d*x]^2)*\text{Sqrt}[b*(\text{Cosh}[c] - \text{Sinh}[c])^4])$$

fricas [B] time = 0.46, size = 2705, normalized size = 31.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\frac{1}{6}*(6*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^6 + 36*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*d*x*\sinh(d*x + c)^6 - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c)^4 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 - 6*a*b)*\sinh(d*x + c)^4 + 24*(5*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 6*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*\cosh(d*x + c)^2 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c)^2 + 4*a^2 + 8*a*b)*\sinh(d*x + c)^2 + 3*(b^2*\cosh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + b^2*\sinh(d*x + c)^6 - 3*b^2*\cosh(d*x + c)^4 + 3*(5*b^2*\cosh(d*x + c)^2 - b^2)*\sinh(d*x + c)^4 + 3*b^2*\cosh(d*x + c)^2 + 4*(5*b^2*\cosh(d*x + c)^3 - 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*b^2*\cosh(d*x + c)^4 - 6*b^2*\cosh(d*x + c)^2 + b^2)*\sinh(d*x + c)^2 - b^2 + 6*(b^2*\cosh(d*x + c)^5 - 2*b^2*\cosh(d*x + c)^3 + b^2*\cosh(d*x + c))*\sinh(d*x + c))*\text{sqrt}(b/(a + b))*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\text{sqrt}(b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a) - 16*a^2 - 28*a*b + 12*(3*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*\cosh(d*x + c))*\sinh(d*x + c))/(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*d*\sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^4 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d)*\sinh(d*x + c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^2 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^4 - 6*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d)*\sinh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/3*(3*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^6 + 18*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2)*d*x*\sinh(d*x + c)^6 - 3*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c)^4 + 3*(15*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 - 6*a*b)*\sinh(d*x + c)^4 + 12*(5*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*\cosh(d*x + c)^2 + 3*(15*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*\cosh(d*x + c)^2 + 4*a^2 + 8*a*$$

$b) \sinh(dx + c)^2 - 3(b^2 \cosh(dx + c)^6 + 6b^2 \cosh(dx + c) \sinh(dx + c)^5 + b^2 \sinh(dx + c)^6 - 3b^2 \cosh(dx + c)^4 + 3(5b^2 \cosh(dx + c)^2 - b^2) \sinh(dx + c)^4 + 3b^2 \cosh(dx + c)^2 + 4(5b^2 \cosh(dx + c)^3 - 3b^2 \cosh(dx + c)) \sinh(dx + c)^3 + 3(5b^2 \cosh(dx + c)^4 - 6b^2 \cosh(dx + c)^2 + b^2) \sinh(dx + c)^2 - b^2 + 6(b^2 \cosh(dx + c)^5 - 2b^2 \cosh(dx + c)^3 + b^2 \cosh(dx + c)) \sinh(dx + c) \sqrt{-b/(a + b)}) \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-b/(a + b)})/b - 8a^2 - 14a^2 b + 6(3(a^2 + 2ab + b^2) dx \cosh(dx + c)^5 - 2(3(a^2 + 2ab + b^2) dx + 4a^2 + 6ab) \cosh(dx + c)^3 + (3(a^2 + 2ab + b^2) dx + 4a^2 + 8ab) \cosh(dx + c)) \sinh(dx + c) / ((a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^6 + 6(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c) \sinh(dx + c)^5 + (a^3 + 2a^2 b + ab^2) dx \sinh(dx + c)^6 - 3(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^4 + 3(5(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^2 - (a^3 + 2a^2 b + ab^2) d) \sinh(dx + c)^4 + 3(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^2 + 4(5(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^3 - 3(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)) \sinh(dx + c)^3 + 3(5(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^4 - 6(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^2 + (a^3 + 2a^2 b + ab^2) d) \sinh(dx + c)^2 - (a^3 + 2a^2 b + ab^2) d + 6((a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^5 - 2(a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)^3 + (a^3 + 2a^2 b + ab^2) dx \cosh(dx + c)) \sinh(dx + c))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^4/(a+b*sech(dx+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueEvaluation time: 0.68Done

maple [B] time = 0.49, size = 298, normalized size = 3.43

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{24d(a+b)^2} - \frac{\left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b}{24d(a+b)^2} - \frac{5a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{8d(a+b)^2} - \frac{9 \tanh \left(\frac{dx}{2} + \frac{c}{2} \right) b}{8d(a+b)^2} - \frac{\ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)}{da} + \frac{\ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^4/(a+b*sech(dx+c)^2),x)

[Out] $-1/24/d/(a+b)^2 a \tanh(1/2 dx + 1/2 c)^3 - 1/24/d/(a+b)^2 a \tanh(1/2 dx + 1/2 c) - 3b - 5/8/d/(a+b)^2 a \tanh(1/2 dx + 1/2 c) - 9/8/d/(a+b)^2 \tanh(1/2 dx + 1/2 c) * b - 1/d/a \ln(\tanh(1/2 dx + 1/2 c) - 1) + 1/d/a \ln(\tanh(1/2 dx + 1/2 c) + 1) + 1/2/d * b^{(5/2)}/a/(a+b)^{(5/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2 dx + 1/2 c)^2 - 2b^{(1/2)} * \tanh(1/2 dx + 1/2 c) + (a+b)^{(1/2)}) - 1/2/d * b^{(5/2)}/a/(a+b)^{(5/2)} * \ln((a+b)^{(1/2)} * \tanh(1/2 dx + 1/2 c)^2 + 2b^{(1/2)} * \tanh(1/2 dx + 1/2 c) + (a+b)^{(1/2)}) - 1/24/d/(a+b) / \tanh(1/2 dx + 1/2 c)^3 - 5/8/d/(a+b)^2 / \tanh(1/2 dx + 1/2 c) * a - 9/8/d/(a+b)^2 / \tanh(1/2 dx + 1/2 c) * b$

maxima [B] time = 0.63, size = 1435, normalized size = 16.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{3}{16}ab \log\left(\frac{(ae^{-2dx} - 2c) + a + 2b - 2\sqrt{(a+b)b}}{(ae^{-2dx} - 2c) + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^2 + 2ab + b^2)\sqrt{(a+b)b}d) + \frac{1}{8}(ab + 2b^2) \log\left(\frac{ae^{4dx} + 4c + 2(a + 2b)e^{2dx} + 2c + a}{(a^3 + 2a^2b + ab^2)d} - \frac{1}{4}b \log\left(\frac{ae^{4dx} + 4c + 2(a + 2b)e^{2dx} + 2c + a}{(a^2 + 2ab + b^2)d} - \frac{1}{8}(ab + 2b^2) \log\left(\frac{2(a + 2b)e^{-2dx} - 2c + ae^{-4dx} - 4c + a}{(a^3 + 2a^2b + ab^2)d} + \frac{1}{4}b \log\left(\frac{2(a + 2b)e^{-2dx} - 2c + ae^{-4dx} - 4c + a}{(a^2 + 2ab + b^2)d} + \frac{1}{4}(2a + 3b) \log\left(\frac{e^{2dx} + 2c - 1}{(a^2 + 2ab + b^2)d} + \frac{1}{2}b \log\left(\frac{e^{2dx} + 2c - 1}{(a^2 + 2ab + b^2)d} - \frac{1}{4}(2a + 3b) \log\left(\frac{e^{-2dx} - 2c - 1}{(a^2 + 2ab + b^2)d} - \frac{1}{2}b \log\left(\frac{e^{-2dx} - 2c - 1}{(a^2 + 2ab + b^2)d} - \frac{1}{32}(a^2b + 8ab^2 + 8b^3) \log\left(\frac{(ae^{2dx} + 2c) + a + 2b - 2\sqrt{(a+b)b}}{(ae^{2dx} + 2c) + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^3 + 2a^2b + ab^2)\sqrt{(a+b)b}d) + \frac{1}{8}(ab + 2b^2) \log\left(\frac{(ae^{2dx} + 2c) + a + 2b - 2\sqrt{(a+b)b}}{(ae^{2dx} + 2c) + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^2 + 2ab + b^2)\sqrt{(a+b)b}d) + \frac{1}{32}(a^2b + 8ab^2 + 8b^3) \log\left(\frac{(ae^{-2dx} - 2c) + a + 2b - 2\sqrt{(a+b)b}}{(ae^{-2dx} - 2c) + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^3 + 2a^2b + ab^2)\sqrt{(a+b)b}d) - \frac{1}{8}(ab + 2b^2) \log\left(\frac{(ae^{-2dx} - 2c) + a + 2b - 2\sqrt{(a+b)b}}{(ae^{-2dx} - 2c) + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^2 + 2ab + b^2)\sqrt{(a+b)b}d) + \frac{1}{24}(3(12a + 13b)e^{4dx} + 4c) - 6(9a + 10b)e^{2dx} + 2c + 25b) / ((a^2 + 2ab + b^2 - (a^2 + 2ab + b^2)e^{6dx} + 6c) + 3(a^2 + 2ab + b^2)e^{4dx} + 4c) - 3(a^2 + 2ab + b^2)e^{2dx} + 2c) * d) + \frac{1}{6}(3(4a + 5b)e^{4dx} + 4c) - 6(2a + 3b)e^{2dx} + 4a + 7b) / ((a^2 + 2ab + b^2 - (a^2 + 2ab + b^2)e^{6dx} + 6c) + 3(a^2 + 2ab + b^2)e^{4dx} + 4c) - 3(a^2 + 2ab + b^2)e^{2dx} + 2c) * d) + \frac{1}{24}(6(9a + 10b)e^{-2dx} - 2c) - 3(12a + 13b)e^{-4dx} - 4c) - 22a - 25b) / ((a^2 + 2ab + b^2 - 3(a^2 + 2ab + b^2)e^{-2dx} - 2c) + 3(a^2 + 2ab + b^2)e^{-4dx} - 4c) - (a^2 + 2ab + b^2)e^{-6dx} - 6c) * d) + \frac{1}{6}(6(2a + 3b)e^{-2dx} - 2c) - 3(4a + 5b)e^{-4dx} - 4c) - 4a - 7b) / ((a^2 + 2ab + b^2 - 3(a^2 + 2ab + b^2)e^{-2dx} - 2c) + 3(a^2 + 2ab + b^2)e^{-4dx} - 4c) - (a^2 + 2ab + b^2)e^{-6dx} - 6c) * d) - \frac{1}{4}(6ae^{-2dx} - 2c) + 3be^{-4dx} - 4c) - 2a + b) / ((a^2 + 2ab + b^2 - 3(a^2 + 2ab + b^2)e^{-2dx} - 2c) + 3(a^2 + 2ab + b^2)e^{-4dx} - 4c) - (a^2 + 2ab + b^2)e^{-6dx} - 6c) * d)$

mupad [B] time = 4.15, size = 779, normalized size = 8.95

$$\frac{x}{a} - \frac{8}{3(ad + bd) (3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1)} - \operatorname{atan}\left(\left(e^{2c} e^{2dx} \left(\frac{4b^3}{a^3 d (a+b)^2 \sqrt{b^5 (a^3 + 2a^2 b + ab^2)}} + \frac{1}{a^2 b^3 (a^3 + 2a^2 b + ab^2)}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)

[Out] $\frac{x}{a} - \frac{8}{(3(ad + bd)(3\exp(2c + 2dx) - 3\exp(4c + 4dx) + \exp(6c + 6dx) - 1))} - \operatorname{atan}\left(\frac{\exp(2c)\exp(2dx)\left(\frac{4b^3}{a^3 d(a+b)^2(b^5)^{1/2}} + \frac{2ab^3 d(b^5)^{1/2} + 4a^3 b d(b^5)^{1/2} + 5a^2 b^2 d(b^5)^{1/2}}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}}\right) + \frac{(a + 2b)(a^4 d(b^5)^{1/2} + 2a^3 b d(b^5)^{1/2} + a^2 b^2 d(b^5)^{1/2})}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}} - \frac{5a^6 b d^2 - a^2 b^5 d^2 - 5a^3 b^4 d^2 - 10a^4 b^3 d^2 - 10a^5 b^2 d^2}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}} + \frac{(a + 2b)(a^4 d(b^5)^{1/2} + 2a^3 b d(b^5)^{1/2} + a^2 b^2 d(b^5)^{1/2})}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}} - \frac{5a^6 b d^2 - a^2 b^5 d^2 - 5a^3 b^4 d^2 - 10a^4 b^3 d^2 - 10a^5 b^2 d^2}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}}\right) / 2 + \frac{(a^2 b^2 (-a^7 d^2 - 5a^6 b d^2 - a^2 b^5 d^2 - 5a^3 b^4 d^2 - 10a^4 b^3 d^2 - 10a^5 b^2 d^2)^{1/2}}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}} - \frac{5a^6 b d^2 - a^2 b^5 d^2 - 5a^3 b^4 d^2 - 10a^4 b^3 d^2 - 10a^5 b^2 d^2}{a^2 b^3 (a^3 + 2a^2 b + ab^2)(-a^2 d^2 (a+b)^5)^{1/2}}\right) / 2$

$$\frac{(2 + a^3 b (-a^7 d^2 - 5 a^6 b d^2 - a^2 b^5 d^2 - 5 a^3 b^4 d^2 - 10 a^4 b^3 d^2 - 10 a^5 b^2 d^2)^{1/2}) (b^5)^{1/2}}{(-a^7 d^2 - 5 a^6 b d^2 - a^2 b^5 d^2 - 5 a^3 b^4 d^2 - 10 a^4 b^3 d^2 - 10 a^5 b^2 d^2)^{1/2} - (4 (a b + a^2)) / (a (a + b) (a d + b d) (\exp(4 c + 4 d x) - 2 \exp(2 c + 2 d x) + 1)) - (2 (3 a b + 2 a^2)) / (a (\exp(2 c + 2 d x) - 1) (a + b) (a d + b d))}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2), x)

[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2), x)

$$3.148 \quad \int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=76

$$\frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \log(a \cosh^2(c+dx) + b)}{2d} + \frac{(a+b)^2}{2a^2bd(a \cosh^2(c+dx) + b)} + \frac{\log(\cosh(c+dx))}{b^2d}$$

[Out] 1/2*(a+b)^2/a^2/b/d/(b+a*cosh(d*x+c)^2)+ln(cosh(d*x+c))/b^2/d+1/2*(1/a^2-1/b^2)*ln(b+a*cosh(d*x+c)^2)/d

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \log(a \cosh^2(c+dx) + b)}{2d} + \frac{(a+b)^2}{2a^2bd(a \cosh^2(c+dx) + b)} + \frac{\log(\cosh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]

[Out] (a + b)^2/(2*a^2*b*d*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]]/(b^2*d) + ((a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{(a+b)^2}{2a^2bd(b+a\cosh^2(c+dx))} + \frac{\log(\cosh(c+dx))}{b^2d} + \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\log(b+a\cosh^2)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.45, size = 109, normalized size = 1.43

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log(a\cosh^2(c+dx)+b)+\frac{(a+b)^2}{a^2b(a\cosh^2(c+dx)+b)}+\frac{2\log(\cosh)}{b^2}\right)}{8d(a+b\operatorname{sech}^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^2)) + (2*Log[Cosh[c + d*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])*Sech[c + d*x]^4)/(8*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.50, size = 853, normalized size = 11.22

$$\frac{2ab^2dx\cosh(dx+c)^4 + 8ab^2dx\cosh(dx+c)\sinh(dx+c)^3 + 2ab^2dx\sinh(dx+c)^4 + 2ab^2dx - 4(a^2b + 2ab^2)}{8d(a+b\operatorname{sech}^2(c+dx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*a*b^2*d*x*cosh(d*x + c)^4 + 8*a*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b^2*d*x*sinh(d*x + c)^4 + 2*a*b^2*d*x - 4*(a^2*b + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*a*b^2*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3)*cosh(d*x + c)^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3 + 3*(a^3 - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a*b^2*d*x*cosh(d*x + c)^3 - (a^2*b + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b^2*d*cosh(d*x + c)^4 + 4*a^3*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b^2*d*sinh(d*x + c)^4 + a^3*b^2*d + 2*(a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2

$$+ 2*(3*a^3*b^2*d*cosh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^3*b^2*d*cosh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)$$

giac [B] time = 1.38, size = 208, normalized size = 2.74

$$\frac{\frac{2 dx}{a^2} - \frac{2 \log(e^{(2dx+2c)+1})}{b^2} + \frac{(a^2-b^2) \log(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}})}{a^2b^2}}{2d} - \frac{a^2e^{(4dx+4c)}-b^2e^{(4dx+4c)+2a^2e^{(2dx+2c)+8abe^{(2dx+2c)+6bae^{(2dx+2c)+a}}}}{(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}})})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*(2*d*x/a^2 - 2*log(e^(2*d*x + 2*c) + 1)/b^2 + (a^2 - b^2)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/(a^2*b^2) - (a^2*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*c) + 6*b^2*e^(2*d*x + 2*c) + a^2 - b^2)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a*b^2))/d

maple [B] time = 0.37, size = 351, normalized size = 4.62

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d a^2} - \frac{2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{db\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)

[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/b*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/2/d/b^2*ln(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-2/d/a*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)

maxima [B] time = 0.45, size = 154, normalized size = 2.03

$$\frac{2(a^2 + 2ab + b^2)e^{(-2dx-2c)}}{(a^3be^{(-4dx-4c)} + a^3b + 2(a^3b + 2a^2b^2)e^{(-2dx-2c)})d} + \frac{dx + c}{a^2d} + \frac{\log(e^{(-2dx-2c)} + 1)}{b^2d} - \frac{(a^2 - b^2) \log(2(a + 2b)e^{(-2dx-2c)})}{2a^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 2*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)/((a^3*b*e^(-4*d*x - 4*c) + a^3*b + 2*(a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d) - 1/2*(a^2 - b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*b^2*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \tanh(c + dx)^5}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^2,x)`

[Out] `int((cosh(c + d*x)^4*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)`

$$3.149 \quad \int \frac{\tanh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=91

$$\frac{(a-2b)\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} + \frac{x}{a^2} - \frac{(a+b) \tanh(c+dx)}{2abd(a-b \tanh^2(c+dx)+b)}$$

[Out] x/a^2+1/2*(a-2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/b^(3/2)/d-1/2*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.19, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4141, 1975, 470, 522, 206, 208}

$$\frac{(a-2b)\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} + \frac{x}{a^2} - \frac{(a+b) \tanh(c+dx)}{2abd(a-b \tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2), x]

[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*b^(3/2)*d) - ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975


```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^(p))/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a+b)\tanh(c+dx)}{2abd(a+b-b\tanh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{a+b+(-a+b)x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{2abd} \\ &= -\frac{(a+b)\tanh(c+dx)}{2abd(a+b-b\tanh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{a^2d} + \frac{((a-b)\tanh(c+dx))}{2abd(a+b-b\tanh^2(c+dx))} \\ &= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} - \frac{(a+b)\tanh(c+dx)}{2abd(a+b-b\tanh^2(c+dx))} \end{aligned}$$

Mathematica [B] time = 2.13, size = 228, normalized size = 2.51

$$\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{(a^2 - ab - 2b^2)(\cosh(2c) - \sinh(2c))(a \cosh(2(c + dx)) + a + 2b) \tanh^{-1}\left(\frac{(\cosh(2c) - \sinh(2c)) \operatorname{sech}(c)}{2\sqrt{a+b}\sqrt{b}}\right)}{bd\sqrt{a+b}\sqrt{b(\cosh(c) - \sinh(c))^4}} \right)$$

$$8a^2(a + b \operatorname{sech}^2(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2, x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(c + d*x)]) + ((a^2 - a*b - 2*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])]*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*d))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.45, size = 1479, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 - ((a^2 - 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a^2 + 4*a*b + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*sinh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^3*b*d*cosh(d*x + c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*d*cosh(d*x + c)^3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cosh(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x + c)^4 + 2*a*b*d*x + 2*(2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 + ((a^2 - 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) + 2*a^2 + 2*a*b + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*sinh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^3*b*d*cosh(d*x + c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*d*cosh(d*x + c)^3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [B] time = 1.16, size = 187, normalized size = 2.05

$$\frac{\frac{2 dx}{a^2} + \frac{(a^2 e^{(2c)} - a b e^{(2c)} - 2 b^2 e^{(2c)}) \arctan\left(\frac{a e^{(2dx+2c)} + a + 2b}{2 \sqrt{-ab - b^2}}\right) e^{(-2c)}}{\sqrt{-ab - b^2} a^2 b}}{2d} + \frac{2(a^2 e^{(2dx+2c)} + 3 a b e^{(2dx+2c)} + 2 b^2 e^{(2dx+2c)} + a^2 + ab)}{(a e^{(4dx+4c)} + 2 a e^{(2dx+2c)} + 4 b e^{(2dx+2c)} + a) a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(2*d*x/a^2 + (a^2*e^(2*c) - a*b*e^(2*c) - 2*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2*b) + 2*(a^2*e^(2*d*x + 2*c) + 3*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a^2 + a*b)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a^2*b))/d

maple [B] time = 0.40, size = 676, normalized size = 7.43

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d a^2} - \frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*tanh(1/2*d*x+1/2*c)^3-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*tanh(1/2*d*x+1/2*c)-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d/a^2*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^2*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$

maxima [B] time = 0.84, size = 1053, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3*b + a^2*b^2)*\sqrt{(a + b)*b}*d) + 1/16*a*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*(a*b + b^2)*d) - 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3*b + a^2*b^2)*\sqrt{(a + b)*b}*d) - 3/32*(a + 2*b)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*(a*b + b^2)*d) - 1/16*a*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/(\sqrt{(a + b)*b}*(a*b + b^2)*d) + 1/16*(a^3 + 8*a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^{(2*d*x + 2*c)})/((a^4*b + a^3*b^2 + (a^4*b + a^3*b^2)*e^{(4*d*x + 4*c)} + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^{(2*d*x + 2*c)})*d) - 1/16*(a^3 + 8*a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^{(-2*d*x - 2*c)})/((a^4*b + a^3*b^2 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^{(-2*d*x - 2*c)} + (a^4*b + a^3*b^2)*e^{(-4*d*x - 4*c)})*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^{(2*d*x + 2*c)})/((a^3*b + a^2*b^2 + (a^3*b + a^2*b^2)*e^{(4*d*x + 4*c)} + 2*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^{(2*d*x + 2*c)})*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^{(-2*d*x - 2*c)})/((a^3*b + a^2*b^2 + 2*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^{(-2*d*x - 2*c)} + (a^3*b + a^2*b^2)*e^{(-4*d*x - 4*c)})*d) - 3/8*((a + 2*b)*e^{(-2*d*x - 2*c)} + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*e^{(-2*d*x - 2*c)} + (a^2*b + a*b^2)*e^{(-4*d*x - 4*c)})*d) + 1/4*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/(a^2*d) - 1/4*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\cosh(c + dx)^2 - 1)^2}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int((cosh(c + d*x)^2 - 1)^2/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)

$$3.150 \quad \int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^2d}$$

[Out] 1/2*(a+b)/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 444, 43}

$$\frac{a+b}{2a^2d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] (a + b)/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{1-x}{(b+ax)^2} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2} - \frac{1}{a(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{a+b}{2a^2d(b+a\cosh^2(c+dx))} + \frac{\log(b+a\cosh^2(c+dx))}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 81, normalized size = 1.59

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2(a+b)}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)]))

fricas [B] time = 0.42, size = 485, normalized size = 9.51

$$2\,ad\,x\cosh(dx+c)^4 + 8\,ad\,x\cosh(dx+c)\sinh(dx+c)^3 + 2\,ad\,x\sinh(dx+c)^4 + 2\,ad\,x + 4((a+2b)dx - a - b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - a - b)*cosh(d*x + c)^2 + 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - a - b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3 + ((a + 2*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c)/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.89, size = 121, normalized size = 2.37

$$\frac{2\,dx}{a^2} + \frac{e^{(4\,dx+4\,c)} - 2e^{(2\,dx+2\,c)} + 1}{(ae^{(4\,dx+4\,c)} + 2ae^{(2\,dx+2\,c)} + 4be^{(2\,dx+2\,c)} + a)a} - \frac{\log(ae^{(4\,dx+4\,c)} + 2ae^{(2\,dx+2\,c)} + 4be^{(2\,dx+2\,c)} + a)}{a^2}$$

$$2\,d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2*(2*d*x/a^2 + (e^{(4*d*x + 4*c)} - 2*e^{(2*d*x + 2*c)} + 1)/((a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)*a) - \log(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)/a^2)/d$

maple [B] time = 0.35, size = 186, normalized size = 3.65

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d a^2} - \frac{2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + b \left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-2/d/a^2*\tanh(1/2*d*x+1/2*c)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)$

maxima [B] time = 0.38, size = 108, normalized size = 2.12

$$\frac{2(a+b)e^{(-2dx-2c)}}{(a^3e^{(-4dx-4c)} + a^3 + 2(a^3 + 2a^2b)e^{(-2dx-2c)})d} + \frac{dx+c}{a^2d} + \frac{\log(2(a+2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $2*(a + b)*e^{(-2*d*x - 2*c)/((a^3*e^{(-4*d*x - 4*c)} + a^3 + 2*(a^3 + 2*a^2*b)*e^{(-2*d*x - 2*c)})*d) + (d*x + c)/(a^2*d) + 1/2*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + dx)^4 \tanh(c + dx)^3}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int((cosh(c + d*x)^4*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.151 \quad \int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=85

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}d\sqrt{a+b}} + \frac{x}{a^2} - \frac{\tanh(c+dx)}{2ad(a-b\tanh^2(c+dx)+b)}$$

[Out] $x/a^2 - 1/2*(a+2*b)*\operatorname{arctanh}(b^{1/2}*\tanh(d*x+c)/(a+b)^{1/2})/a^2/d/b^{1/2}/(a+b)^{1/2} - 1/2*\tanh(d*x+c)/a/d/(a+b*b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4141, 1975, 471, 522, 206, 208}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}d\sqrt{a+b}} + \frac{x}{a^2} - \frac{\tanh(c+dx)}{2ad(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[c + d*x]^2/(a + b*\operatorname{Sech}[c + d*x]^2)^2, x]$

[Out] $x/a^2 - ((a + 2*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])])/(2*a^2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[a + b]*d) - \operatorname{Tanh}[c + d*x]/(2*a*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2))$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 471

$\operatorname{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}))^{(q_)}), x_Symbol] \rightarrow \operatorname{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(n*(b*c-a*d)*(p+1)), x] - \operatorname{Dist}[e^n/(n*(b*c-a*d)*(p+1)), \operatorname{Int}[(e*x)^{(m-n)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^q*\operatorname{Simp}[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, q\}, x] \ \&\& \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{GeQ}[n, m-n+1] \ \&\& \operatorname{GtQ}[m-n+1, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 522

$\operatorname{Int}[(e_ + (f_)*(x_)^{(n_)})/((a_ + (b_)*(x_)^{(n_)})*((c_ + (d_)*(x_)^{(n_)})^{(n_)})), x_Symbol] \rightarrow \operatorname{Dist}[(b*e-a*f)/(b*c-a*d), \operatorname{Int}[1/(a+b*x^n), x], x] - \operatorname{Dist}[(d*e-c*f)/(b*c-a*d), \operatorname{Int}[1/(c+d*x^n), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 1975

$\operatorname{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_)^{(m_)}), x_Symbol] \rightarrow \operatorname{Int}[(e*x)^m*\operatorname{ExpandToSum}[u, x]^p*\operatorname{ExpandToSum}[v, x]^q, x] /; \operatorname{FreeQ}\{e, m, p, q\}, x] \ \&\& \operatorname{BinomialQ}\{u, v\}, x] \ \&\& \operatorname{EqQ}[\operatorname{BinomialDegree}[u, x] - \operatorname{BinomialDegree}[v, x], 0] \ \&\& !$

BinomialMatchQ[{u, v}, x]

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{\tanh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2ad}$$

$$= -\frac{\tanh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{a^2d} - \frac{(a + 2b)}{a^2}$$

$$= \frac{x}{a^2} - \frac{(a + 2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}d} - \frac{\tanh(c + dx)}{2ad(a + b - b \tanh^2(c + dx))}$$

Mathematica [B] time = 4.63, size = 326, normalized size = 3.84

$$\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b)^2 \left(\frac{(a^2 + 8ab + 8b^2) \operatorname{sech}(2c)((a+2b) \sinh(2c) - a \sinh(2dx))}{a^2bd(a+b)(a \cosh(2(c+dx)) + a + 2b)} + \frac{16x}{a^2} + \frac{(a^3 - 6a^2b - 24ab^2 - 16b^3)}{a^2} \right)$$

64(a + b)

Warning: Unable to verify antiderivative.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((16*x)/a^2 - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) + ((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])*(Cosh[2*c] - Sinh[2*c])]/(a^2*b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])) + (a*Sinh[2*(c + d*x)]/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(64*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.47, size = 1846, normalized size = 21.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b + 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 2*a^2*b + 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [A] time = 0.72, size = 147, normalized size = 1.73

$$\frac{(ae^{(2c)}+2be^{(2c)})\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)e^{(-2c)}}{\sqrt{-ab-b^2}a^2} - \frac{2dx}{a^2} - \frac{2(ae^{(2dx+2c)}+2be^{(2dx+2c)}+a)}{(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)a^2}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((a*e^(2*c) + 2*b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2) - 2*d*x/a^2 - 2*(a*e^(2*d*x

$$\frac{+ 2*c) + 2*b*e^{(2*d*x + 2*c) + a}/((a*e^{(4*d*x + 4*c) + 2*a*e^{(2*d*x + 2*c)}) + 4*b*e^{(2*d*x + 2*c) + a}*a^2))/d$$

maple [B] time = 0.35, size = 411, normalized size = 4.84

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^2} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d a^2} - \frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d a \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + b \left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 2 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)

[Out] $-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^3-1/d/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)+1/4/d/a/b^{(1/2)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-1/4/d/a/b^{(1/2)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})+1/2/d/a^2*b^{(1/2)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-1/2/d/a^2*b^{(1/2)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})}$

maxima [B] time = 0.55, size = 597, normalized size = 7.02

$$\frac{(a^2 + 6ab + 4b^2) \log\left(\frac{ae^{(2dx+2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(2dx+2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^3 + a^2b)\sqrt{(a+b)b}d} + \frac{(a^2 + 6ab + 4b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{16(a^3 + a^2b)\sqrt{(a+b)b}d} + \frac{1}{4(a^4 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/16*(a^2 + 6*a*b + 4*b^2)*\log((a*e^{(2*d*x + 2*c) + a + 2*b - 2*\sqrt{(a + b)*b}})/(a*e^{(2*d*x + 2*c) + a + 2*b + 2*\sqrt{(a + b)*b}}))/((a^3 + a^2*b)*\sqrt{(a + b)*b}*d) + 1/16*(a^2 + 6*a*b + 4*b^2)*\log((a*e^{(-2*d*x - 2*c) + a + 2*b - 2*\sqrt{(a + b)*b}})/(a*e^{(-2*d*x - 2*c) + a + 2*b + 2*\sqrt{(a + b)*b}}))/((a^3 + a^2*b)*\sqrt{(a + b)*b}*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^{(2*d*x + 2*c)})/((a^4 + a^3*b + (a^4 + a^3*b)*e^{(4*d*x + 4*c) + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^{(2*d*x + 2*c)}})*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^{(-2*d*x - 2*c)})/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^{(-2*d*x - 2*c) + (a^4 + a^3*b)*e^{(-4*d*x - 4*c)}})*d) - 1/2*((a + 2*b)*e^{(-2*d*x - 2*c) + a}/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2)*e^{(-2*d*x - 2*c) + (a^3 + a^2*b)*e^{(-4*d*x - 4*c)}})*d) + 1/8*\log((a*e^{(-2*d*x - 2*c) + a + 2*b - 2*\sqrt{(a + b)*b}})/(a*e^{(-2*d*x - 2*c) + a + 2*b + 2*\sqrt{(a + b)*b}}))/(\sqrt{(a + b)*b}*(a + b)*d) + 1/4*\log(a*e^{(4*d*x + 4*c) + 2*(a + 2*b)*e^{(2*d*x + 2*c) + a}}/(a^2*d) - 1/4*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c) + a}*e^{(-4*d*x - 4*c) + a})/(a^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^2 (\cosh(c + dx)^2 - 1)}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)

[Out] int((cosh(c + d*x)^2*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)

$$3.152 \quad \int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=49

$$\frac{b}{2a^2d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^2d}$$

[Out] 1/2*b/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 266, 43}

$$\frac{b}{2a^2d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]

[Out] b/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3}{(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{(b+ax)^2} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2} + \frac{1}{a(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{b}{2a^2d(b+a\cosh^2(c+dx))} + \frac{\log(b+a\cosh^2(c+dx))}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 79, normalized size = 1.61

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2b}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] (2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)]))

fricas [B] time = 0.42, size = 476, normalized size = 9.71

$$2\,ad\,x\cosh(dx+c)^4 + 8\,ad\,x\cosh(dx+c)\sinh(dx+c)^3 + 2\,ad\,x\sinh(dx+c)^4 + 2\,ad\,x + 4((a+2b)dx-b)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - b)*cosh(d*x + c)^2 + 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3 + ((a + 2*b)*d*x - b)*cosh(d*x + c))*sinh(d*x + c)/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.42, size = 121, normalized size = 2.47

$$\frac{2\,dx}{a^2} + \frac{e^{(4\,dx+4\,c)}+2\,e^{(2\,dx+2\,c)}+1}{(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)a} - \frac{\log(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)}{a^2}$$

$$2\,d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out]
$$-1/2*(2*d*x/a^2 + (e^{(4*d*x + 4*c)} + 2*e^{(2*d*x + 2*c)} + 1)/((a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)*a) - \log(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)/a^2)/d$$

maple [A] time = 0.18, size = 60, normalized size = 1.22

$$\frac{\ln(a + b \operatorname{sech}(dx + c)^2)}{2d a^2} - \frac{1}{2da(a + b \operatorname{sech}(dx + c)^2)} - \frac{\ln(\operatorname{sech}(dx + c))}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)

[Out]
$$1/2/d/a^2*\ln(a+b*\operatorname{sech}(d*x+c)^2)-1/2/d/a/(a+b*\operatorname{sech}(d*x+c)^2)-1/d/a^2*\ln(\operatorname{sech}(d*x+c))$$

maxima [B] time = 0.42, size = 106, normalized size = 2.16

$$\frac{2be^{(-2dx-2c)}}{(a^3e^{(-4dx-4c)} + a^3 + 2(a^3 + 2a^2b)e^{(-2dx-2c)})d} + \frac{dx + c}{a^2d} + \frac{\log(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$2*b*e^{(-2*d*x - 2*c)} / ((a^3*e^{(-4*d*x - 4*c)} + a^3 + 2*(a^3 + 2*a^2*b)*e^{(-2*d*x - 2*c)})*d) + (d*x + c) / (a^2*d) + 1/2*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a) / (a^2*d)$$

mupad [B] time = 1.73, size = 53, normalized size = 1.08

$$\frac{\ln\left(\cosh(c + dx)^2 \left(a + \frac{b}{\cosh(c+dx)^2}\right)\right)}{2a^2d} - \frac{1}{2ad \left(a + \frac{b}{\cosh(c+dx)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)

[Out]
$$\log(\cosh(c + dx)^2*(a + b/\cosh(c + dx)^2))/(2*a^2*d) - 1/(2*a*d*(a + b/\cosh(c + dx)^2))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)

[Out] Timed out

$$3.153 \quad \int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=93

$$-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}} + \frac{x}{a^2} - \frac{b\tanh(c+dx)}{2ad(a+b)(a-b\tanh^2(c+dx)+b)}$$

[Out] $x/a^2 - 1/2*(3*a+2*b)*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})*b^{(1/2)}/a^2/(a+b)^{(3/2)}/d - 1/2*b*\tanh(d*x+c)/a/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4128, 414, 522, 206, 208}

$$-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}} + \frac{x}{a^2} - \frac{b\tanh(c+dx)}{2ad(a+b)(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-2), x]

[Out] $x/a^2 - (\operatorname{Sqrt}[b]*(3*a + 2*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a + b]])/(2*a^2*(a + b)^{(3/2)*d} - (b*\operatorname{Tanh}[c + d*x])/(2*a*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 4128

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/

$(1 + ff^2*x^2), x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \& \& \text{NeQ}[a + b, 0] \& \& \text{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b \tanh(c + dx)}{2a(a+b)d(a+b-b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a+b)d} \\ &= -\frac{b \tanh(c + dx)}{2a(a+b)d(a+b-b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{a^2 d} \\ &= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} - \frac{b \tanh(c + dx)}{2a(a+b)d(a+b-b \tanh^2(c + dx))} \end{aligned}$$

Mathematica [B] time = 2.04, size = 221, normalized size = 2.38

$$\frac{\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(2x(a \cosh(2(c + dx)) + a + 2b) + \frac{b \operatorname{sech}(2c)((a+2b) \sinh(2c) - a \sinh(2dx))}{d(a+b)} \right)}{8a^2(a + b \operatorname{sech}^2(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^(-2), x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(c + d*x)]) - (b*(3*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (b*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.46, size = 1690, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*sinh(d*x + c)^2 + ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +

b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a*b + 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*cosh(d*x + c)^2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d + 4*((a^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a^2 + a*b)*d*x + 2*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*sinh(d*x + c)^2 - ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 2*a*b + 4*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*cosh(d*x + c)^2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d + 4*((a^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [A] time = 0.44, size = 163, normalized size = 1.75

$$\frac{(3ab+2b^2)\arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^3+a^2b)\sqrt{-ab-b^2}} - \frac{2(abe^{(2dx+2c)+2b^2e^{(2dx+2c)+ab}})}{(a^3+a^2b)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}})}} - \frac{2(dx+c)}{a^2}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((3*a*b + 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a*b)/((a^3 + a^2*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) - 2*(d*x + c)/a^2)/d

maple [B] time = 0.38, size = 423, normalized size = 4.55

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^2} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} - \frac{b\left(\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c)^2)^2,x)

[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a*b/(tanh(1/2*d*x+1/2*c)^4+a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*

$$a^{-2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2b+a+b} / (a+b) \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{3-1/d} a^b / \left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{4a+b} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{4+2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2a-2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2b+a+b} / (a+b) \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3/4/d/a^b \right)^{1/2} / (a+b)^{3/2} \ln\left((a+b)^{1/2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2-2b^{1/2}} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right) + (a+b)^{1/2}\right) - 3/4/d/a^b \right)^{1/2} / (a+b)^{3/2} \ln\left((a+b)^{1/2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2+2b^{1/2}} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right) + (a+b)^{1/2}\right) + 1/2/d/a^{2b^{3/2}} / (a+b)^{3/2} \ln\left((a+b)^{1/2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2-2b^{1/2}} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right) + (a+b)^{1/2}\right) - 1/2/d/a^{2b^{3/2}} / (a+b)^{3/2} \ln\left((a+b)^{1/2} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2+2b^{1/2}} \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right) + (a+b)^{1/2}\right)$$

maxima [B] time = 0.69, size = 187, normalized size = 2.01

$$\frac{(3ab + 2b^2) \log\left(\frac{ae^{(-2dx-2c)+a+2b-2\sqrt{(a+b)b}}}{ae^{(-2dx-2c)+a+2b+2\sqrt{(a+b)b}}}\right)}{4(a^3 + a^2b)\sqrt{(a+b)bd}} - \frac{ab + (ab + 2b^2)e^{(-2dx-2c)}}{(a^4 + a^3b + 2(a^4 + 3a^3b + 2a^2b^2))e^{(-2dx-2c)} + (a^4 + a^3b)e^{(-4dx-4c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{4} (3ab + 2b^2) \log\left(\frac{ae^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{ae^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right) / ((a^3 + a^2b)\sqrt{(a+b)b}) - (ab + (ab + 2b^2)e^{(-2dx-2c)}) / ((a^4 + a^3b + 2(a^4 + 3a^3b + 2a^2b^2))e^{(-2dx-2c)} + (a^4 + a^3b)e^{(-4dx-4c)}) + (dx + c) / (a^2d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cosh(c + d*x)^2)^2,x)

[Out] int(1/(a + b/cosh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)**2)**2,x)

[Out] Integral((a + b*sech(c + d*x)**2)**(-2), x)

$$3.154 \quad \int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2d(a+b)(a\cosh^2(c+dx)+b)} + \frac{b(2a+b)\log(a\cosh^2(c+dx)+b)}{2a^2d(a+b)^2} + \frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

[Out] $1/2*b^2/a^2/(a+b)/d/(b+a*\cosh(d*x+c)^2)+1/2*b*(2*a+b)*\ln(b+a*\cosh(d*x+c)^2)/a^2/(a+b)^2/d+\ln(\sinh(d*x+c))/(a+b)^2/d$

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 88}

$$\frac{b^2}{2a^2d(a+b)(a\cosh^2(c+dx)+b)} + \frac{b(2a+b)\log(a\cosh^2(c+dx)+b)}{2a^2d(a+b)^2} + \frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2), x]

[Out] $b^2/(2*a^2*(a+b)*d*(b+a*Cosh[c+d*x]^2)) + (b*(2*a+b)*Log[b+a*Cosh[c+d*x]^2])/(2*a^2*(a+b)^2*d) + Log[Sinh[c+d*x]]/((a+b)^2*d)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b^2}{a(a+b)(b+ax)^2} - \frac{b(2a+b)}{a(a+b)^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{b^2}{2a^2(a+b)d(b+a\cosh^2(c+dx))} + \frac{b(2a+b)\log(b+a\cosh^2(c+dx))}{2a^2(a+b)^2d} + \frac{\log}{2a^2(a+b)^2d}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 115, normalized size = 1.39

$$\frac{a \sinh^2(c+dx) (2a^2 \log(\sinh(c+dx)) + b(2a+b) \log(a \sinh^2(c+dx) + a+b)) + (a+b) (2a^2 \log(\sinh(c+dx)) + b(2a+b) \log(a \sinh^2(c+dx) + a+b))}{a^2 d (a+b)^2 (a \cosh(2(c+dx)) + a+2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2, x]

[Out] ((a + b)*(2*a^2*Log[Sinh[c + d*x]] + b*(b + (2*a + b)*Log[a + b + a*Sinh[c + d*x]^2])) + a*(2*a^2*Log[Sinh[c + d*x]] + b*(2*a + b)*Log[a + b + a*Sinh[c + d*x]^2])*Sinh[c + d*x]^2)/(a^2*(a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x)]))

fricas [B] time = 0.55, size = 1031, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2, x, algorithm="fricas")

[Out] -1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(a*b^2 + b^3 - (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 - a*b^2 - b^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 - ((2*a^2*b + a*b^2)*cosh(d*x + c)^4 + 4*(2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*sinh(d*x + c)^4 + 2*a^2*b + a*b^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3 + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*((a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (a*b^2 + b^3 - (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)

$\sinh(dx + c)^2 + (a^5 + 2a^4b + a^3b^2)*d + 4*((a^5 + 2a^4b + a^3b^2) * d * \cosh(dx + c)^3 + (a^5 + 4a^4b + 5a^3b^2 + 2a^2b^3) * d * \cosh(dx + c)) * \sinh(dx + c)$

giac [B] time = 0.56, size = 246, normalized size = 2.96

$$\frac{(2ab+b^2)\log(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c}+a)}{a^4+2a^3b+a^2b^2} + \frac{2e^{2c}\log(-e^{2dx+2c}+1)}{a^2e^{2c}+2abe^{2c}+b^2e^{2c}} - \frac{2dx}{a^2} - \frac{2abe^{4dx+4c}+b^2e^{4dx+4c}+4abe^{2dx+2c}+6b^2e^{2dx+2c}}{(a^3+2a^2b+ab^2)(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c}+a)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * ((2ab + b^2) * \log(ae^{4dx+4c} + 2ae^{2dx+2c} + 4be^{2dx+2c} + a) / (a^4 + 2a^3b + a^2b^2) + 2e^{2c} * \log(-e^{2dx+2c} + 1)) / (a^2e^{2c} + 2abe^{2c} + b^2e^{2c}) - 2dx/a^2 - (2ab * e^{4dx+4c} + b^2 * e^{4dx+4c} + 4ab * e^{2dx+2c} + 6b^2 * e^{2dx+2c}) / ((a^3 + 2a^2b + ab^2) * (ae^{4dx+4c} + 2ae^{2dx+2c} + 4be^{2dx+2c} + a)) / d$

maple [B] time = 0.42, size = 292, normalized size = 3.52

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^2} \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} \frac{2b^2 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{c}{2}\right)}{da(a+b)^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{c}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)/(a+b*sech(dx+c)^2)^2,x)

[Out] $-1/d/a^2 * \ln(\tanh(1/2*dx+1/2*c)-1) - 1/d/a^2 * \ln(\tanh(1/2*dx+1/2*c)+1) - 2/d*b^2/a/(a+b)^2 * \tanh(1/2*dx+1/2*c)^2 / (\tanh(1/2*dx+1/2*c)^4 * a + b * \tanh(1/2*dx+1/2*c)^4 + 2 * \tanh(1/2*dx+1/2*c)^2 * a - 2 * \tanh(1/2*dx+1/2*c)^2 * b + a + b) + 1/d*b/a/(a+b)^2 * \ln(\tanh(1/2*dx+1/2*c)^4 * a + b * \tanh(1/2*dx+1/2*c)^4 + 2 * \tanh(1/2*dx+1/2*c)^2 * a - 2 * \tanh(1/2*dx+1/2*c)^2 * b + a + b) + 1/2/d*b^2/a^2/(a+b)^2 * \ln(\tanh(1/2*dx+1/2*c)^4 * a + b * \tanh(1/2*dx+1/2*c)^4 + 2 * \tanh(1/2*dx+1/2*c)^2 * a - 2 * \tanh(1/2*dx+1/2*c)^2 * b + a + b) + 1/d/(a+b)^2 * \ln(\tanh(1/2*dx+1/2*c))$

maxima [B] time = 0.35, size = 209, normalized size = 2.52

$$\frac{2b^2e^{(-2dx-2c)}}{(a^4+a^3b+2(a^4+3a^3b+2a^2b^2)e^{(-2dx-2c)}+(a^4+a^3b)e^{(-4dx-4c)})d} + \frac{(2ab+b^2)\log(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)})}{2(a^4+2a^3b+a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)/(a+b*sech(dx+c)^2)^2,x, algorithm="maxima")

[Out] $2b^2 * e^{(-2dx-2c)} / ((a^4 + a^3b + 2(a^4 + 3a^3b + 2a^2b^2) * e^{(-2dx-2c)} + (a^4 + a^3b) * e^{(-4dx-4c)}) * d) + 1/2 * (2ab + b^2) * \log(2 * (a + 2b) * e^{(-2dx-2c)} + a * e^{(-4dx-4c)} + a) / ((a^4 + 2a^3b + a^2b^2) * d) + \log(e^{(-dx-c)} + 1) / ((a^2 + 2ab + b^2) * d) + \log(e^{(-dx-c)} - 1) / ((a^2 + 2ab + b^2) * d) + (dx + c) / (a^2 * d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^4 \coth(c+dx)}{(a \cosh(c+dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)/(a + b/cosh(c + d*x)^2), x)`

[Out] `int((cosh(c + d*x)^4*coth(c + d*x))/(b + a*cosh(c + d*x)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**2, x)`

[Out] `Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)`

$$3.155 \quad \int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=121

$$\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}} + \frac{x}{a^2} - \frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2} - \frac{b\coth(c+dx)}{2ad(a+b)(a-b\tanh^2(c+dx)+b)}$$

[Out] x/a^2-1/2*b^(3/2)*(5*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a+b)^(5/2)/d-1/2*(2*a-b)*coth(d*x+c)/a/(a+b)^2/d-1/2*b*coth(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.28, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}} + \frac{x}{a^2} - \frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2} - \frac{b\coth(c+dx)}{2ad(a+b)(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2), x]

[Out] x/a^2 - (b^(3/2)*(5*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(5/2)*d) - ((2*a - b)*Coth[c + d*x])/(2*a*(a + b)^2*d) - (b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583


```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b \coth(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{-2a+b-3bx^2}{x^2(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= -\frac{(2a - b) \coth(c + dx)}{2a(a + b)^2d} - \frac{b \coth(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{2a^2+b^2}{(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= -\frac{(2a - b) \coth(c + dx)}{2a(a + b)^2d} - \frac{b \coth(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= \frac{x}{a^2} - \frac{b^{3/2}(5a + 2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a + b}}\right)}{2a^2(a + b)^{5/2}d} - \frac{(2a - b) \coth(c + dx)}{2a(a + b)^2d} - \frac{1}{2a(a + b)d} \end{aligned}$$

Mathematica [B] time = 2.82, size = 268, normalized size = 2.21

$$\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{b^2 \operatorname{sech}(2c)((a + 2b) \sinh(2c) - a \sinh(2dx))}{a^2 d(a + b)^2} - \frac{b^2(5a + 2b)(\cosh(2c) - \sinh(2c))(a \cosh(2(c + dx)) + a + 2b)}{a^2} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((2*x*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])])*(a + 2*b + a*Cosh[2*(c + d*x)]*(Cosh[2*c] - Sinh[2*c]))/(a^2*(a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^2*d) + (b^2*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^2*d))/(8*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.51, size = 3624, normalized size = 29.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^6 + 24*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + 2*a^2*b + a*b^2)*d*x*sinh(d*x + c)^6 - 4*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*sinh(d*x + c)^4 + 16*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*a^3 - 4*a*b^2 - 4*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a^3 - 8*a^2*b - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^3 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a) + 8*(3*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*si
```

$$\begin{aligned} & \text{nh}(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^4 + 6*(a^5 + \\ & 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c)^2 - (a^5 + 6*a^4*b + 9*a^3 \\ & *b^2 + 4*a^2*b^3)*d)*\sinh(d*x + c)^2 - (a^5 + 2*a^4*b + a^3*b^2)*d + 2*(3*(\\ & a^5 + 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + \\ & 4*a^2*b^3)*d*\cosh(d*x + c)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*c \\ & \text{osh}(d*x + c))*\sinh(d*x + c)), 1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*\cosh(d*x + \\ & c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^5 + 2*(a \\ & ^3 + 2*a^2*b + a*b^2)*d*x*\sinh(d*x + c)^6 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3 \\ & + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*\cosh(d*x + c)^4 + 2*(15*(a^3 + 2*a^2*b + \\ & a*b^2)*d*x*\cosh(d*x + c)^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a* \\ & b^2 + 4*b^3)*d*x)*\sinh(d*x + c)^4 + 8*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*\cosh(d \\ & *x + c)^3 - (2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x) \\ & *\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*a^3 - 2*a*b^2 - 2*(a^3 + 2*a^2*b + a*b^ \\ & 2)*d*x - 2*(4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x \\ &)*\cosh(d*x + c)^2 + 2*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*\cosh(d*x + c)^4 - 4*a \\ & ^3 - 8*a^2*b - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a \\ & *b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh \\ & (d*x + c)^2 - ((5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)* \\ & \cosh(d*x + c)*\sinh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*\sinh(d*x + c)^6 + (5*a^ \\ & 2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15* \\ & (5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a* \\ & b^2)*\cosh(d*x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c))*\sinh(d*x \\ & + c)^3 - 5*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c)^2 \\ & + (15*(5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6* \\ & (5*a^2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(3*(5*a^2 \\ & *b + 2*a*b^2)*\cosh(d*x + c)^5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c \\ &)^3 - (5*a^2*b + 22*a*b^2 + 8*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a \\ & + b))*\arctan(1/2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a* \\ & \sinh(d*x + c)^2 + a + 2*b)*\sqrt{-b/(a + b)})/b) + 4*(3*(a^3 + 2*a^2*b + a*b^ \\ & 2)*d*x*\cosh(d*x + c)^5 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^ \\ & 2 + 4*b^3)*d*x)*\cosh(d*x + c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b \\ & + 9*a*b^2 + 4*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5 + 2*a^4*b + a^ \\ & 3*b^2)*d*\cosh(d*x + c)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)*\sinh \\ & (d*x + c)^5 + (a^5 + 2*a^4*b + a^3*b^2)*d*\sinh(d*x + c)^6 + (a^5 + 6*a^4*b \\ & + 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)* \\ & d*\cosh(d*x + c)^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*\sinh(d*x + c \\ &)^4 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c)^2 + 4*(5*(a^5 \\ & + 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^ \\ & 2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*c \\ & \text{osh}(d*x + c)^4 + 6*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c)^ \\ & 2 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*\sinh(d*x + c)^2 - (a^5 + 2*a \\ & ^4*b + a^3*b^2)*d + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^5 + 2*(a \\ & ^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c)^3 - (a^5 + 6*a^4*b + \\ & 9*a^3*b^2 + 4*a^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))] \end{aligned}$$

giac [B] time = 0.96, size = 282, normalized size = 2.33

$$\frac{(5ab^2e^{2c} + 2b^3e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + a + 2b}{2\sqrt{-ab-b^2}}\right)e^{(-2c)}}{(a^4 + 2a^3b + a^2b^2)\sqrt{-ab-b^2}} + \frac{2dx}{a^2} - \frac{2(2a^3e^{4dx+4c} - ab^2e^{4dx+4c} - 2b^3e^{4dx+4c} + 4a^3e^{2dx+2c} + 8a^2be^{2dx+2c} + 2b^3e^{2dx+2c})}{(a^4 + 2a^3b + a^2b^2)(ae^{6dx+6c} + ae^{4dx+4c} + 4be^{4dx+4c} - ae^{2dx+2c} - 4b^3e^{2dx+2c})}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] 1/2*((5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*arctan(-1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(-a*b - b^2)) + 2*d*x/a^2 - 2*(2*a^3*e^(4*d*x + 4*c) - a*b^2*e^(4*d*x + 4*c) - 2*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 8*a^2*b*e^(2*d*x + 2*c) + 2*b^3*e^(2*d*x + 2*c) + 2*a^3 + a*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c)

) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) - a))/d

maple [B] time = 0.46, size = 481, normalized size = 3.98

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d(a^2 + 2ab + b^2)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^2} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} - \frac{da(a+b)^2\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh\right)}{da(a+b)^2\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh\right)}\right)}{da(a+b)^2\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)

[Out] -1/2/d/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4+a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4+a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)+5/4/d*b^(3/2)/a/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-5/4/d*b^(3/2)/a/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(5/2)/a^2/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*b^(5/2)/a^2/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^2/tanh(1/2*d*x+1/2*c)

maxima [B] time = 0.92, size = 1070, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/4*(2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/4*(2*a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/16*(3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) - 3/8*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/4*(2*a^3 + a^2*b + 2*a*b^2 + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e^(4*d*x + 4*c) + 2*(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 2*a^4*b + a^3*b^2)*e^(6*d*x + 6*c) - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + a^2*b + 2*a*b^2 + 2*(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(-2*d*x - 2*c) + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e^(-4*d*x - 4*c))/((a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*d*x - 4*c) - (a^5 + 2*a^4*b + a^3*b^2)*e^(-6*d*x - 6*c))*d) - 1/2*(2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 + a*b + 2*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \coth(c + dx)^2}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2, x)

[Out] int((cosh(c + d*x)^4*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)

$$3.156 \quad \int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=110

$$\frac{b^3}{2a^2d(a+b)^2(a\cosh^2(c+dx)+b)} + \frac{b^2(3a+b)\log(a\cosh^2(c+dx)+b)}{2a^2d(a+b)^3} - \frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2} + \frac{(a+3b)\log(\sinh(c+dx))}{d(a+b)^3}$$

[Out] 1/2*b^3/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^2/d+1/2*b^2*(3*a+b)*ln(b+a*cosh(d*x+c)^2)/a^2/(a+b)^3/d+(a+3*b)*ln(sinh(d*x+c))/d/(a+b)^3

Rubi [A] time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$\frac{b^3}{2a^2d(a+b)^2(a\cosh^2(c+dx)+b)} + \frac{b^2(3a+b)\log(a\cosh^2(c+dx)+b)}{2a^2d(a+b)^3} - \frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2} + \frac{(a+3b)\log(\sinh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] b^3/(2*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b)^2*d) + (b^2*(3*a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a^2*(a + b)^3*d) + (a + 3*b)*Log[Sinh[c + d*x]]/((a + b)^3*d)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^m + n*p - 1)^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^7}{(1-x^2)^2(b+ax^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2} + \frac{a+3b}{(a+b)^3(-1+x)} - \frac{b^3}{a(a+b)^2(b+ax)^2} + \frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{b^3}{2a^2(a+b)^2d(b+a\cosh^2(c+dx))} - \frac{\operatorname{csch}^2(c+dx)}{2(a+b)^2d} + \frac{b^2(3a+b)\log(b+a\cosh^2(c+dx))}{2a^2(a+b)^3d}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 130, normalized size = 1.18

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{b^3(a+b)}{a^2(a\sinh^2(c+dx)+a+b)} + \frac{b^2(3a+b)\log(a\sinh^2(c+dx)+a+b)}{a^2}\right) - (a+b)\operatorname{csch}^2(c+dx)}{8d(a+b)^3(a+b\operatorname{sech}^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*(-((a + b)*Csch[c + d*x]^2) + 2*(a + 3*b)*Log[Sinh[c + d*x]] + (b^2*(3*a + b)*Log[a + b + a*Sinh[c + d*x]^2])/(a^2 + (b^3*(a + b))/(a^2*(a + b + a*Sinh[c + d*x]^2))))/(8*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.72, size = 3624, normalized size = 32.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^8 + 4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^6 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 + a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 + 2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x + 4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)

$$\begin{aligned}
&)^6 + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4) \\
& *d*x)*\cosh(d*x + c)^4 + a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + \\
& 3*a*b^3 + b^4)*d*x + 6*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 \\
& + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*\cosh(d*x + c)^2*\sinh(d*x \\
& + c)^2 - ((3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^8 + 8*(3*a^2*b^2 + a*b^3)*\cosh \\
& (d*x + c)*\sinh(d*x + c)^7 + (3*a^2*b^2 + a*b^3)*\sinh(d*x + c)^8 + 4*(3*a*b^3 \\
& + b^4)*\cosh(d*x + c)^6 + 4*(3*a*b^3 + b^4 + 7*(3*a^2*b^2 + a*b^3)*\cosh(d* \\
& x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^3 + 3*(3 \\
& *a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(3*a^2*b^2 + 13*a*b^3 + 4* \\
& b^4)*\cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^4 - 3*a^2*b^2 \\
& - 13*a*b^3 - 4*b^4 + 30*(3*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 \\
& + 3*a^2*b^2 + a*b^3 + 8*(7*(3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^5 + 10*(3*a*b^3 \\
& + b^4)*\cosh(d*x + c)^3 - (3*a^2*b^2 + 13*a*b^3 + 4*b^4)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^3 + 4*(3*a*b^3 + b^4)*\cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + a*b^3 \\
&)*\cosh(d*x + c)^6 + 15*(3*a*b^3 + b^4)*\cosh(d*x + c)^4 + 3*a*b^3 + b^4 - 3* \\
& (3*a^2*b^2 + 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^2 \\
& *b^2 + a*b^3)*\cosh(d*x + c)^7 + 3*(3*a*b^3 + b^4)*\cosh(d*x + c)^5 - (3*a^2* \\
& b^2 + 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^3 + (3*a*b^3 + b^4)*\cosh(d*x + c))*\si \\
& nh(d*x + c))*\log(2*(a*\cosh(d*x + c)^2 + a*\sinh(d*x + c)^2 + a + 2*b)/(cosh(\\
& d*x + c)^2 - 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2)) - 2*((a^4 + \\
& 3*a^3*b)*\cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b)*\cosh(d*x + c)*\sinh(d*x + c)^7 \\
& + (a^4 + 3*a^3*b)*\sinh(d*x + c)^8 + 4*(a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^6 + \\
& 4*(a^3*b + 3*a^2*b^2 + 7*(a^4 + 3*a^3*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 \\
& + 8*(7*(a^4 + 3*a^3*b)*\cosh(d*x + c)^3 + 3*(a^3*b + 3*a^2*b^2)*\cosh(d*x + c \\
&))*\sinh(d*x + c)^5 - 2*(a^4 + 7*a^3*b + 12*a^2*b^2)*\cosh(d*x + c)^4 + 2*(35 \\
& *(a^4 + 3*a^3*b)*\cosh(d*x + c)^4 - a^4 - 7*a^3*b - 12*a^2*b^2 + 30*(a^3*b + \\
& 3*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 3*a^3*b + 8*(7*(a^4 + \\
& 3*a^3*b)*\cosh(d*x + c)^5 + 10*(a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^3 - (a^4 + \\
& 7*a^3*b + 12*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^3*b + 3*a^2*b^2 \\
&)*\cosh(d*x + c)^2 + 4*(7*(a^4 + 3*a^3*b)*\cosh(d*x + c)^6 + 15*(a^3*b + 3*a^ \\
& 2*b^2)*\cosh(d*x + c)^4 + a^3*b + 3*a^2*b^2 - 3*(a^4 + 7*a^3*b + 12*a^2*b^2) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 + 3*a^3*b)*\cosh(d*x + c)^7 + 3* \\
& (a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 12*a^2*b^2)*\cosh(d*x \\
& + c)^3 + (a^3*b + 3*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*\sinh(d*x \\
& + c)/(cosh(d*x + c) - \sinh(d*x + c))) + 8*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a \\
& *b^3)*d*x*\cosh(d*x + c)^7 + 3*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2 \\
& *b^2 + 3*a*b^3 + b^4)*d*x)*\cosh(d*x + c)^5 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2 \\
& + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*c \\
& osh(d*x + c)^3 + (a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 \\
& + b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a \\
& ^3*b^3)*d*\cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(\\
& d*x + c)*\sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\sinh(d*x \\
& + c)^8 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^6 + 4 \\
& *(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^2 + (a^5*b + 3*a^ \\
& 4*b^2 + 3*a^3*b^3 + a^2*b^4)*d)*\sinh(d*x + c)^6 - 2*(a^6 + 7*a^5*b + 15*a^4 \\
& *b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3* \\
& a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^3 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a \\
& ^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 \\
& + a^3*b^3)*d*\cosh(d*x + c)^4 + 30*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4 \\
&)*d*\cosh(d*x + c)^2 - (a^6 + 7*a^5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4) \\
& *d)*\sinh(d*x + c)^4 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d* \\
& x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^5 + 1 \\
& 0*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^3 - (a^6 + 7*a^ \\
& 5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^6 + 15*(a^5*b \\
& + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^4 - 3*(a^6 + 7*a^5*b + \\
& 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*\cosh(d*x + c)^2 + (a^5*b + 3*a^4*b^2 \\
& + 3*a^3*b^3 + a^2*b^4)*d)*\sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a \\
& ^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^7 + 3*
\end{aligned}$$

$$(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^5 - (a^6 + 7*a^5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*\cosh(d*x + c)^3 + (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)$$

giac [B] time = 1.32, size = 383, normalized size = 3.48

$$\frac{(3ab^2+b^3)\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^5+3a^4b+3a^3b^2+a^2b^3} + \frac{2(ae^{(2c)}+3be^{(2c)})\log(|-e^{(2dx+2c)}+1|)}{a^3e^{(2c)}+3a^2be^{(2c)}+3ab^2e^{(2c)}+b^3e^{(2c)}} - \frac{2dx}{a^2} - \frac{3ab^2e^{(4dx+4c)}+b^3e^{(4dx+4c)}+6ab^2e^{(2dx+2c)}}{(a^4+3a^3b+3a^2b^2+ab^3)(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*((3*a*b^2 + b^3)*\log(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) + 2*(a*e^{(2*c)} + 3*b*e^{(2*c)})*\log(\text{abs}(-e^{(2*d*x + 2*c)} + 1))/(a^3*e^{(2*c)} + 3*a^2*b*e^{(2*c)} + 3*a*b^2*e^{(2*c)} + b^3*e^{(2*c)}) - 2*d*x/a^2 - (3*a*b^2*e^{(4*d*x + 4*c)} + b^3*e^{(4*d*x + 4*c)} + 6*a*b^2*e^{(2*d*x + 2*c)} + 10*b^3*e^{(2*d*x + 2*c)} + 3*a*b^2 + b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)) - (3*a*e^{(4*d*x + 4*c)} + 9*b*e^{(4*d*x + 4*c)} - 2*a*e^{(2*d*x + 2*c)} - 14*b*e^{(2*d*x + 2*c)} + 3*a + 9*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^{(2*d*x + 2*c)} - 1)^2))/d$

maple [B] time = 0.53, size = 367, normalized size = 3.34

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d(a^2 + 2ab + b^2)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{da^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} - \frac{da(a+b)^3\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\right)}{da(a+b)^3\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)

[Out] $-1/8/d*\tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-2/d*b^3/a/(a+b)^3*\tanh(1/2*d*x+1/2*c)^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*b^2/a/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c))^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^3/a^2/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c))^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a+b)^2/\tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c))*a+3/d/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c))*b$

maxima [B] time = 0.47, size = 384, normalized size = 3.49

$$\frac{(3ab^2 + b^3)\log\left(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a\right)}{2(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)d} + \frac{(a + 3b)\log\left(e^{(-dx-c)} + 1\right)}{(a^3 + 3a^2b + 3ab^2 + b^3)d} + \frac{(a + 3b)\log\left(e^{(-dx-c)} - 1\right)}{(a^3 + 3a^2b + 3ab^2 + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(3*a*b^2 + b^3)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + (a + 3*b)*\log(e^{(-d*x - c)} + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (a + 3*b)*\log(e^{(-d*x - c)} - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^3 - b^3)*e^{(-2*d*x - 2*c)} + 2*(a^3 + 2*a^2*b + b^3)*e^{(-4*d*x - 4*c)} + (a^3 - b^3)*e^{(-6*d*x - 6*c)})/((a^5 + 2*a^4*b + a^3*b^2 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^{(-2*d*x - 2*c)} - 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^{(-4*d*x - 4*c)} + 4*(a^4*b + 2*a^3*b^2 + 3*a^2*b^2 + a*b^3)*e^{(-6*d*x - 6*c)})/d$

$3*b^2 + a^2*b^3)*e^{(-6*d*x - 6*c)} + (a^5 + 2*a^4*b + a^3*b^2)*e^{(-8*d*x - 8*c)} + (d*x + c)/(a^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \coth(c + dx)^3}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)`

[Out] `int((cosh(c + d*x)^4*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)`

[Out] `Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)`

$$3.157 \quad \int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal. Leaf size=161

$$\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}} - \frac{(2a^2+6ab-b^2)\coth(c+dx)}{2ad(a+b)^3} + \frac{x}{a^2} - \frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2} - \frac{1}{2ad(a+b)}$$

[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a+b)^(7/2)/d-1/2*(2*a^2+6*a*b-b^2)*coth(d*x+c)/a/(a+b)^3/d-1/6*(2*a-3*b)*coth(d*x+c)^3/a/(a+b)^2/d-1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.41, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}} - \frac{(2a^2+6ab-b^2)\coth(c+dx)}{2ad(a+b)^3} + \frac{x}{a^2} - \frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2} - \frac{1}{2ad(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(7/2)*d) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/(2*a*(a + b)^3*d) - ((2*a - 3*b)*Coth[c + d*x]^3)/(6*a*(a + b)^2*d) - (b*Coth[c + d*x]^3)/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{b \coth^3(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{-2a+3b-5bx^2}{x^4(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d}$$

$$= -\frac{(2a - 3b) \coth^3(c + dx)}{6a(a + b)^2d} - \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{3(2a^2 - 3ab + b^2)x^2}{x^4(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d}$$

$$= -\frac{(2a^2 + 6ab - b^2) \coth(c + dx)}{2a(a + b)^3d} - \frac{(2a - 3b) \coth^3(c + dx)}{6a(a + b)^2d} - \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))}$$

$$= -\frac{(2a^2 + 6ab - b^2) \coth(c + dx)}{2a(a + b)^3d} - \frac{(2a - 3b) \coth^3(c + dx)}{6a(a + b)^2d} - \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b - b \tanh^2(c + dx))}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a + 2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a + b)^{7/2}d} - \frac{(2a^2 + 6ab - b^2) \coth(c + dx)}{2a(a + b)^3d} - \frac{(2a - 3b) \coth^3(c + dx)}{6a(a + b)^2d}$$

Mathematica [B] time = 5.09, size = 350, normalized size = 2.17

$$\operatorname{sech}^4(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{3b^3 \operatorname{sech}(2c)((a+2b) \sinh(2c) - a \sinh(2dx))}{a^2 d(a+b)^3} - \frac{3b^3(7a+2b)(\cosh(2c) - \sinh(2c))(a \cosh(2c) + a + 2b)}{a^2 d(a+b)^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((6*x*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 - (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]^2)/((a + b)^2*d) - (3*b^3*(7*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(a^2*(a + b)^(7/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*(2*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^3*d) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x])/((a + b)^2*d) + (3*b^3*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^3*d))/((24*(a + b*Sech[c + d*x]^2)^2)

fricas [B] time = 0.61, size = 9849, normalized size = 61.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^10 + 120*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^10 - 12*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^8 + 12*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 24*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^2*b^2 - a*b^3 - 3*b^4 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 + 8*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 80*a^3*b - 12*a*b^3 + 32*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b

$$\begin{aligned}
& *b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^3 + (2*a^4 + 38*a^3*b \\
& + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)* \\
& d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 \\
&)*d*x - 4*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a^3*b \\
& - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c)^2 + 4*(135*(a^4 + 3*a^3 \\
& *b + 3*a^2*b^2 + a*b^3)*d*x*\cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - a*b^3 - \\
& 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c)^6 \\
& - 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^ \\
& ^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^4 - 4*a^4 - 36*a^3*b - 80*a^2 \\
& *b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x \\
& + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^ \\
& 2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 3*((7*a^2*b^2 \\
& + 2*a*b^3)*\cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)*\sinh(\\
& d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*\sinh(d*x + c)^10 - (7*a^2*b^2 - 26*a*b^3 \\
& - 8*b^4)*\cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4 - 45*(7*a^2*b^2 + \\
& 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*\co \\
& sh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 \\
& + 2*a*b^3)*\cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 12*b^4 - 14*(7*a^2*b^2 \\
& - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 2 \\
& *a*b^3)*\cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^3 \\
& - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(7* \\
& a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 2*a*b^3) \\
& *\cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^4 + 7*a^ \\
& 2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c \\
&)^2)*\sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*\co \\
& sh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^5 - 5*(7*a^2 \\
& *b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + (7*a^2*b^2 + 44*a*b^3 + 12*b^4) \\
& *\cosh(d*x + c))*\sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + \\
& c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^8 - 28*(7*a^2*b^2 - 26*a*b^ \\
& 3 - 8*b^4)*\cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + \\
& c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^9 \\
& - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^7 - 6*(7*a^2*b^2 + 44*a*b \\
& ^3 + 12*b^4)*\cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + \\
& c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b \\
& / (a + b))*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + \\
& c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d* \\
& x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*((a^2 + a*b)*\cosh \\
& (d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(\\
& d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + \\
& c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + \\
& c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 8*(15*(a^4 + 3*a^3*b \\
& + 3*a^2*b^2 + a*b^3)*d*x*\cosh(d*x + c)^9 - 12*(4*a^4 + 8*a^3*b - a*b^3 - 2 \\
& *b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c)^7 - \\
& 18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2 \\
& *b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^5 + 4*(2*a^4 + 38*a^3*b + 72*a^ \\
& 2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh \\
& (d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a \\
& ^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a \\
& ^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^10 + 10*(a^6 + 3*a^5*b \\
& + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^6 + 3*a^5*b + 3 \\
& *a^4*b^2 + a^3*b^3)*d*\sinh(d*x + c)^10 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3* \\
& b^3 - 4*a^2*b^4)*d*\cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b \\
& ^3)*d*\cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)* \\
& d)*\sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4) \\
&)*d*\cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*
\end{aligned}$$

$$\begin{aligned}
& x + c)^3 - (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c) * \sinh(dx + c)^7 + 2 * (105 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^4 - 14 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^2 - (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d) * \sinh(dx + c)^6 + 2 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^4 + 4 * (63 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^5 - 14 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^3 - 3 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (105 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^6 - 35 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^4 - 15 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^2 + (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d) * \sinh(dx + c)^4 + (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^2 + 8 * (15 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^7 - 7 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^5 - 5 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^3 + (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^8 - 28 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^6 - 30 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^4 + 12 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^2 + (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d) * \sinh(dx + c)^2 - (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d + 2 * (5 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^9 - 4 * (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)^7 - 6 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^5 + 4 * (a^6 + 9a^5b + 21a^4b^2 + 19a^3b^3 + 6a^2b^4) * d * \cosh(dx + c)^3 + (a^6 - a^5b - 9a^4b^2 - 11a^3b^3 - 4a^2b^4) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/6 * (6 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^10 + 60 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c) * \sinh(dx + c)^9 + 6 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \sinh(dx + c)^10 - 6 * (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)^8 + 6 * (45 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^2 - 4a^4 - 8a^3b + ab^3 + 2b^4 - (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \sinh(dx + c)^8 + 48 * (15 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^3 - (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)) * \sinh(dx + c)^7 - 12 * (2a^4 + 10a^3b + 16a^2b^2 + ab^3 + 3b^4 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)^6 + 12 * (105 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^4 - 2a^4 - 10a^3b - 16a^2b^2 - ab^3 - 3b^4 - (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x - 14 * (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 24 * (63 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^5 - 14 * (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)^3 - 3 * (2a^4 + 10a^3b + 16a^2b^2 + ab^3 + 3b^4 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)) * \sinh(dx + c)^5 + 4 * (2a^4 + 38a^3b + 72a^2b^2 + 9b^4 + 3 * (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)^4 + 4 * (315 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^6 - 105 * (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)^4 + 2a^4 + 38a^3b + 72a^2b^2 + 9b^4 + 3 * (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x - 45 * (2a^4 + 10a^3b + 16a^2b^2 + ab^3 + 3b^4 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)^2) * \sinh(dx + c)^4 - 16a^4 - 40a^3b - 6ab^3 + 16 * (45 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * d * x * \cosh(dx + c)^7 - 21 * (4a^4 + 8a^3b - ab^3 - 2b^4 + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4) * d * x) * \cosh(dx + c)^5 - 15 * (2a^4 + 10a^3b + 16a^2b^2 + ab^3 + 3b^4 + (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)^3 + (2a^4 + 38a^3b + 72a^2b^2 + 9b^4 + 3 * (a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4) * d * x) * \cosh(dx + c)) * \sinh(dx + c)^3 - 6 * (a^4 + 3a^3b
\end{aligned}$$

$$\begin{aligned}
& *b + 3*a^2*b^2 + a*b^3)*d*x - 2*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + \\
& 6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c)^2 \\
& + 2*(135*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*\cosh(d*x + c)^8 - 84*(4*a^4 \\
& + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4) \\
& *d*x)*\cosh(d*x + c)^6 - 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + \\
& (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^4 - 4*a^4 \\
& - 36*a^3*b - 80*a^2*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 - \\
& 11*a*b^3 - 4*b^4)*d*x + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 \\
& + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^2 - 3*((7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^3) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*\sinh(d*x + c)^10 - \\
& (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8 \\
& *b^4 - 45*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(7 \\
& *a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d \\
& *x + c))*\sinh(d*x + c)^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^6 \\
& + 2*(105*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 1 \\
& 2*b^4 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 \\
& + 4*(63*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 - \\
& 8*b^4)*\cosh(d*x + c)^3 - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c))*\s \\
& \sinh(d*x + c)^5 + 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^4 + 2*(105 \\
& *(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)* \\
& \cosh(d*x + c)^4 + 7*a^2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3 \\
& + 12*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7 \\
& *a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh \\
& (d*x + c)^5 - 5*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^3 + (7*a^2*b^ \\
& 2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b \\
& ^3 - 8*b^4)*\cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^8 - 2 \\
& 8*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3 \\
& + 12*b^4)*\cosh(d*x + c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 + \\
& 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2 \\
& *a*b^3)*\cosh(d*x + c)^9 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c)^7 - \\
& 6*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*\cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^ \\
& 3 + 12*b^4)*\cosh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*\cosh(d*x + c) \\
& *\sinh(d*x + c))*\sqrt{-b/(a + b)}*\arctan(1/2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d \\
& *x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b)*\sqrt{-b/(a + b)})/b + \\
& 4*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*\cosh(d*x + c)^9 - 12*(4*a^4 + \\
& 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x) \\
&)*\cosh(d*x + c)^7 - 18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 \\
& + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*\cosh(d*x + c)^5 + 4*(2*a^4 \\
& + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^ \\
& 3 + 6*b^4)*d*x)*\cosh(d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 \\
& + 6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*\cosh(d*x + c) \\
&)*\sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^10 \\
& + 10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 \\
& + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\sinh(d*x + c)^10 - (a^6 - a^5*b - \\
& 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*\cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b \\
& + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a \\
& ^3*b^3 - 4*a^2*b^4)*d)*\sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19 \\
& *a^3*b^3 + 6*a^2*b^4)*d*\cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 \\
& + a^3*b^3)*d*\cosh(d*x + c)^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^ \\
& 2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 \\
& + a^3*b^3)*d*\cosh(d*x + c)^4 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - \\
& 4*a^2*b^4)*d*\cosh(d*x + c)^2 - (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6 \\
& *a^2*b^4)*d)*\sinh(d*x + c)^6 + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + \\
& 6*a^2*b^4)*d*\cosh(d*x + c)^4 + 4*(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) \\
& *d*\cosh(d*x + c)^5 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)* \\
& d*\cosh(d*x + c)^3 - 3*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4) \\
& *d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3 \\
& *b^3)*d*\cosh(d*x + c)^6 - 35*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*
\end{aligned}$$

$$b^4 * d * \cosh(dx + c)^4 - 15 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^2 + (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \sinh(dx + c)^4 + (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d * \cosh(dx + c)^2 + 8 * (15 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^7 - 7 * (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d * \cosh(dx + c)^5 - 5 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^3 + (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^8 - 28 * (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d * \cosh(dx + c)^6 - 30 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^4 + 12 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^2 + (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d) * \sinh(dx + c)^2 - (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d + 2 * (5 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^9 - 4 * (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d * \cosh(dx + c)^7 - 6 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^5 + 4 * (a^6 + 9 * a^5 * b + 21 * a^4 * b^2 + 19 * a^3 * b^3 + 6 * a^2 * b^4) * d * \cosh(dx + c)^3 + (a^6 - a^5 * b - 9 * a^4 * b^2 - 11 * a^3 * b^3 - 4 * a^2 * b^4) * d * \cosh(dx + c)) * \sinh(dx + c))]$$

giac [B] time = 1.60, size = 302, normalized size = 1.88

$$\frac{3(7ab^3e^{(2c)+2b^4e^{(2c)}} \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)e^{-2c}}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{-ab-b^2}} - \frac{6dx}{a^2} - \frac{6(ab^3e^{(2dx+2c)+2b^4e^{(2dx+2c)+ab^3}})}{(a^5+3a^4b+3a^3b^2+a^2b^3)(ae^{(4dx+4c)+2ae^{(2dx+2c)+4be^{(2dx+2c)+a}})} + \frac{8(3a}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^4/(a+b*sech(dx+c)^2)^2,x, algorithm="giac")

[Out] $-1/6 * (3 * (7 * a * b^3 * e^{(2 * c)} + 2 * b^4 * e^{(2 * c)}) * \arctan(1/2 * (a * e^{(2 * d * x + 2 * c)} + a + 2 * b) / \sqrt{-a * b - b^2})) * e^{(-2 * c)} / ((a^5 + 3 * a^4 * b + 3 * a^3 * b^2 + a^2 * b^3) * \sqrt{-a * b - b^2}) - 6 * d * x / a^2 - 6 * (a * b^3 * e^{(2 * d * x + 2 * c)} + 2 * b^4 * e^{(2 * d * x + 2 * c)} + a * b^3) / ((a^5 + 3 * a^4 * b + 3 * a^3 * b^2 + a^2 * b^3) * (a * e^{(4 * d * x + 4 * c)} + 2 * a * e^{(2 * d * x + 2 * c)} + 4 * b * e^{(2 * d * x + 2 * c)} + a)) + 8 * (3 * a * e^{(4 * d * x + 4 * c)} + 6 * b * e^{(4 * d * x + 4 * c)} - 3 * a * e^{(2 * d * x + 2 * c)} - 9 * b * e^{(2 * d * x + 2 * c)} + 2 * a + 5 * b) / ((a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * (e^{(2 * d * x + 2 * c)} - 1)^3) / d$

maple [B] time = 0.52, size = 634, normalized size = 3.94

$$\frac{a \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{24d(a^2 + 2ab + b^2)(a + b)} - \frac{\left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b}{24d(a^2 + 2ab + b^2)(a + b)} - \frac{5a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{8d(a^2 + 2ab + b^2)(a + b)} - \frac{13 \tanh \left(\frac{dx}{2} + \frac{c}{2} \right) b}{8d(a^2 + 2ab + b^2)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^4/(a+b*sech(dx+c)^2)^2,x)

[Out] $-1/24/d/(a^2+2*a*b+b^2)/(a+b)*a*\tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^2+2*a*b+b^2)/(a+b)*\tanh(1/2*d*x+1/2*c)^3*b-5/8/d/(a^2+2*a*b+b^2)/(a+b)*a*\tanh(1/2*d*x+1/2*c)-13/8/d/(a^2+2*a*b+b^2)/(a+b)*\tanh(1/2*d*x+1/2*c)*b-1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d*b^3/a/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)^3-1/d*b^3/a/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)*\tanh(1/2*d*x+1/2*c)+7/4/d*b^(5/2)/a/(a+b)^(7/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-7/4/d*b^(5/2)/a/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^2/(a+b)^(7/2)*\ln(-(a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*b^(7/2)/a^2/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d*b^(7/2)/a^2/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$

$$(1/2*d*x+1/2*c)+(a+b)^{(1/2)}-1/24/d/(a+b)^2/\tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+b)^3/\tanh(1/2*d*x+1/2*c)*a-13/8/d/(a+b)^3/\tanh(1/2*d*x+1/2*c)*b$$

maxima [B] time = 1.41, size = 2961, normalized size = 18.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/4*(a^2*b + 3*a*b^2 + b^3)*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) - 1/2*b*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) \\ & - 1/4*(a^2*b + 3*a*b^2 + b^3)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + 1/2*b*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) \\ & + 1/2*(a + 2*b)*\log(e^{(2*d*x + 2*c)} - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + b*\log(e^{(2*d*x + 2*c)} - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) \\ & - 1/2*(a + 2*b)*\log(e^{(-2*d*x - 2*c)} - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - b*\log(e^{(-2*d*x - 2*c)} - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) \\ & - 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\sqrt{(a + b)*b}*d) \\ & + 1/16*(3*a*b + 8*b^2)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{(a + b)*b}*d) \\ & + 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\sqrt{(a + b)*b}*d) \\ & - 1/16*(3*a*b + 8*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{(a + b)*b}*d) \\ & + 3/32*(3*a*b - 2*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{(a + b)*b}*d) \\ & + 1/48*(44*a^4 + 59*a^3*b + 24*a^2*b^2 + 24*a*b^3 + 3*(24*a^4 + 27*a^3*b - 18*a^2*b^2 - 48*a*b^3 - 32*b^4)*e^{(8*d*x + 8*c)} + 6*(6*a^4 + 55*a^3*b + 79*a^2*b^2 + 68*a*b^3 + 48*b^4)*e^{(6*d*x + 6*c)} - 2*(50*a^4 + 278*a^3*b + 309*a^2*b^2 + 180*a*b^3 + 144*b^4)*e^{(4*d*x + 4*c)} - 2*(10*a^4 - 75*a^3*b - 103*a^2*b^2 - 36*a*b^3 - 48*b^4)*e^{(2*d*x + 2*c)})/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^{(10*d*x + 10*c)} + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^{(8*d*x + 8*c)} + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^{(6*d*x + 6*c)} - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^{(4*d*x + 4*c)} - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^{(2*d*x + 2*c)})*d) \\ & - 1/48*(44*a^4 + 59*a^3*b + 24*a^2*b^2 + 24*a*b^3 - 2*(10*a^4 - 75*a^3*b - 103*a^2*b^2 - 36*a*b^3 - 48*b^4)*e^{(-2*d*x - 2*c)} - 2*(50*a^4 + 278*a^3*b + 309*a^2*b^2 + 180*a*b^3 + 144*b^4)*e^{(-4*d*x - 4*c)} + 6*(6*a^4 + 55*a^3*b + 79*a^2*b^2 + 68*a*b^3 + 48*b^4)*e^{(-6*d*x - 6*c)} + 3*(24*a^4 + 27*a^3*b - 18*a^2*b^2 - 48*a*b^3 - 32*b^4)*e^{(-8*d*x - 8*c)})/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^{(-2*d*x - 2*c)} - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^{(-4*d*x - 4*c)} + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^{(-6*d*x - 6*c)} + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^{(-8*d*x - 8*c)} - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^{(-10*d*x - 10*c)})*d) \\ & + 1/12*(8*a^3 + 17*a^2*b - 6*a*b^2 + 3*(8*a^3 + 13*a^2*b + 8*a*b^2 + 8*b^3)*e^{(8*d*x + 8*c)} + 6*(4*a^3 + 19*a^2*b + 13*a*b^2 - 12*b^3)*e^{(6*d*x + 6*c)} - 2*(8*a^3 + 68*a^2*b + 69*a*b^2 - 36*b^3)*e^{(4*d*x + 4*c)} - 2*(4*a^3 - 15*a^2*b - 37*a*b^2 + 12*b^3)*e^{(2*d*x + 2*c)})/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^{(10*d*x + 10*c)} + (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*e^{(8*d*x + 8*c)} + 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b^4)*e^{(6*d*x + 6*c)} - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b^4)*e^{(4*d*x + 4*c)} - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b^4)*e^{(2*d*x + 2*c)})*d) \end{aligned}$$

$$\begin{aligned}
 & *b^4)e^{(4dx + 4c)} - (a^5 - a^4b - 9a^3b^2 - 11a^2b^3 - 4ab^4)e^{(2dx + 2c)} *d - 1/12(8a^3 + 17a^2b - 6ab^2 - 2(4a^3 - 15a^2b - 37ab^2 + 12b^3))e^{(-2dx - 2c)} - 2(8a^3 + 68a^2b + 69ab^2 - 36b^3)e^{(-4dx - 4c)} + 6(4a^3 + 19a^2b + 13ab^2 - 12b^3)e^{(-6dx - 6c)} + 3(8a^3 + 13a^2b + 8ab^2 + 8b^3)e^{(-8dx - 8c)} / ((a^5 + 3a^4b + 3a^3b^2 + a^2b^3 - (a^5 - a^4b - 9a^3b^2 - 11a^2b^3 - 4ab^4))e^{(-2dx - 2c)} - 2(a^5 + 9a^4b + 21a^3b^2 + 19a^2b^3 + 6ab^4)e^{(-4dx - 4c)} + 2(a^5 + 9a^4b + 21a^3b^2 + 19a^2b^3 + 6ab^4))e^{(-6dx - 6c)} + (a^5 - a^4b - 9a^3b^2 - 11a^2b^3 - 4ab^4)e^{(-8dx - 8c)} - (a^5 + 3a^4b + 3a^3b^2 + a^2b^3))e^{(-10dx - 10c)}) *d \\
 & + 1/8(4a^2 - 11ab - 2(2a^2 - 9ab + 19b^2))e^{(-2dx - 2c)} - 2(10a^2 + 22ab - 33b^2)e^{(-4dx - 4c)} - 6(2a^2 + 3ab + 11b^2)e^{(-6dx - 6c)} - 3(3ab - 2b^2)e^{(-8dx - 8c)} / ((a^4 + 3a^3b + 3a^2b^2 + ab^3 - (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4))e^{(-2dx - 2c)} - 2(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4))e^{(-4dx - 4c)} + 2(a^4 + 9a^3b + 21a^2b^2 + 19ab^3 + 6b^4))e^{(-6dx - 6c)} + (a^4 - a^3b - 9a^2b^2 - 11ab^3 - 4b^4))e^{(-8dx - 8c)} - (a^4 + 3a^3b + 3a^2b^2 + ab^3))e^{(-10dx - 10c)}) *d
 \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 \coth(c + dx)^4}{(a \cosh(c + dx)^2 + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2, x)

[Out] int((cosh(c + d*x)^4*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**2, x)

[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)

$$3.158 \quad \int \frac{\tanh^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=148

$$\frac{x}{a^3} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d(a-b\tanh^2(c+dx)+b)} - \frac{\sqrt{a+b}(3a^2-4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} - \frac{(a+b)\tanh^3(c+dx)}{4abd(a-b\tanh^2(c+dx))}$$

[Out] x/a^3-1/8*(3*a^2-4*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a^3*b^(5/2)/d-1/4*(a+b)*tanh(d*x+c)^3/a/b/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(3*a-4*b)*(a+b)*tanh(d*x+c)/a^2/b^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.32, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4141, 1975, 470, 578, 522, 206, 208}

$$\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d(a-b\tanh^2(c+dx)+b)} - \frac{\sqrt{a+b}(3a^2-4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} + \frac{x}{a^3} - \frac{(a+b)\tanh^3(c+dx)}{4abd(a-b\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]

[Out] x/a^3 - (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^3*b^(5/2)*d) - ((a + b)*Tanh[c + d*x]^3)/(4*a*b*d*(a + b - b*Tanh[c + d*x]^2)^2) + ((3*a - 4*b)*(a + b)*Tanh[c + d*x])/(8*a^2*b^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n-1)*(e*x)^(m-2*n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(b*n*(b*c-a*d)*(p+1)), x] + Dist[e^(2*n)/(b*n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-2*n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[a*c*(m-2*n+1)+(a*d*(m-n+n*q+1)+b*c*n*(p+1)]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m-n+1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

Int[((g_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 1975

Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_.))^(m_.), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \frac{\tanh^6(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(a+b(1-x^2))^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a + b) \tanh^3(c + dx)}{4abd (a + b - b \tanh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3(a+b)+(-3a+b)x^2)}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{4abd} \\ &= -\frac{(a + b) \tanh^3(c + dx)}{4abd (a + b - b \tanh^2(c + dx))^2} + \frac{(3a - 4b)(a + b) \tanh(c + dx)}{8a^2b^2d (a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{4abd} \\ &= -\frac{(a + b) \tanh^3(c + dx)}{4abd (a + b - b \tanh^2(c + dx))^2} + \frac{(3a - 4b)(a + b) \tanh(c + dx)}{8a^2b^2d (a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{4abd} \\ &= \frac{x}{a^3} - \frac{\sqrt{a + b} (3a^2 - 4ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a + b}}\right)}{8a^3b^{5/2}d} - \frac{(a + b) \tanh^3(c + dx)}{4abd (a + b - b \tanh^2(c + dx))^2} \end{aligned}$$

$$\begin{aligned}
&^2) * \cosh(dx + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3) * \cosh(dx + c) * \sinh(dx \\
&x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4) * \cosh(dx + \\
&c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^4 + 9*a^4 + 12*a^ \\
&3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^3) * \cosh \\
&(dx + c)^2) * \sinh(dx + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 - \\
&4*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^3) * \cosh \\
&(dx + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4) * \cosh(dx \\
&+ c) * \sinh(dx + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3) * \cosh(dx + c)^2 + 4* \\
&(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^6 + 15*(3*a^4 + 2*a^3*b + 16 \\
&*a*b^3) * \cosh(dx + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12*a^3*b \\
&+ 16*a^2*b^2 + 32*a*b^3 + 64*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8*((3* \\
&a^4 - 4*a^3*b + 8*a^2*b^2) * \cosh(dx + c)^7 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3) \\
&* \cosh(dx + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4) * \cosh \\
&(dx + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3) * \cosh(dx + c) * \sinh(dx + c)) * \text{sq} \\
&\text{rt}((a + b)/b) * \log((a^2 * \cosh(dx + c)^4 + 4*a^2 * \cosh(dx + c) * \sinh(dx + c)^ \\
&3 + a^2 * \sinh(dx + c)^4 + 2*(a^2 + 2*a*b) * \cosh(dx + c)^2 + 2*(3*a^2 * \cosh(dx \\
&*x + c)^2 + a^2 + 2*a*b) * \sinh(dx + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2 * \cosh \\
&h(dx + c)^3 + (a^2 + 2*a*b) * \cosh(dx + c)) * \sinh(dx + c) + 4*(a*b * \cosh(dx \\
&+ c)^2 + 2*a*b * \cosh(dx + c) * \sinh(dx + c) + a*b * \sinh(dx + c)^2 + a*b + 2 \\
&*b^2) * \text{sqrt}((a + b)/b)) / (a * \cosh(dx + c)^4 + 4*a * \cosh(dx + c) * \sinh(dx + c) \\
&^3 + a * \sinh(dx + c)^4 + 2*(a + 2*b) * \cosh(dx + c)^2 + 2*(3*a * \cosh(dx + c) \\
&^2 + a + 2*b) * \sinh(dx + c)^2 + 4*(a * \cosh(dx + c)^3 + (a + 2*b) * \cosh(dx + \\
&c)) * \sinh(dx + c) + a) + 8*(16*a^2*b^2*d*x * \cosh(dx + c)^7 - 3*(3*a^4 - a \\
&^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \cosh(dx + c)^5 \\
&- 2*(9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a \\
&*b^3 + 8*b^4)*d*x) * \cosh(dx + c)^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a* \\
&b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \cosh(dx + c)) * \sinh(dx + c)) / (a^5*b^2*d* \\
&\cosh(dx + c)^8 + 8*a^5*b^2*d * \cosh(dx + c) * \sinh(dx + c)^7 + a^5*b^2*d * \text{sin} \\
&\text{h}(dx + c)^8 + a^5*b^2*d + 4*(a^5*b^2 + 2*a^4*b^3)*d * \cosh(dx + c)^6 + 4*(7 \\
&*a^5*b^2*d * \cosh(dx + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d) * \sinh(dx + c)^6 + 2*(\\
&3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d * \cosh(dx + c)^4 + 8*(7*a^5*b^2*d * \cosh(dx \\
&+ c)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(\\
&35*a^5*b^2*d * \cosh(dx + c)^4 + 30*(a^5*b^2 + 2*a^4*b^3)*d * \cosh(dx + c)^2 + \\
&(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d) * \sinh(dx + c)^4 + 4*(a^5*b^2 + 2*a^ \\
&4*b^3)*d * \cosh(dx + c)^2 + 8*(7*a^5*b^2*d * \cosh(dx + c)^5 + 10*(a^5*b^2 + 2 \\
&*a^4*b^3)*d * \cosh(dx + c)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d * \cosh(dx \\
&x + c)) * \sinh(dx + c)^3 + 4*(7*a^5*b^2*d * \cosh(dx + c)^6 + 15*(a^5*b^2 + 2* \\
&a^4*b^3)*d * \cosh(dx + c)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d * \cosh(dx \\
&*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d) * \sinh(dx + c)^2 + 8*(a^5*b^2*d * \cosh(dx \\
&x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*d * \cosh(dx + c)^5 + (3*a^5*b^2 + 8*a^4*b \\
&^3 + 8*a^3*b^4)*d * \cosh(dx + c)^3 + (a^5*b^2 + 2*a^4*b^3)*d * \cosh(dx + c)) * \\
&\sinh(dx + c)), 1/8*(8*a^2*b^2*d*x * \cosh(dx + c)^8 + 64*a^2*b^2*d*x * \cosh(dx \\
&x + c) * \sinh(dx + c)^7 + 8*a^2*b^2*d*x * \sinh(dx + c)^8 - 2*(3*a^4 - a^3*b - \\
&20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \cosh(dx + c)^6 + 2*(1 \\
&12*a^2*b^2*d*x * \cosh(dx + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16 \\
&*(a^2*b^2 + 2*a*b^3)*d*x) * \sinh(dx + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2* \\
&d*x * \cosh(dx + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^ \\
&2 + 2*a*b^3)*d*x) * \cosh(dx + c)) * \sinh(dx + c)^5 - 2*(9*a^4 + 15*a^3*b - 18 \\
&*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x) * \cosh(dx \\
&x + c)^4 + 2*(280*a^2*b^2*d*x * \cosh(dx + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b \\
&^2 + 72*a*b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 - \\
&a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \cosh(dx + c)^2 \\
&)* \sinh(dx + c)^4 - 6*a^4 + 6*a^3*b + 12*a^2*b^2 + 8*(56*a^2*b^2*d*x * \cosh(dx \\
&*x + c)^5 - 5*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^ \\
&3)*d*x) * \cosh(dx + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^ \\
&4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x) * \cosh(dx + c)) * \sinh(dx + c)^3 - 2 \\
&*(9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \text{co} \\
&\text{sh}(dx + c)^2 + 2*(112*a^2*b^2*d*x * \cosh(dx + c)^6 - 15*(3*a^4 - a^3*b - 20 \\
&*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x) * \cosh(dx + c)^4 - 9*a^4 -
\end{aligned}$$

$$\begin{aligned}
& 13a^3b + 28a^2b^2 + 32ab^3 + 16(a^2b^2 + 2ab^3)d^2x - 6(9a^4 + 15a^3b - 18a^2b^2 - 72ab^3 - 48b^4 - 8(3a^2b^2 + 8ab^3 + 8b^4) \\
&)d^2x)cosh(dx + c)^2sinh(dx + c)^2 - ((3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^8 + 8(3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)sinh(dx + c) \\
& ^7 + (3a^4 - 4a^3b + 8a^2b^2)sinh(dx + c)^8 + 4(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)^6 + 4(3a^4 + 2a^3b + 16ab^3 + 7(3a^4 - 4a^3b \\
& *b + 8a^2b^2)cosh(dx + c)^2sinh(dx + c)^6 + 8(7(3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^3 + 3(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)) \\
& sinh(dx + c)^5 + 2(9a^4 + 12a^3b + 16a^2b^2 + 32ab^3 + 64b^4)cosh(dx + c)^4 + 2(35(3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^4 + 9a^4 \\
& + 12a^3b + 16a^2b^2 + 32ab^3 + 64b^4 + 30(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)^2sinh(dx + c)^4 + 3a^4 - 4a^3b + 8a^2b^2 + 8(7(3 \\
& a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^5 + 10(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)^3 + (9a^4 + 12a^3b + 16a^2b^2 + 32ab^3 + 64b^4)cosh(dx + c) \\
&)sinh(dx + c)^3 + 4(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)^2 + 4(7(3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^6 + 15(3a^4 + 2a^3b \\
& *b + 16ab^3)cosh(dx + c)^4 + 3a^4 + 2a^3b + 16ab^3 + 3(9a^4 + 12a^3b + 16a^2b^2 + 32ab^3 + 64b^4)cosh(dx + c)^2sinh(dx + c)^2 + \\
& 8((3a^4 - 4a^3b + 8a^2b^2)cosh(dx + c)^7 + 3(3a^4 + 2a^3b + 16ab^3)cosh(dx + c)^5 + (9a^4 + 12a^3b + 16a^2b^2 + 32ab^3 + 64b^4)cosh(dx + c)^3 + (3a^4 + 2a^3b + 16ab^3)cosh(dx + c)sinh(dx + c)) \\
&)sqrt(-(a + b)/b)arctan(1/2*(acosh(dx + c)^2 + 2acosh(dx + c)sinh(dx + c) + asinh(dx + c)^2 + a + 2b)sqrt(-(a + b)/b)/(a + b)) + 4(16 \\
& a^2b^2d^2x*cosh(dx + c)^7 - 3(3a^4 - a^3b - 20a^2b^2 - 16ab^3 - 16(a^2b^2 + 2ab^3)d^2x)cosh(dx + c)^5 - 2(9a^4 + 15a^3b - 18a^2b^2 - 72ab^3 - 48b^4 - 8(3a^2b^2 + 8ab^3 + 8b^4)d^2x)cosh(dx + c)^3 - (9a^4 + 13a^3b - 28a^2b^2 - 32ab^3 - 16(a^2b^2 + 2ab^3)d^2x)cosh(dx + c)sinh(dx + c))/(a^5b^2d^2cosh(dx + c)^8 + 8a^5b^2d^2cosh(dx + c)sinh(dx + c)^7 + a^5b^2d^2sinh(dx + c)^8 + a^5b^2d + 4(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^6 + 4(7a^5b^2d^2cosh(dx + c)^2 + (a^5b^2 + 2a^4b^3)d)sinh(dx + c)^6 + 2(3a^5b^2 + 8a^4b^3 + 8a^3b^4)d^2cosh(dx + c)^4 + 8(7a^5b^2d^2cosh(dx + c)^3 + 3(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)sinh(dx + c)^5 + 2(35a^5b^2d^2cosh(dx + c)^4 + 30(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^2 + (3a^5b^2 + 8a^4b^3 + 8a^3b^4)d)sinh(dx + c)^4 + 4(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^2 + 8(7a^5b^2d^2cosh(dx + c)^5 + 10(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^3 + (3a^5b^2 + 8a^4b^3 + 8a^3b^4)d^2cosh(dx + c)sinh(dx + c)^3 + 4(7a^5b^2d^2cosh(dx + c)^6 + 15(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^4 + 3(3a^5b^2 + 8a^4b^3 + 8a^3b^4)d^2cosh(dx + c)^2 + (a^5b^2 + 2a^4b^3)d)sinh(dx + c)^2 + 8(a^5b^2d^2cosh(dx + c)^7 + 3(a^5b^2 + 2a^4b^3)d^2cosh(dx + c)^5 + (3a^5b^2 + 8a^4b^3 + 8a^3b^4)d^2cosh(dx + c)^3 + (a^5b^2 + 2a^4b^3)d^2cosh(dx + c)sinh(dx + c))]
\end{aligned}$$

giac [B] time = 2.25, size = 371, normalized size = 2.51

$$\frac{8dx}{a^3} \frac{(3a^3e^{2c} - a^2be^{2c} + 4ab^2e^{2c} + 8b^3e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + a+2b}{2\sqrt{-ab-b^2}}\right) e^{-(2c)}}{\sqrt{-ab-b^2} a^3 b^2} - 2(3a^4e^{6dx+6c} - a^3be^{6dx+6c} - 20a^2b^2e^{6dx+6c} - 16ab^3e^{6dx+6c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^6/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/8*(8d^2x/a^3 - (3a^3e^{2c} - a^2b^2e^{2c} + 4a^3b^2e^{2c} + 8b^3e^{2c}) * arctan(1/2*(a*e^{2dx+2c} + a + 2b)/sqrt(-a*b - b^2)) * e^{-2c}) / (sqrt(-a*b - b^2) * a^3 * b^2) - 2*(3a^4e^{6dx+6c} - a^3b^2e^{6dx+6c} - 20a^2b^2e^{6dx+6c} - 16a^3b^3e^{6dx+6c} + 9a^4e^{4dx+4c} + 15a^3b^2e^{4dx+4c} - 18a^2b^2e^{4dx+4c} - 72a^3b^3e^{4dx+4c} - 48b^4e^{4dx+4c} + 9a^4e^{2dx+2c} + 13a^3b^2e^{2dx+2c} - 28a^2b^2e^{2dx+2c} - 32a^3b^3e^{2dx+2c} +

$$\frac{3a^4 - 3a^3b - 6a^2b^2}{((ae^{4dx+4c} + 2ae^{2dx+2c}) + 4be^{2dx+2c} + a)^2 a^3 b^2)} / d$$

maple [B] time = 0.45, size = 1713, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^6/(a+b*sech(dx+c)^2)^3,x)

[Out] $1/d/a^3 \ln(\tanh(1/2 dx + 1/2 c) + 1) - 1/d/a^3 \ln(\tanh(1/2 dx + 1/2 c) - 1) - 5/4/d/a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^7 - 19/4/d/a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^5 - 19/4/d/a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^3 - 5/4/d/a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c) + 1/2/d / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b \tanh(1/2 dx + 1/2 c)^7 - 7/2/d / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b \tanh(1/2 dx + 1/2 c)^5 - 7/2/d / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b \tanh(1/2 dx + 1/2 c)^3 + 1/2/d / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b \tanh(1/2 dx + 1/2 c) + 3/16/d/b^{5/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 - 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 3/16/d/b^{5/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) + 1/4/d/a^2/b^{1/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 - 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) + (a+b)^{1/2}) + 1/16/d/a/b^{3/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 1/4/d/a^2/b^{1/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) + 3/4/d*a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b^2 \tanh(1/2 dx + 1/2 c)^7 + 9/4/d*a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b^2 \tanh(1/2 dx + 1/2 c)^5 + 9/4/d*a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b^2 \tanh(1/2 dx + 1/2 c)^3 + 3/4/d*a / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 / b^2 \tanh(1/2 dx + 1/2 c) - 1/16/d/a/b^{3/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 - 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 1/2/d/a^3 b^{1/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 + 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) + 1/d/a^2 b / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^5 + 1/2/d/a^3 b^{1/2} / (a+b)^{1/2} \ln((a+b)^{1/2} \tanh(1/2 dx + 1/2 c)^2 - 2b^{1/2} \tanh(1/2 dx + 1/2 c) + (a+b)^{1/2}) - 1/d/a^2 b / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)^3 - 1/d/a^2 b / (\tanh(1/2 dx + 1/2 c)^4 a + b \tanh(1/2 dx + 1/2 c)^4 + 2 \tanh(1/2 dx + 1/2 c)^2 a - 2 \tanh(1/2 dx + 1/2 c)^2 b + a + b)^2 \tanh(1/2 dx + 1/2 c)$

maxima [B] time = 1.99, size = 3239, normalized size = 21.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^6/(a+b*sech(dx+c)^2)^3,x, algorithm="maxima")

```
[Out] -45/1024*(a + 2*b)*a*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^
4)*sqrt((a + b)*b)*d) - 9/512*a^2*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 +
2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 45/1024*(a + 2*b)*a*log((a*e^(-2*d*x -
2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt
((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 9/512*a^2*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d)
- 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5
)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c)
+ a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*sqrt((a +
b)*b)*d) + 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4
+ 256*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)
*sqrt((a + b)*b)*d) + 5/256*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c)
+ a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a +
b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/256*(3*a^6 - 12*
a^5*b - 204*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (3*a^6 - 10*a^5*b - 560*a
^4*b^2 - 2080*a^3*b^3 - 2560*a^2*b^4 - 1024*a*b^5)*e^(6*d*x + 6*c) + (9*a^6
- 12*a^5*b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 30
72*b^6)*e^(4*d*x + 4*c) + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3136*a^3*b^3 -
3840*a^2*b^4 - 1536*a*b^5)*e^(2*d*x + 2*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4
+ (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(8*d*x + 8*c) + 4*(a^7*b^2 + 4*a^6*b^3
+ 5*a^5*b^4 + 2*a^4*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7*b^2 + 14*a^6*b^3 + 27*
a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 4*a^6*b^3
+ 5*a^5*b^4 + 2*a^4*b^5)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 - 12*a^5*b - 20
4*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3
136*a^3*b^3 - 3840*a^2*b^4 - 1536*a*b^5)*e^(-2*d*x - 2*c) + (9*a^6 - 12*a^5
*b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*e
^(-4*d*x - 4*c) + (3*a^6 - 10*a^5*b - 560*a^4*b^2 - 2080*a^3*b^3 - 2560*a^2
*b^4 - 1024*a*b^5)*e^(-6*d*x - 6*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4 + 4*(a
^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7*b^2
+ 14*a^6*b^3 + 27*a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(-4*d*x - 4*c) + 4*(
a^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-6*d*x - 6*c) + (a^7*b^2 +
2*a^6*b^3 + a^5*b^4)*e^(-8*d*x - 8*c))*d) - 3/128*(3*a^5 - 2*a^4*b - 24*a^3
*b^2 - 16*a^2*b^3 + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(6*d*
x + 6*c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256*
b^5)*e^(4*d*x + 4*c) + (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128*
a*b^4)*e^(2*d*x + 2*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + (a^6*b^2 + 2*a^5*
b^3 + a^4*b^4)*e^(8*d*x + 8*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3
*b^5)*e^(6*d*x + 6*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4 + 24*a^3*b^5
+ 8*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3*
b^5)*e^(2*d*x + 2*c))*d) + 3/128*(3*a^5 - 2*a^4*b - 24*a^3*b^2 - 16*a^2*b^3
+ (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-2*d*x - 2
*c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256*b^5)*
e^(-4*d*x - 4*c) + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-6*d*
x - 6*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*
b^4 + 2*a^3*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4
+ 24*a^3*b^5 + 8*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4
*b^4 + 2*a^3*b^5)*e^(-6*d*x - 6*c) + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*e^(-8*
d*x - 8*c))*d) - 15/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + 3*(a^4 + 2*a^3*b)*e^
(6*d*x + 6*c) + (9*a^4 + 36*a^3*b + 100*a^2*b^2 + 128*a*b^3 + 64*b^4)*e^(4*
d*x + 4*c) + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*b^3)*e^(2*d*x + 2*c))/((
a^5*b^2 + 2*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*e^(8*d*x +
8*c) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(6*d*x + 6*c) + 2*
(3*a^5*b^2 + 14*a^4*b^3 + 27*a^3*b^4 + 24*a^2*b^5 + 8*a*b^6)*e^(4*d*x + 4*c
) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(2*d*x + 2*c))*d) + 1
5/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*
```

$b^3)e^{(-2dx - 2c)} + (9a^4 + 36a^3b + 100a^2b^2 + 128ab^3 + 64b^4)e^{(-4dx - 4c)} + 3(a^4 + 2a^3b)e^{(-6dx - 6c)} / ((a^5b^2 + 2a^4b^3 + a^3b^4 + 4(a^5b^2 + 4a^4b^3 + 5a^3b^4 + 2a^2b^5)e^{(-2dx - 2c)} + 2(3a^5b^2 + 14a^4b^3 + 27a^3b^4 + 24a^2b^5 + 8ab^6)e^{(-4dx - 4c)} + 4(a^5b^2 + 4a^4b^3 + 5a^3b^4 + 2a^2b^5)e^{(-6dx - 6c)} + (a^5b^2 + 2a^4b^3 + a^3b^4)e^{(-8dx - 8c)})d) + 5/64(3a^3 + 6a^2b + (9a^3 + 40a^2b + 40ab^2)e^{(-2dx - 2c)} + 3(3a^3 + 14a^2b + 24ab^2 + 16b^3)e^{(-4dx - 4c)} + (3a^3 + 8a^2b + 8ab^2)e^{(-6dx - 6c)}) / ((a^4b^2 + 2a^3b^3 + a^2b^4 + 4(a^4b^2 + 4a^3b^3 + 5a^2b^4 + 2ab^5)e^{(-2dx - 2c)} + 2(3a^4b^2 + 14a^3b^3 + 27a^2b^4 + 24ab^5 + 8b^6)e^{(-4dx - 4c)} + 4(a^4b^2 + 4a^3b^3 + 5a^2b^4 + 2ab^5)e^{(-6dx - 6c)} + (a^4b^2 + 2a^3b^3 + a^2b^4)e^{(-8dx - 8c)})d) + 1/4 \log(ae^{(4dx + 4c)} + 2(a + 2b)e^{(2dx + 2c)} + a) / (a^3d) - 1/4 \log(2(a + 2b)e^{(-2dx - 2c)} + ae^{(-4dx - 4c)} + a) / (a^3d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(\cosh(c + dx)^2 - 1)^3}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^6/(a + b/cosh(c + d*x)^2)^3, x)

[Out] int((cosh(c + d*x)^2 - 1)^3/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**6/(a+b*sech(d*x+c)**2)**3, x)

[Out] Timed out

$$3.159 \quad \int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=77

$$-\frac{(a+b)^2}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{a+b}{a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

[Out] $-1/4*(a+b)^2/a^3/d/(b+a*\cosh(d*x+c)^2)^2+(a+b)/a^3/d/(b+a*\cosh(d*x+c)^2)+1/2*\ln(b+a*\cosh(d*x+c)^2)/a^3/d$

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 444, 43}

$$-\frac{(a+b)^2}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{a+b}{a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-(a+b)^2/(4*a^3*d*(b+a*Cosh[c+d*x]^2)^2) + (a+b)/(a^3*d*(b+a*Cosh[c+d*x]^2)) + \text{Log}[b+a*Cosh[c+d*x]^2]/(2*a^3*d)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x(1-x^2)^2}{(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+b)^2}{4a^3d(b+a\cosh^2(c+dx))^2} + \frac{a+b}{a^3d(b+a\cosh^2(c+dx))} + \frac{\log(b+a\cosh^2(c+dx))}{2a^3d}
\end{aligned}$$

Mathematica [A] time = 2.17, size = 136, normalized size = 1.77

$$\frac{2(a^2 + 4ab + 3b^2) + a^2 \cosh^2(2(c+dx)) \log(a \cosh(2(c+dx)) + a + 2b) + (a + 2b)^2 \log(a \cosh(2(c+dx)) + a + 2b)}{2a^3d(a \cosh(2(c+dx)) + a + 2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.46, size = 1741, normalized size = 22.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a^2 - a*b)*sinh(d*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 - 8*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a^2 + 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*c

osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 1.88, size = 187, normalized size = 2.43

$$\frac{\frac{4 dx}{a^3} - \frac{2 \log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^3} + \frac{3ae^{(8dx+8c)}-4ae^{(6dx+6c)}+8be^{(6dx+6c)}+2ae^{(4dx+4c)}-16be^{(4dx+4c)}-4ae^{(2dx+2c)}+8be^{(2dx+2c)}+a)}{(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)^2} a^2}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) - 4*a*e^(6*d*x + 6*c) + 8*b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 16*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d

maple [B] time = 0.42, size = 579, normalized size = 7.52

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d a^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d a^3} - \frac{2\left(\tanh^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)

[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-2/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-8/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+4/d/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4*b-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-2/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)

maxima [B] time = 0.42, size = 206, normalized size = 2.68

$$\frac{4\left((a^2 + ab)e^{(-2dx-2c)} + (a^2 + 4ab + 3b^2)e^{(-4dx-4c)} + (a^2 + ab)e^{(-6dx-6c)}\right)}{(a^5e^{(-8dx-8c)} + a^5 + 4(a^5 + 2a^4b)e^{(-2dx-2c)} + 2(3a^5 + 8a^4b + 8a^3b^2)e^{(-4dx-4c)} + 4(a^5 + 2a^4b)e^{(-6dx-6c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 4*((a^2 + a*b)*e^(-2*d*x - 2*c) + (a^2 + 4*a*b + 3*b^2)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \tanh(c + dx)^5}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.160 \quad \int \frac{\tanh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=139

$$\frac{x}{a^3} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd(a-b\tanh^2(c+dx)+b)} + \frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}} - \frac{(a+b)\tanh(c+dx)}{4abd(a-b\tanh^2(c+dx)+b)^2}$$

[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/b^(3/2)/d/(a+b)^(1/2)-1/4*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(a-4*b)*tanh(d*x+c)/a^2/b/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, number of rules / integrand size = 0.304, Rules used = {4141, 1975, 470, 527, 522, 206, 208}

$$\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd(a-b\tanh^2(c+dx)+b)} + \frac{x}{a^3} - \frac{(a+b)\tanh(c+dx)}{4abd(a-b\tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^3*b^(3/2)*Sqrt[a + b]*d) - ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b - b*Tanh[c + d*x]^2)^2) + ((a - 4*b)*Tanh[c + d*x])/(8*a^2*b*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_.)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_)*((d_.)*tan[(e_.) + (f
_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a+b)\tanh(c+dx)}{4abd(a+b-b\tanh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{a+b+(-a+3b)x^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{4abd} \\ &= -\frac{(a+b)\tanh(c+dx)}{4abd(a+b-b\tanh^2(c+dx))^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd(a+b-b\tanh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{4abd} \\ &= -\frac{(a+b)\tanh(c+dx)}{4abd(a+b-b\tanh^2(c+dx))^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd(a+b-b\tanh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c+dx)\right)}{4abd} \\ &= \frac{x}{a^3} + \frac{(a^2 - 4ab - 8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}d} - \frac{(a+b)\tanh(c+dx)}{4abd(a+b-b\tanh^2(c+dx))^2} \end{aligned}$$

Mathematica [B] time = 13.09, size = 1457, normalized size = 10.48

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3, x]

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b^2)
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x
)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(1024*b^(5/2)*d*(a + b*
Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((
-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2)
+ (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)
)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c +
d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cos
h[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480
*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*
((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4]))*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh
[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2
*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[
2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh
[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh
[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*
c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c +
4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Si
nh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sin
h[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14
*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2
560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)
] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^
3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*
c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1
056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*
Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(4096*a^3*b^2*(a +
b)^2*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech
[c + d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*S
inh[d*x] - a*Sinh[2*c + d*x])]/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]
)]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (
a*Sech[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d
*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4
- 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b
- 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b
+ a*Cosh[2*(c + d*x)])^2)))/(2048*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3
)
```

fricas [B] time = 0.57, size = 6464, normalized size = 46.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + a^2*b^3)*d*x*sinh(d*x +
c)^8 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + a^2*b^3)*d*x*cosh
(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 +
3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^4*b + 28*a^
3*b^2 + 24*a^2*b^3 + 4*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*
b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
4*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 3*a^4*b + 29*a^3*b^2 + 82*
a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)
)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^
```

$$\begin{aligned}
& 2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 16*(56*(a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^5 + 5*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^3 + (3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 16*(a^3*b^2 + a^2*b^3)*d*x + 4*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*\sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(d*x + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^3 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^2 + 4*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt(a*b + b^2)*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c))^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b)*\sqrt(a*b + b^2))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 8*(16*(a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*\sinh(d*x + c)^8 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^2 + (3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^3 + (3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)
\end{aligned}$$

$$\begin{aligned}
& *d*\cosh(dx + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*\sinh(dx + c)^2 + \\
& (a^6*b^2 + a^5*b^3)*d + 8*((a^6*b^2 + a^5*b^3)*d*\cosh(dx + c)^7 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(dx + c)^5 + (3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*\cosh(dx + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(dx + c))*\sinh(dx + c)), 1/8*(8*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^8 + 64*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)*\sinh(dx + c)^7 + 8*(a^3*b^2 + a^2*b^3)*d*x*\sinh(dx + c)^8 + 2*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^6 + 2*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^2 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\sinh(dx + c)^6 + 4*(112*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c))*\sinh(dx + c)^5 + 2*a^4*b + 14*a^3*b^2 + 12*a^2*b^3 + 2*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(dx + c)^4 + 2*(280*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^4 + 3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^2)*\sinh(dx + c)^4 + 8*(56*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^5 + 5*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^3 + (3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(dx + c))*\sinh(dx + c)^3 + 8*(a^3*b^2 + a^2*b^3)*d*x + 2*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^2 + 2*(112*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + ((a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)*\sinh(dx + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*\sinh(dx + c)^8 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(dx + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2))*\cosh(dx + c)^2)*\sinh(dx + c)^6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^3 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(dx + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3))*\cosh(dx + c)^2)*\sinh(dx + c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3))*\cosh(dx + c)^3 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(dx + c)^2 + 4*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3))*\cosh(dx + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*\cosh(dx + c)^7 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(dx + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*\cosh(dx + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*\cosh(dx + c))*\sinh(dx + c))*\sqrt{-a*b - b^2)*\arctan(1/2*(a*\cosh(dx + c)^2 + 2*a*\cosh(dx + c)*\sinh(dx + c) + a*\sinh(dx + c)^2 + a + 2*b)*\sqrt{-a*b - b^2}/(a*b + b^2)) + 4*(16*(a^3*b^2 + a^2*b^3)*d*x*\cosh(dx + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c)^5 + 2*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*\cosh(dx + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(dx + c))*\sinh(dx + c)))/((a^6*b^2 + a^5*b^3)*d*\cosh(dx + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*\cosh(dx + c)*\sinh(dx + c)^7 + (a^6*b^2 + a^5*b^3)*d*\sinh(dx + c)^8 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*\cosh(dx + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*co
\end{aligned}$$

$$\begin{aligned}
& b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b \\
& + (a+b)^2 / b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 5/4 \cdot d/a / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 \\
& + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + (a+b)^2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) \\
& - 1/d/a^2 \cdot b / (\tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 \cdot a + b \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot a \\
& - 2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot b + (a+b)^2 \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1/16 \cdot d/a/b^{3/2} / (a+b)^{1/2} \cdot \ln(-(a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \\
& \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - (a+b)^{1/2}) + 1/4 \cdot d/a^2/b^{1/2} / (a+b)^{1/2} \cdot \ln(-(a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) \\
& - (a+b)^{1/2}) + 1/2 \cdot d/a^3 \cdot b^{1/2} / (a+b)^{1/2} \cdot \ln(-(a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) - (a+b)^{1/2}) \\
& + 1/16 \cdot d/a/b^{3/2} / (a+b)^{1/2} \cdot \ln((a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{1/2}) - 1/4 \cdot d/a^2/b^{1/2} / (a+b)^{1/2} \cdot \ln((a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{1/2}) \\
& - 1/2 \cdot d/a^3 \cdot b^{1/2} / (a+b)^{1/2} \cdot \ln((a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{1/2}) \\
& + 1/16 \cdot d/a/b^{3/2} / (a+b)^{1/2} \cdot \ln((a+b)^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 2 \cdot b^{1/2} \cdot \tanh(1/2 \cdot d \cdot x + 1/2 \cdot c) + (a+b)^{1/2})
\end{aligned}$$

maxima [B] time = 1.02, size = 2201, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $1/256 \cdot (a^4 - 20 \cdot a^3 \cdot b - 120 \cdot a^2 \cdot b^2 - 160 \cdot a \cdot b^3 - 64 \cdot b^4) \cdot \log((a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a+b) \cdot b}) / (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a+b) \cdot b})) / ((a^5 \cdot b + 2 \cdot a^4 \cdot b^2 + a^3 \cdot b^3) \cdot \sqrt{(a+b) \cdot b} \cdot d) + 1/64 \cdot (a - 2 \cdot b) \cdot \log((a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a+b) \cdot b}) / (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a+b) \cdot b})) / ((a^2 \cdot b + 2 \cdot a \cdot b^2 + b^3) \cdot \sqrt{(a+b) \cdot b} \cdot d) - 1/256 \cdot (a^4 - 20 \cdot a^3 \cdot b - 120 \cdot a^2 \cdot b^2 - 160 \cdot a \cdot b^3 - 64 \cdot b^4) \cdot \log((a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a+b) \cdot b}) / (a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a+b) \cdot b})) / ((a^5 \cdot b + 2 \cdot a^4 \cdot b^2 + a^3 \cdot b^3) \cdot \sqrt{(a+b) \cdot b} \cdot d) - 3/128 \cdot (a + 4 \cdot b) \cdot \log((a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a+b) \cdot b}) / (a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a+b) \cdot b})) / ((a^2 \cdot b + 2 \cdot a \cdot b^2 + b^3) \cdot \sqrt{(a+b) \cdot b} \cdot d) - 1/64 \cdot (a - 2 \cdot b) \cdot \log((a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b - 2 \cdot \sqrt{(a+b) \cdot b}) / (a \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a + 2 \cdot b + 2 \cdot \sqrt{(a+b) \cdot b})) / ((a^2 \cdot b + 2 \cdot a \cdot b^2 + b^3) \cdot \sqrt{(a+b) \cdot b} \cdot d) + 1/64 \cdot (a^5 + 38 \cdot a^4 \cdot b + 88 \cdot a^3 \cdot b^2 + 48 \cdot a^2 \cdot b^3 + (a^5 + 76 \cdot a^4 \cdot b + 392 \cdot a^3 \cdot b^2 + 576 \cdot a^2 \cdot b^3 + 256 \cdot a \cdot b^4) \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + (3 \cdot a^5 + 186 \cdot a^4 \cdot b + 1024 \cdot a^3 \cdot b^2 + 2240 \cdot a^2 \cdot b^3 + 2176 \cdot a \cdot b^4 + 768 \cdot b^5) \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + (3 \cdot a^5 + 148 \cdot a^4 \cdot b + 648 \cdot a^3 \cdot b^2 + 896 \cdot a^2 \cdot b^3 + 384 \cdot a \cdot b^4) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) / ((a^7 \cdot b + 2 \cdot a^6 \cdot b^2 + a^5 \cdot b^3 + (a^7 \cdot b + 2 \cdot a^6 \cdot b^2 + a^5 \cdot b^3) \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} + 4 \cdot (a^7 \cdot b + 4 \cdot a^6 \cdot b^2 + 5 \cdot a^5 \cdot b^3 + 2 \cdot a^4 \cdot b^4) \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 2 \cdot (3 \cdot a^7 \cdot b + 14 \cdot a^6 \cdot b^2 + 27 \cdot a^5 \cdot b^3 + 24 \cdot a^4 \cdot b^4 + 8 \cdot a^3 \cdot b^5) \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 4 \cdot (a^7 \cdot b + 4 \cdot a^6 \cdot b^2 + 5 \cdot a^5 \cdot b^3 + 2 \cdot a^4 \cdot b^4) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) \cdot d) - 1/64 \cdot (a^5 + 38 \cdot a^4 \cdot b + 88 \cdot a^3 \cdot b^2 + 48 \cdot a^2 \cdot b^3 + (3 \cdot a^5 + 148 \cdot a^4 \cdot b + 648 \cdot a^3 \cdot b^2 + 896 \cdot a^2 \cdot b^3 + 384 \cdot a \cdot b^4) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + (3 \cdot a^5 + 186 \cdot a^4 \cdot b + 1024 \cdot a^3 \cdot b^2 + 2240 \cdot a^2 \cdot b^3 + 2176 \cdot a \cdot b^4 + 768 \cdot b^5) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + (a^5 + 76 \cdot a^4 \cdot b + 392 \cdot a^3 \cdot b^2 + 576 \cdot a^2 \cdot b^3 + 256 \cdot a \cdot b^4) \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)}) / ((a^7 \cdot b + 2 \cdot a^6 \cdot b^2 + a^5 \cdot b^3 + 4 \cdot (a^7 \cdot b + 4 \cdot a^6 \cdot b^2 + 5 \cdot a^5 \cdot b^3 + 2 \cdot a^4 \cdot b^4) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 2 \cdot (3 \cdot a^7 \cdot b + 14 \cdot a^6 \cdot b^2 + 27 \cdot a^5 \cdot b^3 + 24 \cdot a^4 \cdot b^4 + 8 \cdot a^3 \cdot b^5) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 4 \cdot (a^7 \cdot b + 4 \cdot a^6 \cdot b^2 + 5 \cdot a^5 \cdot b^3 + 2 \cdot a^4 \cdot b^4) \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)}) \cdot d) + 1/16 \cdot (a^4 + 8 \cdot a^3 \cdot b + 4 \cdot a^2 \cdot b^2 + (a^4 + 30 \cdot a^3 \cdot b + 64 \cdot a^2 \cdot b^2 + 32 \cdot a \cdot b^3) \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + (3 \cdot a^4 + 64 \cdot a^3 \cdot b + 180 \cdot a^2 \cdot b^2 + 192 \cdot a \cdot b^3 + 64 \cdot b^4) \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + (3 \cdot a^4 + 42 \cdot a^3 \cdot b + 80 \cdot a^2 \cdot b^2 + 32 \cdot a \cdot b^3) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) / ((a^6 \cdot b + 2 \cdot a^5 \cdot b^2 + a^4 \cdot b^3 + (a^6 \cdot b + 2 \cdot a^5 \cdot b^2 + a^4 \cdot b^3) \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} + 4 \cdot (a^6 \cdot b + 4 \cdot a^5 \cdot b^2 + 5 \cdot a^4 \cdot b^3 + 2 \cdot a^3 \cdot b^4) \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 2 \cdot (3 \cdot a^6 \cdot b + 14 \cdot a^5 \cdot b^2 + 27 \cdot a^4 \cdot b^3 + 24 \cdot a^3 \cdot b^4 + 8 \cdot a^2 \cdot b^5) \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 4 \cdot (a^6 \cdot b + 4 \cdot a^5 \cdot b^2 + 5 \cdot a^4 \cdot b^3 + 2 \cdot a^3 \cdot b^4) \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) \cdot d) - 1/16 \cdot (a^4 + 8 \cdot a^3 \cdot b + 4 \cdot a^2 \cdot b^2 + (3 \cdot a^4 + 42 \cdot a^3 \cdot b + 80 \cdot a^2 \cdot b^2 + 32 \cdot a \cdot b^3) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + (3 \cdot a^4 + 64 \cdot a^3 \cdot b + 180 \cdot a^2 \cdot b^2 + 192 \cdot a \cdot b^3 + 64 \cdot b^4) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + (a^4 + 30 \cdot a^3 \cdot b + 64 \cdot a^2 \cdot b^2 + 32 \cdot a \cdot b^3) \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)})$

$$\begin{aligned}
 & - 6*c)) / ((a^6*b + 2*a^5*b^2 + a^4*b^3 + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + \\
 & 2*a^3*b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^6*b + 14*a^5*b^2 + 27*a^4*b^3 + 24*a^3 \\
 & *b^4 + 8*a^2*b^5)*e^{(-4*d*x - 4*c)} + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a \\
 & ^3*b^4)*e^{(-6*d*x - 6*c)} + (a^6*b + 2*a^5*b^2 + a^4*b^3)*e^{(-8*d*x - 8*c)}) * \\
 & d) - 3/32*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^{(-2*d*x - 2*c)} + \\
 & (3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^{(-4*d*x - 4*c)} + (a^3 + 4*a^2*b)*e^{(\\
 & -6*d*x - 6*c)}) / ((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^4*b^2 + 5*a^3 \\
 & *b^3 + 2*a^2*b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^5*b + 14*a^4*b^2 + 27*a^3*b^3 + \\
 & 24*a^2*b^4 + 8*a*b^5)*e^{(-4*d*x - 4*c)} + 4*(a^5*b + 4*a^4*b^2 + 5*a^3*b^3 \\
 & + 2*a^2*b^4)*e^{(-6*d*x - 6*c)} + (a^5*b + 2*a^4*b^2 + a^3*b^3)*e^{(-8*d*x - 8 \\
 & *c)}) * d) + 1/4*log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/(a^3 \\
 & *d) - 1/4*log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/(a^3*d \\
 &)
 \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \tanh(c + dx)^4}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.161 \quad \int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{a+2b}{2a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

[Out] $-1/4*b*(a+b)/a^3/d/(b+a*\cosh(d*x+c)^2)^2+1/2*(a+2*b)/a^3/d/(b+a*\cosh(d*x+c)^2)+1/2*\ln(b+a*\cosh(d*x+c)^2)/a^3/d$

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 77}

$$-\frac{b(a+b)}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{a+2b}{2a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-(b*(a+b))/(4*a^3*d*(b+a*Cosh[c+d*x]^2)^2) + (a+2*b)/(2*a^3*d*(b+a*Cosh[c+d*x]^2)) + \text{Log}[b+a*Cosh[c+d*x]^2]/(2*a^3*d)$

Rule 77

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3} + \frac{a+2b}{a^2(b+ax)^2} - \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{b(a+b)}{4a^3d(b+a\cosh^2(c+dx))^2} + \frac{a+2b}{2a^3d(b+a\cosh^2(c+dx))} + \frac{\log(b+a\cosh^2(c+dx))}{2a^3d}
\end{aligned}$$

Mathematica [A] time = 1.57, size = 131, normalized size = 1.62

$$\frac{2(a^2 + 3ab + 3b^2) + a^2 \cosh^2(2(c+dx)) \log(a \cosh(2(c+dx))) + a + 2b + (a+2b)^2 \log(a \cosh(2(c+dx)))}{2a^3d(a \cosh(2(c+dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*(a + 2*b)*Cosh[2*(c + d*x)]*(1 + Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

fricas [B] time = 0.45, size = 1753, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*a^2*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 4*(14*a^2*d*x*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(14*a^2*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 2*a^2 - 6*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 + 4*(14*a^2*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^4 + 2*(a^2 + 2*a*b)*d*x + 6*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)

$$\begin{aligned} &)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c) \\ &)^2 + a^2 + 8*(a^2*\cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*\cosh(d*x + c)^5 + (3*a \\ &^2 + 8*a*b + 8*b^2)*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x \\ &+ c))*\log(2*(a*\cosh(d*x + c)^2 + a*\sinh(d*x + c)^2 + a + 2*b)/(\cosh(d*x + \\ &c)^2 - 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2)) + 8*(2*a^2*d*x*\cos \\ &h(d*x + c)^7 + 3*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*\cosh(d*x + c)^5 + 2*((\\ &3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*\cosh(d*x + c)^3 + (2*(a \\ &^2 + 2*a*b)*d*x - a^2 - 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c))/(a^5*d*\cosh(d \\ &x + c)^8 + 8*a^5*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^5*d*\sinh(d*x + c)^8 + \\ &4*(a^5 + 2*a^4*b)*d*\cosh(d*x + c)^6 + 4*(7*a^5*d*\cosh(d*x + c)^2 + (a^5 + 2 \\ &a^4*b)*d)*\sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh \\ &(d*x + c)^4 + 8*(7*a^5*d*\cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*\cosh(d*x + c \\ &))*\sinh(d*x + c)^5 + 2*(35*a^5*d*\cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*\cos \\ &h(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*\sinh(d*x + c)^4 + 4*(a^5 + \\ &2*a^4*b)*d*\cosh(d*x + c)^2 + 8*(7*a^5*d*\cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b \\ &)*d*\cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c))*\sinh(d \\ &x + c)^3 + 4*(7*a^5*d*\cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*\cosh(d*x + c) \\ &^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d \\ &*\sinh(d*x + c)^2 + 8*(a^5*d*\cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*\cosh(d*x \\ &+ c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*\cosh(d*x + c)^3 + (a^5 + 2*a^4*b)* \\ &d*\cosh(d*x + c))*\sinh(d*x + c)) \end{aligned}$$

giac [B] time = 1.22, size = 175, normalized size = 2.16

$$\frac{\frac{4dx}{a^3} - \frac{2 \log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^3} + \frac{3ae^{(8dx+8c)} + 4ae^{(6dx+6c)} + 8be^{(6dx+6c)} + 2ae^{(4dx+4c)} + 4ae^{(2dx+2c)} + 8be^{(2dx+2c)} + 3a}{(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)^2} a^2}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-1/4*(4*d*x/a^3 - 2*\log(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)/a^3 + (3*a*e^{(8*d*x + 8*c)} + 4*a*e^{(6*d*x + 6*c)} + 8*b*e^{(6*d*x + 6*c)} + 2*a*e^{(4*d*x + 4*c)} + 4*a*e^{(2*d*x + 2*c)} + 8*b*e^{(2*d*x + 2*c)} + 3*a)/((a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)^2*a^2))/d$$

maple [B] time = 0.40, size = 672, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$\begin{aligned} &-1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)+1)-2/d/a/ \\ &(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a- \\ &2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6-2/d/a^2*b/(\tanh(1/2* \\ &d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2 \\ &*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6-4/d/(\tanh(1/2*d*x+1/2*c)^4*a+b \\ &*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+ \\ &a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^4-4/d/a*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(\\ &1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2 \\ &/ (a+b)*\tanh(1/2*d*x+1/2*c)^4+4/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/ \\ &2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/ \\ &(a+b)*\tanh(1/2*d*x+1/2*c)^4-2/d/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/ \\ &2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2* \\ &d*x+1/2*c)^2-2/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*t \\ &anh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c) \\ &^2+1/2/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2* \\ &d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b) \end{aligned}$$

maxima [B] time = 0.34, size = 209, normalized size = 2.58

$$\frac{2\left((a^2 + 2ab)e^{(-2dx-2c)} + 2(a^2 + 3ab + 3b^2)e^{(-4dx-4c)} + (a^2 + 2ab)e^{(-6dx-6c)}\right)}{(a^5e^{(-8dx-8c)} + a^5 + 4(a^5 + 2a^4b)e^{(-2dx-2c)} + 2(3a^5 + 8a^4b + 8a^3b^2)e^{(-4dx-4c)} + 4(a^5 + 2a^4b)e^{(-6dx-6c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 2*((a^2 + 2*a*b)*e^(-2*d*x - 2*c) + 2*(a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c) + (a^2 + 2*a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \tanh(c + dx)^3}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)

[Out] Timed out

$$3.162 \quad \int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=139

$$\frac{x}{a^3} - \frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)(a-b\tanh^2(c+dx)+b)} - \frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}d(a+b)^{3/2}} - \frac{\tanh(c+dx)}{4ad(a-b\tanh^2(c+dx)+b)}$$

[Out] $x/a^3 - 1/8*(3*a^2+12*a*b+8*b^2)*\operatorname{arctanh}(b^{1/2}*\tanh(d*x+c)/(a+b)^{1/2})/a^3$
 $/ (a+b)^{3/2}/d/b^{1/2} - 1/4*\tanh(d*x+c)/a/d/(a+b-b*\tanh(d*x+c)^2)^2 - 1/8*(3*a$
 $+4*b)*\tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, number of rules / integrand size = 0.304, Rules used = {4141, 1975, 471, 527, 522, 206, 208}

$$\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}d(a+b)^{3/2}} - \frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)(a-b\tanh^2(c+dx)+b)} + \frac{x}{a^3} - \frac{\tanh(c+dx)}{4ad(a-b\tanh^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3, x]

[Out] $x/a^3 - ((3*a^2 + 12*a*b + 8*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])])/(8*a^3*\operatorname{Sqrt}[b]*(a + b)^{3/2}*d) - \operatorname{Tanh}[c + d*x]/(4*a*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^2) - ((3*a + 4*b)*\operatorname{Tanh}[c + d*x])/((8*a^2*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)^(n_)]^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)^(n_)])^(m_.)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{\tanh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= -\frac{\tanh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{(3a + 4b) \tanh(c + dx)}{8a^2(a + b)d(a + b - b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= -\frac{\tanh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))^2} - \frac{(3a + 4b) \tanh(c + dx)}{8a^2(a + b)d(a + b - b \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= \frac{x}{a^3} - \frac{(3a^2 + 12ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a + b)^{3/2}d} - \frac{\tanh(c + dx)}{4ad(a + b - b \tanh^2(c + dx))} \end{aligned}$$

Mathematica [B] time = 12.06, size = 1457, normalized size = 10.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/1024*((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((3*a^2 + 8*a*b
+ 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*
```

$$\begin{aligned} & \text{Sqrt}[b] \cdot (3a^2 + 16ab + 16b^2 + 3a(a + 2b) \cdot \text{Cosh}[2(c + dx)]) \cdot \text{Sinh}[2(c + dx)] / ((a + b)^{2(a + 2b + a \cdot \text{Cosh}[2(c + dx)])^2}) / (b^{5/2} \cdot d \cdot (a + b \cdot \text{Sech}[c + dx]^2)^3 - ((a + 2b + a \cdot \text{Cosh}[2c + 2dx])^3 \cdot \text{Sech}[c + dx]^6 \cdot ((-3a(a + 2b) \cdot \text{ArcTanh}[(\text{Sqrt}[b] \cdot \text{Tanh}[c + dx]) / \text{Sqrt}[a + b]]) / (a + b)^{5/2} + (\text{Sqrt}[b] \cdot (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \cdot \text{Cosh}[2(c + dx)]) \cdot \text{Sinh}[2(c + dx)] / ((a + b)^{2(a + 2b + a \cdot \text{Cosh}[2(c + dx)])^2})) / (2048b^{5/2} \cdot d \cdot (a + b \cdot \text{Sech}[c + dx]^2)^3 + ((a + 2b + a \cdot \text{Cosh}[2c + 2dx])^3 \cdot \text{Sech}[c + dx]^6 \cdot ((-2(3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \cdot \text{ArcTanh}[(\text{Sech}[dx] \cdot (\text{Cosh}[2c] - \text{Sinh}[2c]) \cdot ((a + 2b) \cdot \text{Sinh}[dx] - a \cdot \text{Sinh}[2c + dx])) / (2 \cdot \text{Sqrt}[a + b] \cdot \text{Sqrt}[b \cdot (\text{Cosh}[c] - \text{Sinh}[c])^4])]) \cdot (\text{Cosh}[2c] - \text{Sinh}[2c])) / (\text{Sqrt}[a + b] \cdot \text{Sqrt}[b \cdot (\text{Cosh}[c] - \text{Sinh}[c])^4]) + (\text{Sech}[2c] \cdot (256b^2(a + b)^2(3a^2 + 8ab + 8b^2) \cdot dx \cdot \text{Cosh}[2c] + 512ab^2(a + b)^2(a + 2b) \cdot dx \cdot \text{Cosh}[2dx] + 128a^4b^2 \cdot dx \cdot \text{Cosh}[2(c + 2dx)] + 256a^3b^3 \cdot dx \cdot \text{Cosh}[2(c + 2dx)] + 128a^2b^4 \cdot dx \cdot \text{Cosh}[2(c + 2dx)] + 512a^4b^2 \cdot dx \cdot \text{Cosh}[4c + 2dx] + 2048a^3b^3 \cdot dx \cdot \text{Cosh}[4c + 2dx] + 2560a^2b^4 \cdot dx \cdot \text{Cosh}[4c + 2dx] + 1024ab^5 \cdot dx \cdot \text{Cosh}[4c + 2dx] + 128a^4b^2 \cdot dx \cdot \text{Cosh}[6c + 4dx] + 256a^3b^3 \cdot dx \cdot \text{Cosh}[6c + 4dx] + 128a^2b^4 \cdot dx \cdot \text{Cosh}[6c + 4dx] - 9a^6 \cdot \text{Sinh}[2c] + 12a^5b \cdot \text{Sinh}[2c] + 684a^4b^2 \cdot \text{Sinh}[2c] + 2880a^3b^3 \cdot \text{Sinh}[2c] + 5280a^2b^4 \cdot \text{Sinh}[2c] + 4608ab^5 \cdot \text{Sinh}[2c] + 1536b^6 \cdot \text{Sinh}[2c] + 9a^6 \cdot \text{Sinh}[2dx] - 14a^5b \cdot \text{Sinh}[2dx] - 608a^4b^2 \cdot \text{Sinh}[2dx] - 2112a^3b^3 \cdot \text{Sinh}[2dx] - 2560a^2b^4 \cdot \text{Sinh}[2dx] - 1024ab^5 \cdot \text{Sinh}[2dx] + 3a^6 \cdot \text{Sinh}[2(c + 2dx)] - 12a^5b \cdot \text{Sinh}[2(c + 2dx)] - 204a^4b^2 \cdot \text{Sinh}[2(c + 2dx)] - 384a^3b^3 \cdot \text{Sinh}[2(c + 2dx)] - 192a^2b^4 \cdot \text{Sinh}[2(c + 2dx)] - 3a^6 \cdot \text{Sinh}[4c + 2dx] + 10a^5b \cdot \text{Sinh}[4c + 2dx] + 304a^4b^2 \cdot \text{Sinh}[4c + 2dx] + 1056a^3b^3 \cdot \text{Sinh}[4c + 2dx] + 1280a^2b^4 \cdot \text{Sinh}[4c + 2dx] + 512ab^5 \cdot \text{Sinh}[4c + 2dx])) / (a + 2b + a \cdot \text{Cosh}[2(c + dx)])^2) / (4096a^3b^2(a + b)^2 \cdot d \cdot (a + b \cdot \text{Sech}[c + dx]^2)^3 + ((a + 2b + a \cdot \text{Cosh}[2c + 2dx])^3 \cdot \text{Sech}[c + dx]^6 \cdot ((6a^2 \cdot \text{ArcTanh}[(\text{Sech}[dx] \cdot (\text{Cosh}[2c] - \text{Sinh}[2c]) \cdot ((a + 2b) \cdot \text{Sinh}[dx] - a \cdot \text{Sinh}[2c + dx])) / (2 \cdot \text{Sqrt}[a + b] \cdot \text{Sqrt}[b \cdot (\text{Cosh}[c] - \text{Sinh}[c])^4])]) \cdot (\text{Cosh}[2c] - \text{Sinh}[2c])) / (\text{Sqrt}[a + b] \cdot \text{Sqrt}[b \cdot (\text{Cosh}[c] - \text{Sinh}[c])^4]) + (a \cdot \text{Sech}[2c] \cdot ((-9a^4 - 16a^3b + 48a^2b^2 + 128ab^3 + 64b^4) \cdot \text{Sinh}[2dx] + a \cdot (-3a^3 + 2a^2b + 24ab^2 + 16b^3) \cdot \text{Sinh}[2(c + 2dx)] + (3a^4 - 64a^2b^2 - 128ab^3 - 64b^4) \cdot \text{Sinh}[4c + 2dx]) + (9a^5 + 18a^4b - 64a^3b^2 - 256a^2b^3 - 320ab^4 - 128b^5) \cdot \text{Tanh}[2c]) / (a^2(a + 2b + a \cdot \text{Cosh}[2(c + dx)])^2))) / (2048b^2(a + b)^2 \cdot d \cdot (a + b \cdot \text{Sech}[c + dx]^2)^3) \end{aligned}$$

fricas [B] time = 0.57, size = 7158, normalized size = 51.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^2/(a+b*sech(dx+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(dx + c)^8 + 128*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(dx + c)*sinh(dx + c)^7 + 16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sinh(dx + c)^8 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(dx + c)^6 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(dx + c)^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*sinh(dx + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(dx + c)^3 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(dx + c))*sinh(dx + c)^5 + 20*a^4*b + 44*a^3*b^2 + 24*a^2*b^3 + 4*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(dx + c)^4 + 4*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(dx + c)^4 + 15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4

$$\begin{aligned}
&) * d * x) * \cosh(d * x + c)^2 * \sinh(d * x + c)^4 + 16 * (56 * (a^4 * b + 2 * a^3 * b^2 + a^2 * b^3) * d * x * \cosh(d * x + c)^5 + 5 * (5 * a^4 * b + 25 * a^3 * b^2 + 36 * a^2 * b^3 + 16 * a * b^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x) * \cosh(d * x + c)^3 + (15 * a^4 * b + 73 * a^3 * b^2 + 146 * a^2 * b^3 + 136 * a * b^4 + 48 * b^5 + 8 * (3 * a^4 * b + 14 * a^3 * b^2 + 27 * a^2 * b^3 + 24 * a * b^4 + 8 * b^5) * d * x) * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 16 * (a^4 * b + 2 * a^3 * b^2 + a^2 * b^3) * d * x + 4 * (15 * a^4 * b + 59 * a^3 * b^2 + 76 * a^2 * b^3 + 32 * a * b^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x) * \cosh(d * x + c)^2 + 4 * (112 * (a^4 * b + 2 * a^3 * b^2 + a^2 * b^3) * d * x * \cosh(d * x + c)^6 + 15 * a^4 * b + 59 * a^3 * b^2 + 76 * a^2 * b^3 + 32 * a * b^4 + 15 * (5 * a^4 * b + 25 * a^3 * b^2 + 36 * a^2 * b^3 + 16 * a * b^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x) * \cosh(d * x + c)^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x + 6 * (15 * a^4 * b + 73 * a^3 * b^2 + 146 * a^2 * b^3 + 136 * a * b^4 + 48 * b^5 + 8 * (3 * a^4 * b + 14 * a^3 * b^2 + 27 * a^2 * b^3 + 24 * a * b^4 + 8 * b^5) * d * x) * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 + ((3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^8 + 8 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c) * \sinh(d * x + c)^7 + (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \sinh(d * x + c)^8 + 4 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^6 + 4 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3 + 7 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^2) * \sinh(d * x + c)^6 + 8 * (7 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^3 + 3 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (9 * a^4 + 60 * a^3 * b + 144 * a^2 * b^2 + 160 * a * b^3 + 64 * b^4) * \cosh(d * x + c)^4 + 2 * (35 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^4 + 9 * a^4 + 60 * a^3 * b + 144 * a^2 * b^2 + 160 * a * b^3 + 64 * b^4 + 30 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^2) * \sinh(d * x + c)^4 + 3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2 + 8 * (7 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^5 + 10 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^3 + (9 * a^4 + 60 * a^3 * b + 144 * a^2 * b^2 + 160 * a * b^3 + 64 * b^4) * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 4 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^2 + 4 * (7 * (3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^6 + 15 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^4 + 3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3 + 3 * (9 * a^4 + 60 * a^3 * b + 144 * a^2 * b^2 + 160 * a * b^3 + 64 * b^4) * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 + 8 * ((3 * a^4 + 12 * a^3 * b + 8 * a^2 * b^2) * \cosh(d * x + c)^7 + 3 * (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)^5 + (9 * a^4 + 60 * a^3 * b + 144 * a^2 * b^2 + 160 * a * b^3 + 64 * b^4) * \cosh(d * x + c)^3 + (3 * a^4 + 18 * a^3 * b + 32 * a^2 * b^2 + 16 * a * b^3) * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{a * b + b^2} * \log((a^2 * \cosh(d * x + c)^4 + 4 * a^2 * \cosh(d * x + c) * \sinh(d * x + c)^3 + a^2 * \sinh(d * x + c)^4 + 2 * (a^2 + 2 * a * b) * \cosh(d * x + c)^2 + 2 * (3 * a^2 * \cosh(d * x + c)^2 + a^2 + 2 * a * b) * \sinh(d * x + c)^2 + a^2 + 8 * a * b + 8 * b^2 + 4 * (a^2 * \cosh(d * x + c)^3 + (a^2 + 2 * a * b) * \cosh(d * x + c)) * \sinh(d * x + c) + 4 * (a * \cosh(d * x + c)^2 + 2 * a * \cosh(d * x + c) * \sinh(d * x + c) + a * \sinh(d * x + c)^2 + a + 2 * b) * \sqrt{a * b + b^2})) / (a * \cosh(d * x + c)^4 + 4 * a * \cosh(d * x + c) * \sinh(d * x + c)^3 + a * \sinh(d * x + c)^4 + 2 * (a + 2 * b) * \cosh(d * x + c)^2 + 2 * (3 * a * \cosh(d * x + c)^2 + a + 2 * b) * \sinh(d * x + c)^2 + 4 * (a * \cosh(d * x + c)^3 + (a + 2 * b) * \cosh(d * x + c)) * \sinh(d * x + c) + a) + 8 * (16 * (a^4 * b + 2 * a^3 * b^2 + a^2 * b^3) * d * x * \cosh(d * x + c)^7 + 3 * (5 * a^4 * b + 25 * a^3 * b^2 + 36 * a^2 * b^3 + 16 * a * b^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x) * \cosh(d * x + c)^5 + 2 * (15 * a^4 * b + 73 * a^3 * b^2 + 146 * a^2 * b^3 + 136 * a * b^4 + 48 * b^5 + 8 * (3 * a^4 * b + 14 * a^3 * b^2 + 27 * a^2 * b^3 + 24 * a * b^4 + 8 * b^5) * d * x) * \cosh(d * x + c)^3 + (15 * a^4 * b + 59 * a^3 * b^2 + 76 * a^2 * b^3 + 32 * a * b^4 + 16 * (a^4 * b + 4 * a^3 * b^2 + 5 * a^2 * b^3 + 2 * a * b^4) * d * x) * \cosh(d * x + c)) * \sinh(d * x + c)) / ((a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \cosh(d * x + c)^8 + 8 * (a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \cosh(d * x + c) * \sinh(d * x + c)^7 + (a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \sinh(d * x + c)^8 + 4 * (a^7 * b + 4 * a^6 * b^2 + 5 * a^5 * b^3 + 2 * a^4 * b^4) * d * \cosh(d * x + c)^6 + 4 * (7 * (a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \cosh(d * x + c)^2 + (a^7 * b + 4 * a^6 * b^2 + 5 * a^5 * b^3 + 2 * a^4 * b^4) * d) * \sinh(d * x + c)^6 + 2 * (3 * a^7 * b + 14 * a^6 * b^2 + 27 * a^5 * b^3 + 24 * a^4 * b^4 + 8 * a^3 * b^5) * d * \cosh(d * x + c)^4 + 8 * (7 * (a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \cosh(d * x + c)^3 + 3 * (a^7 * b + 4 * a^6 * b^2 + 5 * a^5 * b^3 + 2 * a^4 * b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (35 * (a^7 * b + 2 * a^6 * b^2 + a^5 * b^3) * d * \cosh(d * x + c)^4 + 30 * (a^7 * b + 4 * a^6 * b^2 + 5 * a^5 * b^3 + 2 * a^4 * b^4) * d * \cosh(d * x + c)^2 + (3 * a^7 * b + 14 * a^6 * b^2 + 27 * a^5 * b^3 + 24 * a^4 * b^4 + 8 * a^3 * b^5) * d) * \sinh(d * x + c)^4 + 4 * (a^7 * b + 4 * a^6 * b^2 + 5 * a^5 * b^3 + 2 * a^4 * b^4) * d * \cosh(d * x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d + 8*((a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 64*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 2*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 2*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*a^4*b + 22*a^3*b^2 + 12*a^2*b^3 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 2*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x) + 6*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
\end{aligned}$$

$$\begin{aligned} & (x + c) \sqrt{-ab - b^2} \arctan\left(\frac{1}{2}(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-ab - b^2} / (ab + b^2)\right) \\ & + 4(16(a^4b + 2a^3b^2 + a^2b^3) dx \cosh(dx + c)^7 + 3(5a^4b + 25a^3b^2 + 36a^2b^3 + 16ab^4 + 16(a^4b + 4a^3b^2 + 5a^2b^3 + 2ab^4) dx) \cosh(dx + c)^5 \\ & + 2(15a^4b + 73a^3b^2 + 146a^2b^3 + 136ab^4 + 48b^5 + 8(3a^4b + 14a^3b^2 + 27a^2b^3 + 24ab^4 + 8b^5) dx) \cosh(dx + c)^3 \\ & + (15a^4b + 59a^3b^2 + 76a^2b^3 + 32ab^4 + 16(a^4b + 4a^3b^2 + 5a^2b^3 + 2ab^4) dx) \cosh(dx + c) \sinh(dx + c)) / \\ & ((a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^8 + 8(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c) \sinh(dx + c)^7 \\ & + (a^7b + 2a^6b^2 + a^5b^3) d \sinh(dx + c)^8 + 4(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^6 \\ & + 4(7(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^2 + (a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d) \sinh(dx + c)^6 \\ & + 2(3a^7b + 14a^6b^2 + 27a^5b^3 + 24a^4b^4 + 8a^3b^5) d \cosh(dx + c)^4 + 8(7(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^3 \\ & + 3(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^4 \\ & + 30(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^2 + (3a^7b + 14a^6b^2 + 27a^5b^3 + 24a^4b^4 + 8a^3b^5) d) \sinh(dx + c)^4 \\ & + 4(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^2 + 8(7(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^5 + 10(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^3 \\ & + (3a^7b + 14a^6b^2 + 27a^5b^3 + 24a^4b^4 + 8a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^6 \\ & + 15(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^4 + 3(3a^7b + 14a^6b^2 + 27a^5b^3 + 24a^4b^4 + 8a^3b^5) d \cosh(dx + c)^2 \\ & + (a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d) \sinh(dx + c)^2 + (a^7b + 2a^6b^2 + a^5b^3) d + 8((a^7b + 2a^6b^2 + a^5b^3) d \cosh(dx + c)^7 \\ & + 3(a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)^5 + (3a^7b + 14a^6b^2 + 27a^5b^3 + 24a^4b^4 + 8a^3b^5) d \cosh(dx + c)^3 \\ & + (a^7b + 4a^6b^2 + 5a^5b^3 + 2a^4b^4) d \cosh(dx + c)) \sinh(dx + c) \end{aligned}$$

giac [B] time = 0.97, size = 309, normalized size = 2.22

$$\frac{(3a^2e^{2c} + 12abe^{2c} + 8b^2e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + a + 2b}{2\sqrt{-ab-b^2}}\right) e^{-2c}}{(a^4 + a^3b)\sqrt{-ab-b^2}} - \frac{8dx}{a^3} - \frac{2(5a^3e^{6dx+6c} + 20a^2be^{6dx+6c} + 16ab^2e^{6dx+6c} + 15a^3e^{4dx+4c} + 15a^3e^{4dx+4c} + 15a^3e^{4dx+4c})}{(a^4 + a^3b)\sqrt{-ab-b^2}}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^2/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8 * ((3a^2e^{(2*c)} + 12*a*b*e^{(2*c)} + 8*b^2*e^{(2*c)}) * \arctan(1/2 * (a * e^{(2*d*x + 2*c)} + a + 2*b) / \sqrt{-a*b - b^2})) * e^{(-2*c)} / ((a^4 + a^3*b) * \sqrt{-a*b - b^2}) \\ & - 8*d*x/a^3 - 2 * (5*a^3*e^{(6*d*x + 6*c)} + 20*a^2*b*e^{(6*d*x + 6*c)} + 16*a*b^2*e^{(6*d*x + 6*c)} + 15*a^3*e^{(4*d*x + 4*c)} + 58*a^2*b*e^{(4*d*x + 4*c)} \\ & + 88*a*b^2*e^{(4*d*x + 4*c)} + 48*b^3*e^{(4*d*x + 4*c)} + 15*a^3*e^{(2*d*x + 2*c)} + 44*a^2*b*e^{(2*d*x + 2*c)} + 32*a*b^2*e^{(2*d*x + 2*c)} + 5*a^3 + 6*a^2*b) / ((a^4 + a^3*b) * (a * e^{(4*d*x + 4*c)} + 2*a * e^{(2*d*x + 2*c)} + 4*b * e^{(2*d*x + 2*c)} + a^2)) / d \end{aligned}$$

maple [B] time = 0.43, size = 1173, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^2/(a+b*sech(dx+c)^2)^3,x)

[Out]
$$\begin{aligned} & -1/d/a^3 * \ln(\tanh(1/2*d*x + 1/2*c) - 1) + 1/d/a^3 * \ln(\tanh(1/2*d*x + 1/2*c) + 1) - 5/4/d/a \\ & / (\tanh(1/2*d*x + 1/2*c)^4 + b * \tanh(1/2*d*x + 1/2*c)^4 + 2 * \tanh(1/2*d*x + 1/2*c)^2 * a - 2 * \tanh(1/2*d*x + 1/2*c)^2 * b + a + b)^2 * \tanh(1/2*d*x + 1/2*c)^7 - 1/d/a^2 * b / (\tanh(1/ \end{aligned}$$

$$2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-15/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-15/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+3/16/d/a/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+3/4/d/a^2/(a+b)^(3/2)*b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^3/(a+b)^(3/2)*b^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-3/16/d/a/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-3/4/d*b^(1/2)/a^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$$

maxima [B] time = 0.93, size = 1255, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\frac{-1/64*(3*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^5 + 2*a^4*b + a^3*b^2)*\sqrt{(a + b)*b}*d) + 1/64*(3*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^5 + 2*a^4*b + a^3*b^2)*\sqrt{(a + b)*b}*d) + 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (5*a^4 + 66*a^3*b + 128*a^2*b^2 + 64*a*b^3)*e^{(6*d*x + 6*c)} + (15*a^4 + 164*a^3*b + 460*a^2*b^2 + 512*a*b^3 + 192*b^4)*e^{(4*d*x + 4*c)} + (15*a^4 + 118*a^3*b + 208*a^2*b^2 + 96*a*b^3)*e^{(2*d*x + 2*c)})/((a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 2*a^6*b + a^5*b^2)*e^{(8*d*x + 8*c)} + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(6*d*x + 6*c)} + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^{(4*d*x + 4*c)} + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(2*d*x + 2*c)})*d) - 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (15*a^4 + 118*a^3*b + 208*a^2*b^2 + 96*a*b^3)*e^{(-2*d*x - 2*c)} + (15*a^4 + 164*a^3*b + 460*a^2*b^2 + 512*a*b^3 + 192*b^4)*e^{(-4*d*x - 4*c)} + (5*a^4 + 66*a^3*b + 128*a^2*b^2 + 64*a*b^3)*e^{(-6*d*x - 6*c)})/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(-2*d*x - 2*c)} + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(-6*d*x - 6*c)} + (a^7 + 2*a^6*b + a^5*b^2)*e^{(-8*d*x - 8*c)})*d) - 1/8*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^{(-2*d*x - 2*c)} + (15*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^{(-4*d*x - 4*c)} + (5*a^3 + 16*a^2*b + 8*a*b^2)*e^{(-6*d*x - 6*c)})/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*e^{(-2*d*x - 2*c)} + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2 + 24*a^3*b^3 + 8*a^2*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*e^{(-6*d*x - 6*c)} + (a^6 + 2*a^5*b + a^4*b^2)*e^{(-8*d*x - 8*c)})*d) + 3/32*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*$$

```
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)
)*b)*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3
*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^4 (\cosh(c + dx)^2 - 1)}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

```
[Out] int((cosh(c + d*x)^4*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3, x)
```

```
[Out] Timed out
```

$$3.163 \quad \int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=73

$$-\frac{b^2}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{b}{a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

[Out] $-1/4*b^2/a^3/d/(b+a*\cosh(d*x+c)^2)^2+b/a^3/d/(b+a*\cosh(d*x+c)^2)+1/2*\ln(b+a*\cosh(d*x+c)^2)/a^3/d$

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 266, 43}

$$-\frac{b^2}{4a^3d(a\cosh^2(c+dx)+b)^2} + \frac{b}{a^3d(a\cosh^2(c+dx)+b)} + \frac{\log(a\cosh^2(c+dx)+b)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] $-b^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + b/(a^3*d*(b + a*Cosh[c + d*x]^2)) + \text{Log}[b + a*Cosh[c + d*x]^2]/(2*a^3*d)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(b+ax)^3} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{b^2}{4a^3d(b+a\cosh^2(c+dx))^2} + \frac{b}{a^3d(b+a\cosh^2(c+dx))} + \frac{\log(b+a\cosh^2(c+dx))}{2a^3d}
\end{aligned}$$

Mathematica [A] time = 2.01, size = 129, normalized size = 1.77

$$\frac{a^2 \cosh^2(2(c+dx)) \log(a \cosh(2(c+dx)) + a + 2b) + (a + 2b)^2 \log(a \cosh(2(c+dx)) + a + 2b) + 2a \cosh(2(c+dx))}{2a^3d(a \cosh(2(c+dx)) + a + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] (2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x)]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b + a*Cosh[2*(c + d*x)]^2)

fricas [B] time = 0.45, size = 1666, normalized size = 22.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a*b)*sinh(d*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 - 4*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^4 + (a^2 + 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c))^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3

*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.53, size = 187, normalized size = 2.56

$$\frac{\frac{4 dx}{a^3} - \frac{2 \log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^3} + \frac{3ae^{(8dx+8c)} + 12ae^{(6dx+6c)} + 8be^{(6dx+6c)} + 18ae^{(4dx+4c)} + 16be^{(4dx+4c)} + 12ae^{(2dx+2c)} + 8a^2}{(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)^2 a^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) + 12*a*e^(6*d*x + 6*c) + 8*b*e^(6*d*x + 6*c) + 18*a*e^(4*d*x + 4*c) + 16*b*e^(4*d*x + 4*c) + 12*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d

maple [A] time = 0.21, size = 82, normalized size = 1.12

$$\frac{\ln(a + b \operatorname{sech}(dx + c)^2)}{2d a^3} - \frac{1}{4da (a + b \operatorname{sech}(dx + c)^2)^2} - \frac{1}{2d a^2 (a + b \operatorname{sech}(dx + c)^2)} - \frac{\ln(\operatorname{sech}(dx + c))}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)

[Out] 1/2/d/a^3*ln(a+b*sech(d*x+c)^2)-1/4/d/a/(a+b*sech(d*x+c)^2)^2-1/2/d/a^2/(a+b*sech(d*x+c)^2)-1/d/a^3*ln(sech(d*x+c))

maxima [B] time = 0.42, size = 193, normalized size = 2.64

$$\frac{4 (abe^{(-2 dx-2c)} + abe^{(-6 dx-6c)} + (2 ab + 3 b^2)e^{(-4 dx-4c)})}{(a^5 e^{(-8 dx-8c)} + a^5 + 4(a^5 + 2 a^4 b)e^{(-2 dx-2c)} + 2(3 a^5 + 8 a^4 b + 8 a^3 b^2)e^{(-4 dx-4c)} + 4(a^5 + 2 a^4 b)e^{(-6 dx-6c)})d} + \frac{dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 4*(a*b*e^(-2*d*x - 2*c) + a*b*e^(-6*d*x - 6*c) + (2*a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6

$(d*x - 6*c)*d) + (d*x + c)/(a^3*d) + 1/2*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)}$
 $+ a*e^{(-4*d*x - 4*c)} + a)/(a^3*d)$

mupad [B] time = 1.60, size = 94, normalized size = 1.29

$$\frac{\ln\left(\cosh(c+dx)^2\left(a + \frac{b}{\cosh(c+dx)^2}\right)\right)}{2a^3d} - \frac{b^2}{4a^3d\cosh(c+dx)^4\left(a + \frac{b}{\cosh(c+dx)^2}\right)^2} + \frac{b}{a^3d\cosh(c+dx)^2\left(a + \frac{b}{\cosh(c+dx)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)

[Out] $\log(\cosh(c + d*x)^2*(a + b/\cosh(c + d*x)^2))/(2*a^3*d) - b^2/(4*a^3*d*\cosh(c + d*x)^4*(a + b/\cosh(c + d*x)^2)^2) + b/(a^3*d*\cosh(c + d*x)^2*(a + b/\cosh(c + d*x)^2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**3, x)

[Out] Timed out

$$3.164 \quad \int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=146

$$\frac{x}{a^3} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2(a-b\tanh^2(c+dx)+b)} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}} - \frac{b\tanh(c)}{4ad(a+b)(a-b\tanh^2(c+dx))}$$

[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^3/(a+b)^(5/2)/d-1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(7*a+4*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4128, 414, 527, 522, 206, 208}

$$\frac{\sqrt{b}(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2(a-b\tanh^2(c+dx)+b)} + \frac{x}{a^3} - \frac{b\tanh(c)}{4ad(a+b)(a-b\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-3), x]

[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) - (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{b \tanh(c + dx)}{4a(a+b)d(a+b-b \tanh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-4a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c + dx)\right)}{4a(a+b)d}$$

$$= -\frac{b \tanh(c + dx)}{4a(a+b)d(a+b-b \tanh^2(c + dx))^2} - \frac{b(7a+4b) \tanh(c + dx)}{8a^2(a+b)^2d(a+b-b \tanh^2(c + dx))}$$

$$= -\frac{b \tanh(c + dx)}{4a(a+b)d(a+b-b \tanh^2(c + dx))^2} - \frac{b(7a+4b) \tanh(c + dx)}{8a^2(a+b)^2d(a+b-b \tanh^2(c + dx))}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}(15a^2 + 20ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} - \frac{b \tanh(c + dx)}{4a(a+b)d(a+b-b \tanh^2(c + dx))}$$

Mathematica [B] time = 6.21, size = 301, normalized size = 2.06

$$\operatorname{sech}^6(c + dx)(a \cosh(2(c + dx)) + a + 2b) \left(\frac{b \operatorname{sech}(2c)((9a^2 + 28ab + 16b^2) \sinh(2c) - 3a(3a + 2b) \sinh(2dx))(a \cosh(2(c + dx)) + a + 2b)}{d(a+b)^2} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-3), x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(8*x*(a + 2*b + a*Cosh[2*(
c + d*x)])^2 - (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] -
Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x])])/(2*Sqrt[a + b]*Sqrt[
b*(Cosh[c] - Sinh[c])^4]))*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - S
inh[2*c]))/((a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*b^2*Sech[2*
c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d) + (b*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[2*c]*((9*a^2 + 28*a*b + 16*b^2)*Sinh[2*c] - 3*a*(3*a +
2*b)*Sinh[2*d*x]))/((a + b)^2*d))/(64*a^3*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.57, size = 6538, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*sinh(d*x + c)^8 + 4*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 36*a^3*b + 24*a^2*b^2 + 16*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x + 4*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3

$$\begin{aligned}
& + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 \\
& + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c \\
&))*\sinh(d*x + c) + a)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\cosh(d*x + c)^7 \\
& + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2* \\
& a*b^3)*d*x)*\cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 \\
& + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*\cosh(d*x + c)^3 \\
& + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a \\
& *b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(\\
& d*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + \\
& (a^7 + 2*a^6*b + a^5*b^2)*d*\sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 \\
& + 2*a^4*b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(d*x \\
& + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*\sinh(d*x + c)^6 + 2*(3* \\
& a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*\cosh(d*x + c)^4 + 8 \\
& *(7*(a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5* \\
& b^2 + 2*a^4*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^7 + 2*a^6*b + \\
& a^5*b^2)*d*\cosh(d*x + c)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*c \\
& osh(d*x + c)^2 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d \\
&)*\sinh(d*x + c)^4 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*\cosh(d*x + \\
& c)^2 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*\cosh(d*x + c)^5 + 10*(a^7 + 4*a^6*b \\
& + 5*a^5*b^2 + 2*a^4*b^3)*d*\cosh(d*x + c)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^ \\
& 2 + 24*a^4*b^3 + 8*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7 + \\
& 2*a^6*b + a^5*b^2)*d*\cosh(d*x + c)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^ \\
& 4*b^3)*d*\cosh(d*x + c)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + \\
& 8*a^3*b^4)*d*\cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*s \\
& inh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d + 8*((a^7 + 2*a^6*b + a^5*b^2) \\
& *d*\cosh(d*x + c)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*\cosh(d*x + \\
& c)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*\cosh(d*x \\
& + c)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)), 1/8*(8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\cosh(d*x + c)^8 + 64*(a^4 + 2* \\
& a^3*b + a^2*b^2)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + 8*(a^4 + 2*a^3*b + a^2 \\
& *b^2)*d*x*\sinh(d*x + c)^8 + 2*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + \\
& 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*\cosh(d*x + c)^6 + 2*(112*(a^4 + 2*a^3*b \\
& + a^2*b^2)*d*x*\cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 \\
& + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*\sinh(d*x + c)^6 + 4*(112*(a^4 + 2*a^ \\
& 3*b + a^2*b^2)*d*x*\cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 1 \\
& 6*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 \\
& + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27 \\
& *a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*\cosh(d*x + c)^4 + 2*(280*(a^4 + 2*a^3*b + \\
& a^2*b^2)*d*x*\cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 \\
& + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + \\
& 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*\cosh(\\
& d*x + c)^2)*\sinh(d*x + c)^4 + 18*a^3*b + 12*a^2*b^2 + 8*(56*(a^4 + 2*a^3*b \\
& + a^2*b^2)*d*x*\cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a \\
& ^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*\cosh(d*x + c)^3 + (27*a^3*b + 90*a \\
& ^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + \\
& 8*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d \\
& *x + 2*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + \\
& 2*a*b^3)*d*x)*\cosh(d*x + c)^2 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\cosh(d \\
& *x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2 \\
& *b^2 + 2*a*b^3)*d*x)*\cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 1 \\
& 6*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 12 \\
& 0*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*\cosh(\\
& d*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 \\
& + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*\sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b \\
& + 48*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^ \\
& 2 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*\cosh(d*x + c)^3 + 3*(15*a^4 + \\
& 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(45*a
\end{aligned}$$

$$\begin{aligned} &^4 + 180a^3b + 304a^2b^2 + 224ab^3 + 64b^4) \cosh(dx + c)^4 + 2(35(15a^4 + 20a^3b + 8a^2b^2) \cosh(dx + c)^4 + 45a^4 + 180a^3b + 304a^2b^2 + 224ab^3 + 64b^4 + 30(15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 15a^4 + 20a^3b + 8a^2b^2 + 8(7(15a^4 + 20a^3b + 8a^2b^2) \cosh(dx + c)^5 + 10(15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)^3 + (45a^4 + 180a^3b + 304a^2b^2 + 224ab^3 + 64b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)^2 + 4(7(15a^4 + 20a^3b + 8a^2b^2) \cosh(dx + c)^6 + 15(15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)^4 + 15a^4 + 50a^3b + 48a^2b^2 + 16ab^3 + 3(45a^4 + 180a^3b + 304a^2b^2 + 224ab^3 + 64b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((15a^4 + 20a^3b + 8a^2b^2) \cosh(dx + c)^7 + 3(15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)^5 + (45a^4 + 180a^3b + 304a^2b^2 + 224ab^3 + 64b^4) \cosh(dx + c)^3 + (15a^4 + 50a^3b + 48a^2b^2 + 16ab^3) \cosh(dx + c)) \sinh(dx + c)) \sqrt{-b/(a + b)} \arctan(1/2(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a + 2b) \sqrt{-b/(a + b)})/b) + 4(16(a^4 + 2a^3b + a^2b^2) dx \cosh(dx + c)^7 + 3(9a^3b + 28a^2b^2 + 16ab^3 + 16(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) dx) \cosh(dx + c)^5 + 2(27a^3b + 90a^2b^2 + 120ab^3 + 48b^4 + 8(3a^4 + 14a^3b + 27a^2b^2 + 24ab^3 + 8b^4) dx) \cosh(dx + c)^3 + (27a^3b + 68a^2b^2 + 32ab^3 + 16(a^4 + 4a^3b + 5a^2b^2 + 2ab^3) dx) \cosh(dx + c)) \sinh(dx + c)) / ((a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^8 + 8(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c) \sinh(dx + c)^7 + (a^7 + 2a^6b + a^5b^2) d \sinh(dx + c)^8 + 4(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^6 + 4(7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^2 + (a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d) \sinh(dx + c)^6 + 2(3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) d \cosh(dx + c)^4 + 8(7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^3 + 3(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^4 + 30(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^2 + (3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) d) \sinh(dx + c)^4 + 4(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^2 + 8(7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^5 + 10(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^3 + (3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^6 + 15(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^4 + 3(3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) d \cosh(dx + c)^2 + (a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d) \sinh(dx + c)^2 + (a^7 + 2a^6b + a^5b^2) d + 8((a^7 + 2a^6b + a^5b^2) d \cosh(dx + c)^7 + 3(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)^5 + (3a^7 + 14a^6b + 27a^5b^2 + 24a^4b^3 + 8a^3b^4) d \cosh(dx + c)^3 + (a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3) d \cosh(dx + c)) \sinh(dx + c))] \end{aligned}$$

giac [B] time = 0.86, size = 327, normalized size = 2.24

$$\frac{(15a^2b + 20ab^2 + 8b^3) \arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^5 + 2a^4b + a^3b^2)\sqrt{-ab-b^2}} - \frac{2(9a^3be^{(6dx+6c)} + 28a^2b^2e^{(6dx+6c)} + 16ab^3e^{(6dx+6c)} + 27a^3be^{(4dx+4c)} + 90a^2b^2e^{(4dx+4c)} + 120ab^3e^{(4dx+4c)} + 48b^4e^{(4dx+4c)})}{(a^5 + 2a^4b + a^3b^2)(ae^{(4dx+4c)} + 2a^2e^{(4dx+4c)} + a^3e^{(4dx+4c)})}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] $-1/8 * ((15a^2b + 20ab^2 + 8b^3) \arctan(1/2(ae^{(2dx+2c)} + a + 2b) / \sqrt{-ab - b^2}) / ((a^5 + 2a^4b + a^3b^2) \sqrt{-ab - b^2}) - 2(9a^3b e^{(6dx+6c)} + 28a^2b^2 e^{(6dx+6c)} + 16ab^3 e^{(6dx+6c)} + 27a^3b e^{(4dx+4c)} + 90a^2b^2 e^{(4dx+4c)} + 120ab^3 e^{(4dx+4c)} + 48b^4 e^{(4dx+4c)} + 27a^3b e^{(2dx+2c)} + 68a^2b^2 e^{(2dx+2c)} + 32ab^3 e^{(2dx+2c)} + 9a^3b + 6a^2b^2) / ((a^5 + 2a^4b + a^3b^2)(ae^{(4dx+4c)} + 2a^2e^{(4dx+4c)} + a^3e^{(4dx+4c)}))$

$$a^4*b + a^3*b^2)*(a*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} + 4*b*e^{(2*d*x + 2*c)} + a)^2) - 8*(d*x + c)/a^3)/d$$

maple [B] time = 0.41, size = 1283, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c))^2)^3,x)

[Out]
$$-1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)+1)-9/4/d/a*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^7-1/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)^7-27/4/d*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5-11/4/d/a*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5+1/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^5-27/4/d*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3-11/4/d/a*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*\tanh(1/2*d*x+1/2*c)^3-9/4/d/a*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)-1/d/a^2*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*\tanh(1/2*d*x+1/2*c)+15/16/d/a*b^(1/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+5/4/d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-15/16/d/a*b^(1/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-5/4/d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))$$

maxima [B] time = 0.53, size = 402, normalized size = 2.75

$$\frac{(15a^2b + 20ab^2 + 8b^3) \log\left(\frac{ae^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{ae^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right)}{16(a^5 + 2a^4b + a^3b^2)\sqrt{(a+b)b}d} - \frac{9a^3b + 6a^2b^2 + (27a^3b^3)}{4(a^7 + 2a^6b + a^5b^2 + 4(a^7 + 4a^6b + 5a^5b^2 + 2a^4b^3))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c))^2)^3,x, algorithm="maxima")

[Out]
$$1/16*(15*a^2*b + 20*a*b^2 + 8*b^3)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^5 + 2*a^4*b + a^3*b^2)*\sqrt{(a + b)*b}*d) - 1/4*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^{(-2*d*x - 2*c)} + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^{(-4*d*x - 4*c)} + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^{(-6*d*x - 6*c)})/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(-2*d*x - 2*c)} + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^{(-$$

$6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^{(-8*d*x - 8*c)}*d) + (d*x + c)/(a^3*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b/cosh(c + d*x)^2)^3, x)`

[Out] `int(1/(a + b/cosh(c + d*x)^2)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{sech}^2(c + dx)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(d*x+c)**2)**3, x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**(-3), x)`

$$3.165 \quad \int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3d(a+b)(a\cosh^2(c+dx)+b)^2} + \frac{b^2(3a+2b)}{2a^3d(a+b)^2(a\cosh^2(c+dx)+b)} + \frac{b(3a^2+3ab+b^2)\log(a\cosh^2(c+dx)+b)}{2a^3d(a+b)^3}$$

[Out] $-1/4*b^3/a^3/(a+b)/d/(b+a*\cosh(d*x+c)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(b+a*\cosh(d*x+c)^2)+1/2*b*(3*a^2+3*a*b+b^2)*\ln(b+a*\cosh(d*x+c)^2)/a^3/(a+b)^3/d+\ln(\sinh(d*x+c))/d/(a+b)^3$

Rubi [A] time = 0.19, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4138, 446, 88}

$$-\frac{b^3}{4a^3d(a+b)(a\cosh^2(c+dx)+b)^2} + \frac{b^2(3a+2b)}{2a^3d(a+b)^2(a\cosh^2(c+dx)+b)} + \frac{b(3a^2+3ab+b^2)\log(a\cosh^2(c+dx)+b)}{2a^3d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-b^3/(4*a^3*(a+b)*d*(b+a*Cosh[c+d*x]^2)^2) + (b^2*(3*a+2*b))/(2*a^3*(a+b)^2*d*(b+a*Cosh[c+d*x]^2)) + (b*(3*a^2+3*a*b+b^2)*Log[b+a*Cosh[c+d*x]^2])/(2*a^3*(a+b)^3*d) + Log[Sinh[c+d*x]]/((a+b)^3*d)$

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.))^(n_.)*((c_.) + (d_.)*(x_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] := Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{(1-x^2)(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3} dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b(3a^2+3ab+b^2)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{b^3}{4a^3(a+b)d(b+a\cosh^2(c+dx))^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d(b+a\cosh^2(c+dx))} + \frac{b(3a^2+3ab+b^2)}{a^3 \log(a\sinh^2(c+dx)+a+b)}$$

Mathematica [A] time = 1.06, size = 155, normalized size = 1.19

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3 \left(-\frac{b^3(a+b)^2}{a^3(a\sinh^2(c+dx)+a+b)^2} + \frac{2b^2(a+b)(3a+2b)}{a^3(a\sinh^2(c+dx)+a+b)} + \frac{2b(3a^2+3ab+b^2)\log(a\sinh^2(c+dx)+a+b)}{a^3} \right)}{32d(a+b)^3(a+b\operatorname{sech}^2(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(4*Log[Sinh[c + d*x]] + (2*b*(3*a^2 + 3*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 - (b^3*(a + b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*(a + b + a*Sinh[c + d*x]^2))))/(32*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 0.85, size = 4132, normalized size = 31.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3, x, algorithm="fricas")

[Out] -1/2*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(3*5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 - 2*0*a^2*b^3 - 20*a*b^4 - 6*b^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)

$$\begin{aligned}
& \text{sh}(d*x + c) * \sinh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x \\
& - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 \\
& *b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^2 + 4*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *d*x*\cosh(d*x + c)^6 - 3*a^3*b^2 - 5*a^2*b^3 - 2*a*b^4 - 15*(3*a^3*b^2 \\
& ^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4) \\
& *d*x)*\cosh(d*x + c)^4 + 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4) \\
& *d*x - 6*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b \\
& + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^2*\sinh(d*x \\
& + c)^2 - ((3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^8 + 8*(3*a^4*b + \\
& 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^4*b + 3*a^3*b^2 + \\
& a^2*b^3)*\sinh(d*x + c)^8 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*\cosh \\
& \cosh(d*x + c)^6 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(3*a^4*b \\
& + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 3 \\
& *a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^3 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + \\
& 2*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2 \\
& *(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*\cosh(d*x + c)^4 + 2 \\
& *(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5 + 35*(3*a^4*b + 3*a^3*b^2 \\
& + a^2*b^3)*\cosh(d*x + c)^4 + 30*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a \\
& *b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *\cosh(d*x + c)^5 + 10*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*\cosh(d \\
& *x + c)^3 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^3 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*\cosh \\
& (d*x + c)^2 + 4*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^6 + 3*a^4*b \\
& + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 15*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + \\
& 2*a*b^4)*\cosh(d*x + c)^4 + 3*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 \\
& + 8*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *\cosh(d*x + c)^7 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*\cosh(d \\
& *x + c)^5 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*\cosh(d*x \\
& + c)^3 + (3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*\cosh(d*x + c))*\sinh(d \\
& *x + c))*\log(2*(a*\cosh(d*x + c)^2 + a*\sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x \\
& + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^5*cosh(d*x \\
& + c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*(a^5 \\
& + 2*a^4*b)*\cosh(d*x + c)^6 + 4*(7*a^5*cosh(d*x + c)^2 + a^5 + 2*a^4*b)*s \\
& inh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(d*x + c)^4 + \\
& 2*(35*a^5*cosh(d*x + c)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*(a^5 + \\
& 2*a^4*b)*\cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(d*x + c))*\sin \\
& h(d*x + c)^3 + 4*(a^5 + 2*a^4*b)*\cosh(d*x + c)^2 + 4*(7*a^5*cosh(d*x + c)^6 \\
& + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*\cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b \\
& + 8*a^3*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(a^5*cosh(d*x + c)^7 + 3* \\
& (a^5 + 2*a^4*b)*\cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(d*x + \\
& c)^3 + (a^5 + 2*a^4*b)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*\sinh(d*x + c)/(c \\
& osh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *d*x*\cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b \\
& + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)^5 - 2*(6*a^3*b^2 + \\
& 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 \\
& + 32*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^3 - (3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 \\
& - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*\cosh(d*x + c)) \\
& *\sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + \\
& 8*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*\sinh(d*x + c)^7 + (\\
& a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + \\
& 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b \\
& + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7* \\
& a^5*b^3 + 2*a^4*b^4)*d)*\sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 \\
& + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7 \\
& *b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 \\
& + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^8 + 3 \\
& *a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6
\end{aligned}$$

*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.72, size = 475, normalized size = 3.65

$$\frac{2(3a^2b+3ab^2+b^3)\log(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c}+a)}{a^6+3a^5b+3a^4b^2+a^3b^3} + \frac{4e^{2c}\log(|-e^{2dx+2c}+1|)}{a^3e^{2c}+3a^2be^{2c}+3ab^2e^{2c}+b^3e^{2c}} - \frac{4dx}{a^3} - \frac{9a^3be^{8dx+8c}+9a^2b^2e^{8dx+8c}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 4*e^(2*c)*log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c)) - 4*d*x/a^3 - (9*a^3*b*e^(8*d*x + 8*c) + 9*a^2*b^2*e^(8*d*x + 8*c) + 3*a*b^3*e^(8*d*x + 8*c) + 36*a^3*b*e^(6*d*x + 6*c) + 84*a^2*b^2*e^(6*d*x + 6*c) + 44*a*b^3*e^(6*d*x + 6*c) + 8*b^4*e^(6*d*x + 6*c) + 54*a^3*b*e^(4*d*x + 4*c) + 150*a^2*b^2*e^(4*d*x + 4*c) + 146*a*b^3*e^(4*d*x + 4*c) + 32*b^4*e^(4*d*x + 4*c) + 36*a^3*b*e^(2*d*x + 2*c) + 84*a^2*b^2*e^(2*d*x + 2*c) + 44*a*b^3*e^(2*d*x + 2*c) + 8*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d

maple [B] time = 0.47, size = 1046, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)

[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-8/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6-2/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^6-12/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^4-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4+a*b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-8/d*b^3/(a+b)^3/(tanh

$$\begin{aligned} & (1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh \\ & \tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^2-2/d*b^4/(a+b)^3/(\tanh(1 \\ & /2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(\\ & 1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*\tanh(1/2*d*x+1/2*c)^2+3/2/d*b/a/(a+b)^3*\ln(\tanh \\ & (1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh \\ & \tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*b^2/a^2/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c)^4* \\ & a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2 \\ & *b+a+b)+1/2/d*b^3/a^3/(a+b)^3*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2 \\ & *c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)^3* \\ & \ln(\tanh(1/2*d*x+1/2*c)) \end{aligned}$$

maxima [B] time = 0.72, size = 419, normalized size = 3.22

$$\frac{(3a^2b + 3ab^2 + b^3) \log(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2(a^6 + 3a^5b + 3a^4b^2 + a^3b^3)d} + \frac{1}{(a^7 + 2a^6b + a^5b^2 + 4(a^7 + 4a^6b + 5a^5b^2 + \dots))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 2*((3*a^2*b^2 + 2*a*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^2 + 2*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) + log(e^(-d*x - c) + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + log(e^(-d*x - c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (d*x + c)/(a^3*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \coth(c + dx)}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)

$$3.166 \quad \int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=182

$$\frac{x}{a^3} - \frac{(8a^2 - 11ab - 4b^2) \coth(c + dx)}{8a^2d(a + b)^3} - \frac{b(9a + 4b) \coth(c + dx)}{8a^2d(a + b)^2 (a - b \tanh^2(c + dx) + b)} - \frac{b^{3/2} (35a^2 + 28ab + 8b^2) \tanh^{-1}(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}})}{8a^3d(a + b)^{7/2}}$$

[Out] x/a^3-1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/(a+b)^(7/2)/d-1/8*(8*a^2-11*a*b-4*b^2)*coth(d*x+c)/a^2/(a+b)^3/d-1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(9*a+4*b)*coth(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.40, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$\frac{b^{3/2} (35a^2 + 28ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a + b)^{7/2}} - \frac{(8a^2 - 11ab - 4b^2) \coth(c + dx)}{8a^2d(a + b)^3} - \frac{b(9a + 4b) \coth(c + dx)}{8a^2d(a + b)^2 (a - b \tanh^2(c + dx) + b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

[Out] x/a^3 - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*d) - ((8*a^2 - 11*a*b - 4*b^2)*Coth[c + d*x])/(8*a^2*(a + b)^3*d) - (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(9*a + 4*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b \coth(c+dx)}{4a(a+b)d(a+b-b \tanh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-4a+b-5bx^2}{x^2(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a+b)d} \\
&= -\frac{b \coth(c+dx)}{4a(a+b)d(a+b-b \tanh^2(c+dx))^2} - \frac{b(9a+4b) \coth(c+dx)}{8a^2(a+b)^2d(a+b-b \tanh^2(c+dx))} \\
&= -\frac{(8a^2-11ab-4b^2) \coth(c+dx)}{8a^2(a+b)^3d} - \frac{b \coth(c+dx)}{4a(a+b)d(a+b-b \tanh^2(c+dx))^2} - \frac{b \coth(c+dx)}{8a^2(a+b)^3d} \\
&= -\frac{(8a^2-11ab-4b^2) \coth(c+dx)}{8a^2(a+b)^3d} - \frac{b \coth(c+dx)}{4a(a+b)d(a+b-b \tanh^2(c+dx))^2} - \frac{b \coth(c+dx)}{8a^2(a+b)^3d} \\
&= \frac{x}{a^3} - \frac{b^{3/2}(35a^2+28ab+8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d} - \frac{(8a^2-11ab-4b^2) \coth(c+dx)}{8a^2(a+b)^3d}
\end{aligned}$$

Mathematica [C] time = 7.23, size = 2083, normalized size = 11.45

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]

[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((I/64)*b^2*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]))*Cosh[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^2*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]))*Sinh[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[c + d*x]^6*(8*a^5*d*x*Cosh[d*x] + 56*a^4*b*d*x*Cosh[d*x] + 184*a^3*b^2*d*x*Cosh[d*x] + 296*a^2*b^3*d*x*Cosh[d*x] + 224*a*b^4*d*x*Cosh[d*x] + 64*b^5*d*x*Cosh[d*x] - 12*a^5*d*x*Cosh[3*d*x] - 68*a^4*b*d*x*Cosh[3*d*x] - 132*a^3*b^2*d*x*Cosh[3*d*x] - 108*a^2*b^3*d*x*Cosh[3*d*x] - 32*a*b^4*d*x*Cosh[3*d*x] - 8*a^5*d*x*Cosh[2*c - d*x] - 56*a^4*b*d*x*Cosh[2*c - d*x] - 184*a^3*b^2*d*x*Cosh[2*c - d*x] - 296*a^2*b^3*d*x*Cosh[2*c - d*x] - 224*a*b^4*d*x*Cosh[2*c - d*x] - 64*b^5*d*x*Cosh[2*c - d*x] - 8*a^5*d*x*Cosh[2*c + d*x] - 56*a^4*b*d*x*Cosh[2*c + d*x] - 184*a^3*b^2*d*x*Cosh[2*c + d*x] - 296*a^2*b^3*d*x*Cosh[2*c + d*x] - 224*a*b^4*d*x*Cosh[2*c + d*x] - 64*b^5*d*x*Cosh[2*c + d*x] + 8*a^5*d*x*Cosh[4*c + d*x] + 56*a^4*b*d*x*Cosh[4*c + d*x] + 184*a^3*b^2*d*x*Cosh[4*c + d*x] + 296*a^2*b^3*d*x*Cosh[4*c + d*x] + 224*a*b^4*d*x*C

$$\begin{aligned} & \text{osh}[4*c + d*x] + 64*b^5*d*x*\text{Cosh}[4*c + d*x] + 12*a^5*d*x*\text{Cosh}[2*c + 3*d*x] \\ & + 68*a^4*b*d*x*\text{Cosh}[2*c + 3*d*x] + 132*a^3*b^2*d*x*\text{Cosh}[2*c + 3*d*x] + 108* \\ & a^2*b^3*d*x*\text{Cosh}[2*c + 3*d*x] + 32*a*b^4*d*x*\text{Cosh}[2*c + 3*d*x] - 12*a^5*d*x \\ & *\text{Cosh}[4*c + 3*d*x] - 68*a^4*b*d*x*\text{Cosh}[4*c + 3*d*x] - 132*a^3*b^2*d*x*\text{Cosh}[\\ & 4*c + 3*d*x] - 108*a^2*b^3*d*x*\text{Cosh}[4*c + 3*d*x] - 32*a*b^4*d*x*\text{Cosh}[4*c + \\ & 3*d*x] + 12*a^5*d*x*\text{Cosh}[6*c + 3*d*x] + 68*a^4*b*d*x*\text{Cosh}[6*c + 3*d*x] + 13 \\ & 2*a^3*b^2*d*x*\text{Cosh}[6*c + 3*d*x] + 108*a^2*b^3*d*x*\text{Cosh}[6*c + 3*d*x] + 32*a* \\ & b^4*d*x*\text{Cosh}[6*c + 3*d*x] - 4*a^5*d*x*\text{Cosh}[2*c + 5*d*x] - 12*a^4*b*d*x*\text{Cosh} \\ & [2*c + 5*d*x] - 12*a^3*b^2*d*x*\text{Cosh}[2*c + 5*d*x] - 4*a^2*b^3*d*x*\text{Cosh}[2*c + \\ & 5*d*x] + 4*a^5*d*x*\text{Cosh}[4*c + 5*d*x] + 12*a^4*b*d*x*\text{Cosh}[4*c + 5*d*x] + 12 \\ & *a^3*b^2*d*x*\text{Cosh}[4*c + 5*d*x] + 4*a^2*b^3*d*x*\text{Cosh}[4*c + 5*d*x] - 4*a^5*d* \\ & x*\text{Cosh}[6*c + 5*d*x] - 12*a^4*b*d*x*\text{Cosh}[6*c + 5*d*x] - 12*a^3*b^2*d*x*\text{Cosh}[\\ & 6*c + 5*d*x] - 4*a^2*b^3*d*x*\text{Cosh}[6*c + 5*d*x] + 4*a^5*d*x*\text{Cosh}[8*c + 5*d*x \\ &] + 12*a^4*b*d*x*\text{Cosh}[8*c + 5*d*x] + 12*a^3*b^2*d*x*\text{Cosh}[8*c + 5*d*x] + 4*a \\ & ^2*b^3*d*x*\text{Cosh}[8*c + 5*d*x] - 32*a^5*\text{Sinh}[d*x] - 64*a^4*b*\text{Sinh}[d*x] - 30*a \\ & ^2*b^3*\text{Sinh}[d*x] - 120*a*b^4*\text{Sinh}[d*x] - 48*b^5*\text{Sinh}[d*x] + 32*a^5*\text{Sinh}[3*d \\ & *x] + 64*a^4*b*\text{Sinh}[3*d*x] + 26*a^3*b^2*\text{Sinh}[3*d*x] + 86*a^2*b^3*\text{Sinh}[3*d*x \\ &] + 32*a*b^4*\text{Sinh}[3*d*x] - 48*a^5*\text{Sinh}[2*c - d*x] - 128*a^4*b*\text{Sinh}[2*c - d* \\ & x] - 128*a^3*b^2*\text{Sinh}[2*c - d*x] - 30*a^2*b^3*\text{Sinh}[2*c - d*x] - 120*a*b^4*S \\ & \text{inh}[2*c - d*x] - 48*b^5*\text{Sinh}[2*c - d*x] + 48*a^5*\text{Sinh}[2*c + d*x] + 128*a^4* \\ & b*\text{Sinh}[2*c + d*x] + 102*a^3*b^2*\text{Sinh}[2*c + d*x] - 86*a^2*b^3*\text{Sinh}[2*c + d*x \\ &] - 136*a*b^4*\text{Sinh}[2*c + d*x] - 48*b^5*\text{Sinh}[2*c + d*x] - 32*a^5*\text{Sinh}[4*c + \\ & d*x] - 64*a^4*b*\text{Sinh}[4*c + d*x] + 26*a^3*b^2*\text{Sinh}[4*c + d*x] + 86*a^2*b^3*S \\ & \text{inh}[4*c + d*x] + 136*a*b^4*\text{Sinh}[4*c + d*x] + 48*b^5*\text{Sinh}[4*c + d*x] - 8*a^5 \\ & *\text{Sinh}[2*c + 3*d*x] - 26*a^3*b^2*\text{Sinh}[2*c + 3*d*x] - 86*a^2*b^3*\text{Sinh}[2*c + 3 \\ & *d*x] - 32*a*b^4*\text{Sinh}[2*c + 3*d*x] + 32*a^5*\text{Sinh}[4*c + 3*d*x] + 64*a^4*b*S \\ & \text{inh}[4*c + 3*d*x] - 13*a^3*b^2*\text{Sinh}[4*c + 3*d*x] - 36*a^2*b^3*\text{Sinh}[4*c + 3*d* \\ & x] - 16*a*b^4*\text{Sinh}[4*c + 3*d*x] - 8*a^5*\text{Sinh}[6*c + 3*d*x] + 13*a^3*b^2*\text{Sinh} \\ & [6*c + 3*d*x] + 36*a^2*b^3*\text{Sinh}[6*c + 3*d*x] + 16*a*b^4*\text{Sinh}[6*c + 3*d*x] + \\ & 8*a^5*\text{Sinh}[2*c + 5*d*x] + 13*a^3*b^2*\text{Sinh}[2*c + 5*d*x] + 6*a^2*b^3*\text{Sinh}[2* \\ & c + 5*d*x] - 13*a^3*b^2*\text{Sinh}[4*c + 5*d*x] - 6*a^2*b^3*\text{Sinh}[4*c + 5*d*x] + 8 \\ & *a^5*\text{Sinh}[6*c + 5*d*x]))/(512*a^3*(a + b)^3*d*(a + b*\text{Sech}[c + d*x]^2)^3) \end{aligned}$$

fricas [B] time = 0.63, size = 11606, normalized size = 63.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 160*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^8 - 4*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 180*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x + c)^8 + 32*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - (8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 8*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 -

$$\begin{aligned}
& 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x \\
& x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 32*a^5 - 52*a^3*b^2 - 24*a^2*b^3 - 8*(2 \\
& 4*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7 \\
& *a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^4 + \\
& 8*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^6 - 24*a^5 \\
& - 64*a^4*b - 64*a^3*b^2 - 15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a \\
& ^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2* \\
& b^3 + 8*a*b^4)*d*x)*\cosh(d*x + c)^4 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^ \\
& 2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2 \\
& *b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28* \\
& a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 32*(60*(a^5 + 3*a^4* \\
& b + 3*a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a \\
& ^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 \\
&)*d*x)*\cosh(d*x + c)^5 - 5*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 6 \\
& 8*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + \\
& 8*b^5)*d*x)*\cosh(d*x + c)^3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 \\
& + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 \\
& + 8*b^5)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 16*(a^5 + 3*a^4*b + 3*a^3*b \\
& ^2 + a^2*b^3)*d*x - 8*(16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b \\
& ^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*\cosh(d*x \\
& + c)^2 + 8*(90*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^8 - \\
& 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33* \\
& a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*\cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b - \\
& 13*a^3*b^2 - 43*a^2*b^3 - 16*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 4 \\
& 3*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 \\
& + 28*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 \\
& + 27*a^2*b^3 + 8*a*b^4)*d*x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b \\
& ^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a* \\
& b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((35*a^4*b + 28*a^3*b^ \\
& 2 + 8*a^2*b^3)*\cosh(d*x + c)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*co \\
& sh(d*x + c)*\sinh(d*x + c)^9 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\sinh(d*x \\
& + c)^10 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^ \\
& 8 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a \\
& ^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28 \\
& *a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2* \\
& b^3 + 64*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2 \\
& + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3 \\
& *b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^ \\
& 2*b^3)*\cosh(d*x + c)^4 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b \\
& ^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2 \\
& *b^3)*\cosh(d*x + c)^5 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^ \\
& 4)*\cosh(d*x + c)^3 + 3*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + \\
& 64*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3 \\
& - 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + \\
& c)^4 + 2*(105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^6 - 35*a^4*b \\
& - 168*a^3*b^2 - 400*a^2*b^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^ \\
& 3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^ \\
& 2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8* \\
& (15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^7 + 7*(105*a^4*b + 36 \\
& 4*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3 \\
& *b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^3 - (35*a^4*b + 168* \\
& a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^2 + (45* \\
& (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a \\
& ^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 \\
& - 248*a^2*b^3 - 64*a*b^4 + 30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a \\
& *b^4 + 64*b^5)*\cosh(d*x + c)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + \\
& 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28 \\
& *a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^
\end{aligned}$$

$$\begin{aligned}
& 2*b^3 + 64*a*b^4)*\cosh(d*x + c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 \\
& + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 24 \\
& 8*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/(a + b))*\log((a^2*\cosh(d*x + c)^4 + 4*a^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*\sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 2*(3*a^2*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*\cosh(d*x + c)^3 + (a^2 + 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*\sqrt{b/(a + b)))/(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 + 2*(a + 2*b)*\cosh(d*x + c)^2 + 2*(3*a*\cosh(d*x + c)^2 + a + 2*b)*\sinh(d*x + c)^2 + 4*(a*\cosh(d*x + c)^3 + (a + 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a)) + 16*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^9 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*\cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*\cosh(d*x + c)^3 - (16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*\sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^4 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^8 + 28*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^6 + 30*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^4 - 12*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^2 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*\sinh(d*x + c)^2 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^9 + 4*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^7 + 6*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^5 - 4*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/8*(8*(a^5 + 3*a^4*b
\end{aligned}$$

$$\begin{aligned}
& b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^{10} + 80(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c) \sinh(dx + c)^9 + 8(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \sinh(dx + c)^{10} - 2(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^8 - 2(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 180(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^2 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \sinh(dx + c)^8 + 16(60(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^3 - (8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)) \sinh(dx + c)^7 - 4(16a^5 + 32a^4b - 13a^3b^2 - 43a^2b^3 - 68ab^4 - 24b^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^6 + 4(420(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^4 - 16a^5 - 32a^4b + 13a^3b^2 + 43a^2b^3 + 68ab^4 + 24b^5 + 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx - 14(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(252(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^5 - 14(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^3 - 3(16a^5 + 32a^4b - 13a^3b^2 - 43a^2b^3 - 68ab^4 - 24b^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)) \sinh(dx + c)^5 - 16a^5 - 26a^3b^2 - 12a^2b^3 - 4(24a^5 + 64a^4b + 64a^3b^2 + 15a^2b^3 + 60ab^4 + 24b^5 + 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^4 + 4(420(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^6 - 24a^5 - 64a^4b - 64a^3b^2 - 15a^2b^3 - 60ab^4 - 24b^5 - 35(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^4 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx - 15(16a^5 + 32a^4b - 13a^3b^2 - 43a^2b^3 - 68ab^4 - 24b^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^2) \sinh(dx + c)^4 + 16(60(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^7 - 7(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^5 - 5(16a^5 + 32a^4b - 13a^3b^2 - 43a^2b^3 - 68ab^4 - 24b^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^3 - (24a^5 + 64a^4b + 64a^3b^2 + 15a^2b^3 + 60ab^4 + 24b^5 + 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)) \sinh(dx + c)^3 - 8(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx - 4(16a^5 + 32a^4b + 13a^3b^2 + 43a^2b^3 + 16ab^4 + 2(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^2 + 4(90(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)dx \cosh(dx + c)^8 - 14(8a^5 - 13a^3b^2 - 36a^2b^3 - 16ab^4 - 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx) \cosh(dx + c)^6 - 16a^5 - 32a^4b - 13a^3b^2 - 43a^2b^3 - 16ab^4 - 15(16a^5 + 32a^4b - 13a^3b^2 - 43a^2b^3 - 68ab^4 - 24b^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^4 - 2(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4)dx - 6(24a^5 + 64a^4b + 64a^3b^2 + 15a^2b^3 + 60ab^4 + 24b^5 + 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5)dx) \cosh(dx + c)^2) \sinh(dx + c)^2 - ((35a^4b + 28a^3b^2 + 8a^2b^3) \cosh(dx + c))^10 + 10(35a^4b + 28a^3b^2 + 8a^2b^3) \cosh(dx + c) \sinh(dx + c)^9 + (35a^4b + 28a^3b^2 + 8a^2b^3) \sinh(dx + c)^10 + (105a^4b + 364a^3b^2 + 248a^2b^3 + 64ab^4) \cosh(dx + c)^8 + (105a^4b + 364a^3b^2 + 248a^2b^3 + 64ab^4 + 45(35a^4b + 28a^3b^2 + 8a^2b^3) \cosh(dx + c)^2) \sinh(dx + c)^8 + 8(15(35a^4b + 28a^3b^2 + 8a^2b^3) \cosh(dx + c)^3 + (105a^4b + 364a^3b^2 + 248a^2b^3 + 64ab^4) \cosh(dx + c)) \sinh(dx + c)^7 + 2(35a^4b + 168a^3b^2 + 400a^2b^3 + 256ab^4 + 64b^5) \cosh(dx + c)^6 + 2(35a^4b + 168a^3b^2 + 400a^2b^3 + 256ab^4 + 64b^5 + 105(35a^4b + 28a^3b^2 + 8a^2b^3) \cosh(dx + c)^4 + 14(105a^4b + 364a^3b^2 + 248a^2b^3 + 64ab^4) \cosh(dx + c)^2) \sinh(dx + c)^5
\end{aligned}$$

$$\begin{aligned}
& *x + c)^6 + 4*(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^5 + 14* \\
& (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^3 + 3*(35* \\
& a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c))*\sinh \\
& (d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3 - 2*(35*a^4*b + 168*a^3*b^2 \\
& + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 2 \\
& 8*a^3*b^2 + 8*a^2*b^3)*\cosh(d*x + c)^6 - 35*a^4*b - 168*a^3*b^2 - 400*a^2*b \\
& ^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a* \\
& b^4)*\cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 \\
& + 64*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 28*a^3*b^2 \\
& + 8*a^2*b^3)*\cosh(d*x + c)^7 + 7*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 6 \\
& 4*a*b^4)*\cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a* \\
& b^4 + 64*b^5)*\cosh(d*x + c)^3 - (35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256 \\
& *a*b^4 + 64*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 \\
& + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x + c)^2 + (45*(35*a^4*b + 28*a^3*b^2 + 8* \\
& a^2*b^3)*\cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a \\
& *b^4)*\cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 - 248*a^2*b^3 - 64*a*b^4 + \\
& 30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d*x + c \\
&)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*\cosh(\\
& d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(d*x \\
& + c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*\cosh \\
& (d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5) \\
& *\cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*\cosh(\\
& d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a + b))*\arctan(1/2*(a*\cosh(d*x + c)^2 + 2 \\
& *a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a + 2*b))*\sqrt{-b/(a + \\
& b))/b) + 8*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*\cosh(d*x + c)^9 - \\
& 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^ \\
& 3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x))*\cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b - \\
& 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^ \\
& 2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x))*\cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^ \\
& 4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a \\
& ^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x))*\cosh(d*x + c)^3 - (16*a^5 + 32 \\
& *a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3* \\
& b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x))*\cosh(d*x + c))*\sinh(d*x + c))/((a^8 + 3*a^ \\
& 7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b \\
& ^2 + a^5*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 \\
& + a^5*b^3)*d*\sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 \\
& + 8*a^4*b^4)*d*\cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3) \\
& *d*\cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^ \\
& 4)*d)*\sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4 \\
& *b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^ \\
& 5*b^3)*d*\cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8* \\
& a^4*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b \\
& ^2 + a^5*b^3)*d*\cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^ \\
& 5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5 \\
& *b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a \\
& ^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^4 + 4*(63*(a^ \\
& 8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b \\
& + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^3 + 3*(a^8 + 7*a^7* \\
& b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*\cosh(d*x + c)^ \\
& 6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*\cosh(d*x \\
& + c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b \\
& ^5)*d*\cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b \\
& ^4 + 8*a^3*b^5)*d)*\sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^ \\
& 5*b^3 + 8*a^4*b^4)*d*\cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a \\
& ^5*b^3)*d*\cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + \\
& 8*a^4*b^4)*d*\cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 \\
& + 28*a^4*b^4 + 8*a^3*b^5)*d*\cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 +
\end{aligned}$$

$$37a^5b^3 + 28a^4b^4 + 8a^3b^5) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (45(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * d * \cosh(dx + c)^8 + 28(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^6 + 30(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * d * \cosh(dx + c)^4 - 12(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * d * \cosh(dx + c)^2 - (3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * d) * \sinh(dx + c)^2 - (a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * d + 2(5(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * d * \cosh(dx + c)^9 + 4(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)^7 + 6(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * d * \cosh(dx + c)^5 - 4(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * d * \cosh(dx + c)^3 - (3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * d * \cosh(dx + c)) * \sinh(dx + c)]$$

giac [B] time = 1.27, size = 402, normalized size = 2.21

$$\frac{(35a^2b^2e^{2c} + 28ab^3e^{2c} + 8b^4e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + a+2b}{2\sqrt{-ab-b^2}}\right) e^{-2c}}{(a^6 + 3a^5b + 3a^4b^2 + a^3b^3)\sqrt{-ab-b^2}} - \frac{8dx}{a^3} - \frac{2(13a^3b^2e^{6dx+6c} + 36a^2b^3e^{6dx+6c} + 16ab^4e^{6dx+6c} + 39a^3b^2e^{4dx+4c} + 122a^2b^3e^{4dx+4c} + 152a^2b^4e^{4dx+4c} + 48b^5e^{4dx+4c} + 39a^3b^2e^{2dx+2c} + 92a^2b^3e^{2dx+2c} + 32a^2b^4e^{2dx+2c} + 13a^3b^2 + 6a^2b^3)}{(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * (ae^{4dx+4c} + 2ae^{2dx+2c} + 4b * e^{2dx+2c} + a)^2} + \frac{16}{(a^3 + 3a^2b + 3ab^2 + b^3) * (e^{2dx+2c} - 1)} \Big/ d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^2/(a+b*sech(dx+c)^2)^3,x, algorithm="giac")

[Out] -1/8*((35*a^2*b^2*e^(2*c) + 28*a*b^3*e^(2*c) + 8*b^4*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(-a*b - b^2)) - 8*d*x/a^3 - 2*(13*a^3*b^2*e^(6*d*x + 6*c) + 36*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 122*a^2*b^3*e^(4*d*x + 4*c) + 152*a*b^4*e^(4*d*x + 4*c) + 48*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 92*a^2*b^3*e^(2*d*x + 2*c) + 32*a^2*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 6*a^2*b^3)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) + 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1))/d

maple [B] time = 0.51, size = 1433, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^2/(a+b*sech(dx+c)^2)^3,x)

[Out] -1/2/d/(a^3+3a^2b+3ab^2+b^3)*tanh(1/2*d*x+1/2*c)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-17/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3

$$\begin{aligned}
& *x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)-17/4/d*b^3/a/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)-1/d*b^4/a^2/(a+b)^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)+35/16/d*b^(3/2)/a/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-35/16/d*b^(3/2)/a/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+7/4/d*b^(5/2)/a^2/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-7/4/d*b^(5/2)/a^2/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^3/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(7/2)/a^3/(a+b)^(7/2)*\ln((a+b)^(1/2)*\tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*\tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^3/\tanh(1/2*d*x+1/2*c)
\end{aligned}$$

maxima [B] time = 0.94, size = 1971, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& 1/4*(3*a^2*b + 3*a*b^2 + b^3)*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/4*(3*a^2*b + 3*a*b^2 + b^3)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\sqrt{(a + b)*b}*d) + 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\sqrt{(a + b)*b}*d) - 15/32*b*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sqrt{(a + b)*b}*d) + 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 + 12*a^2*b^3 + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b^4)*e^{(8*d*x + 8*c)} + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 96*b^5)*e^{(6*d*x + 6*c)} + 2*(24*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 + 272*a*b^4 + 96*b^5)*e^{(4*d*x + 4*c)} + 2*(16*a^5 + 41*a^4*b + 77*a^3*b^2 + 130*a^2*b^3 + 48*a*b^4)*e^{(2*d*x + 2*c)})/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^{(10*d*x + 10*c)} - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^{(8*d*x + 8*c)} - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^{(6*d*x + 6*c)} + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^{(4*d*x + 4*c)} + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^{(2*d*x + 2*c)})*d) - 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 + 12*a^2*b^3 + 2*(16*a^5 + 41*a^4*b + 77*a^3*b^2 + 130*a^2*b^3 + 48*a*b^4)*e^{(-2*d*x - 2*c)} + 2*(24*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 + 272*a*b^4 + 96*b^5)*e^{(-4*d*x - 4*c)} + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 96*b^5)*e^{(-6*d*x - 6*c)} + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b^4)*e^{(-8*d*x - 8*c)})/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^{(-2*d*x - 2*c)} + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^{(-4*d*x - 4*c)} - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^{(-6*d*x - 6*c)} - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^{(-8*d*x - 8*c)} - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^{(-10*d*x - 10*c)})*d) - 1/8*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*e^{(-2*d*x - 2*c)} + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8
\end{aligned}$$

```

*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b
^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*e^(-8*d*x -
8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) - 2*(a^7 + 7*a^6
*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-6*d*x - 6*c) - (
3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-8*d*x - 8*c) -
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*d) + 1/2*log(e^(2
*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*log(e^(-2*d*x -
2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \coth(c + dx)^2}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)

$$3.167 \quad \int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=152

$$-\frac{b^4}{4a^3d(a+b)^2(a\cosh^2(c+dx)+b)^2} + \frac{b^3(2a+b)}{a^3d(a+b)^3(a\cosh^2(c+dx)+b)} + \frac{b^2(6a^2+4ab+b^2)\log(a\cosh^2(c+dx)+b)}{2a^3d(a+b)^4}$$

[Out] $-1/4*b^4/a^3/(a+b)^2/d/(b+a*\cosh(d*x+c)^2)^2+b^3*(2*a+b)/a^3/(a+b)^3/d/(b+a*\cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/d/(a+b)^3+1/2*b^2*(6*a^2+4*a*b+b^2)*\ln(b+a*\cosh(d*x+c)^2)/a^3/(a+b)^4/d+(a+4*b)*\ln(\sinh(d*x+c))/d/(a+b)^4$

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4138, 446, 88}

$$-\frac{b^4}{4a^3d(a+b)^2(a\cosh^2(c+dx)+b)^2} + \frac{b^3(2a+b)}{a^3d(a+b)^3(a\cosh^2(c+dx)+b)} + \frac{b^2(6a^2+4ab+b^2)\log(a\cosh^2(c+dx)+b)}{2a^3d(a+b)^4}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $-b^4/(4*a^3*(a+b)^2*d*(b+a*Cosh[c+d*x]^2)^2) + (b^3*(2*a+b))/(a^3*(a+b)^3*d*(b+a*Cosh[c+d*x]^2)) - CsCh[c+d*x]^2/(2*(a+b)^3*d) + (b^2*(6*a^2+4*a*b+b^2)*Log[b+a*Cosh[c+d*x]^2])/(2*a^3*(a+b)^4*d) + ((a+4*b)*Log[Sinh[c+d*x]])/((a+b)^4*d)$

Rule 88

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4138

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*tan[(e_) + (f_)*(x_)^(m_)], x_Symbol] := Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p/x^(m + n*p), x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^9}{(1-x^2)^2(b+ax^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3} dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2(6a^2+4ab+b^2)}{a^2(a+b)^4(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{b^4}{4a^3(a+b)^2d(b+a\cosh^2(c+dx))^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d(b+a\cosh^2(c+dx))} - \frac{\operatorname{csch}^2(c+dx)}{2(a+b)}
\end{aligned}$$

Mathematica [A] time = 1.92, size = 172, normalized size = 1.13

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx)) + a + 2b)^3 \left(\frac{b^4(a+b)^2}{a^3(a\sinh^2(c+dx)+a+b)^2} - \frac{4b^3(a+b)(2a+b)}{a^3(a\sinh^2(c+dx)+a+b)} - \frac{2b^2(6a^2+4ab+b^2)\log(a\sinh^2(c+dx))}{a^3} \right)}{32d(a+b)^4(a+b\operatorname{sech}^2(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] -1/32*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(2*(a + b)*Csch[c + d*x]^2 - 4*(a + 4*b)*Log[Sinh[c + d*x]] - (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 + (b^4*(a + b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) - (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b + a*Sinh[c + d*x]^2)))/(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)

fricas [B] time = 1.34, size = 10255, normalized size = 67.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^12 + 24*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*sinh(d*x + c)^12 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^10 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 33*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^2 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*sinh(d*x + c)^10 + 40*(11*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 90*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*sinh(d*x + c)^3)/(a + b)^4*d*(a + b*sech(d*x+c)^2)^3)

$$\begin{aligned}
& d*x + c)^2)*\sinh(d*x + c)^8 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 \\
& + a^2*b^4)*d*x*\cosh(d*x + c)^5 + 30*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - \\
& 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5) \\
& *d*x)*\cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a* \\
& b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a* \\
& *b^5 - 16*b^6)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 8*(3*a^6 + 11*a^5*b + \\
& 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + \\
& 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*\cosh(d*x + c) \\
& ^6 + 8*(231*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*\cosh(d*x \\
& + c)^6 + 3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 \\
& + 6*b^6 + 105*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^ \\
& 5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*\cosh(d*x + c)^4 \\
& - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6) \\
& *d*x + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - \\
& (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)* \\
& d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + \\
& 4*a^3*b^3 + a^2*b^4)*d*x*\cosh(d*x + c)^7 + 63*(a^6 + a^5*b - 4*a^3*b^3 - 6 \\
& *a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 \\
& + 4*a*b^5)*d*x)*\cosh(d*x + c)^5 + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2 \\
& *b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^ \\
& 2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*\cosh(d*x + c)^3 + 3*(3*a^6 + 11*a^5*b + 16* \\
& a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30* \\
& a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*\cosh(d*x + c))*s \\
& \sinh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - \\
& 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 \\
& - 16*b^6)*d*x)*\cosh(d*x + c)^4 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3* \\
& b^3 + a^2*b^4)*d*x*\cosh(d*x + c)^8 + 420*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b \\
& ^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a* \\
& b^5)*d*x)*\cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40 \\
& *a*b^5 - 12*b^6 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 \\
& - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 \\
& - 16*b^6)*d*x)*\cosh(d*x + c)^4 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 \\
& - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 60*(3*a^6 + 11*a^5*b + 16*a^4*b^2 \\
& + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 \\
& + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*\cosh(d*x + c)^2)*\sinh(d* \\
& x + c)^4 + 8*(55*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*\cosh \\
& (d*x + c)^9 + 60*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8* \\
& a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*\cosh(d*x + c)^ \\
& 7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a \\
& ^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d* \\
& x)*\cosh(d*x + c)^5 + 20*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^ \\
& 2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^ \\
& 2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*\cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4 \\
& *b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^ \\
& 3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x + 4*(a^6 + a^5*b \\
& - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b \\
& ^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*\cosh(d*x + c)^2 + 4*(33*(a^6 + 4*a^5*b + 6* \\
& a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*\cosh(d*x + c)^10 + 45*(a^6 + a^5*b - 4*a \\
& ^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 1 \\
& 7*a^2*b^4 + 4*a*b^5)*d*x)*\cosh(d*x + c)^8 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b \\
& ^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3* \\
& b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*\cosh(d*x + c)^6 + a^6 + a^5*b - \\
& 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 30*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^ \\
& 3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^ \\
& 3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*\cosh(d*x + c)^4 + (a^6 + 8*a^5* \\
& b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x + 3*(8*a^6 + 24*a^5 \\
& *b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4* \\
& b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*\cosh(d*x + c)^2)*\si
\end{aligned}$$

$$\begin{aligned}
& \text{nh}(d*x + c)^2 - ((6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^{12} + 12*(6* \\
& a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + (6*a^4*b^2 \\
& + 4*a^3*b^3 + a^2*b^4)*\sinh(d*x + c)^{12} + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 \\
& + 4*a*b^5)*\cosh(d*x + c)^{10} + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 \\
& + 4*a*b^5 + 33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^{10} + 20*(11*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^3 + (6*a^4 \\
& b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^9 - \\
& (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^8 - \\
& (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6 - 495*(6*a^4*b^2 + \\
& 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^4 - 90*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2 \\
& *b^4 + 4*a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*(6*a^4*b^2 + 4*a^3 \\
& *b^3 + a^2*b^4)*\cosh(d*x + c)^5 + 30*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + \\
& 4*a*b^5)*\cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 \\
& - 16*b^6)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 4*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2 \\
& *b^4 + 36*a*b^5 + 8*b^6)*\cosh(d*x + c)^6 + 4*(231*(6*a^4*b^2 + 4*a^3*b^3 \\
& + a^2*b^4)*\cosh(d*x + c)^6 - 6*a^4*b^2 - 28*a^3*b^3 - 65*a^2*b^4 - 36*a*b^5 \\
& - 8*b^6 + 105*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh(d*x + c \\
&)^4 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^6 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 8*(99*(6*a^4*b^2 \\
& + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^7 + 63*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2 \\
& *b^4 + 4*a*b^5)*\cosh(d*x + c)^5 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - \\
& 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^3 - 3*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 \\
& + 36*a*b^5 + 8*b^6)*\cosh(d*x + c))*\sinh(d*x + c)^5 - (6*a^4*b^2 + 4*a^3*b \\
& ^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^4 + (495*(6*a^4*b^2 + 4* \\
& a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^8 + 420*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b \\
& ^4 + 4*a*b^5)*\cosh(d*x + c)^6 - 6*a^4*b^2 - 4*a^3*b^3 + 95*a^2*b^4 + 64*a*b \\
& ^5 + 16*b^6 - 70*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c \\
& osh(d*x + c)^4 - 60*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(55*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 \\
&)*\cosh(d*x + c)^9 + 60*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh \\
& (d*x + c)^7 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c \\
& osh(d*x + c)^5 - 20*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6 \\
&)*\cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6 \\
&)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + \\
& 4*a*b^5)*\cosh(d*x + c)^2 + 2*(33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d* \\
& x + c)^{10} + 45*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh(d*x + c \\
&)^8 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x \\
& + c)^6 + 6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5 - 30*(6*a^4*b^2 + 28 \\
& *a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*\cosh(d*x + c)^4 - 3*(6*a^4*b^2 + \\
& 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
& 2 + 4*(3*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*\cosh(d*x + c)^{11} + 5*(6*a^4*b^2 \\
& + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh(d*x + c)^9 - 2*(6*a^4*b^2 + 4*a^3 \\
& *b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^7 - 6*(6*a^4*b^2 + 28* \\
& a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*\cosh(d*x + c)^5 - (6*a^4*b^2 + 4*a \\
& ^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*\cosh(d*x + c)^3 + (6*a^4*b^2 + 28* \\
& a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*(a*\cosh \\
& (d*x + c)^2 + a*\sinh(d*x + c)^2 + a + 2*b)/(\cosh(d*x + c)^2 - 2*\cosh(d*x + \\
& c)*\sinh(d*x + c) + \sinh(d*x + c)^2)) - 2*((a^6 + 4*a^5*b)*\cosh(d*x + c)^{12} \\
& + 12*(a^6 + 4*a^5*b)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + (a^6 + 4*a^5*b)*\sinh(\\
& d*x + c)^{12} + 2*(a^6 + 8*a^5*b + 16*a^4*b^2)*\cosh(d*x + c)^{10} + 2*(a^6 + 8* \\
& a^5*b + 16*a^4*b^2 + 33*(a^6 + 4*a^5*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + \\
& 20*(11*(a^6 + 4*a^5*b)*\cosh(d*x + c)^3 + (a^6 + 8*a^5*b + 16*a^4*b^2)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^9 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*\cosh \\
& (d*x + c)^8 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3 - 495*(a^6 + 4*a^5*b \\
&)*\cosh(d*x + c)^4 - 90*(a^6 + 8*a^5*b + 16*a^4*b^2)*\cosh(d*x + c)^2)*\sinh(d \\
& *x + c)^8 + 8*(99*(a^6 + 4*a^5*b)*\cosh(d*x + c)^5 + 30*(a^6 + 8*a^5*b + 16* \\
& a^4*b^2)*\cosh(d*x + c)^3 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*\cosh(d \\
& *x + c))*\sinh(d*x + c)^7 - 4*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*\cosh \\
& (d*x + c)^6 + 4*(231*(a^6 + 4*a^5*b)*\cosh(d*x + c)^6 - a^6 - 8*a^5*b - 24*a
\end{aligned}$$

$$\begin{aligned}
& ^4b^2 - 32a^3b^3 + 105*(a^6 + 8a^5b + 16a^4b^2)*\cosh(dx + c)^4 - 7*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^2*\sinh(dx + c)^6 \\
& + a^6 + 4a^5b + 8*(99*(a^6 + 4a^5b)*\cosh(dx + c)^7 + 63*(a^6 + 8a^5b + 16a^4b^2)*\cosh(dx + c)^5 - 7*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3) \\
&)*\cosh(dx + c)^3 - 3*(a^6 + 8a^5b + 24a^4b^2 + 32a^3b^3)*\cosh(dx + c))*\sinh(dx + c)^5 - (a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^4 \\
& + (495*(a^6 + 4a^5b)*\cosh(dx + c)^8 + 420*(a^6 + 8a^5b + 16a^4b^2)*\cosh(dx + c)^6 - a^6 - 4a^5b + 16a^4b^2 + 64a^3b^3 - 70*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3) \\
&)*\cosh(dx + c)^4 - 60*(a^6 + 8a^5b + 24a^4b^2 + 32a^3b^3)*\cosh(dx + c)^2*\sinh(dx + c)^4 + 4*(55*(a^6 + 4a^5b)*\cosh(dx + c)^9 + 60*(a^6 + 8a^5b + 16a^4b^2) \\
&)*\cosh(dx + c)^7 - 14*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^5 - 20*(a^6 + 8a^5b + 24a^4b^2 + 32a^3b^3)*\cosh(dx + c)^3 - (a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3) \\
&)*\cosh(dx + c))*\sinh(dx + c)^3 + 2*(a^6 + 8a^5b + 16a^4b^2)*\cosh(dx + c)^2 + 2*(33*(a^6 + 4a^5b)*\cosh(dx + c)^10 + 45*(a^6 + 8a^5b + 16a^4b^2) \\
&)*\cosh(dx + c)^8 - 14*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^6 + a^6 + 8a^5b + 16a^4b^2 - 30*(a^6 + 8a^5b + 24a^4b^2 + 32a^3b^3) \\
&)*\cosh(dx + c)^4 - 3*(a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(3*(a^6 + 4a^5b)*\cosh(dx + c)^11 + 5*(a^6 + 8a^5b + 16a^4b^2) \\
&)*\cosh(dx + c)^9 - 2*(a^6 + 4a^5b + 16a^4b^2 - 64a^3b^3)*\cosh(dx + c)^7 - 6*(a^6 + 8a^5b + 24a^4b^2 + 32a^3b^3)*\cosh(dx + c)^5 - (a^6 + 4a^5b - 16a^4b^2 - 64a^3b^3) \\
&)*\cosh(dx + c)^3 + (a^6 + 8a^5b + 16a^4b^2)*\cosh(dx + c))*\sinh(dx + c))*\log(2*\sinh(dx + c)/(cosh(dx + c) - sinh(dx + c))) + 8*(3*(a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4) \\
&)*dx*\cosh(dx + c)^11 + 5*(a^6 + a^5b + 4a^4b^2 - 6a^3b^3 - 6a^2b^4 - 2ab^5 + (a^6 + 8a^5b + 22a^4b^2 + 28a^3b^3 + 17a^2b^4 + 4ab^5)*dx)*\cosh(dx + c)^9 + 2*(8a^6 + 24a^5b + 16a^4b^2 - 28a^2b^4 - 40ab^5 - 12b^6 - (a^6 + 4a^5b - 10a^4b^2 - 60a^3b^3 - 95a^2b^4 - 64ab^5 - 16b^6)*dx)*\cosh(dx + c)^7 + 6*(3a^6 + 11a^5b + 16a^4b^2 + 12a^3b^3 + 20a^2b^4 + 22ab^5 + 6b^6 - (a^6 + 8a^5b + 30a^4b^2 + 60a^3b^3 + 65a^2b^4 + 36ab^5 + 8b^6)*dx)*\cosh(dx + c)^5 + (8a^6 + 24a^5b + 16a^4b^2 - 28a^2b^4 - 40ab^5 - 12b^6 - (a^6 + 4a^5b - 10a^4b^2 - 60a^3b^3 - 95a^2b^4 - 64ab^5 - 16b^6)*dx)*\cosh(dx + c)^3 + (a^6 + a^5b - 4a^3b^3 - 6a^2b^4 - 2ab^5 + (a^6 + 8a^5b + 22a^4b^2 + 28a^3b^3 + 17a^2b^4 + 4ab^5)*dx)*\cosh(dx + c))*\sinh(dx + c))/((a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^12 + 12*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c))*\sinh(dx + c)^11 + (a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\sinh(dx + c)^12 + 2*(a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d*\cosh(dx + c)^10 + 2*(33*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^2 + (a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d)*\sinh(dx + c)^10 - (a^9 + 4a^8b - 10a^7b^2 - 60a^6b^3 - 95a^5b^4 - 64a^4b^5 - 16a^3b^6)*d*\cosh(dx + c)^8 + 20*(11*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^3 + (a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d*\cosh(dx + c))*\sinh(dx + c)^9 + (495*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^4 + 90*(a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d*\cosh(dx + c)^2 - (a^9 + 4a^8b + 6a^7b^2 - 10a^7b^2 - 60a^6b^3 - 95a^5b^4 - 64a^4b^5 - 16a^3b^6)*d)*\sinh(dx + c)^8 - 4*(a^9 + 8a^8b + 30a^7b^2 + 60a^6b^3 + 65a^5b^4 + 36a^4b^5 + 8a^3b^6)*d*\cosh(dx + c)^6 + 8*(99*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^5 + 30*(a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d*\cosh(dx + c)^3 - (a^9 + 4a^8b - 10a^7b^2 - 60a^6b^3 - 95a^5b^4 - 64a^4b^5 - 16a^3b^6)*d*\cosh(dx + c))*\sinh(dx + c)^7 + 4*(231*(a^9 + 4a^8b + 6a^7b^2 + 4a^6b^3 + a^5b^4)*d*\cosh(dx + c)^6 + 105*(a^9 + 8a^8b + 22a^7b^2 + 28a^6b^3 + 17a^5b^4 + 4a^4b^5)*d*\cosh(dx + c)^4 - 7*(a^9 + 4a^8b - 10a^7b^2 - 60a^6b^3 - 95a^5b^4 - 64a^4b^5 - 16a^3b^6)*d*\cosh(dx + c)^2 - (a^9 + 8a^8b + 30a^7b^2 + 60a^6b^3 + 65a^5b^4 + 36a^4b^5 + 8a^3b^6)*
\end{aligned}$$

$$4*d*x + 4*c) + 13*a^3*b^2*e^{(4*d*x + 4*c)} + 47*a^2*b^3*e^{(4*d*x + 4*c)} - 8*a*b^4*e^{(4*d*x + 4*c)} - 8*b^5*e^{(4*d*x + 4*c)} + 6*a^5*e^{(2*d*x + 2*c)} + 14*a^4*b*e^{(2*d*x + 2*c)} + 30*a^3*b^2*e^{(2*d*x + 2*c)} + 10*a^2*b^3*e^{(2*d*x + 2*c)} + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^{(6*d*x + 6*c)} + a*e^{(4*d*x + 4*c)} + 4*b*e^{(4*d*x + 4*c)} - a*e^{(2*d*x + 2*c)} - 4*b*e^{(2*d*x + 2*c)} - a)^2))/d$$

maple [B] time = 0.57, size = 1128, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)

[Out]
$$-1/8/d*\tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c)+1)-8/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6-10/d*b^4/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^6-2/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^6-16/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^4+8/d*b^4/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^4+4/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^4-8/d*b^3/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^2-10/d*b^4/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*\tanh(1/2*d*x+1/2*c)^2-2/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^2+3/d*b^2/(a+b)^4/a*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+2/d*b^3/(a+b)^4/a^2*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^4/(a+b)^4/a^3*\ln(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a+b)^3/\tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^4*\ln(\tanh(1/2*d*x+1/2*c))*a+4/d/(a+b)^4*\ln(\tanh(1/2*d*x+1/2*c))*b$$

maxima [B] time = 0.54, size = 692, normalized size = 4.55

$$\frac{(6a^2b^2 + 4ab^3 + b^4) \log(2(a + 2b)e^{(-2dx-2c)} + ae^{(-4dx-4c)} + a)}{2(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4)d} + \frac{(a + 4b) \log(e^{(-dx-c)} + 1)}{(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)d} + \frac{(a + 4b) \log(e^{(-dx-c)} - 1)}{(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + (a + 4*b)*\log(e^{(-d*x - c)} + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + (a + 4*b)*\log(e^{(-d*x - c)} - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 2*((a^5 - 4*a^2*b^3 - 2*a*b^4)*e^{(-2*d*x - 2*c)} + 2*(2*a^5 + 4*a^4*b - 7*a*b^4 - 3*b^5)*e^{(-4*d*x - 4*c)} + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2 + 4*a^2*b^3 + 16*a*b^4 + 6*b^5)*e^{(-6*d*x - 6*c)} + 2*(2*a^5 + 4*a^4*b - 7*a*b^4 - 3*b^5)*e^{(-8*d*x - 8*c)} + (a^5 - 4*a^2*b^3 - 2*a*b^4)*e^{(-10*d*x - 10*c)})/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^8 + 7*a^7*b + 15*a^6*b^2$$

+ 13*a^5*b^3 + 4*a^4*b^4)*e^(-2*d*x - 2*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + 7*a^7*b + 15*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4)*e^(-10*d*x - 10*c) + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-12*d*x - 12*c))*d) + (d*x + c)/(a^3*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^6 \coth(c + dx)^3}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3, x)

[Out] int((cosh(c + d*x)^6*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**3, x)

[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)

$$3.168 \quad \int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal. Leaf size=232

$$\frac{x}{a^3} - \frac{(8a^2 - 39ab - 12b^2) \coth^3(c + dx)}{24a^2d(a + b)^3} - \frac{b(11a + 4b) \coth^3(c + dx)}{8a^2d(a + b)^2(a - b \tanh^2(c + dx) + b)} - \frac{b^{5/2} (63a^2 + 36ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a + b)^{9/2}}$$

[Out] $x/a^3 - 1/8*b^{(5/2)}*(63*a^2+36*a*b+8*b^2)*\operatorname{arctanh}(b^{(1/2)}*\tanh(d*x+c)/(a+b)^{(1/2)})/a^3/(a+b)^{(9/2)}/d - 1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*\coth(d*x+c)/a^2/(a+b)^4/d - 1/24*(8*a^2-39*a*b-12*b^2)*\coth(d*x+c)^3/a^2/(a+b)^3/d - 1/4*b*\coth(d*x+c)^3/a/(a+b)/d/(a+b-b*\tanh(d*x+c)^2)^2 - 1/8*b*(11*a+4*b)*\coth(d*x+c)^3/a^2/(a+b)^2/d/(a+b-b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.51, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$\frac{b^{5/2} (63a^2 + 36ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a + b)^{9/2}} - \frac{(8a^2 - 39ab - 12b^2) \coth^3(c + dx)}{24a^2d(a + b)^3} - \frac{(32a^2b + 8a^3 - 15ab^2 - 4b^3)}{8a^2d(a + b)^2}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] $x/a^3 - (b^{(5/2)}*(63*a^2 + 36*a*b + 8*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[a + b])])/(8*a^3*(a + b)^{(9/2)*d}) - ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3)*\operatorname{Coth}[c + d*x])/(8*a^2*(a + b)^4*d) - ((8*a^2 - 39*a*b - 12*b^2)*\operatorname{Coth}[c + d*x]^3)/(24*a^2*(a + b)^3*d) - (b*\operatorname{Coth}[c + d*x]^3)/(4*a*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^2) - (b*(11*a + 4*b)*\operatorname{Coth}[c + d*x]^3)/(8*a^2*(a + b)^2*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b\coth^3(c+dx)}{4a(a+b)d(a+b-b\tanh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-4a+3b-7bx^2}{x^4(1-x^2)(a+b-bx^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a+b)d} \\
&= -\frac{b\coth^3(c+dx)}{4a(a+b)d(a+b-b\tanh^2(c+dx))^2} - \frac{b(11a+4b)\coth^3(c+dx)}{8a^2(a+b)^2d(a+b-b\tanh^2(c+dx))} \\
&= -\frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d} - \frac{b\coth^3(c+dx)}{4a(a+b)d(a+b-b\tanh^2(c+dx))^2} \\
&= -\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d} \\
&= -\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d} \\
&= \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d} - \frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d}
\end{aligned}$$

Mathematica [C] time = 7.74, size = 3334, normalized size = 14.37

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3, x]

[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((I/64)*b^3*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]))*Cosh[2*c]/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^3*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])*Sinh[2*c]/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^4*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]^3*Sech[2*c]*Sech[c + d*x]^6*(-36*a^6*d*x*Cosh[d*x] - 336*a^5*b*d*x*Cosh[d*x] - 1560*a^4*b^2*d*x*Cosh[d*x] - 3600*a^3*b^3*d*x*Cosh[d*x] - 4260*a^2*b^4*d*x*Cosh[d*x] - 2496*a*b^5*d*x*Cosh[d*x] - 576*b^6*d*x*Cosh[d*x] + 36*a^6*d*x*Cosh[3*d*x] + 240*a^5*b*d*x*Cosh[3*d*x] + 408*a^4*b^2*d*x*Cosh[3*d*x] - 48*a^3*b^3*d*x*Cosh[3*d*x] - 732*a^2*b^4*d*x*Cosh[3*d*x] - 672*a*b^5*d*x*Cosh[3*d*x] - 192*b^6*d*x*Cosh[3*d*x] + 36*a^6*d*x*Cosh[2*c - d*x] + 336*a^5*b*d*x*Cosh[2*c - d*x] + 1560*a^4*b^2*d*x*Cosh[2*c - d*x] + 3600*a^3*b^3*d*x*Cosh[2*c -

$$\begin{aligned}
& d*x] + 4260*a^2*b^4*d*x*Cosh[2*c - d*x] + 2496*a*b^5*d*x*Cosh[2*c - d*x] + \\
& 576*b^6*d*x*Cosh[2*c - d*x] + 36*a^6*d*x*Cosh[2*c + d*x] + 336*a^5*b*d*x*C \\
& osh[2*c + d*x] + 1560*a^4*b^2*d*x*Cosh[2*c + d*x] + 3600*a^3*b^3*d*x*Cosh[2 \\
& *c + d*x] + 4260*a^2*b^4*d*x*Cosh[2*c + d*x] + 2496*a*b^5*d*x*Cosh[2*c + d* \\
& x] + 576*b^6*d*x*Cosh[2*c + d*x] - 36*a^6*d*x*Cosh[4*c + d*x] - 336*a^5*b*d \\
& *x*Cosh[4*c + d*x] - 1560*a^4*b^2*d*x*Cosh[4*c + d*x] - 3600*a^3*b^3*d*x*Co \\
& sh[4*c + d*x] - 4260*a^2*b^4*d*x*Cosh[4*c + d*x] - 2496*a*b^5*d*x*Cosh[4*c \\
& + d*x] - 576*b^6*d*x*Cosh[4*c + d*x] - 36*a^6*d*x*Cosh[2*c + 3*d*x] - 240*a \\
& ^5*b*d*x*Cosh[2*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh[2*c + 3*d*x] + 48*a^3*b^3 \\
& *d*x*Cosh[2*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[2*c + 3*d*x] + 672*a*b^5*d*x* \\
& Cosh[2*c + 3*d*x] + 192*b^6*d*x*Cosh[2*c + 3*d*x] + 36*a^6*d*x*Cosh[4*c + 3 \\
& *d*x] + 240*a^5*b*d*x*Cosh[4*c + 3*d*x] + 408*a^4*b^2*d*x*Cosh[4*c + 3*d*x] \\
& - 48*a^3*b^3*d*x*Cosh[4*c + 3*d*x] - 732*a^2*b^4*d*x*Cosh[4*c + 3*d*x] - 6 \\
& 72*a*b^5*d*x*Cosh[4*c + 3*d*x] - 192*b^6*d*x*Cosh[4*c + 3*d*x] - 36*a^6*d*x \\
& *Cosh[6*c + 3*d*x] - 240*a^5*b*d*x*Cosh[6*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh \\
& [6*c + 3*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[6*c \\
& + 3*d*x] + 672*a*b^5*d*x*Cosh[6*c + 3*d*x] + 192*b^6*d*x*Cosh[6*c + 3*d*x] \\
& - 12*a^6*d*x*Cosh[2*c + 5*d*x] - 144*a^5*b*d*x*Cosh[2*c + 5*d*x] - 456*a^4 \\
& *b^2*d*x*Cosh[2*c + 5*d*x] - 624*a^3*b^3*d*x*Cosh[2*c + 5*d*x] - 396*a^2*b^ \\
& 4*d*x*Cosh[2*c + 5*d*x] - 96*a*b^5*d*x*Cosh[2*c + 5*d*x] + 12*a^6*d*x*Cosh[\\
& 4*c + 5*d*x] + 144*a^5*b*d*x*Cosh[4*c + 5*d*x] + 456*a^4*b^2*d*x*Cosh[4*c + \\
& 5*d*x] + 624*a^3*b^3*d*x*Cosh[4*c + 5*d*x] + 396*a^2*b^4*d*x*Cosh[4*c + 5* \\
& d*x] + 96*a*b^5*d*x*Cosh[4*c + 5*d*x] - 12*a^6*d*x*Cosh[6*c + 5*d*x] - 144* \\
& a^5*b*d*x*Cosh[6*c + 5*d*x] - 456*a^4*b^2*d*x*Cosh[6*c + 5*d*x] - 624*a^3*b \\
& ^3*d*x*Cosh[6*c + 5*d*x] - 396*a^2*b^4*d*x*Cosh[6*c + 5*d*x] - 96*a*b^5*d*x \\
& *Cosh[6*c + 5*d*x] + 12*a^6*d*x*Cosh[8*c + 5*d*x] + 144*a^5*b*d*x*Cosh[8*c \\
& + 5*d*x] + 456*a^4*b^2*d*x*Cosh[8*c + 5*d*x] + 624*a^3*b^3*d*x*Cosh[8*c + 5 \\
& *d*x] + 396*a^2*b^4*d*x*Cosh[8*c + 5*d*x] + 96*a*b^5*d*x*Cosh[8*c + 5*d*x] \\
& - 12*a^6*d*x*Cosh[4*c + 7*d*x] - 48*a^5*b*d*x*Cosh[4*c + 7*d*x] - 72*a^4*b^ \\
& 2*d*x*Cosh[4*c + 7*d*x] - 48*a^3*b^3*d*x*Cosh[4*c + 7*d*x] - 12*a^2*b^4*d*x \\
& *Cosh[4*c + 7*d*x] + 12*a^6*d*x*Cosh[6*c + 7*d*x] + 48*a^5*b*d*x*Cosh[6*c + \\
& 7*d*x] + 72*a^4*b^2*d*x*Cosh[6*c + 7*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 7*d* \\
& x] + 12*a^2*b^4*d*x*Cosh[6*c + 7*d*x] - 12*a^6*d*x*Cosh[8*c + 7*d*x] - 48*a \\
& ^5*b*d*x*Cosh[8*c + 7*d*x] - 72*a^4*b^2*d*x*Cosh[8*c + 7*d*x] - 48*a^3*b^3* \\
& d*x*Cosh[8*c + 7*d*x] - 12*a^2*b^4*d*x*Cosh[8*c + 7*d*x] + 12*a^6*d*x*Cosh[\\
& 10*c + 7*d*x] + 48*a^5*b*d*x*Cosh[10*c + 7*d*x] + 72*a^4*b^2*d*x*Cosh[10*c \\
& + 7*d*x] + 48*a^3*b^3*d*x*Cosh[10*c + 7*d*x] + 12*a^2*b^4*d*x*Cosh[10*c + 7 \\
& *d*x] - 128*a^6*Sinh[d*x] - 440*a^5*b*Sinh[d*x] - 1152*a^4*b^2*Sinh[d*x] - \\
& 1920*a^3*b^3*Sinh[d*x] + 228*a^2*b^4*Sinh[d*x] + 1320*a*b^5*Sinh[d*x] + 432 \\
& *b^6*Sinh[d*x] + 48*a^6*Sinh[3*d*x] + 104*a^5*b*Sinh[3*d*x] + 640*a^4*b^2*S \\
& inh[3*d*x] + 1511*a^3*b^3*Sinh[3*d*x] - 528*a^2*b^4*Sinh[3*d*x] + 264*a*b^5 \\
& *Sinh[3*d*x] + 144*b^6*Sinh[3*d*x] - 32*a^6*Sinh[2*c - d*x] + 384*a^5*b*Sinh \\
& [2*c - d*x] + 2048*a^4*b^2*Sinh[2*c - d*x] + 3072*a^3*b^3*Sinh[2*c - d*x] \\
& + 228*a^2*b^4*Sinh[2*c - d*x] + 1320*a*b^5*Sinh[2*c - d*x] + 432*b^6*Sinh[2 \\
& *c - d*x] + 32*a^6*Sinh[2*c + d*x] - 384*a^5*b*Sinh[2*c + d*x] - 2048*a^4*b \\
& ^2*Sinh[2*c + d*x] - 2919*a^3*b^3*Sinh[2*c + d*x] + 642*a^2*b^4*Sinh[2*c + \\
& d*x] + 1416*a*b^5*Sinh[2*c + d*x] + 432*b^6*Sinh[2*c + d*x] - 128*a^6*Sinh[\\
& 4*c + d*x] - 440*a^5*b*Sinh[4*c + d*x] - 1152*a^4*b^2*Sinh[4*c + d*x] - 207 \\
& 3*a^3*b^3*Sinh[4*c + d*x] - 642*a^2*b^4*Sinh[4*c + d*x] - 1416*a*b^5*Sinh[4 \\
& *c + d*x] - 432*b^6*Sinh[4*c + d*x] - 144*a^6*Sinh[2*c + 3*d*x] - 672*a^5*b \\
& *Sinh[2*c + 3*d*x] - 960*a^4*b^2*Sinh[2*c + 3*d*x] + 153*a^3*b^3*Sinh[2*c + \\
& 3*d*x] + 528*a^2*b^4*Sinh[2*c + 3*d*x] - 264*a*b^5*Sinh[2*c + 3*d*x] - 144 \\
& *b^6*Sinh[2*c + 3*d*x] + 48*a^6*Sinh[4*c + 3*d*x] + 104*a^5*b*Sinh[4*c + 3* \\
& d*x] + 640*a^4*b^2*Sinh[4*c + 3*d*x] + 1664*a^3*b^3*Sinh[4*c + 3*d*x] - 66* \\
& a^2*b^4*Sinh[4*c + 3*d*x] - 408*a*b^5*Sinh[4*c + 3*d*x] - 144*b^6*Sinh[4*c \\
& + 3*d*x] - 144*a^6*Sinh[6*c + 3*d*x] - 672*a^5*b*Sinh[6*c + 3*d*x] - 960*a^ \\
& 4*b^2*Sinh[6*c + 3*d*x] + 66*a^2*b^4*Sinh[6*c + 3*d*x] + 408*a*b^5*Sinh[6*c \\
& + 3*d*x] + 144*b^6*Sinh[6*c + 3*d*x] + 80*a^6*Sinh[2*c + 5*d*x] + 480*a^5* \\
& b*Sinh[2*c + 5*d*x] + 832*a^4*b^2*Sinh[2*c + 5*d*x] + 294*a^2*b^4*Sinh[2*c
\end{aligned}$$

```
+ 5*d*x] + 96*a*b^5*Sinh[2*c + 5*d*x] - 48*a^6*Sinh[4*c + 5*d*x] - 120*a^5*
b*Sinh[4*c + 5*d*x] - 294*a^2*b^4*Sinh[4*c + 5*d*x] - 96*a*b^5*Sinh[4*c + 5
*d*x] + 80*a^6*Sinh[6*c + 5*d*x] + 480*a^5*b*Sinh[6*c + 5*d*x] + 832*a^4*b^
2*Sinh[6*c + 5*d*x] - 51*a^3*b^3*Sinh[6*c + 5*d*x] - 132*a^2*b^4*Sinh[6*c +
5*d*x] - 48*a*b^5*Sinh[6*c + 5*d*x] - 48*a^6*Sinh[8*c + 5*d*x] - 120*a^5*b
*Sinh[8*c + 5*d*x] + 51*a^3*b^3*Sinh[8*c + 5*d*x] + 132*a^2*b^4*Sinh[8*c +
5*d*x] + 48*a*b^5*Sinh[8*c + 5*d*x] + 32*a^6*Sinh[4*c + 7*d*x] + 104*a^5*b*
Sinh[4*c + 7*d*x] + 51*a^3*b^3*Sinh[4*c + 7*d*x] + 18*a^2*b^4*Sinh[4*c + 7*
d*x] - 51*a^3*b^3*Sinh[6*c + 7*d*x] - 18*a^2*b^4*Sinh[6*c + 7*d*x] + 32*a^6
*Sinh[8*c + 7*d*x] + 104*a^5*b*Sinh[8*c + 7*d*x]))/(6144*a^3*(a + b)^4*d*(a
+ b*Sech[c + d*x]^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 2.59, size = 482, normalized size = 2.08

$$\frac{3(63a^2b^3e^{2c}+36ab^4e^{2c}+8b^5e^{2c})\arctan\left(\frac{ae^{2dx+2c}+a+2b}{2\sqrt{-ab-b^2}}\right)e^{(-2c)}}{(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4)\sqrt{-ab-b^2}} - \frac{24dx}{a^3} - \frac{6(17a^3b^3e^{6dx+6c}+44a^2b^4e^{6dx+6c}+16ab^5e^{6dx+6c}+51a^3)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3*e^(2*c) + 36*a*b^4*e^(2*c) + 8*b^5*e^(2*c))*arctan(1/2
*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^7 + 4*a^6*b +
6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt(-a*b - b^2)) - 24*d*x/a^3 - 6*(17*a^
3*b^3*e^(6*d*x + 6*c) + 44*a^2*b^4*e^(6*d*x + 6*c) + 16*a*b^5*e^(6*d*x + 6*
c) + 51*a^3*b^3*e^(4*d*x + 4*c) + 154*a^2*b^4*e^(4*d*x + 4*c) + 184*a*b^5*e
^(4*d*x + 4*c) + 48*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x + 2*c) + 116*
a^2*b^4*e^(2*d*x + 2*c) + 32*a*b^5*e^(2*d*x + 2*c) + 17*a^3*b^3 + 6*a^2*b^4
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) + 16*(6*a*e^(4*d*x + 4*c)
+ 15*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 24*b*e^(2*d*x + 2*c) + 4*a +
13*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) - 1)^3
))/d
```

maple [B] time = 0.57, size = 1610, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-5/8/d/(
a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*a*tanh(1/2*d*x+1/2*c)-17/8/d/(a^3+3*a^2*b+3*
a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)*b-1/24/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^3
-5/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*a-17/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*b-1
/2/d*b^(9/2)/(a+b)^(9/2)/a^3*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)
*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+63/16/d*b^(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-63/16/
d*b^(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh
```

$$\begin{aligned} & (1/2*d*x+1/2*c)+(a+b)^{(1/2)}+9/4/d*b^{(7/2)}/(a+b)^{(9/2)}/a^2*\ln((a+b)^{(1/2)}* \\ & \operatorname{anh}(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-9/4/d*b^{(7/ \\ & 2)}/(a+b)^{(9/2)}/a^2*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2* \\ & d*x+1/2*c)+(a+b)^{(1/2)})+1/2/d*b^{(9/2)}/(a+b)^{(9/2)}/a^3*\ln((a+b)^{(1/2)}*\tanh(1 \\ & /2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)})-17/4/d*b^3/(a+b) \\ & ^4/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2 \\ & *a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)-17/4/d*b^3/(a+b)^4/ \\ & (\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a- \\ & 2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^7-51/4/d*b^3/(a+b)^4/(\\ & \tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2 \\ & *\tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^5-51/4/d*b^3/(a+b)^4/(\\ & \operatorname{anh}(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2* \\ & \tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*\tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3+3*a^2*b+3* \\ & a*b^2+b^3)/(a+b)*a*\tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(\\ & a+b)*\tanh(1/2*d*x+1/2*c)^3*b-1/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b \\ & *\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+ \\ & a+b)^2*\tanh(1/2*d*x+1/2*c)-3/4/d*b^4/(a+b)^4/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*t \\ & \operatorname{anh}(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & b)^2*\tanh(1/2*d*x+1/2*c)^3-3/4/d*b^4/(a+b)^4/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*t \\ & \operatorname{anh}(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & b)^2*\tanh(1/2*d*x+1/2*c)^5-21/4/d*b^4/(a+b)^4/a/(\tanh(1/2*d*x+1/2*c)^4*a+b* \\ & \tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & +b)^2*\tanh(1/2*d*x+1/2*c)^7-1/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b* \\ & \tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & +b)^2*\tanh(1/2*d*x+1/2*c)^7+1/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b* \\ & \tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & +b)^2*\tanh(1/2*d*x+1/2*c)^5+1/d*b^5/(a+b)^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b* \\ & \tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & +b)^2*\tanh(1/2*d*x+1/2*c)^3-21/4/d*b^4/(a+b)^4/a/(\tanh(1/2*d*x+1/2*c)^4*a+b \\ & *\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+ \\ & a+b)^2*\tanh(1/2*d*x+1/2*c) \end{aligned}$$

maxima [B] time = 1.84, size = 4920, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{8}*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) - \frac{3}{4}*b*\log(a*e^{(4*d*x + 4*c)} + 2*(a + 2*b)*e^{(2*d*x + 2*c)} + a)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - \frac{1}{8}*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + \frac{3}{4}*b*\log(2*(a + 2*b)*e^{(-2*d*x - 2*c)} + a*e^{(-4*d*x - 4*c)} + a)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + \frac{1}{4}*(2*a + 5*b)*\log(e^{(2*d*x + 2*c)} - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + \frac{3}{2}*b*\log(e^{(2*d*x + 2*c)} - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - \frac{1}{4}*(2*a + 5*b)*\log(e^{(-2*d*x - 2*c)} - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - \frac{3}{2}*b*\log(e^{(-2*d*x - 2*c)} - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - \frac{1}{256}*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*\sqrt{(a + b)*b}*d) + \frac{5}{64}*(3*a*b + 10*b^2)*\log((a*e^{(2*d*x + 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(2*d*x + 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\sqrt{(a + b)*b}*d) + \frac{1}{256}*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*\sqrt{(a + b)*b}*d) - \frac{5}{$

$$\begin{aligned}
& 64*(3*a*b + 10*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/ \\
& (a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^4 + 4*a^3*b + 6*a^2*b^2 \\
& *b^2 + 4*a*b^3 + b^4)*\sqrt{(a + b)*b}*d) + 15/128*(3*a*b - 4*b^2)*\log((a*e^{(-2*d*x - 2*c)} + a + 2*b - 2*\sqrt{(a + b)*b})/(a*e^{(-2*d*x - 2*c)} + a + 2*b + 2*\sqrt{(a + b)*b}))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\sqrt{(a + b)*b}*d) + 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*a^2*b^4 + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3 - 832*a^2*b^4 - 256*a*b^5)*e^{(12*d*x + 12*c)} + 6*(120*a^6 + 528*a^5*b + 525*a^4*b^2 - 52*a^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*b^6)*e^{(10*d*x + 10*c)} + (176*a^6 + 1337*a^5*b + 7554*a^4*b^2 + 16416*a^3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 + 6912*b^6)*e^{(8*d*x + 8*c)} - 4*(184*a^6 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1728*b^6)*e^{(6*d*x + 6*c)} - (384*a^6 + 1177*a^5*b - 736*a^4*b^2 + 112*a^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b^6)*e^{(4*d*x + 4*c)} + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3 + 1896*a^2*b^4 + 576*a*b^5)*e^{(2*d*x + 2*c)}))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4 - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^{(14*d*x + 14*c)} - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^{(12*d*x + 12*c)} + (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^{(10*d*x + 10*c)} + (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^{(8*d*x + 8*c)} - (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^{(6*d*x + 6*c)} - (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^{(4*d*x + 4*c)} + (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^{(2*d*x + 2*c)}))*d) - 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*a^2*b^4 + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3 + 1896*a^2*b^4 + 576*a*b^5)*e^{(-2*d*x - 2*c)} - (384*a^6 + 1177*a^5*b - 736*a^4*b^2 + 112*a^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b^6)*e^{(-4*d*x - 4*c)} - 4*(184*a^6 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1728*b^6)*e^{(-6*d*x - 6*c)} + (176*a^6 + 1337*a^5*b + 7554*a^4*b^2 + 16416*a^3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 + 6912*b^6)*e^{(-8*d*x - 8*c)} + 6*(120*a^6 + 528*a^5*b + 525*a^4*b^2 - 52*a^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*b^6)*e^{(-10*d*x - 10*c)} + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3 - 832*a^2*b^4 - 256*a*b^5)*e^{(-12*d*x - 12*c)}))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4 + (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^{(-2*d*x - 2*c)} - (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^{(-4*d*x - 4*c)} - (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^{(-6*d*x - 6*c)} + (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^{(-8*d*x - 8*c)} + (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^{(-10*d*x - 10*c)} - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^{(-12*d*x - 12*c)} - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^{(-14*d*x - 14*c)}))*d) + 1/48*(32*a^5 + 77*a^4*b - 72*a^3*b^2 - 12*a^2*b^3 + 3*(32*a^5 + 65*a^4*b + 94*a^3*b^2 + 128*a^2*b^3 + 32*a*b^4)*e^{(12*d*x + 12*c)} + 6*(48*a^5 + 200*a^4*b + 203*a^3*b^2 + 90*a^2*b^3 + 176*a*b^4 + 32*b^5)*e^{(10*d*x + 10*c)} + (224*a^5 + 839*a^4*b + 1500*a^3*b^2 - 612*a^2*b^3 - 3648*a*b^4 - 576*b^5)*e^{(8*d*x + 8*c)} - 4*(16*a^5 + 216*a^4*b + 695*a^3*b^2 + 252*a^2*b^3 - 912*a*b^4 - 144*b^5)*e^{(6*d*x + 6*c)} - (96*a^5 + 343*a^4*b - 850*a^3*b^2 - 1808*a^2*b^3 + 1056*a*b^4 + 192*b^5)*e^{(4*d*x + 4*c)} + 2*(16*a^5 + 216*a^4*b + 269*a^3*b^2 - 294*a^2*b^3 - 48*a*b^4)*e^{(2*d*x + 2*c)}))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 - (a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*e^{(14*d*x + 14*c)} - (a^8 + 12*a^7*b + 38*a^6*b^2 + 52*a^5*b^3 + 33*a^4*b^4 + 8*a^3*b^5)*e^{(12*d*x + 12*c)} + (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^4*b^4 - 56*a^3*b^5 - 16*a^2*b^6)*e^{(10*d*x + 10*c)} + (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^{(8*d*x + 8*c)} - (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^{(6*d*x + 6*c)} - (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^4*b^4 - 56*a^3*b^5 - 16*a^2*b^6)*e^{(4*d*x + 4*c)} + (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^{(2*d*x + 2*c)} - (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^{(0*d*x + 0*c)}))
\end{aligned}$$

$$\begin{aligned}
& b^6)e^{(4dx + 4c)} + (a^8 + 12a^7b + 38a^6b^2 + 52a^5b^3 + 33a^4b^4 + 8a^3b^5)e^{(2dx + 2c)}d - 1/48(32a^5 + 77a^4b - 72a^3b^2 \\
& - 12a^2b^3 + 2(16a^5 + 216a^4b + 269a^3b^2 - 294a^2b^3 - 48ab^4) \\
&)e^{(-2dx - 2c)} - (96a^5 + 343a^4b - 850a^3b^2 - 1808a^2b^3 + 105 \\
& 6ab^4 + 192b^5)e^{(-4dx - 4c)} - 4(16a^5 + 216a^4b + 695a^3b^2 + \\
& 252a^2b^3 - 912ab^4 - 144b^5)e^{(-6dx - 6c)} + (224a^5 + 839a^4b \\
& + 1500a^3b^2 - 612a^2b^3 - 3648ab^4 - 576b^5)e^{(-8dx - 8c)} + 6 \\
& (48a^5 + 200a^4b + 203a^3b^2 + 90a^2b^3 + 176ab^4 + 32b^5)e^{(-10 \\
& dx - 10c)} + 3(32a^5 + 65a^4b + 94a^3b^2 + 128a^2b^3 + 32ab^4) \\
& e^{(-12dx - 12c)}/((a^8 + 4a^7b + 6a^6b^2 + 4a^5b^3 + a^4b^4 + (a^8 \\
& + 12a^7b + 38a^6b^2 + 52a^5b^3 + 33a^4b^4 + 8a^3b^5)e^{(-2dx \\
& - 2c)} - (3a^8 + 20a^7b + 34a^6b^2 - 4a^5b^3 - 61a^4b^4 - 56a^3b^5 \\
& ^5 - 16a^2b^6)e^{(-4dx - 4c)} - (3a^8 + 28a^7b + 130a^6b^2 + 300a^5 \\
& ^5b^3 + 355a^4b^4 + 208a^3b^5 + 48a^2b^6)e^{(-6dx - 6c)} + (3a^8 \\
& + 28a^7b + 130a^6b^2 + 300a^5b^3 + 355a^4b^4 + 208a^3b^5 + 48a^2 \\
& ^5b^6)e^{(-8dx - 8c)} + (3a^8 + 20a^7b + 34a^6b^2 - 4a^5b^3 - 61a^4 \\
& ^4b^4 - 56a^3b^5 - 16a^2b^6)e^{(-10dx - 10c)} - (a^8 + 12a^7b + 38 \\
& ^6b^2 + 52a^5b^3 + 33a^4b^4 + 8a^3b^5)e^{(-12dx - 12c)} - (a^8 + \\
& 4a^7b + 6a^6b^2 + 4a^5b^3 + a^4b^4)e^{(-14dx - 14c)}d) + 1/32(1 \\
& 6a^4 - 83a^3b + 6a^2b^2 + 2(8a^4 - 299a^2b^2 + 24ab^3)e^{(-2dx \\
& - 2c)} - (96a^4 + 71a^3b - 344a^2b^2 + 1208ab^3 - 48b^4)e^{(-4dx \\
& - 4c)} - 4(56a^4 + 144a^3b + 31a^2b^2 - 546ab^3 + 36b^4)e^{(-6dx \\
& x - 6c)} - (176a^4 + 569a^3b + 666a^2b^2 + 1704ab^3 - 144b^4)e^{(-8 \\
& dx - 8c)} - 6(8a^4 + 32a^3b + 93a^2b^2 - 28ab^3 + 8b^4)e^{(-10dx \\
& *x - 10c)} - 15(3a^3b - 4a^2b^2)e^{(-12dx - 12c)})/((a^7 + 4a^6b + \\
& 6a^5b^2 + 4a^4b^3 + a^3b^4 + (a^7 + 12a^6b + 38a^5b^2 + 52a^4b^3 \\
& + 33a^3b^4 + 8a^2b^5)e^{(-2dx - 2c)} - (3a^7 + 20a^6b + 34a^5b^2 \\
& ^2 - 4a^4b^3 - 61a^3b^4 - 56a^2b^5 - 16ab^6)e^{(-4dx - 4c)} - (3 \\
& ^7 + 28a^6b + 130a^5b^2 + 300a^4b^3 + 355a^3b^4 + 208a^2b^5 + 48 \\
& ^5b^6)e^{(-6dx - 6c)} + (3a^7 + 28a^6b + 130a^5b^2 + 300a^4b^3 + \\
& 355a^3b^4 + 208a^2b^5 + 48ab^6)e^{(-8dx - 8c)} + (3a^7 + 20a^6b \\
& + 34a^5b^2 - 4a^4b^3 - 61a^3b^4 - 56a^2b^5 - 16ab^6)e^{(-10dx - \\
& 10c)} - (a^7 + 12a^6b + 38a^5b^2 + 52a^4b^3 + 33a^3b^4 + 8a^2b^5) \\
&)e^{(-12dx - 12c)} - (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4)e^{ \\
& (-14dx - 14c)}d)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + dx)^6 \coth(c + dx)^4}{(a \cosh(c + dx)^2 + b)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)

[Out] int((cosh(c + d*x)^6*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)

[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)

$$3.169 \quad \int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^4} dx$$

Optimal. Leaf size=207

$$\frac{x}{a^4} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2d(a+b)^2(a-b\tanh^2(c+dx)+b)^2} - \frac{b(19a^2+22ab+8b^2)\tanh(c+dx)}{16a^3d(a+b)^3(a-b\tanh^2(c+dx)+b)} - \frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{24a^2d(a+b)^2(a-b\tanh^2(c+dx)+b)^2}$$

[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^3-1/24*b*(11*a+6*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.35, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4128, 414, 527, 522, 206, 208}

$$\frac{\sqrt{b}(70a^2b+35a^3+56ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}} - \frac{b(19a^2+22ab+8b^2)\tanh(c+dx)}{16a^3d(a+b)^3(a-b\tanh^2(c+dx)+b)} - \frac{x}{24a^2d(a+b)^2(a-b\tanh^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-4), x]

[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tanh[c + d*x])/(6*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tanh[c + d*x])/(24*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,

c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^4} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^4} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{b \tanh(c + dx)}{6a(a + b)d (a + b - b \tanh^2(c + dx))^3} - \frac{\operatorname{Subst}\left(\int \frac{-6a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3} dx, x, \tanh(c + dx)\right)}{6a(a + b)d}$$

$$= -\frac{b \tanh(c + dx)}{6a(a + b)d (a + b - b \tanh^2(c + dx))^3} - \frac{b(11a + 6b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b - b \tanh^2(c + dx))^2}$$

$$= -\frac{b \tanh(c + dx)}{6a(a + b)d (a + b - b \tanh^2(c + dx))^3} - \frac{b(11a + 6b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b - b \tanh^2(c + dx))^2}$$

$$= -\frac{b \tanh(c + dx)}{6a(a + b)d (a + b - b \tanh^2(c + dx))^3} - \frac{b(11a + 6b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b - b \tanh^2(c + dx))^2}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b} (35a^3 + 70a^2b + 56ab^2 + 16b^3) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a + b)^{7/2}d} - \frac{b \tanh(c + dx)}{6a(a + b)d (a + b - b \tanh^2(c + dx))^3}$$

Mathematica [C] time = 6.93, size = 1405, normalized size = 6.79

$$(35a^3 + 70ba^2 + 56b^2a + 16b^3) (\cosh(2c + 2dx)a + a + 2b)^4 \left(\frac{ib \tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i \sinh(2c)}{2\sqrt{a+b} \sqrt{b \cosh(4c)-b \sinh(4c)}} - \frac{i \cosh(2c)}{2\sqrt{a+b} \sqrt{b \cosh(4c)-b \sinh(4c)}}\right)\right)}{256a^4 \sqrt{a+b} d \sqrt{b \cosh(4c)-b \sinh(4c)}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-4), x]
```



```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^4*
Sech[c + d*x]^8*((I/256)*b*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a
+ b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt
[b*Cosh[4*c] - b*Sinh[4*c]])))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c
+ d*x])*Cosh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - (
(I/256)*b*ArcTan[Sech[d*x]*((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4
*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*S
inh[4*c]])))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])*Sinh[2*c]
)/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*S
ech[c + d*x]^2)^4) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[c + d*
x]^8*(480*a^6*d*x*Cosh[2*c] + 3168*a^5*b*d*x*Cosh[2*c] + 8928*a^4*b^2*d*x*C
osh[2*c] + 14112*a^3*b^3*d*x*Cosh[2*c] + 13248*a^2*b^4*d*x*Cosh[2*c] + 6912
*a*b^5*d*x*Cosh[2*c] + 1536*b^6*d*x*Cosh[2*c] + 360*a^6*d*x*Cosh[2*d*x] + 2
232*a^5*b*d*x*Cosh[2*d*x] + 5688*a^4*b^2*d*x*Cosh[2*d*x] + 7272*a^3*b^3*d*x
*Cosh[2*d*x] + 4608*a^2*b^4*d*x*Cosh[2*d*x] + 1152*a*b^5*d*x*Cosh[2*d*x] +
360*a^6*d*x*Cosh[4*c + 2*d*x] + 2232*a^5*b*d*x*Cosh[4*c + 2*d*x] + 5688*a^4
*b^2*d*x*Cosh[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 4608*a^2*
b^4*d*x*Cosh[4*c + 2*d*x] + 1152*a*b^5*d*x*Cosh[4*c + 2*d*x] + 144*a^6*d*x*
Cosh[2*c + 4*d*x] + 720*a^5*b*d*x*Cosh[2*c + 4*d*x] + 1296*a^4*b^2*d*x*Cosh
[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[2*c + 4*d*x] + 288*a^2*b^4*d*x*Cosh[2
*c + 4*d*x] + 144*a^6*d*x*Cosh[6*c + 4*d*x] + 720*a^5*b*d*x*Cosh[6*c + 4*d*
x] + 1296*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[6*c + 4*d*x
] + 288*a^2*b^4*d*x*Cosh[6*c + 4*d*x] + 24*a^6*d*x*Cosh[4*c + 6*d*x] + 72*a
^5*b*d*x*Cosh[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[4*c + 6*d*x] + 24*a^3*b^3*
d*x*Cosh[4*c + 6*d*x] + 24*a^6*d*x*Cosh[8*c + 6*d*x] + 72*a^5*b*d*x*Cosh[8*
c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cosh[8*c + 6
*d*x] + 870*a^5*b*Sinh[2*c] + 4292*a^4*b^2*Sinh[2*c] + 8792*a^3*b^3*Sinh[2*
c] + 9936*a^2*b^4*Sinh[2*c] + 5824*a*b^5*Sinh[2*c] + 1408*b^6*Sinh[2*c] - 8
70*a^5*b*Sinh[2*d*x] - 3792*a^4*b^2*Sinh[2*d*x] - 6432*a^3*b^3*Sinh[2*d*x]
- 4608*a^2*b^4*Sinh[2*d*x] - 1248*a*b^5*Sinh[2*d*x] + 435*a^5*b*Sinh[4*c +
2*d*x] + 2124*a^4*b^2*Sinh[4*c + 2*d*x] + 3972*a^3*b^3*Sinh[4*c + 2*d*x] +
3072*a^2*b^4*Sinh[4*c + 2*d*x] + 864*a*b^5*Sinh[4*c + 2*d*x] - 435*a^5*b*Si
nh[2*c + 4*d*x] - 1374*a^4*b^2*Sinh[2*c + 4*d*x] - 1248*a^3*b^3*Sinh[2*c +
4*d*x] - 384*a^2*b^4*Sinh[2*c + 4*d*x] + 87*a^5*b*Sinh[6*c + 4*d*x] + 366*a
^4*b^2*Sinh[6*c + 4*d*x] + 408*a^3*b^3*Sinh[6*c + 4*d*x] + 144*a^2*b^4*Sinh
[6*c + 4*d*x] - 87*a^5*b*Sinh[4*c + 6*d*x] - 116*a^4*b^2*Sinh[4*c + 6*d*x]
- 44*a^3*b^3*Sinh[4*c + 6*d*x]))/(3072*a^4*(a + b)^3*d*(a + b*Sech[c + d*x]
^2)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.85, size = 594, normalized size = 2.87

$$\frac{3(35a^3b+70a^2b^2+56ab^3+16b^4)\arctan\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^7+3a^6b+3a^5b^2+a^4b^3)\sqrt{-ab-b^2}} - \frac{2(87a^5be^{(10dx+10c)}+366a^4b^2e^{(10dx+10c)}+408a^3b^3e^{(10dx+10c)}+144a^2b^4e^{(10dx+10c)})}{(a^7+3a^6b+3a^5b^2+a^4b^3)\sqrt{-ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(1/2*(a*e^(2*d*x
+ 2*c) + a + 2*b)/sqrt(-a*b - b^2)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
```

$$\begin{aligned} & \sqrt{-ab - b^2}) - 2(87a^5b^2e^{(10dx + 10c)} + 366a^4b^2e^{(10dx + 10c)} + 408a^3b^3e^{(10dx + 10c)} + 144a^2b^4e^{(10dx + 10c)} + 4 \\ & 35a^5b^2e^{(8dx + 8c)} + 2124a^4b^2e^{(8dx + 8c)} + 3972a^3b^3e^{(8dx + 8c)} + 3072a^2b^4e^{(8dx + 8c)} + 864ab^5e^{(8dx + 8c)} + 87 \\ & 0a^5b^2e^{(6dx + 6c)} + 4292a^4b^2e^{(6dx + 6c)} + 8792a^3b^3e^{(6dx + 6c)} + 9936a^2b^4e^{(6dx + 6c)} + 5824ab^5e^{(6dx + 6c)} + 14 \\ & 08b^6e^{(6dx + 6c)} + 870a^5b^2e^{(4dx + 4c)} + 3792a^4b^2e^{(4dx + 4c)} + 6432a^3b^3e^{(4dx + 4c)} + 4608a^2b^4e^{(4dx + 4c)} + 1248 \\ & ab^5e^{(4dx + 4c)} + 435a^5b^2e^{(2dx + 2c)} + 1374a^4b^2e^{(2dx + 2c)} + 1248a^3b^3e^{(2dx + 2c)} + 384a^2b^4e^{(2dx + 2c)} + 87a^5 \\ & 5b + 116a^4b^2 + 44a^3b^3) / ((a^7 + 3a^6b + 3a^5b^2 + a^4b^3)(a^{4dx + 4c} + 2ae^{(2dx + 2c)} + 4b^2e^{(2dx + 2c)} + a^3) - 48(dx + c)/a^4) / d \end{aligned}$$

maple [B] time = 0.45, size = 2880, normalized size = 13.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c))^2)^4,x)

[Out]
$$\begin{aligned} & -2/d*b^5/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7-37/4/d*b^3/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7-1/2/d*b^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7-145/4/d*b*a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7-145/8/d*b/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3-37/d*b^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*\tanh(1/2*d*x+1/2*c)^5-13/4/d*b^2/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)^11-1/d*b^3/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)^11-281/24/d*b^2/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+11/4/d*b^3/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+3/d*b^4/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9-37/4/d*b^3/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*\tanh(1/2*d*x+1/2*c)^5-1/2/d*b^4/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*\tanh(1/2*d*x+1/2*c)^5-2/d*b^5/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*\tanh(1/2*d*x+1/2*c)^5-29/8/d*b/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)^11-1/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)-145/4/d*b*a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*\tanh(1/2*d*x+1/2*c)^5-29/8/d*b/a/(\tanh(1/2*d*x+1/2*c)^4*a+b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)-35/16/d*b^(3/2)/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a \end{aligned}$$

$(+b)^{(1/2)} - 7/4/d*b^{(5/2)}/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) - 37/d*b^2/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+35/32/d*b^{(1/2)}/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) - 1/2/d*b^{(7/2)}/a^4/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) + 35/16/d*b^{(3/2)}/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) + 1/2/d*b^{(7/2)}/a^4/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) - 35/32/d*b^{(1/2)}/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2+2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) + 7/4/d*b^{(5/2)}/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^{(1/2)}*\ln((a+b)^{(1/2)}*\tanh(1/2*d*x+1/2*c)^2-2*b^{(1/2)}*\tanh(1/2*d*x+1/2*c)+(a+b)^{(1/2)}) - 281/24/d*b^2/a/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+11/4/d*b^3/a^2/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+3/d*b^4/a^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/a^2/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)-1/d*b^3/a^3/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*\tanh(1/2*d*x+1/2*c)-145/8/d*b/(\tanh(1/2*d*x+1/2*c)^4+a*b*\tanh(1/2*d*x+1/2*c)^4+2*\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9$

maxima [B] time = 0.84, size = 718, normalized size = 3.47

$$\frac{(35 a^3 b + 70 a^2 b^2 + 56 a b^3 + 16 b^4) \log\left(\frac{a e^{(-2 d x - 2 c) + a + 2 b - 2 \sqrt{(a+b)b}}}{a e^{(-2 d x - 2 c) + a + 2 b + 2 \sqrt{(a+b)b}}}\right)}{32 (a^7 + 3 a^6 b + 3 a^5 b^2 + a^4 b^3) \sqrt{(a+b)b} d} - \frac{87 a^5 b + 116 a^4 b^2 + \dots}{24 (a^{10} + 3 a^9 b + 3 a^8 b^2 + a^7 b^3 + 6 (a^{10} + \dots))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c))^2)^4,x, algorithm="maxima")

[Out] 1/32*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)*d) - 1/24*(87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3 + 3*(145*a^5*b + 458*a^4*b^2 + 416*a^3*b^3 + 128*a^2*b^4)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 632*a^4*b^2 + 1072*a^3*b^3 + 768*a^2*b^4 + 208*a*b^5)*e^(-4*d*x - 4*c) + 2*(435*a^5*b + 2146*a^4*b^2 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6)*e^(-6*d*x - 6*c) + 3*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4)*e^(-10*d*x - 10*c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3 + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-2*d*x - 2*c) + 3*(5*a^10 + 31*a^9*b + 79*a^8*b^2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-4*d*x - 4*c) + 4*(5*a^10 + 33*a^9*b + 93*a^8*b^2 + 147*a^7*b^3 + 138*a^6*b^4 + 72*a^5*b^5 + 16*a^4*b^6)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 31*a^9*b + 79*a^8*b^2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-8*d*x - 8*c) + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-10*d*x - 10*c) + (a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*e^(-12*d*x - 12*c))*d) + (d*x + c)/(a^4*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^4,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^4, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

$$3.170 \quad \int (1 - \operatorname{sech}^2(x))^{3/2} dx$$

Optimal. Leaf size=29

$$\sqrt{\tanh^2(x) \operatorname{coth}(x) \log(\cosh(x))} - \frac{1}{2} \tanh^2(x)^{3/2} \operatorname{coth}(x)$$

[Out] $\operatorname{coth}(x) \ln(\cosh(x)) (\tanh(x)^2)^{(1/2)} - 1/2 \operatorname{coth}(x) (\tanh(x)^2)^{(3/2)}$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4121, 3658, 3473, 3475}

$$\sqrt{\tanh^2(x) \operatorname{coth}(x) \log(\cosh(x))} - \frac{1}{2} \tanh^2(x)^{3/2} \operatorname{coth}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sech[x]^2)^(3/2), x]

[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2] - (Coth[x]*(Tanh[x]^2)^(3/2))/2

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3658

Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 4121

Int[(u_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rubi steps

$$\begin{aligned} \int (1 - \operatorname{sech}^2(x))^{3/2} dx &= \int \tanh^2(x)^{3/2} dx \\ &= \left(\operatorname{coth}(x) \sqrt{\tanh^2(x)} \right) \int \tanh^3(x) dx \\ &= -\frac{1}{2} \operatorname{coth}(x) \tanh^2(x)^{3/2} + \left(\operatorname{coth}(x) \sqrt{\tanh^2(x)} \right) \int \tanh(x) dx \\ &= \operatorname{coth}(x) \log(\cosh(x)) \sqrt{\tanh^2(x)} - \frac{1}{2} \operatorname{coth}(x) \tanh^2(x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{1}{2}\sqrt{\tanh^2(x)(\operatorname{csch}(x)\operatorname{sech}(x) + 2\coth(x)\log(\cosh(x)))}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sech[x]^2)^(3/2), x]

[Out] ((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[Tanh[x]^2])/2

fricas [B] time = 0.67, size = 183, normalized size = 6.31

$$\frac{x \cosh(x)^4 + 4x \cosh(x) \sinh(x)^3 + x \sinh(x)^4 + 2(x-1) \cosh(x)^2 + 2(3x \cosh(x)^2 + x-1) \sinh(x)^2 - (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1) \log(2 \cosh(x) / (\cosh(x) - \sinh(x))) + 4(x \cosh(x)^3 + (x-1) \cosh(x)) \sinh(x) + x}{\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="fricas")

[Out] -(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 + 2*(x - 1)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + x - 1)*sinh(x)^2 - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(x*cosh(x)^3 + (x - 1)*cosh(x))*sinh(x) + x)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)

giac [B] time = 0.13, size = 72, normalized size = 2.48

$$-x \operatorname{sgn}(e^{4x} - 1) + \log(e^{2x} + 1) \operatorname{sgn}(e^{4x} - 1) - \frac{3e^{4x} \operatorname{sgn}(e^{4x} - 1) + 2e^{2x} \operatorname{sgn}(e^{4x} - 1) + 3 \operatorname{sgn}(e^{4x} - 1)}{2(e^{2x} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="giac")

[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 1/2*(3*e^(4*x)*sgn(e^(4*x) - 1) + 2*e^(2*x)*sgn(e^(4*x) - 1) + 3*sgn(e^(4*x) - 1))/(e^(2*x) + 1)^2

maple [B] time = 0.28, size = 120, normalized size = 4.14

$$-\frac{(1 + e^{2x}) \sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}}{e^{2x} - 1} x + \frac{2 \sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}}{(e^{2x} - 1)(1 + e^{2x})} e^{2x} + \frac{(1 + e^{2x}) \sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}}{e^{2x} - 1} \ln(1 + e^{2x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-sech(x)^2)^(3/2), x)

[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+2/(exp(2*x)-1)/(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)+1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))

maxima [A] time = 0.46, size = 33, normalized size = 1.14

$$-x - \frac{2e^{-2x}}{2e^{-2x} + e^{-4x} + 1} - \log(e^{-2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] $-x - 2e^{-2x}/(2e^{-2x} + e^{-4x} + 1) - \log(e^{-2x} + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(1 - \frac{1}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - 1/cosh(x)^2)^(3/2), x)

[Out] int((1 - 1/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - \operatorname{sech}^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)**2)**(3/2), x)

[Out] Integral((1 - sech(x)**2)**(3/2), x)

$$3.171 \quad \int \sqrt{1 - \operatorname{sech}^2(x)} dx$$

Optimal. Leaf size=14

$$\sqrt{\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

[Out] $\operatorname{coth}(x) * \ln(\cosh(x)) * (\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4121, 3658, 3475}

$$\sqrt{\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[1 - Sech[x]^2], x]`

[Out] `Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]`

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3658

`Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]`

Rule 4121

`Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rubi steps

$$\begin{aligned} \int \sqrt{1 - \operatorname{sech}^2(x)} dx &= \int \sqrt{\tanh^2(x)} dx \\ &= \left(\operatorname{coth}(x) \sqrt{\tanh^2(x)} \right) \int \tanh(x) dx \\ &= \operatorname{coth}(x) \log(\cosh(x)) \sqrt{\tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sqrt{\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[1 - Sech[x]^2], x]`

[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]

fricas [A] time = 0.77, size = 18, normalized size = 1.29

$$-x + \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))

giac [B] time = 0.12, size = 26, normalized size = 1.86

$$-x \operatorname{sgn}(e^{4x} - 1) + \log(e^{2x} + 1) \operatorname{sgn}(e^{4x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="giac")

[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1)

maple [B] time = 0.30, size = 79, normalized size = 5.64

$$-\frac{(1 + e^{2x}) \sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} x}{e^{2x} - 1} + \frac{(1 + e^{2x}) \sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} \ln(1 + e^{2x})}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-sech(x)^2)^(1/2),x)

[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))

maxima [A] time = 0.61, size = 13, normalized size = 0.93

$$-x - \log(e^{-2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] -x - log(e^(-2*x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \sqrt{1 - \frac{1}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - 1/cosh(x)^2)^(1/2),x)

[Out] int((1 - 1/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \operatorname{sech}^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sech(x)**2)**(1/2),x)

[Out] Integral(sqrt(1 - sech(x)**2), x)

$$3.172 \quad \int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=14

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

[Out] $\ln(\sinh(x)) * \tanh(x) / (\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[1 - Sech[x]^2], x]`

[Out] `(Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]`

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3658

`Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]`

Rule 4121

`Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}} dx &= \int \frac{1}{\sqrt{\tanh^2(x)}} dx \\ &= \frac{\tanh(x) \int \operatorname{coth}(x) dx}{\sqrt{\tanh^2(x)}} \\ &= \frac{\log(\sinh(x)) \tanh(x)}{\sqrt{\tanh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - Sech[x]^2], x]

[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]

fricas [A] time = 0.53, size = 18, normalized size = 1.29

$$-x + \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))

giac [B] time = 0.13, size = 31, normalized size = 2.21

$$-\frac{x}{\operatorname{sgn}(e^{4x} - 1)} + \frac{\log(|e^{2x} - 1|)}{\operatorname{sgn}(e^{4x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sech(x)^2)^(1/2), x, algorithm="giac")

[Out] -x/sgn(e^(4*x) - 1) + log(abs(e^(2*x) - 1))/sgn(e^(4*x) - 1)

maple [B] time = 0.26, size = 79, normalized size = 5.64

$$-\frac{(e^{2x} - 1)x}{\sqrt{\frac{(e^{2x} - 1)^2}{(1 + e^{2x})^2}} (1 + e^{2x})} + \frac{(e^{2x} - 1) \ln(e^{2x} - 1)}{\sqrt{\frac{(e^{2x} - 1)^2}{(1 + e^{2x})^2}} (1 + e^{2x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sech(x)^2)^(1/2), x)

[Out] -1/((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1)

maxima [A] time = 0.67, size = 22, normalized size = 1.57

$$-x - \log(e^{(-x)} + 1) - \log(e^{(-x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sech(x)^2)^(1/2), x, algorithm="maxima")

[Out] -x - log(e^(-x) + 1) - log(e^(-x) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{1 - \frac{1}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1 - 1/cosh(x)^2)^(1/2), x)

[Out] int(1/(1 - 1/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1 - \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sech(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(1 - sech(x)**2), x)

3.173 $\int (-1 + \operatorname{sech}^2(x))^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x)} - \sqrt{-\tanh^2(x)} \coth(x) \log(\cosh(x))$$

[Out] $-\coth(x) \cdot \ln(\cosh(x)) \cdot (-\tanh(x)^2)^{(1/2)} + 1/2 \cdot (-\tanh(x)^2)^{(1/2)} \cdot \tanh(x)$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4121, 3658, 3473, 3475}

$$\frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x)} - \sqrt{-\tanh^2(x)} \coth(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sech[x]^2)^(3/2), x]

[Out] $-(\operatorname{Coth}[x] \cdot \operatorname{Log}[\operatorname{Cosh}[x]] \cdot \operatorname{Sqrt}[-\operatorname{Tanh}[x]^2]) + (\operatorname{Tanh}[x] \cdot \operatorname{Sqrt}[-\operatorname{Tanh}[x]^2])/2$

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3658

Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 4121

Int[(u_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rubi steps

$$\begin{aligned} \int (-1 + \operatorname{sech}^2(x))^{3/2} dx &= \int (-\tanh^2(x))^{3/2} dx \\ &= -\left(\left(\coth(x) \sqrt{-\tanh^2(x)} \right) \int \tanh^3(x) dx \right) \\ &= \frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x)} - \left(\coth(x) \sqrt{-\tanh^2(x)} \right) \int \tanh(x) dx \\ &= -\coth(x) \log(\cosh(x)) \sqrt{-\tanh^2(x)} + \frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{1}{2}\sqrt{-\tanh^2(x)}(\operatorname{csch}(x)\operatorname{sech}(x) + 2\operatorname{coth}(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sech[x]^2)^(3/2), x]

[Out] -1/2*((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[-Tanh[x]^2])

fricas [A] time = 0.68, size = 1, normalized size = 0.03

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(3/2), x, algorithm="fricas")

[Out] 0

giac [C] time = 0.14, size = 83, normalized size = 2.44

$$-ix\operatorname{sgn}(-e^{4x} + 1) + i\log(e^{2x} + 1)\operatorname{sgn}(-e^{4x} + 1) - \frac{i(3e^{4x}\operatorname{sgn}(-e^{4x} + 1) + 2e^{2x}\operatorname{sgn}(-e^{4x} + 1) + 3\operatorname{sgn}(-e^{4x} + 1))}{2(e^{2x} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(3/2), x, algorithm="giac")

[Out] -I*x*sgn(-e^(4*x) + 1) + I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1) - 1/2*I*(3*e^(4*x)*sgn(-e^(4*x) + 1) + 2*e^(2*x)*sgn(-e^(4*x) + 1) + 3*sgn(-e^(4*x) + 1))/(e^(2*x) + 1)^2

maple [B] time = 0.32, size = 123, normalized size = 3.62

$$\frac{(1 + e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}x}{e^{2x}-1} - \frac{2\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}e^{2x}}{(e^{2x}-1)(1+e^{2x})} - \frac{(1 + e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\ln(1 + e^{2x})}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sech(x)^2)^(3/2), x)

[Out] 1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x-2/(exp(2*x)-1)/(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)-1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))

maxima [C] time = 0.49, size = 33, normalized size = 0.97

$$ix + \frac{2ie^{(-2x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + i\log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(3/2), x, algorithm="maxima")

[Out] I*x + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + I*log(e^(-2*x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cosh(x)^2} - 1 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/cosh(x)^2 - 1)^(3/2), x)`

[Out] `int((1/cosh(x)^2 - 1)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\operatorname{sech}^2(x) - 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+sech(x)**2)**(3/2), x)`

[Out] `Integral((sech(x)**2 - 1)**(3/2), x)`

$$3.174 \quad \int \sqrt{-1 + \operatorname{sech}^2(x)} dx$$

Optimal. Leaf size=16

$$\sqrt{-\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

[Out] $\operatorname{coth}(x) * \ln(\cosh(x)) * (-\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4121, 3658, 3475}

$$\sqrt{-\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Sech[x]^2], x]

[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3658

Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 4121

Int[(u_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{-1 + \operatorname{sech}^2(x)} dx &= \int \sqrt{-\tanh^2(x)} dx \\ &= \left(\operatorname{coth}(x) \sqrt{-\tanh^2(x)} \right) \int \tanh(x) dx \\ &= \operatorname{coth}(x) \log(\cosh(x)) \sqrt{-\tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\sqrt{-\tanh^2(x)} \operatorname{coth}(x) \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Sech[x]^2], x]

[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]

fricas [A] time = 0.51, size = 1, normalized size = 0.06

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] 0

giac [C] time = 0.13, size = 31, normalized size = 1.94

$$ix \operatorname{sgn}(-e^{4x} + 1) - i \log(e^{2x} + 1) \operatorname{sgn}(-e^{4x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="giac")

[Out] I*x*sgn(-e^(4*x) + 1) - I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1)

maple [B] time = 0.32, size = 81, normalized size = 5.06

$$-\frac{(1 + e^{2x}) \sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} x}{e^{2x} - 1} + \frac{(1 + e^{2x}) \sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} \ln(1 + e^{2x})}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sech(x)^2)^(1/2),x)

[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))

maxima [C] time = 0.55, size = 13, normalized size = 0.81

$$-ix - i \log(e^{-2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] -I*x - I*log(e^(-2*x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\frac{1}{\cosh(x)^2} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cosh(x)^2 - 1)^(1/2),x)

[Out] int((1/cosh(x)^2 - 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sech(x)**2)**(1/2),x)

[Out] Integral(sqrt(sech(x)**2 - 1), x)

$$3.175 \quad \int \frac{1}{\sqrt{-1 + \operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=16

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

[Out] $\ln(\sinh(x)) * \tanh(x) / (-\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[-1 + Sech[x]^2], x]`

[Out] `(Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]`

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3658

`Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]`

Rule 4121

`Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1 + \operatorname{sech}^2(x)}} dx &= \int \frac{1}{\sqrt{-\tanh^2(x)}} dx \\ &= \frac{\tanh(x) \int \operatorname{coth}(x) dx}{\sqrt{-\tanh^2(x)}} \\ &= \frac{\log(\sinh(x)) \tanh(x)}{\sqrt{-\tanh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh(x) \log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-1 + Sech[x]^2], x]

[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]

fricas [A] time = 0.49, size = 1, normalized size = 0.06

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] 0

giac [C] time = 0.14, size = 37, normalized size = 2.31

$$-\frac{ix}{\operatorname{sgn}(-e^{4x} + 1)} + \frac{i \log(-ie^{2x} + i)}{\operatorname{sgn}(-e^{4x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+sech(x)^2)^(1/2), x, algorithm="giac")

[Out] -I*x/sgn(-e^(4*x) + 1) + I*log(-I*e^(2*x) + I)/sgn(-e^(4*x) + 1)

maple [B] time = 0.30, size = 81, normalized size = 5.06

$$-\frac{(e^{2x} - 1)x}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} (1 + e^{2x})} + \frac{(e^{2x} - 1) \ln(e^{2x} - 1)}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}} (1 + e^{2x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+sech(x)^2)^(1/2), x)

[Out] -1/(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1)

maxima [C] time = 0.44, size = 22, normalized size = 1.38

$$ix + i \log(e^{-x} + 1) + i \log(e^{-x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+sech(x)^2)^(1/2), x, algorithm="maxima")

[Out] I*x + I*log(e^(-x) + 1) + I*log(e^(-x) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\frac{1}{\cosh(x)^2} - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cosh(x)^2 - 1)^(1/2), x)

[Out] int(1/(1/cosh(x)^2 - 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(x)**2 - 1), x)
```

3.176 $\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^5(x) dx$

Optimal. Leaf size=83

$$-\frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b^2} + \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} - \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)$$

[Out] 1/3*(a+2*b)*(a+b*sech(x)^2)^(3/2)/b^2-1/5*(a+b*sech(x)^2)^(5/2)/b^2+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 88, 50, 63, 208}

$$-\frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b^2} + \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} - \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + ((a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (a + b*Sech[x]^2)^(5/2)/(5*b^2)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x]]

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 4139

$\text{Int}[\{(a_) + (b_)*((c_)*\text{sec}[e_] + (f_)*(x_))\}^{(n_)}\}^{(p_)}*\text{tan}[e_] + (f_)*(x_)]^{(m_)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Sec}[e + f*x], x]\}, \text{Dist}[1/ff, \text{Subst}[\text{Int}[((-1 + ff^2*x^2)^{(m-1)/2}*(a + b*(c*ff*x)^n)^p)/x, x], x, \text{Sec}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2] \ \&\& \ (\text{GtQ}[m, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ \text{IGtQ}[p, 0] \ || \ \text{IntegersQ}[2*n, p])$

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^5(x) dx &= -\operatorname{Subst}\left(\int \frac{(-1 + x^2)^2 \sqrt{a + bx^2}}{x} dx, x, \operatorname{sech}(x)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{(-1 + x)^2 \sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \left(\frac{(-a - 2b)\sqrt{a + bx}}{b} + \frac{\sqrt{a + bx}}{x} + \frac{(a + bx)^{3/2}}{b}\right) dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} - \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b^2} - \frac{1}{2} \operatorname{Subst}\left(\int \frac{\sqrt{a + bx}}{x} dx, x\right) \\ &= -\sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} - \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b^2} - \frac{1}{2} a \operatorname{Subst}\left(\int \frac{1}{x} dx, x\right) \\ &= -\sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} - \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b^2} - \frac{1}{2} a \operatorname{Subst}\left(\int \frac{1}{x} dx, x\right) \\ &= \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + 2b)(a + b \operatorname{sech}^2(x))^{3/2}}{3b^2} \end{aligned}$$

Mathematica [A] time = 0.75, size = 114, normalized size = 1.37

$$\frac{1}{15} \cosh(x) \sqrt{a + b \operatorname{sech}^2(x)} \left(\left(\frac{2a^2}{b^2} + \frac{10a}{b} - 15 \right) \operatorname{sech}(x) + \left(10 - \frac{a}{b} \right) \operatorname{sech}^3(x) + \frac{15\sqrt{2} \sqrt{a} \log\left(\sqrt{a} \cosh(2x) + a + \sqrt{a} \cosh(2x) + a\right)}{\sqrt{a} \cosh(2x) + a} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]

[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((15*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-15 + (2*a^2)/b^2 + (10*a)/b)*Sech[x] + (10 - a/b)*Sech[x]^3 - 3*Sech[x]^5))/15

fricas [B] time = 1.33, size = 4594, normalized size = 55.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^8 + 8*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15*b^2)*sinh(x)^8 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^3 + 3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^4 + 30*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sinh(x)^4 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^3 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x)
```

$$\begin{aligned}
&)^2 + 2a^2 + 10ab - 15b^2 + 8((2a^2 + 10ab - 15b^2)\cosh(x)^7 + 3(\\
& (2a^2 + 9ab - 5b^2)\cosh(x)^5 + (6a^2 + 26ab - 29b^2)\cosh(x)^3 + (\\
& 2a^2 + 9ab - 5b^2)\cosh(x))\sinh(x))\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + \\
& a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(b^2\cosh(x)^{10} + 10 \\
& b^2\cosh(x)\sinh(x)^9 + b^2\sinh(x)^{10} + 5b^2\cosh(x)^8 + 5(9b^2\cosh(x) \\
&)^2 + b^2)\sinh(x)^8 + 10b^2\cosh(x)^6 + 40(3b^2\cosh(x)^3 + b^2\cosh(x) \\
&)\sinh(x)^7 + 10(21b^2\cosh(x)^4 + 14b^2\cosh(x)^2 + b^2)\sinh(x)^6 + 10 \\
& b^2\cosh(x)^4 + 4(63b^2\cosh(x)^5 + 70b^2\cosh(x)^3 + 15b^2\cosh(x))\s \\
& \sinh(x)^5 + 10(21b^2\cosh(x)^6 + 35b^2\cosh(x)^4 + 15b^2\cosh(x)^2 + b^2 \\
&)\sinh(x)^4 + 5b^2\cosh(x)^2 + 40(3b^2\cosh(x)^7 + 7b^2\cosh(x)^5 + 5b \\
& ^2\cosh(x)^3 + b^2\cosh(x))\sinh(x)^3 + 5(9b^2\cosh(x)^8 + 28b^2\cosh(x) \\
& ^6 + 30b^2\cosh(x)^4 + 12b^2\cosh(x)^2 + b^2)\sinh(x)^2 + b^2 + 10(b^2\c \\
& \osh(x)^9 + 4b^2\cosh(x)^7 + 6b^2\cosh(x)^5 + 4b^2\cosh(x)^3 + b^2\cosh(x) \\
&)\sinh(x)), -1/30(15(b^2\cosh(x)^{10} + 10b^2\cosh(x)\sinh(x)^9 + b^2\sin \\
& h(x)^{10} + 5b^2\cosh(x)^8 + 5(9b^2\cosh(x)^2 + b^2)\sinh(x)^8 + 10b^2\co \\
& sh(x)^6 + 40(3b^2\cosh(x)^3 + b^2\cosh(x))\sinh(x)^7 + 10(21b^2\cosh(x) \\
& ^4 + 14b^2\cosh(x)^2 + b^2)\sinh(x)^6 + 10b^2\cosh(x)^4 + 4(63b^2\cosh(\\
& x)^5 + 70b^2\cosh(x)^3 + 15b^2\cosh(x))\sinh(x)^5 + 10(21b^2\cosh(x)^6 \\
& + 35b^2\cosh(x)^4 + 15b^2\cosh(x)^2 + b^2)\sinh(x)^4 + 5b^2\cosh(x)^2 + \\
& 40(3b^2\cosh(x)^7 + 7b^2\cosh(x)^5 + 5b^2\cosh(x)^3 + b^2\cosh(x))\sinh \\
& (x)^3 + 5(9b^2\cosh(x)^8 + 28b^2\cosh(x)^6 + 30b^2\cosh(x)^4 + 12b^2\c \\
& \osh(x)^2 + b^2)\sinh(x)^2 + b^2 + 10(b^2\cosh(x)^9 + 4b^2\cosh(x)^7 + 6b \\
& ^2\cosh(x)^5 + 4b^2\cosh(x)^3 + b^2\cosh(x))\sinh(x))\sqrt{-a}\arctan(\sqrt{ \\
& 2})((a + b)\cosh(x)^2 + 2(a + b)\cosh(x)\sinh(x) + (a + b)\sinh(x)^2 + a) \\
& \sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x) \\
&)\sinh(x) + \sinh(x)^2))/((a^2 + ab)\cosh(x)^4 + 4(a^2 + ab)\cosh(x)\sinh(\\
& x)^3 + (a^2 + ab)\sinh(x)^4 + (2a^2 + 3ab)\cosh(x)^2 + (6(a^2 + ab)\c \\
& \osh(x)^2 + 2a^2 + 3ab)\sinh(x)^2 + a^2 + 2(2(a^2 + ab)\cosh(x)^3 + (2 \\
& a^2 + 3ab)\cosh(x))\sinh(x))) + 15(b^2\cosh(x)^{10} + 10b^2\cosh(x)\sinh \\
& (x)^9 + b^2\sinh(x)^{10} + 5b^2\cosh(x)^8 + 5(9b^2\cosh(x)^2 + b^2)\sinh(x) \\
&)^8 + 10b^2\cosh(x)^6 + 40(3b^2\cosh(x)^3 + b^2\cosh(x))\sinh(x)^7 + 10 \\
& (21b^2\cosh(x)^4 + 14b^2\cosh(x)^2 + b^2)\sinh(x)^6 + 10b^2\cosh(x)^4 + \\
& 4(63b^2\cosh(x)^5 + 70b^2\cosh(x)^3 + 15b^2\cosh(x))\sinh(x)^5 + 10(21 \\
& b^2\cosh(x)^6 + 35b^2\cosh(x)^4 + 15b^2\cosh(x)^2 + b^2)\sinh(x)^4 + 5b \\
& ^2\cosh(x)^2 + 40(3b^2\cosh(x)^7 + 7b^2\cosh(x)^5 + 5b^2\cosh(x)^3 + b \\
& ^2\cosh(x))\sinh(x)^3 + 5(9b^2\cosh(x)^8 + 28b^2\cosh(x)^6 + 30b^2\cosh(\\
& x)^4 + 12b^2\cosh(x)^2 + b^2)\sinh(x)^2 + b^2 + 10(b^2\cosh(x)^9 + 4b^2 \\
& \cosh(x)^7 + 6b^2\cosh(x)^5 + 4b^2\cosh(x)^3 + b^2\cosh(x))\sinh(x))\sqrt{ \\
& -a}\arctan(\sqrt{2})(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1)\sqrt{-a} \\
& \sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) \\
& + \sinh(x)^2))/(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2(a + 2 \\
& b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + 2b)\sinh(x)^2 + 4(a\cosh(x)^3 + (a \\
& + 2b)\cosh(x))\sinh(x) + a) - 2\sqrt{2})((2a^2 + 10ab - 15b^2)\cosh(\\
& x)^8 + 8(2a^2 + 10ab - 15b^2)\cosh(x)\sinh(x)^7 + (2a^2 + 10ab - 15 \\
& b^2)\sinh(x)^8 + 4(2a^2 + 9ab - 5b^2)\cosh(x)^6 + 4(7(2a^2 + 10a \\
& b - 15b^2)\cosh(x)^2 + 2a^2 + 9ab - 5b^2)\sinh(x)^6 + 8(7(2a^2 + 10 \\
& ab - 15b^2)\cosh(x)^3 + 3(2a^2 + 9ab - 5b^2)\cosh(x))\sinh(x)^5 + 2 \\
& (6a^2 + 26ab - 29b^2)\cosh(x)^4 + 2(35(2a^2 + 10ab - 15b^2)\cosh \\
& (x)^4 + 30(2a^2 + 9ab - 5b^2)\cosh(x)^2 + 6a^2 + 26ab - 29b^2)\sin \\
& h(x)^4 + 8(7(2a^2 + 10ab - 15b^2)\cosh(x)^5 + 10(2a^2 + 9ab - 5b \\
& ^2)\cosh(x)^3 + (6a^2 + 26ab - 29b^2)\cosh(x))\sinh(x)^3 + 4(2a^2 + 9 \\
& ab - 5b^2)\cosh(x)^2 + 4(7(2a^2 + 10ab - 15b^2)\cosh(x)^6 + 15(2 \\
& a^2 + 9ab - 5b^2)\cosh(x)^4 + 3(6a^2 + 26ab - 29b^2)\cosh(x)^2 + 2 \\
& a^2 + 9ab - 5b^2)\sinh(x)^2 + 2a^2 + 10ab - 15b^2 + 8((2a^2 + 10a \\
& b - 15b^2)\cosh(x)^7 + 3(2a^2 + 9ab - 5b^2)\cosh(x)^5 + (6a^2 + 26 \\
& ab - 29b^2)\cosh(x)^3 + (2a^2 + 9ab - 5b^2)\cosh(x))\sinh(x))\sqrt{(a \\
& \cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x) \\
&)^2)))/(b^2\cosh(x)^{10} + 10b^2\cosh(x)\sinh(x)^9 + b^2\sinh(x)^{10} + 5b^2 \\
& \cosh(x)^8 + 5(9b^2\cosh(x)^2 + b^2)\sinh(x)^8 + 10b^2\cosh(x)^6 + 40(3
\end{aligned}$$

$b^2 \cosh(x)^3 + b^2 \cosh(x) \sinh(x)^7 + 10(21b^2 \cosh(x)^4 + 14b^2 \cosh(x)^2 + b^2) \sinh(x)^6 + 10b^2 \cosh(x)^4 + 4(63b^2 \cosh(x)^5 + 70b^2 \cosh(x)^3 + 15b^2 \cosh(x) \sinh(x)^5 + 10(21b^2 \cosh(x)^6 + 35b^2 \cosh(x)^4 + 15b^2 \cosh(x)^2 + b^2) \sinh(x)^4 + 5b^2 \cosh(x)^2 + 40(3b^2 \cosh(x)^7 + 7b^2 \cosh(x)^5 + 5b^2 \cosh(x)^3 + b^2 \cosh(x) \sinh(x)^3 + 5(9b^2 \cosh(x)^8 + 28b^2 \cosh(x)^6 + 30b^2 \cosh(x)^4 + 12b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 10(b^2 \cosh(x)^9 + 4b^2 \cosh(x)^7 + 6b^2 \cosh(x)^5 + 4b^2 \cosh(x)^3 + b^2 \cosh(x) \sinh(x))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 0.47Error: Bad Argument Type

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(x)^2} (\tanh^5(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)

[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \tanh(x)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^5 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2),x)

[Out] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^5(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**5,x)

[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**5, x)

3.177 $\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^4(x) dx$

Optimal. Leaf size=125

$$-\frac{(a^2 + 6ab - 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a-b \tanh^2(x)+b}}\right)}{8b^{3/2}} + \frac{(a-3b) \tanh(x) \sqrt{a-b \tanh^2(x)+b}}{8b} + \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a-b \tanh^2(x)+b}}\right)$$

[Out] $-1/8*(a^2+6*a*b-3*b^2)*\arctan(b^{(1/2)}*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/b^{(3/2)}+\arctanh(a^{(1/2)}*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})*a^{(1/2)}+1/8*(a-3*b)*(a+b-b*\tanh(x)^2)^{(1/2)}*\tanh(x)/b-1/4*(a+b-b*\tanh(x)^2)^{(1/2)}*\tanh(x)^3$

Rubi [A] time = 0.31, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4141, 1975, 478, 582, 523, 217, 203, 377, 206}

$$-\frac{(a^2 + 6ab - 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a-b \tanh^2(x)+b}}\right)}{8b^{3/2}} - \frac{1}{4} \tanh^3(x) \sqrt{a-b \tanh^2(x)+b} + \frac{(a-3b) \tanh(x) \sqrt{a-b \tanh^2(x)+b}}{8b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4, x]

[Out] $-((a^2 + 6*a*b - 3*b^2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[x])/\text{Sqrt}[a + b - b*\text{Tanh}[x]^2]])/(8*b^{(3/2)}) + \text{Sqrt}[a]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Tanh}[x])/\text{Sqrt}[a + b - b*\text{Tanh}[x]^2]] + ((a - 3*b)*\text{Tanh}[x]*\text{Sqrt}[a + b - b*\text{Tanh}[x]^2])/(8*b) - (\text{Tanh}[x]^3*\text{Sqrt}[a + b - b*\text{Tanh}[x]^2])/4$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 478

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^q)/(b*(m+n*(p+q)+1)), x] - Dist[e^n/(b*(m+n*(p+q)+1)), x]

1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n + 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^4(x) dx &= \operatorname{Subst} \left(\int \frac{x^4 \sqrt{a + b(1 - x^2)}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{x^4 \sqrt{a + b - bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= -\frac{1}{4} \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{4} \operatorname{Subst} \left(\int \frac{x^2 (3(a + b) + (a - 3b)x^2)}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
&= \frac{(a - 3b) \tanh(x) \sqrt{a + b - b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} - \frac{1}{4} \operatorname{Subst} \left(\int \frac{x^2 (3(a + b) + (a - 3b)x^2)}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
&= \frac{(a - 3b) \tanh(x) \sqrt{a + b - b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{x^2 (3(a + b) + (a - 3b)x^2)}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
&= \frac{(a - 3b) \tanh(x) \sqrt{a + b - b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{x^2 (3(a + b) + (a - 3b)x^2)}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
&= -\frac{(a^2 + 6ab - 3b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)}{8b^{3/2}} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)
\end{aligned}$$

Mathematica [A] time = 0.47, size = 192, normalized size = 1.54

$$\frac{\cosh(x) \sqrt{a + b \operatorname{sech}^2(x)} \left(\sqrt{2} (a^2 + 6ab - 3b^2) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{b} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) - 8\sqrt{2} \sqrt{a} b^{3/2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) \right)}{8b^{3/2} \sqrt{a \cosh(2x) + a + 2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]

[Out] -1/8*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*Sqrt[a]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - (a - 5*b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] - 2*b^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(b^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])

fricas [B] time = 1.22, size = 8852, normalized size = 70.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")

[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5

$$\begin{aligned}
& + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8 \\
& *a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2* \\
& (14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a \\
& *b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70* \\
& a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin \\
& h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2* \\
& b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14 \\
& *a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4* \\
& a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh \\
& (x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2) \\
& *sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)* \\
& cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - \\
& a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x) \\
&))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x) \\
&)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 \\
& + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(\\
& cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin \\
& h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 \\
& + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (\\
& a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(\\
& a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 \\
& + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 + \\
& 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + \\
& 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8 \\
& *(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + \\
& 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x) \\
&)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh \\
& (x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 + \\
& a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b \\
& - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b \\
& ^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)* \\
& sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x) \\
&)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh \\
& (x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - \\
& 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x)) \\
& *sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh \\
& (x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + \\
& 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x) \\
& ^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh \\
& (x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + \\
& 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 \\
& + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2* \\
& cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b \\
& ^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x) \\
&)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b) \\
& *sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a) \\
&)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) \\
& + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 \\
& + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6* \\
& (a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*c \\
&osh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b \\
& - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh \\
& (x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b \\
& ^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2) \\
&)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x) \\
& ^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 \\
& + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh \\
& (x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh \\
& (x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
\end{aligned}$$

$$\begin{aligned}
& 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)), \\
& -1/8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*cosh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh(
\end{aligned}$$

$$\begin{aligned}
& x)^2 + (15*(a*b - 5*b^2)*\cosh(x)^4 + 6*(a*b + 3*b^2)*\cosh(x)^2 - a*b - 3*b^2) \\
& * \sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*\cosh(x)^5 + 2*(a*b + 3*b^2) \\
& *\cosh(x)^3 - (a*b + 3*b^2)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)} \\
& / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) / (b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 \\
& + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 \\
& + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 \\
& + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 \\
& + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 8*(b^2*\cosh(x)^7 \\
& + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)), -1/16*(8*(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 \\
& + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 \\
& + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 \\
& + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 \\
& + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 8*(b^2*\cosh(x)^7 \\
& + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) \\
& + b*\sinh(x)^2 + a))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)} / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)) / (a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 - (a^2 + 3*a*b)*\cosh(x)^2 \\
& + (6*a*b*\cosh(x)^2 - a^2 - 3*a*b)*\sinh(x)^2 - a^2 + 2*(2*a*b*\cosh(x)^3 - (a^2 + 3*a*b)*\cosh(x))*\sinh(x)) \\
& + 8*(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 \\
& + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 \\
& + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 \\
& + 3*b^2*\cosh(x))*\sinh(x)^3 + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 \\
& + b^2 + 8*(b^2*\cosh(x)^7 + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 \\
& + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)} / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)) / (a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 \\
& + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a) - ((a^2 + 6*a*b - 3*b^2)*\cosh(x)^8 \\
& + 8*(a^2 + 6*a*b - 3*b^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 6*a*b - 3*b^2)*\sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^6 \\
& + 4*(7*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*\sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^3 \\
& + 3*(a^2 + 6*a*b - 3*b^2)*\cosh(x))*\sinh(x)^5 + 6*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^4 \\
& + 30*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*\sinh(x)^4 + 8*(7*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^5 \\
& + 10*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^2 \\
& + 4*(7*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^2 \\
& + a^2 + 6*a*b - 3*b^2)*\sinh(x)^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*\cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^5 \\
& + 3*(a^2 + 6*a*b - 3*b^2)*\cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*\cosh(x))*\sinh(x))*\sqrt{-b}*\log(-((a - b)*\cosh(x)^4 \\
& + 4*(a - b)*\cosh(x)*\sinh(x)^3 + (a - b)*\sinh(x)^4 + 2*(a + 3*b)*\cosh(x)^2 + 2*(3*(a - b)*\cosh(x)^2 + a + 3*b)*\sinh(x)^2 \\
& + 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)} \\
& / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*((a - b)*\cosh(x)^3 + (a + 3*b)*\cosh(x))*\sinh(x) + a - b) / \\
& (\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) \\
& + 1)) - 2*\sqrt{2}*((a*b - 5*b^2)*\cosh(x)^6 + 6*(a*b - 5*b^2)*\cosh(x)*\sinh(x)^5 + (a*b - 5*b^2)*\sinh(x)^6 \\
& + (a*b + 3*b^2)*\cosh(x)^4 + (15*(a*b - 5*b^2)*\cosh(x)^2 + a*b + 3*b^2)*\sinh(x)^4 + 4*(5*(a*b - 5*b^2)*\cosh(x)^3 \\
& + (a*b + 3*b^2)*\cosh(x))*\sinh(x)^3 - (a*b + 3*b^2)*\cosh(x)^2 + (15*(a*b - 5*b^2)*\cosh(x)^4 + 6*(a*b + 3*b^2)*\cosh(x)^2 \\
& - a*b - 3*b^2)*\sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*\cosh(x)^5 + 2*(a*b + 3*b^2)*\cosh(x)^3 - (a*b + 3*b^2)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)} \\
& / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)
\end{aligned}$$

$$\begin{aligned}
&)^2)))/(b^2 \cosh(x)^8 + 8b^2 \cosh(x) \sinh(x)^7 + b^2 \sinh(x)^8 + 4b^2 \cosh(x)^6 + 4(7b^2 \cosh(x)^2 + b^2) \sinh(x)^6 + 6b^2 \cosh(x)^4 + 8(7b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^5 + 2(35b^2 \cosh(x)^4 + 30b^2 \cosh(x)^2 + 3b^2) \sinh(x)^4 + 4b^2 \cosh(x)^2 + 8(7b^2 \cosh(x)^5 + 10b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^3 + 4(7b^2 \cosh(x)^6 + 15b^2 \cosh(x)^4 + 9b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 8(b^2 \cosh(x)^7 + 3b^2 \cosh(x)^5 + 3b^2 \cosh(x)^3 + b^2 \cosh(x)) \sinh(x)), \\
& -1/8(4(b^2 \cosh(x)^8 + 8b^2 \cosh(x) \sinh(x)^7 + b^2 \sinh(x)^8 + 4b^2 \cosh(x)^6 + 4(7b^2 \cosh(x)^2 + b^2) \sinh(x)^6 + 6b^2 \cosh(x)^4 + 8(7b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^5 + 2(35b^2 \cosh(x)^4 + 30b^2 \cosh(x)^2 + 3b^2) \sinh(x)^4 + 4b^2 \cosh(x)^2 + 8(7b^2 \cosh(x)^5 + 10b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^3 + 4(7b^2 \cosh(x)^6 + 15b^2 \cosh(x)^4 + 9b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 8(b^2 \cosh(x)^7 + 3b^2 \cosh(x)^5 + 3b^2 \cosh(x)^3 + b^2 \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a b \cosh(x)^4 + 4 a b \cosh(x) \sinh(x)^3 + a b \sinh(x)^4 - (a^2 + 3 a b) \cosh(x)^2 + (6 a b \cosh(x)^2 - a^2 - 3 a b) \sinh(x)^2 - a^2 + 2(2 a b \cosh(x)^3 - (a^2 + 3 a b) \cosh(x)) \sinh(x))) + 4(b^2 \cosh(x)^8 + 8b^2 \cosh(x) \sinh(x)^7 + b^2 \sinh(x)^8 + 4b^2 \cosh(x)^6 + 4(7b^2 \cosh(x)^2 + b^2) \sinh(x)^6 + 6b^2 \cosh(x)^4 + 8(7b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^5 + 2(35b^2 \cosh(x)^4 + 30b^2 \cosh(x)^2 + 3b^2) \sinh(x)^4 + 4b^2 \cosh(x)^2 + 8(7b^2 \cosh(x)^5 + 10b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^3 + 4(7b^2 \cosh(x)^6 + 15b^2 \cosh(x)^4 + 9b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 8(b^2 \cosh(x)^7 + 3b^2 \cosh(x)^5 + 3b^2 \cosh(x)^3 + b^2 \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^4 + 4 a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3 a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x)) \sinh(x) + a)) + ((a^2 + 6 a b - 3 b^2) \cosh(x)^8 + 8(a^2 + 6 a b - 3 b^2) \cosh(x) \sinh(x)^7 + (a^2 + 6 a b - 3 b^2) \sinh(x)^8 + 4(a^2 + 6 a b - 3 b^2) \cosh(x)^6 + 4(7(a^2 + 6 a b - 3 b^2) \cosh(x)^2 + a^2 + 6 a b - 3 b^2) \sinh(x)^6 + 8(7(a^2 + 6 a b - 3 b^2) \cosh(x)^3 + 3(a^2 + 6 a b - 3 b^2) \cosh(x)) \sinh(x)^5 + 6(a^2 + 6 a b - 3 b^2) \cosh(x)^4 + 2(35(a^2 + 6 a b - 3 b^2) \cosh(x)^4 + 30(a^2 + 6 a b - 3 b^2) \cosh(x)^2 + 3 a^2 + 18 a b - 9 b^2) \sinh(x)^4 + 8(7(a^2 + 6 a b - 3 b^2) \cosh(x)^5 + 10(a^2 + 6 a b - 3 b^2) \cosh(x)^3 + 3(a^2 + 6 a b - 3 b^2) \cosh(x)) \sinh(x)^3 + 4(a^2 + 6 a b - 3 b^2) \cosh(x)^2 + 4(7(a^2 + 6 a b - 3 b^2) \cosh(x)^6 + 15(a^2 + 6 a b - 3 b^2) \cosh(x)^4 + 9(a^2 + 6 a b - 3 b^2) \cosh(x)^2 + a^2 + 6 a b - 3 b^2) \sinh(x)^2 + a^2 + 6 a b - 3 b^2 + 8((a^2 + 6 a b - 3 b^2) \cosh(x)^7 + 3(a^2 + 6 a b - 3 b^2) \cosh(x)^5 + 3(a^2 + 6 a b - 3 b^2) \cosh(x)^3 + (a^2 + 6 a b - 3 b^2) \cosh(x)) \sinh(x)) \sqrt{b} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^4 + 4 a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3 a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x)) \sinh(x) + a)) - \sqrt{2}((a b - 5 b^2) \cosh(x)^6 + 6(a b - 5 b^2) \cosh(x) \sinh(x)^5 + (a b - 5 b^2) \sinh(x)^6 + (a b + 3 b^2) \cosh(x)^4 + (15(a b - 5 b^2) \cosh(x)^2 + a b + 3 b^2) \sinh(x)^4 + 4(5(a b - 5 b^2) \cosh(x)^3 + (a b + 3 b^2) \cosh(x)) \sinh(x)^3 - (a b + 3 b^2) \cosh(x)^2 + (15(a b - 5 b^2) \cosh(x)^4 + 6(a b + 3 b^2) \cosh(x)^2 - a b - 3 b^2) \sinh(x)^2 - a b + 5 b^2 + 2(3(a b - 5 b^2) \cosh(x)^5 + 2(a b + 3 b^2) \cosh(x)^3 - (a b + 3 b^2) \cosh(x)) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)))/ (b^2 \cosh(x)^8 + 8b^2 \cosh(x) \sinh(x)^7 + b^2 \sinh(x)^8 + 4b^2 \cosh(x)^6 + 4(7b^2 \cosh(x)^2 + b^2) \sinh(x)^6 + 6b^2 \cosh(x)^4 + 8(7b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^5 + 2(35b^2 \cosh(x)^4 + 30b^2 \cosh(x)^2 + 3b^2) \sinh(x)^4 + 4b^2 \cosh(x)^2 + 8(7b^2 \cosh(x)^5 + 10b^2 \cosh(x)^3 + 3b^2 \cosh(x)) \sinh(x)^3 + 4(7b^2 \cosh(x)^6 + 15b^2 \cosh(x)^4 + 9b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 8(b^2 \cosh(x)^7 + 3b^2 \cosh(x)^5
\end{aligned}$$

+ 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT>Error: Bad Argument Type

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(x)^2} (\tanh^4(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)

[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \tanh(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^4 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2),x)

[Out] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**4,x)

[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**4, x)

3.178 $\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^3(x) dx$

Optimal. Leaf size=59

$$\frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} - \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)$$

[Out] $1/3*(a+b*\operatorname{sech}(x)^2)^{(3/2)}/b+\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} - \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]`

[Out] `Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + (a + b*Sech[x]^2)^(3/2)/(3*b)`

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 4139

$\text{Int}[(a + b*(c*sec[e + f*x] + (f*x))^n)^p * tan[e + f*x] * (f*x)^m, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sec}[e + f*x], x]\}, \text{Dist}[1/ff, \text{Subst}[\text{Int}[(-1 + ff^2*x^2)^{(m-1)/2} * (a + b*(c*ff*x)^n)^p / x, x], x, \text{Sec}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2] \ \&\& \ (\text{GtQ}[m, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ \text{IGtQ}[p, 0] \ || \ \text{IntegersQ}[2*n, p])$

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^3(x) dx &= \text{Subst} \left(\int \frac{(-1 + x^2) \sqrt{a + bx^2}}{x} dx, x, \operatorname{sech}(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{(-1 + x) \sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x) \right) \\ &= \frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} - \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x) \right) \\ &= -\sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} - \frac{1}{2} a \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\ &= -\sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} - \frac{a \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \operatorname{sech}^2(x)} \right)}{b} \\ &= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a + b \operatorname{sech}^2(x)} + \frac{(a + b \operatorname{sech}^2(x))^{3/2}}{3b} \end{aligned}$$

Mathematica [A] time = 0.40, size = 90, normalized size = 1.53

$$\frac{1}{3} \cosh(x) \sqrt{a + b \operatorname{sech}^2(x)} \left(\left(\frac{a}{b} - 3 \right) \operatorname{sech}(x) + \frac{3\sqrt{2} \sqrt{a} \log(\sqrt{a} \cosh(2x) + a + 2b) + \sqrt{2} \sqrt{a} \cosh(x)}{\sqrt{a} \cosh(2x) + a + 2b} \right) + \operatorname{sech}^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]

[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((3*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-3 + a/b)*Sech[x] + Sech[x]^3))/3

fricas [B] time = 0.73, size = 2394, normalized size = 40.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")

[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)

$$\begin{aligned}
&)^3 + 3*b*\cosh(x)^2 + 3*(5*b*\cosh(x)^4 + 6*b*\cosh(x)^2 + b)*\sinh(x)^2 + 6*(\\
& b*\cosh(x)^5 + 2*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\sqrt{a}*\log(((a^3 + 2 \\
& *a^2*b + a*b^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)*\sinh(x)^7 + (\\
& a^3 + 2*a^2*b + a*b^2)*\sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh \\
& (x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*\cos \\
& h(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^3 + 3*(2*a^3 + 5* \\
& a^2*b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\co \\
& sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^ \\
& 2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 \\
& + 2*a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x) \\
& ^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a \\
& ^2*b)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + 15*(2*a^3 + 5*a \\
& ^2*b + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9 \\
& *a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2*a*b + b^2)*\cosh(x)^6 + 6*(\\
& a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh(x)^6 + 3*(a \\
& ^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 + 2* \\
& a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 + 2*a*b \\
& + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 + (15*(a^2 + 2*a*b + \\
& b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^ \\
& 2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x) \\
&)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(\\
& x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + \\
& 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^5 \\
& + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh(\\
& x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^ \\
& 3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + \\
& 3*(b*\cosh(x)^6 + 6*b*\cosh(x)*\sinh(x)^5 + b*\sinh(x)^6 + 3*b*\cosh(x)^4 + 3*(5 \\
& *b*\cosh(x)^2 + b)*\sinh(x)^4 + 4*(5*b*\cosh(x)^3 + 3*b*\cosh(x))*\sinh(x)^3 + 3 \\
& *b*\cosh(x)^2 + 3*(5*b*\cosh(x)^4 + 6*b*\cosh(x)^2 + b)*\sinh(x)^2 + 6*(b*\cosh(\\
& x)^5 + 2*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\sqrt{a}*\log(-(a*\cosh(x)^4 + \\
& 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b) \\
& *\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a} \\
&)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)} + 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\co \\
& sh(x)*\sinh(x) + \sinh(x)^2)) + 4*\sqrt{2}*((a - 3*b)*\cosh(x)^4 + 4*(a - 3*b)* \\
& \cosh(x)*\sinh(x)^3 + (a - 3*b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a - 3 \\
& *b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a - 3*b)*\cosh(x)^3 + (a - b)*\cosh(x) \\
&)*\sinh(x) + a - 3*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 \\
& - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(b*\cosh(x)^6 + 6*b*\cosh(x)*\sinh(x)^5 + b \\
& *\sinh(x)^6 + 3*b*\cosh(x)^4 + 3*(5*b*\cosh(x)^2 + b)*\sinh(x)^4 + 4*(5*b*\cosh(\\
& x)^3 + 3*b*\cosh(x))*\sinh(x)^3 + 3*b*\cosh(x)^2 + 3*(5*b*\cosh(x)^4 + 6*b*\cosh \\
& (x)^2 + b)*\sinh(x)^2 + 6*(b*\cosh(x)^5 + 2*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) \\
& + b), -1/6*(3*(b*\cosh(x)^6 + 6*b*\cosh(x)*\sinh(x)^5 + b*\sinh(x)^6 + 3*b*\cosh \\
& (x)^4 + 3*(5*b*\cosh(x)^2 + b)*\sinh(x)^4 + 4*(5*b*\cosh(x)^3 + 3*b*\cosh(x))*s \\
& inh(x)^3 + 3*b*\cosh(x)^2 + 3*(5*b*\cosh(x)^4 + 6*b*\cosh(x)^2 + b)*\sinh(x)^2 \\
& + 6*(b*\cosh(x)^5 + 2*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\sqrt{-a}*\arctan(\\
& \sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 \\
& + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cos \\
& h(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\s \\
& inh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a* \\
& b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 \\
& + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x)) + 3*(b*\cosh(x)^6 + 6*b*\cosh(x)*\sinh(x) \\
& ^5 + b*\sinh(x)^6 + 3*b*\cosh(x)^4 + 3*(5*b*\cosh(x)^2 + b)*\sinh(x)^4 + 4*(5*b \\
& *\cosh(x)^3 + 3*b*\cosh(x))*\sinh(x)^3 + 3*b*\cosh(x)^2 + 3*(5*b*\cosh(x)^4 + 6* \\
& b*\cosh(x)^2 + b)*\sinh(x)^2 + 6*(b*\cosh(x)^5 + 2*b*\cosh(x)^3 + b*\cosh(x))*\si \\
& nh(x) + b)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x) \\
& ^2 - 1)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2* \\
& \cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh \\
& (x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(
\end{aligned}$$

```
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((a - 3*b)*cosh(x)^4 + 4*(a - 3*b)*cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - 3*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(x)^2} (\tanh^3(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \tanh(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(x)^3 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**3, x)
```

$$3.179 \quad \int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^2(x) dx$$

Optimal. Leaf size=87

$$-\frac{1}{2} \tanh(x) \sqrt{a - b \tanh^2(x) + b} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \frac{(a - b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)}{2\sqrt{b}}$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/2*(a-b)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.22, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4141, 1975, 478, 523, 217, 203, 377, 206}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \frac{1}{2} \tanh(x) \sqrt{a - b \tanh^2(x) + b} - \frac{(a - b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]

[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*Sqrt[b]) + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 478

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +

1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
 \int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^2(x) dx &= \operatorname{Subst} \left(\int \frac{x^2 \sqrt{a + b(1 - x^2)}}{1 - x^2} dx, x, \tanh(x) \right) \\
 &= \operatorname{Subst} \left(\int \frac{x^2 \sqrt{a + b - bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{2} \tanh(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{2} \operatorname{Subst} \left(\int \frac{a + b + (a - b)x^2}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x \right) \\
 &= -\frac{1}{2} \tanh(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x \right) \\
 &= -\frac{1}{2} \tanh(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) \\
 &= -\frac{(a - b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)}{2\sqrt{b}} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{a + b - b \tanh^2(x)}
 \end{aligned}$$

Mathematica [A] time = 0.29, size = 150, normalized size = 1.72

$$\frac{\cosh(x) \sqrt{a + b \operatorname{sech}^2(x)} \left(\sqrt{2} (a - b) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{b} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) - 2\sqrt{2} \sqrt{a} \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + \sqrt{b} \tanh(x) \right)}{2\sqrt{b} \sqrt{a \cosh(2x) + a + 2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]

[Out]
$$-1/2*(\text{Cosh}[x]*\text{Sqrt}[a + b*\text{Sech}[x]^2]*(\text{Sqrt}[2]*(a - b)*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[b]*\text{Sinh}[x])/\text{Sqrt}[a + 2*b + a*\text{Cosh}[2*x]])] - 2*\text{Sqrt}[2]*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[a]*\text{Sinh}[x])/\text{Sqrt}[a + 2*b + a*\text{Cosh}[2*x]])] + \text{Sqrt}[b]*\text{Sqrt}[a + 2*b + a*\text{Cosh}[2*x]]*\text{Sech}[x]*\text{Tanh}[x])/(\text{Sqrt}[b]*\text{Sqrt}[a + 2*b + a*\text{Cosh}[2*x]])$$

fricas [B] time = 0.89, size = 4316, normalized size = 49.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + \\ & 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)* \\ & \text{sqrt}(a)*\log((a*b^2*\cosh(x)^8 + 8*a*b^2*\cosh(x)*\sinh(x)^7 + a*b^2*\sinh(x)^8 \\ & - 2*(a*b^2 - b^3)*\cosh(x)^6 + 2*(14*a*b^2*\cosh(x)^2 - a*b^2 + b^3)*\sinh(x)^6 \\ & + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\cosh(x))*\sinh(x)^5 + (a^3 + 4*a \\ & ^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - \\ & 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*a*b^2*\cosh(x)^5 - 10*(a*b^2 \\ & - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(\\ & a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x)^6 - 15*(a*b^2 - b^3)*\cosh(x) \\ & ^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \text{sqrt} \\ & (2)*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b^2*\cosh \\ & (x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3*b^2*\cosh \\ & (x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (15*b^2*\cosh(x)^4 - 18*b^2*\cosh \\ & (x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^2*\cosh(x)^5 - 6*b^2*\cosh(x)^3 \\ & - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\text{sqrt}(a)*\text{sqrt}((a*\cosh(x)^2 + a*\sinh(x)^2 \\ & + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(2*a*b^2*\cosh(x) \\ &)^7 - 3*(a*b^2 - b^3)*\cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^3 + (a^3 \\ & + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x) \\ &)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x) \\ &)*\sinh(x)^5 + \sinh(x)^6)) + ((a - b)*\cosh(x)^4 + 4*(a - b)*\cosh(x)*\sinh(x)^3 \\ & + (a - b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a - b)*\cosh(x)^2 + a - b) \\ &)*\sinh(x)^2 + 4*((a - b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a - b)*\text{sqrt} \\ & (-b)*\log(-((a - b)*\cosh(x)^4 + 4*(a - b)*\cosh(x)*\sinh(x)^3 + (a - b)*\sinh(x) \\ &)^4 + 2*(a + 3*b)*\cosh(x)^2 + 2*(3*(a - b)*\cosh(x)^2 + a + 3*b)*\sinh(x)^2 + \\ & 2*\text{sqrt}(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\text{sqrt}(-b)*\text{sqrt}((a \\ &)*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x) \\ &)^2)) + 4*((a - b)*\cosh(x)^3 + (a + 3*b)*\cosh(x))*\sinh(x) + a - b)/(\cosh(x) \\ &)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh \\ & (x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)) + (b*\cosh(x)^4 + 4*b*\cosh(x) \\ &)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 \\ & + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\text{sqrt}(a)*\log(-(a*\cosh(x)^4 + 4*a \\ &)*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + \\ & a + b)*\sinh(x)^2 + \text{sqrt}(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1) \\ &)*\text{sqrt}(a)*\text{sqrt}((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)* \\ &)\sinh(x) + \sinh(x)^2)) + 4*(a*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a)/(\cosh \\ & (x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\text{sqrt}(2)*(b*\cosh(x)^2 + 2*b*\cosh \\ & (x))*\sinh(x) + b*\sinh(x)^2 - b)*\text{sqrt}((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/ \\ & (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh \\ & (x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4 \\ & *(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b), -1/4*(2*((a - b)*\cosh(x)^4 + 4*(a \\ & - b)*\cosh(x)*\sinh(x)^3 + (a - b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a \\ & - b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a - b)*\cosh(x)^3 + (a - b)*\cosh(x) \\ &)*\sinh(x) + a - b)*\text{sqrt}(b)*\arctan(\text{sqrt}(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh \\ & (x)^2 - 1)*\text{sqrt}(b)*\text{sqrt}((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 \\ & - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a \\ &)*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 \end{aligned}$$

$$\begin{aligned}
& + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a) - (b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\sqrt{a}*\log((a*b^2*\cosh(x)^8 + 8*a*b^2*\cosh(x)*\sinh(x)^7 + a*b^2*\sinh(x)^8 - 2*(a*b^2 - b^3)*\cosh(x)^6 + 2*(14*a*b^2*\cosh(x)^2 - a*b^2 + b^3)*\sinh(x)^6 + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\cosh(x))*\sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*a*b^2*\cosh(x)^5 - 10*(a*b^2 - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x)^6 - 15*(a*b^2 - b^3)*\cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (15*b^2*\cosh(x)^4 - 18*b^2*\cosh(x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^2*\cosh(x)^5 - 6*b^2*\cosh(x)^3 - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*\cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (b*\cosh(x)^4 + 4*b*\cosh(x)*sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*sinh(x)^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(b*\cosh(x)^2 + 2*b*\cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/2*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*
\end{aligned}$$

```
(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2
+ 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)
)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (
6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
+ 3*a*b)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh
(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a - b)*cosh
(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)
)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*
b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh
(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)
)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(x)^2} (\tanh^2(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)

[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \tanh(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2), x)`

[Out] `int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**2, x)`

[Out] `Integral(sqrt(a + b*sech(x)**2)*tanh(x)**2, x)`

3.180 $\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh(x) dx$

Optimal. Leaf size=40

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a + b \operatorname{sech}^2(x)}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 266, 50, 63, 208}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a + b \operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]*\operatorname{Tanh}[x], x]$

[Out] $\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]] - \operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p]/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegerQ
```

[2*n, p])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh(x) dx &= -\operatorname{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x} dx, x, \operatorname{sech}(x) \right) \\
&= -\left(\frac{1}{2} \operatorname{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x) \right) \right) \\
&= -\sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\sqrt{a + b \operatorname{sech}^2(x)} - \frac{a \operatorname{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \operatorname{sech}^2(x)} \right)}{b} \\
&= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a + b \operatorname{sech}^2(x)}
\end{aligned}$$

Mathematica [B] time = 0.27, size = 90, normalized size = 2.25

$$\frac{\sqrt{a + b \operatorname{sech}^2(x)} \left(-\sqrt{2} \sqrt{a} \sqrt{b} \cosh(x) \sqrt{\frac{a \cosh(2x) + a + 2b}{b}} \sinh^{-1} \left(\frac{\sqrt{a} \cosh(x)}{\sqrt{b}} \right) + a \cosh(2x) + a + 2b \right)}{a \cosh(2x) + a + 2b}$$

Antiderivative was successfully verified.

`[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x], x]`

```
[Out] -(((a + 2*b + a*Cosh[2*x] - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[a]*Cosh[x])]/Sqrt[b]]*Cosh[x]*Sqrt[(a + 2*b + a*Cosh[2*x])/b])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x]))
```

fricas [B] time = 0.49, size = 1608, normalized size = 40.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x), x, algorithm="fricas")`

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
```

```

+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x)
)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 +
4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*
cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(
2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + (cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2
*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.07, size = 43, normalized size = 1.08

$$-\sqrt{a + b\operatorname{sech}(x)^2} + \sqrt{a} \ln\left(\frac{2a + 2\sqrt{a} \sqrt{a + b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(1/2)*tanh(x),x)

[Out] -(a+b*sech(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sec
h(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)

mupad [B] time = 1.71, size = 32, normalized size = 0.80

$$\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right) - \sqrt{a + \frac{b}{\cosh(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b/cosh(x)^2)^(1/2), x)

[Out] a^(1/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x), x)

[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x), x)

3.181 $\int \sqrt{a + b \operatorname{sech}^2(x)} dx$

Optimal. Leaf size=59

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) + \sqrt{b} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)$$

[Out] $\operatorname{arctanh}(a^{1/2} \tanh(x) / (a + b - b \tanh(x)^2)^{1/2}) * a^{1/2} + \operatorname{arctan}(b^{1/2} \tanh(x) / (a + b - b \tanh(x)^2)^{1/2}) * b^{1/2}$

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4128, 402, 217, 203, 377, 206}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) + \sqrt{b} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*Sech[x]^2], x]`

[Out] `Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]`

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 377

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]`

Rule 402

`Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])`

Rule 4128

`Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/`

$(1 + ff^2*x^2), x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&$
 $\& \text{NeQ}[a + b, 0] \&\& \text{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \operatorname{sech}^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + b - bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= a \text{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) + b \text{Subst} \left(\int \frac{1}{\sqrt{a + b - bx^2}} dx, x, \right. \\ &= a \text{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) + b \text{Subst} \left(\int \frac{1}{1 + bx^2} dx, x, \frac{t}{\sqrt{a + b}} \right) \\ &= \sqrt{b} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) \end{aligned}$$

Mathematica [B] time = 0.74, size = 134, normalized size = 2.27

$$\frac{\sqrt{2} \cosh(x) \sqrt{a + b \operatorname{sech}^2(x)} \left(\sqrt{a} \sqrt{a + b} \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a + b}} \right) \sqrt{\frac{a \cosh(2x) + a + 2b}{a + b}} + \sqrt{b} \sqrt{a \cosh(2x) + a + 2b} \tan^{-1} \left(\frac{\sqrt{b} \sinh(x)}{\sqrt{a + b - b \sinh^2(x)}} \right) \right)}{a \cosh(2x) + a + 2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sech[x]^2], x]

[Out] (Sqrt[2]*Cosh[x]*(Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])

fricas [B] time = 0.58, size = 2949, normalized size = 49.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c

$$\begin{aligned} & \text{osh}(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 1/2 \sqrt{-b} \log(-((a-b) \cosh(x)^4 + 4(a-b) \cosh(x) \sinh(x)^3 + (a-b) \sinh(x)^4 + 2(a+3b) \cosh(x)^2 + 2(3(a-b) \cosh(x)^2 + a+3b) \sinh(x)^2 - 2\sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) + 4((a-b) \cosh(x)^3 + (a+3b) \cosh(x)) \sinh(x) + a - b) / (\cosh(x)^4 + 4\cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1) \sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1)) + 1/4 \sqrt{a} \log(- (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a+b) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) + 4(a \cosh(x)^3 + (a+b) \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2)), \sqrt{b} \arctan(\sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a+2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a+2b) \cosh(x)) \sinh(x) + a)) + 1/4 \sqrt{a} \log((a b^2 \cosh(x)^8 + 8a b^2 \cosh(x) \sinh(x)^7 + a b^2 \sinh(x)^8 - 2(a b^2 - b^3) \cosh(x)^6 + 2(14a b^2 \cosh(x)^2 - a b^2 + b^3) \sinh(x)^6 + 4(14a b^2 \cosh(x)^3 - 3(a b^2 - b^3) \cosh(x)) \sinh(x)^5 + (a^3 + 4a^2 b + 9a b^2) \cosh(x)^4 + (70a b^2 \cosh(x)^4 + a^3 + 4a^2 b + 9a b^2 - 30(a b^2 - b^3) \cosh(x)^2) \sinh(x)^4 + 4(14a b^2 \cosh(x)^5 - 10(a b^2 - b^3) \cosh(x)^3 + (a^3 + 4a^2 b + 9a b^2) \cosh(x)) \sinh(x)^3 + a^3 + 2(a^3 + 3a^2 b) \cosh(x)^2 + 2(14a b^2 \cosh(x)^6 - 15(a b^2 - b^3) \cosh(x)^4 + a^3 + 3a^2 b + 3(a^3 + 4a^2 b + 9a b^2) \cosh(x)^2) \sinh(x)^2 + \sqrt{2}(b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 + 4a b) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 - 4a b) \sinh(x)^2 - a^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 + 4a b) \cosh(x)) \sinh(x)) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) + 4(2a b^2 \cosh(x)^7 - 3(a b^2 - b^3) \cosh(x)^5 + (a^3 + 4a^2 b + 9a b^2) \cosh(x)^3 + (a^3 + 3a^2 b) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6\cosh(x)^5 \sinh(x) + 15\cosh(x)^4 \sinh(x)^2 + 20\cosh(x)^3 \sinh(x)^3 + 15\cosh(x)^2 \sinh(x)^4 + 6\cosh(x) \sinh(x)^5 + \sinh(x)^6) + 1/4 \sqrt{a} \log(- (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a+b) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) + 4(a \cosh(x)^3 + (a+b) \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2)), -1/2 \sqrt{-a} \arctan(\sqrt{2}(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) / (a b \cosh(x)^4 + 4a b \cosh(x) \sinh(x)^3 + a b \sinh(x)^4 - (a^2 + 3a b) \cosh(x)^2 + (6a b \cosh(x)^2 - a^2 - 3a b) \sinh(x)^2 - a^2 + 2(2a b \cosh(x)^3 - (a^2 + 3a b) \cosh(x)) \sinh(x))) - 1/2 \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a+2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a+2b) \cosh(x)) \sinh(x) + a)) + 1/2 \sqrt{-b} \log(-((a-b) \cosh(x)^4 + 4(a-b) \cosh(x) \sinh(x)^3 + (a-b) \sinh(x)^4 + 2(a+3b) \cosh(x)^2 + 2(3(a-b) \cosh(x)^2 + a+3b) \sinh(x)^2 - 2\sqrt{2}(\cosh(x)^2 + 2\cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) + 4((a-b) \cosh(x)^3 + (a+3b) \cosh(x)) \sinh(x) + a - b) / (\cosh(x)^4 + 4\cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1) \sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1)), -1/2 \sqrt{-a} \arctan(\sqrt{2}(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}) / (a b \cosh(x)^4 + 4a b \cosh(x) \sinh(x)^3 + a b \sinh(x)^4 - (a^2 + 3a b) \cosh(x)^2 + (6a b \cosh(x)^2 - a^2 - 3a b) \sinh(x)^2 - a^2 + 2(2a b \cosh(x)^3 - (a^2 + 3a b) \cosh(x)) \sinh(x))) \end{aligned}$$

```

+ 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a
*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3
*a*b)*cosh(x))*sinh(x)) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(b
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by `u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by `u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
`u`, a substitution variable should perhaps be purged.Warning, replacing 0
by `u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by `u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by `u`, a substitution variable should perhaps be purged.Warning,
integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep)]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.Non regu
lar value [0] was discarded and replaced randomly by 0=[-10]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.Non regular value [0] was discarded and replaced randomly by 0=[-3
9]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0] was discarded and replaced
randomly by 0=[35]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.Non regular value [0] was discard
ed and replaced randomly by 0=[90]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.Non regular value
[0] was discarded and replaced randomly by 0=[-98]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
Non regular value [0] was discarded and replaced randomly by 0=[-5]Evaluati
on time: 0.57index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(x)^2)^(1/2),x)

[Out] int((a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sech(x)**2), x)

3.182 $\int \coth(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$

Optimal. Leaf size=56

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/(a+b)^{(1/2))}*(a+b)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 446, 83, 63, 208}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[x]*\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2], x]$

[Out] $\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]] - \operatorname{Sqrt}[a + b]*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a + b]]$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 83

$\operatorname{Int}[(e_. + (f_.)*(x_.))^{(p_.)/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))}, x_Symbol] := \operatorname{Dist}[(b*e - a*f)/(b*c - a*d), \operatorname{Int}[(e + f*x)^{(p-1)}/(a + b*x), x], x] - \operatorname{Dist}[(d*e - c*f)/(b*c - a*d), \operatorname{Int}[(e + f*x)^{(p-1)}/(c + d*x), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{LtQ}[0, p, 1]$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)*((c_.) + (d_.)*(x_.)^{(n_.))^{(q_.)}}, x_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 4139

$\operatorname{Int}[(a_. + (b_.)*((c_.)*\operatorname{sec}[(e_.) + (f_.)*(x_.)])^{(n_.))^{(p_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] := \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/f, \operatorname{Subst}[\operatorname{Int}[((-1 + ff^2*x^2)^{((m-1)/2)}*(a + b*(c*ff*x)^n)^p/x, x], x, \operatorname{Sec}[e + f*x]/ff], x]] /; \operatorname{FreeQ}[\{a, b, c, e, f, n, p\}, x] \&\& \operatorname{IntegerQ}[(m$

$$\begin{aligned}
& \text{osh}(x)^3 + 3*(a^2 + 2*a*b + b^2)*\text{cosh}(x))*\text{sinh}(x)^3 + (3*a^2 + 4*a*b)*\text{cosh}(x)^2 + (15*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^4 + 18*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^2 + 3*a^2 + 4*a*b)*\text{sinh}(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^5 + 6*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^3 + (3*a^2 + 4*a*b)*\text{cosh}(x))*\text{sinh}(x))*\text{sqrt}(a)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\text{cosh}(x)^3 + (2*a^3 + 3*a^2*b)*\text{cosh}(x))*\text{sinh}(x))/(\text{cosh}(x)^6 + 6*\text{cosh}(x)^5*\text{sinh}(x) + 15*\text{cosh}(x)^4*\text{sinh}(x)^2 + 20*\text{cosh}(x)^3*\text{sinh}(x)^3 + 15*\text{cosh}(x)^2*\text{sinh}(x)^4 + 6*\text{cosh}(x)*\text{sinh}(x)^5 + \text{sinh}(x)^6)) + 1/2*\text{sqrt}(a + b)*\text{log}(((2*a + b)*\text{cosh}(x)^4 + 4*(2*a + b)*\text{cosh}(x)*\text{sinh}(x)^3 + (2*a + b)*\text{sinh}(x)^4 + 2*(2*a + 3*b)*\text{cosh}(x)^2 + 2*(3*(2*a + b)*\text{cosh}(x)^2 + 2*a + 3*b)*\text{sinh}(x)^2 - 2*\text{sqrt}(2)*(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2 + 1))*\text{sqrt}(a + b)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)) + 4*((2*a + b)*\text{cosh}(x)^3 + (2*a + 3*b)*\text{cosh}(x))*\text{sinh}(x) + 2*a + b)/(\text{cosh}(x)^4 + 4*\text{cosh}(x)*\text{sinh}(x)^3 + \text{sinh}(x)^4 + 2*(3*\text{cosh}(x)^2 - 1)*\text{sinh}(x)^2 - 2*\text{cosh}(x)^2 + 4*(\text{cosh}(x)^3 - \text{cosh}(x))*\text{sinh}(x) + 1)) + 1/4*\text{sqrt}(a)*\text{log}(-(a*\text{cosh}(x)^4 + 4*a*\text{cosh}(x)*\text{sinh}(x)^3 + a*\text{sinh}(x)^4 + 2*b*\text{cosh}(x)^2 + 2*(3*a*\text{cosh}(x)^2 + b)*\text{sinh}(x)^2 + \text{sqrt}(2)*(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2 - 1))*\text{sqrt}(a)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)) + 4*(a*\text{cosh}(x)^3 + b*\text{cosh}(x))*\text{sinh}(x) + a)/(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)), \text{sqrt}(-a - b)*\text{arctan}(\text{sqrt}(2)*(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2 + 1))*\text{sqrt}(-a - b)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2))/(a*\text{cosh}(x)^4 + 4*a*\text{cosh}(x)*\text{sinh}(x)^3 + a*\text{sinh}(x)^4 + 2*(a + 2*b)*\text{cosh}(x)^2 + 2*(3*a*\text{cosh}(x)^2 + a + 2*b)*\text{sinh}(x)^2 + 4*(a*\text{cosh}(x)^3 + (a + 2*b)*\text{cosh}(x))*\text{sinh}(x) + a)) + 1/4*\text{sqrt}(a)*\text{log}(((a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)*\text{sinh}(x)^7 + (a^3 + 2*a^2*b + a*b^2)*\text{sinh}(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^2)*\text{sinh}(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x))*\text{sinh}(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\text{cosh}(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^2)*\text{sinh}(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\text{cosh}(x))*\text{sinh}(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*\text{cosh}(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*\text{cosh}(x)^2)*\text{sinh}(x)^2 + \text{sqrt}(2)*((a^2 + 2*a*b + b^2)*\text{cosh}(x)^6 + 6*(a^2 + 2*a*b + b^2)*\text{cosh}(x)*\text{sinh}(x)^5 + (a^2 + 2*a*b + b^2)*\text{sinh}(x)^6 + 3*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^2 + a^2 + 2*a*b + b^2)*\text{sinh}(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^3 + 3*(a^2 + 2*a*b + b^2)*\text{cosh}(x))*\text{sinh}(x)^3 + (3*a^2 + 4*a*b)*\text{cosh}(x)^2 + (15*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^4 + 18*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^2 + 3*a^2 + 4*a*b)*\text{sinh}(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^5 + 6*(a^2 + 2*a*b + b^2)*\text{cosh}(x)^3 + (3*a^2 + 4*a*b)*\text{cosh}(x))*\text{sinh}(x))*\text{sqrt}(a)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\text{cosh}(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\text{cosh}(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\text{cosh}(x)^3 + (2*a^3 + 3*a^2*b)*\text{cosh}(x))*\text{sinh}(x))/(\text{cosh}(x)^6 + 6*\text{cosh}(x)^5*\text{sinh}(x) + 15*\text{cosh}(x)^4*\text{sinh}(x)^2 + 20*\text{cosh}(x)^3*\text{sinh}(x)^3 + 15*\text{cosh}(x)^2*\text{sinh}(x)^4 + 6*\text{cosh}(x)*\text{sinh}(x)^5 + \text{sinh}(x)^6)) + 1/4*\text{sqrt}(a)*\text{log}(-(a*\text{cosh}(x)^4 + 4*a*\text{cosh}(x)*\text{sinh}(x)^3 + a*\text{sinh}(x)^4 + 2*b*\text{cosh}(x)^2 + 2*(3*a*\text{cosh}(x)^2 + b)*\text{sinh}(x)^2 + \text{sqrt}(2)*(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2 - 1))*\text{sqrt}(a)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)) + 4*(a*\text{cosh}(x)^3 + b*\text{cosh}(x))*\text{sinh}(x) + a)/(\text{cosh}(x)^2 + 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2)), -1/2*\text{sqrt}(-a)*\text{arctan}(\text{sqrt}(2)*((a + b)*\text{cosh}(x)^2 + 2*(a + b)*\text{cosh}(x)*\text{sinh}(x) + (a + b)*\text{sinh}(x)^2 + a))*\text{sqrt}(-a)*\text{sqrt}((a*\text{cosh}(x)^2 + a*\text{sinh}(x)^2 + a + 2*b)/(\text{cosh}(x)^2 - 2*\text{cosh}(x)*\text{sinh}(x) + \text{sinh}(x)^2))/(a^2 + a*b)*\text{cosh}(x)^4 + 4*(a^2 + a*b)*\text{cosh}(x)*\text{sinh}(x)^3 + (a^2 + a*b)*\text{sinh}(x)^4 + (2*a^2 + 3*a*b)*\text{cos
\end{aligned}$$

[0] was discarded and replaced randomly by 0=[89]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.
 Non regular value [0] was discarded and replaced randomly by 0=[-4]Precision problem choosing root in common_EXT, current precision 14Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.
 Non regular value [0] was discarded and replaced randomly by 0=[16]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.
 Non regular value [0] was discarded and replaced randomly by 0=[-86]Precision problem choosing root in common_EXT, current precision 14
 Evaluation time: 0.62
 index.cc index_m operator + Error: Bad Argument Value

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \coth(x) \sqrt{a + b \operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a+b*sech(x)^2)^(1/2), x)

[Out] int(coth(x)*(a+b*sech(x)^2)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x) \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a + b/cosh(x)^2)^(1/2), x)

[Out] int(coth(x)*(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sech(x)**2)**(1/2), x)

[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x), x)

3.183 $\int \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$

Optimal. Leaf size=48

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \coth(x) \sqrt{a - b \tanh^2(x) + b}$$

[Out] $\operatorname{arctanh}(a^{(1/2)} \tanh(x) / (a + b \tanh(x)^2)^{(1/2)}) * a^{(1/2)} - \coth(x) * (a + b \tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4141, 1975, 475, 12, 377, 206}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \coth(x) \sqrt{a - b \tanh^2(x) + b}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]`

[Out] `Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqrt[a + b - b*Tanh[x]^2]`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 377

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]`

Rule 475

`Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]`

Rule 1975

`Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !`

BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
 \int \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} dx &= \operatorname{Subst} \left(\int \frac{\sqrt{a + b(1 - x^2)}}{x^2(1 - x^2)} dx, x, \tanh(x) \right) \\
 &= \operatorname{Subst} \left(\int \frac{\sqrt{a + b - bx^2}}{x^2(1 - x^2)} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b - b \tanh^2(x)} + \operatorname{Subst} \left(\int \frac{a}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b - b \tanh^2(x)} + a \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) \\
 &= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) - \coth(x) \sqrt{a + b - b \tanh^2(x)}
 \end{aligned}$$

Mathematica [A] time = 0.52, size = 75, normalized size = 1.56

$$\sqrt{a + b \operatorname{sech}^2(x)} \left(\frac{\sqrt{2} \sqrt{a} \cosh(x) \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a+b}} \right)}{\sqrt{a+b} \sqrt{\frac{a \cosh(2x) + a + 2b}{a+b}}} - \coth(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2*Sqrt[a + b*Sech[x]^2], x]

[Out] ((Sqrt[2]*Sqrt[a]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Cosh[x])/(Sqrt[a + b]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]) - Coth[x])*Sqrt[a + b*Sech[x]^2]

fricas [B] time = 0.52, size = 1303, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)

```

x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cos
h(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b +
3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 +
6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(
x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^
2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*
sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh
(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)
*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x)
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log(-(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
+ a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*co
sh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a
*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cos
h(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*
b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
+ 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int (\coth^2(x)) \sqrt{a + b \operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)^2 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2), x)

[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \coth^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(1/2), x)

[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**2, x)

$$3.184 \quad \int \coth^3(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$$

Optimal. Leaf size=83

$$-\frac{1}{2} \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)}{2\sqrt{a+b}}$$

[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*(2*a+b)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 99, 156, 63, 208}

$$-\frac{1}{2} \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3*Sqrt[a + b*Sech[x]^2], x]

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((2*a + b)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]) - (Coth[x]^2*Sqrt[a + b*Sech[x]^2])/2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (
f_)*(x_)^(m_)], x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^(m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \coth^3(x) \sqrt{a + b \operatorname{sech}^2(x)} dx &= -\operatorname{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x(-1 + x^2)^2} dx, x, \operatorname{sech}(x) \right) \\
&= -\left(\frac{1}{2} \operatorname{Subst} \left(\int \frac{\sqrt{a + bx}}{(-1 + x)^2 x} dx, x, \operatorname{sech}^2(x) \right) \right) \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{2} \operatorname{Subst} \left(\int \frac{-a - \frac{bx}{2}}{(-1 + x)x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) - \frac{1}{4} \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{a \operatorname{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \operatorname{sech}^2(x)} \right)}{b} \\
&= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a + b}} \right)}{2\sqrt{a + b}} - \frac{1}{2} \coth^2(x)
\end{aligned}$$

Mathematica [A] time = 0.59, size = 156, normalized size = 1.88

$$\frac{\sqrt{a + b \operatorname{sech}^2(x)} \left(\sqrt{2} (2a + b) \cosh(x) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a + b} \cosh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + \sqrt{a + b} \left(\coth^2(x) \sqrt{a \cosh(2x) + a + 2b} \right) \right)}{2\sqrt{a + b} \sqrt{a \cosh(2x) + a + 2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Sech[x]^2], x]
```

```
[Out] -1/2*((Sqrt[2]*(2*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
+ a*Cosh[2*x]])*Cosh[x] + Sqrt[a + b]*(Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[x]
^2 - 2*Sqrt[2]*Sqrt[a]*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*Sqrt[a + 2*b + a*Cosh[
2*x]])
```

fricas [B] time = 0.63, size = 5247, normalized size = 63.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 \\ & - 2*(a + b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - a - b)*\sinh(x)^2 + 4*((a + \\ & b)*\cosh(x)^3 - (a + b)*\cosh(x))*\sinh(x) + a + b)*\sqrt{a}*\log(((a^3 + 2*a^2 \\ & *b + a*b^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)*\sinh(x)^7 + (a^3 \\ & + 2*a^2*b + a*b^2)*\sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^6 \\ & + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x) \\ & ^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^3 + 3*(2*a^3 + 5*a^2* \\ & b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x) \\ & ^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + \\ & 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + 2* \\ & a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^3 + \\ & (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b \\ &)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + 15*(2*a^3 + 5*a^2*b \\ & + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b \\ & ^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2*a*b + b^2)*\cosh(x)^6 + 6*(a^2 \\ & + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh(x)^6 + 3*(a^2 + \\ & 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 + 2*a*b \\ & + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 + 2*a*b + b^ \\ & 2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 + (15*(a^2 + 2*a*b + b^2) \\ & *\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^2 + \\ & a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)^3 \\ & + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 \\ & + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + 2*a^ \\ & 2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^5 + (6 \\ & *a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/ \\ & (\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 \\ & + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + ((2*a \\ & + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 - 2* \\ & (2*a + b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a - b)*\sinh(x)^2 + 4*((2 \\ & *a + b)*\cosh(x)^3 - (2*a + b)*\cosh(x))*\sinh(x) + 2*a + b)*\sqrt{a + b}*\log(((\\ & 2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 + \\ & 2*(2*a + 3*b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a + 3*b)*\sinh(x)^2 \\ & - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a + b}*\sqrt{ \\ & (a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} \\ & + 4*((2*a + b)*\cosh(x)^3 + (2*a + 3*b)*\cosh(x))*\sinh(x) + 2*a + b \\ &)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x) \\ &)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) + ((a + b)*\cosh(x) \\ & ^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 - 2*(a + b)*\cosh(x)^2 \\ & + 2*(3*(a + b)*\cosh(x)^2 - a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 - (a + \\ & b)*\cosh(x))*\sinh(x) + a + b)*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x) \\ &)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqrt{2} \\ &)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 \\ & + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4 \\ & *(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2) \\ & - 2*\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + \\ & b)*\sinh(x)^2 + a + b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 \\ & - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x) \\ & *\sinh(x)^3 + (a + b)*\sinh(x)^4 - 2*(a + b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x) \\ & ^2 - a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 - (a + b)*\cosh(x))*\sinh(x) + a \\ & + b), 1/4*(2*((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + \\ & b)*\sinh(x)^4 - 2*(2*a + b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a - b) \\ & *\sinh(x)^2 + 4*((2*a + b)*\cosh(x)^3 - (2*a + b)*\cosh(x))*\sinh(x) + 2*a + b) \\ & *\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1) \\ &)*\sqrt{-a - b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x) \\ & *\sinh(x) + \sinh(x)^2)})/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x) \\ &)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a \\ & \end{aligned}$$

$$\begin{aligned}
& \cosh(x)^3 + (a + 2b)\cosh(x)\sinh(x) + a) + ((a + b)\cosh(x)^4 + 4(a + b)\cosh(x)\sinh(x)^3 + (a + b)\sinh(x)^4 - 2(a + b)\cosh(x)^2 + 2(3(a + b)\cosh(x)^2 - a - b)\sinh(x)^2 + 4((a + b)\cosh(x)^3 - (a + b)\cosh(x))\sinh(x) + a + b)\sqrt{a}\log(((a^3 + 2a^2b + ab^2)\cosh(x)^8 + 8(a^3 + 2a^2b + ab^2)\cosh(x)\sinh(x)^7 + (a^3 + 2a^2b + ab^2)\sinh(x)^8 + 2(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x)^6 + 2(2a^3 + 5a^2b + 4ab^2 + b^3 + 14(a^3 + 2a^2b + ab^2)\cosh(x)^2)\sinh(x)^6 + 4(14(a^3 + 2a^2b + ab^2)\cosh(x)^3 + 3(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x))\sinh(x)^5 + (6a^3 + 14a^2b + 9ab^2)\cosh(x)^4 + (70(a^3 + 2a^2b + ab^2)\cosh(x)^4 + 6a^3 + 14a^2b + 9ab^2 + 30(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x)^2)\sinh(x)^4 + 4(14(a^3 + 2a^2b + ab^2)\cosh(x)^5 + 10(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x)^3 + (6a^3 + 14a^2b + 9ab^2)\cosh(x))\sinh(x)^3 + a^3 + 2(2a^3 + 3a^2b)\cosh(x)^2 + 2(14(a^3 + 2a^2b + ab^2)\cosh(x)^6 + 15(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x)^4 + 2a^3 + 3a^2b + 3(6a^3 + 14a^2b + 9ab^2)\cosh(x)^2)\sinh(x)^2 + \sqrt{2}((a^2 + 2ab + b^2)\cosh(x)^6 + 6(a^2 + 2ab + b^2)\cosh(x)\sinh(x)^5 + (a^2 + 2ab + b^2)\sinh(x)^6 + 3(a^2 + 2ab + b^2)\cosh(x)^4 + 3(5(a^2 + 2ab + b^2)\cosh(x)^2 + a^2 + 2ab + b^2)\sinh(x)^4 + 4(5(a^2 + 2ab + b^2)\cosh(x)^3 + 3(a^2 + 2ab + b^2)\cosh(x))\sinh(x)^3 + (3a^2 + 4ab)\cosh(x)^2 + (15(a^2 + 2ab + b^2)\cosh(x)^4 + 18(a^2 + 2ab + b^2)\cosh(x)^2 + 3a^2 + 4ab)\sinh(x)^2 + a^2 + 2(3(a^2 + 2ab + b^2)\cosh(x)^5 + 6(a^2 + 2ab + b^2)\cosh(x)^3 + (3a^2 + 4ab)\cosh(x))\sinh(x))\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(cosh(x)^2 - 2cosh(x)\sinh(x) + sinh(x)^2)) + 4(2(a^3 + 2a^2b + ab^2)\cosh(x)^7 + 3(2a^3 + 5a^2b + 4ab^2 + b^3)\cosh(x)^5 + (6a^3 + 14a^2b + 9ab^2)\cosh(x)^3 + (2a^3 + 3a^2b)\cosh(x))\sinh(x))/(cosh(x)^6 + 6cosh(x)^5\sinh(x) + 15cosh(x)^4\sinh(x)^2 + 20cosh(x)^3\sinh(x)^3 + 15cosh(x)^2\sinh(x)^4 + 6cosh(x)\sinh(x)^5 + sinh(x)^6)) + ((a + b)\cosh(x)^4 + 4(a + b)\cosh(x)\sinh(x)^3 + (a + b)\sinh(x)^4 - 2(a + b)\cosh(x)^2 + 2(3(a + b)\cosh(x)^2 - a - b)\sinh(x)^2 + 4((a + b)\cosh(x)^3 - (a + b)\cosh(x))\sinh(x) + a + b)\sqrt{a}\log(-(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2b\cosh(x)^2 + 2(3a\cosh(x)^2 + b)\sinh(x)^2 + \sqrt{2}(cosh(x)^2 + 2cosh(x)\sinh(x) + sinh(x)^2 - 1)\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(cosh(x)^2 - 2cosh(x)\sinh(x) + sinh(x)^2)) + 4(a\cosh(x)^3 + b\cosh(x))\sinh(x) + a)/(cosh(x)^2 + 2cosh(x)\sinh(x) + sinh(x)^2)) - 2\sqrt{2}((a + b)\cosh(x)^2 + 2(a + b)\cosh(x)\sinh(x) + (a + b)\sinh(x)^2 + a + b)\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(cosh(x)^2 - 2cosh(x)\sinh(x) + sinh(x)^2)))/((a + b)\cosh(x)^4 + 4(a + b)\cosh(x)\sinh(x)^3 + (a + b)\sinh(x)^4 - 2(a + b)\cosh(x)^2 + 2(3(a + b)\cosh(x)^2 - a - b)\sinh(x)^2 + 4((a + b)\cosh(x)^3 - (a + b)\cosh(x))\sinh(x) + a + b), -1/4(2((a + b)\cosh(x)^4 + 4(a + b)\cosh(x)\sinh(x)^3 + (a + b)\sinh(x)^4 - 2(a + b)\cosh(x)^2 + 2(3(a + b)\cosh(x)^2 - a - b)\sinh(x)^2 + 4((a + b)\cosh(x)^3 - (a + b)\cosh(x))\sinh(x) + a + b)\sqrt{-a}\arctan(\sqrt{2}((a + b)\cosh(x)^2 + 2(a + b)\cosh(x)\sinh(x) + (a + b)\sinh(x)^2 + a)\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(cosh(x)^2 - 2cosh(x)\sinh(x) + sinh(x)^2)))/((a^2 + ab)\cosh(x)^4 + 4(a^2 + ab)\cosh(x)\sinh(x)^3 + (a^2 + ab)\sinh(x)^4 + (2a^2 + 3ab)\cosh(x)^2 + (6(a^2 + ab)\cosh(x)^2 + 2a^2 + 3ab)\sinh(x)^2 + a^2 + 2(2(a^2 + ab)\cosh(x)^3 + (2a^2 + 3ab)\cosh(x))\sinh(x))) + 2((a + b)\cosh(x)^4 + 4(a + b)\cosh(x)\sinh(x)^3 + (a + b)\sinh(x)^4 - 2(a + b)\cosh(x)^2 + 2(3(a + b)\cosh(x)^2 - a - b)\sinh(x)^2 + 4((a + b)\cosh(x)^3 - (a + b)\cosh(x))\sinh(x) + a + b)\sqrt{-a}\arctan(\sqrt{2}(cosh(x)^2 + 2cosh(x)\sinh(x) + sinh(x)^2 - 1)\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(cosh(x)^2 - 2cosh(x)\sinh(x) + sinh(x)^2)))/(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2(a + 2b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + 2b)\sinh(x)^2 + 4(a\cosh(x)^3 + (a + 2b)\cosh(x))\sinh(x) + a)) - ((2a + b)\cosh(x)^4 + 4(2a + b)\cosh(x)\sinh(x)^3 + (2a + b)\sinh(x)^4 - 2(2a + b)\cosh(x)^2 + 2(3(2a + b)\cosh(x)^2 - 2a - b)\sinh(x)^2 + 4((2a + b)\cosh(x)^3 - (2a + b)\cosh(x))\sinh(x) + 2a + b)\sqrt{a + b}\log(((2a + b)\cosh(x)^4 + 4(2a + b)\cosh(x)\sinh(x)
\end{aligned}$$

```
(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b), -1/2*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (\coth^3(x)) \sqrt{a + b\operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)

[Out] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \operatorname{coth}(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{coth}(x)^3 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2),x)

[Out] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \operatorname{coth}^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3*(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**3, x)

3.185 $\int \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$

Optimal. Leaf size=84

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \frac{1}{3} \coth^3(x) \sqrt{a - b \tanh^2(x) + b} - \frac{(3a + 2b) \coth(x) \sqrt{a - b \tanh^2(x) + b}}{3(a + b)}$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/3*(3*a+2*b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/(a+b)-1/3*coth(x)^3*(a+b-b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.25, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4141, 1975, 475, 583, 12, 377, 206}

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \frac{1}{3} \coth^3(x) \sqrt{a - b \tanh^2(x) + b} - \frac{(3a + 2b) \coth(x) \sqrt{a - b \tanh^2(x) + b}}{3(a + b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4*Sqrt[a + b*Sech[x]^2], x]

[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*(a + b)) - (Coth[x]^3*Sqrt[a + b - b*Tanh[x]^2])/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 475

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(

$m + 1)$), $\text{Int}[(g*x)^{(m+n)}*(a+b*x^n)^p*(c+d*x^n)^q*\text{Simp}[a*f*c*(m+1) - e*(b*c+a*d)*(m+n+1) - e*n*(b*c*p+a*d*q) - b*e*d*(m+n*(p+q+2) + 1)*x^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, p, q\}, x$ && $\text{IGtQ}[n, 0]$ && $\text{LtQ}[m, -1]$

Rule 1975

$\text{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_))^{(m_)}, x_Symbol] := \text{Int}[(e*x)^m*\text{ExpandToSum}[u, x]^p*\text{ExpandToSum}[v, x]^q, x] /;$ $\text{FreeQ}\{e, m, p, q\}, x$ && $\text{BinomialQ}\{u, v\}, x$ && $\text{EqQ}[\text{BinomialDegree}[u, x] - \text{BinomialDegree}[v, x], 0]$ && ! $\text{BinomialMatchQ}\{u, v\}, x$

Rule 4141

$\text{Int}[(a_ + (b_)*\text{sec}[(e_ + (f_)*(x_)]^{(n_)}])^{(p_)}*((d_)*\text{tan}[(e_ + (f_)*(x_)]^{(m_)}), x_Symbol] := \text{With}\{\text{ff} = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(d*ff*x)^m*(a + b*(1 + ff^2*x^2)^{(n/2)})^p/(1 + ff^2*x^2), x], x, \text{Tan}[e + f*x]/\text{ff}], x] /;$ $\text{FreeQ}\{a, b, d, e, f, m, p\}, x$ && $\text{IntegerQ}[n/2]$ && $(\text{IntegerQ}[m/2] \parallel \text{EqQ}[n, 2])$

Rubi steps

$$\begin{aligned} \int \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + b(1 - x^2)}}{x^4(1 - x^2)} dx, x, \tanh(x) \right) \\ &= \text{Subst} \left(\int \frac{\sqrt{a + b - bx^2}}{x^4(1 - x^2)} dx, x, \tanh(x) \right) \\ &= -\frac{1}{3} \coth^3(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{3} \text{Subst} \left(\int \frac{3a + 2b - 2bx^2}{x^2(1 - x^2) \sqrt{a + b - bx^2}} dx \right) \\ &= -\frac{(3a + 2b) \coth(x) \sqrt{a + b - b \tanh^2(x)}}{3(a + b)} - \frac{1}{3} \coth^3(x) \sqrt{a + b - b \tanh^2(x)} \\ &= -\frac{(3a + 2b) \coth(x) \sqrt{a + b - b \tanh^2(x)}}{3(a + b)} - \frac{1}{3} \coth^3(x) \sqrt{a + b - b \tanh^2(x)} \\ &= -\frac{(3a + 2b) \coth(x) \sqrt{a + b - b \tanh^2(x)}}{3(a + b)} - \frac{1}{3} \coth^3(x) \sqrt{a + b - b \tanh^2(x)} \\ &= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) - \frac{(3a + 2b) \coth(x) \sqrt{a + b - b \tanh^2(x)}}{3(a + b)} \end{aligned}$$

Mathematica [A] time = 0.62, size = 149, normalized size = 1.77

$$\frac{\sqrt{2} \cosh(x) \sqrt{a \sinh^2(x) + a + b} \sqrt{a + b \operatorname{sech}^2(x)} \left(3\sqrt{a} (a + b) \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a + b}} \right) - \sqrt{a + b} \operatorname{csch}(x) \sqrt{\frac{a \sinh^2(x) + a + b}{a + b}} \right)}{3(a + b)^{3/2} \sqrt{\frac{a \sinh^2(x) + a + b}{a + b}} \sqrt{a \cosh(2x) + a + 2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]

[Out] (Sqrt[2]*Cosh[x]*Sqrt[a + b*Sech[x]^2]*Sqrt[a + b + a*Sinh[x]^2]*(3*Sqrt[a]*(a + b)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]] - Sqrt[a + b]*Csch[x]*(4*a + 3*b + (a + b)*Csch[x]^2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/(3*(a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])

fricas [B] time = 0.62, size = 2341, normalized size = 27.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*cosh(x)*sinh(x)^3 + (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + 3*b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b), -1/6*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)

```

- a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*
(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)
)*cosh(x)^3 - 3*(a + b)*cosh(x)*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a
+ b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)
)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x)*sinh(x) - a - b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*c
osh(x)*sinh(x)^3 + (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*
a + 3*b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a +
b)*cosh(x))*sinh(x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a +
b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a +
b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(
x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*co
sh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (
a + b)*cosh(x))*sinh(x) - a - b)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (\coth^4(x)) \sqrt{a + b \operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(x)^2 + a} \coth(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^4 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^4*(a + b/cosh(x)^2)^(1/2), x)`

[Out] `int(coth(x)^4*(a + b/cosh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \operatorname{coth}^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)**4*(a+b*sech(x)**2)**(1/2), x)`

[Out] `Integral(sqrt(a + b*sech(x)**2)*coth(x)**4, x)`

3.186 $\int \coth^5(x) \sqrt{a + b \operatorname{sech}^2(x)} dx$

Optimal. Leaf size=125

$$\frac{(8a^2 + 12ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}} - \frac{1}{4} \coth^4(x) \sqrt{a+b\operatorname{sech}^2(x)} - \frac{(4a+3b) \coth^2(x) \sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)}$$

[Out] $-1/8*(8*a^2+12*a*b+3*b^2)*\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)/(a+b)^{(1/2)})}/(a+b)^{(3/2)}+\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)/a^{(1/2)})}*a^{(1/2)}-1/8*(4*a+3*b)*\coth(x)^2*(a+b*\operatorname{sech}(x)^2)^{(1/2)/(a+b)}-1/4*\coth(x)^4*(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4139, 446, 99, 151, 156, 63, 208}

$$\frac{(8a^2 + 12ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}} - \frac{1}{4} \coth^4(x) \sqrt{a+b\operatorname{sech}^2(x)} - \frac{(4a+3b) \coth^2(x) \sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]^5*Sqrt[a + b*Sech[x]^2], x]`

[Out] `Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((8*a^2 + 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)) - ((4*a + 3*b)*Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(8*(a + b)) - (Coth[x]^4*Sqrt[a + b*Sech[x]^2])/4`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 99

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])`

Rule 151

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]`

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \coth^5(x) \sqrt{a + b \operatorname{sech}^2(x)} dx &= \operatorname{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x(-1 + x^2)^3} dx, x, \operatorname{sech}(x) \right) \\
&= \frac{1}{2} \operatorname{Subst} \left(\int \frac{\sqrt{a + bx}}{(-1 + x)^3 x} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\frac{1}{4} \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} + \frac{1}{4} \operatorname{Subst} \left(\int \frac{-2a - \frac{3bx}{2}}{(-1 + x)^2 x \sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\frac{(4a + 3b) \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)}}{8(a + b)} - \frac{1}{4} \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{\operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right)}{2} \\
&= -\frac{(4a + 3b) \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)}}{8(a + b)} - \frac{1}{4} \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\frac{(4a + 3b) \coth^2(x) \sqrt{a + b \operatorname{sech}^2(x)}}{8(a + b)} - \frac{1}{4} \coth^4(x) \sqrt{a + b \operatorname{sech}^2(x)} - \frac{a \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right)}{2} \\
&= \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - \frac{(8a^2 + 12ab + 3b^2) \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a + b}} \right)}{8(a + b)^{3/2}} - \frac{a \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right)}{2}
\end{aligned}$$

Mathematica [A] time = 1.02, size = 191, normalized size = 1.53

$$\frac{\cosh(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{2}\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a}\cosh(2x)+a+2b}\right)+\sqrt{a+b}\left(\frac{1}{2}\coth(x)\operatorname{csch}^3(x)\sqrt{a}\right)\right)}{8(a+b)^{3/2}\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^5*Sqrt[a + b*Sech[x]^2], x]

[Out] -1/8*(Cosh[x]*(Sqrt[2]*(8*a^2 + 12*a*b + 3*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[a + b]*((Sqrt[a + 2*b + a*Cosh[2*x]]*(-2*a - b + (6*a + 5*b)*Cosh[2*x])*Coth[x]*Csch[x]^3)/2 - 8*Sqrt[2]*Sqrt[a]*(a + b)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])*Sqrt[a + b*Sech[x]^2]))/(a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT>Error: Bad Argument Type

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (\coth^5(x)) \sqrt{a + b\operatorname{sech}(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^5*(a+b*sech(x)^2)^(1/2), x)

[Out] int(coth(x)^5*(a+b*sech(x)^2)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\operatorname{sech}(x)^2 + a} \coth(x)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^5 \sqrt{a + \frac{b}{\cosh(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

```
[Out] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}^2(x)} \operatorname{coth}^5(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*sech(x)**2)**(1/2), x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**5, x)
```

3.187 $\int (a + b \operatorname{sech}^2(x))^{3/2} \tanh^3(x) dx$

Optimal. Leaf size=76

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} - a \sqrt{a + b \operatorname{sech}^2(x)}$$

[Out] $a^{3/2} \operatorname{arctanh}((a+b \operatorname{sech}(x)^2)^{1/2}/a^{1/2}) - 1/3 (a+b \operatorname{sech}(x)^2)^{3/2} + 1/5 (a+b \operatorname{sech}(x)^2)^{5/2}/b - a (a+b \operatorname{sech}(x)^2)^{1/2}$

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 80, 50, 63, 208}

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} - a \sqrt{a + b \operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]

[Out] $a^{3/2} \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]] - a \operatorname{Sqrt}[a + b \operatorname{Sech}[x]^2] - (a + b \operatorname{Sech}[x]^2)^{3/2}/3 + (a + b \operatorname{Sech}[x]^2)^{5/2}/(5*b)$

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p,

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^2(x))^{3/2} \tanh^3(x) dx &= \operatorname{Subst} \left(\int \frac{(-1 + x^2)(a + bx^2)^{3/2}}{x} dx, x, \operatorname{sech}(x) \right) \\
&= \frac{1}{2} \operatorname{Subst} \left(\int \frac{(-1 + x)(a + bx)^{3/2}}{x} dx, x, \operatorname{sech}^2(x) \right) \\
&= \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{1}{2} \operatorname{Subst} \left(\int \frac{(a + bx)^{3/2}}{x} dx, x, \operatorname{sech}^2(x) \right) \\
&= -\frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x) \right) \\
&= -a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{1}{2} a^2 \operatorname{Subst} \left(\int \frac{1}{x} dx, x, \operatorname{sech}^2(x) \right) \\
&= -a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b} - \frac{a^2 \operatorname{Subst} \left(\int \frac{1}{x} dx, x, \operatorname{sech}^2(x) \right)}{2} \\
&= a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} + \frac{(a + b \operatorname{sech}^2(x))^{5/2}}{5b}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 129, normalized size = 1.70

$$\cosh^3(x) (a + b \operatorname{sech}^2(x))^{3/2} \left(\frac{2\sqrt{2} a^{3/2} \log(\sqrt{a} \cosh(2x) + a + 2b) + \sqrt{2} \sqrt{a} \cosh(x)}{(a \cosh(2x) + a + 2b)^{3/2}} + \frac{2(b(6a - 5b) \operatorname{sech}^3(x) + a)}{15b(a \cosh(2x) + a + 2b)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3, x]
```

```
[Out] Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*((2*Sqrt[2]*a^(3/2)*Log[Sqrt[2]*Sqrt[a]*C
osh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])/(a + 2*b + a*Cosh[2*x])^(3/2) + (2*(
a*(3*a - 20*b)*Sech[x] + (6*a - 5*b)*b*Sech[x]^3 + 3*b^2*Sech[x]^5))/(15*b*
(a + 2*b + a*Cosh[2*x])))
```

fricas [B] time = 2.15, size = 4226, normalized size = 55.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="fricas")

[Out] [1/60*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2 - 20*a*b)*cosh(x)*sinh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 20*a*b)*cosh(x)^4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*a*b + 4*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)

$$\begin{aligned}
& * \sinh(x)^2 + 3a^2 - 20ab + 8((3a^2 - 20ab) \cosh(x)^7 + 3(3a^2 - 14 \\
& ab - 5b^2) \cosh(x)^5 + (9a^2 - 36ab + 4b^2) \cosh(x)^3 + (3a^2 - 14a \\
& ab - 5b^2) \cosh(x) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / \\
& (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (b \cosh(x)^{10} + 10b \cosh(x) \sinh(x)^9 + b \sinh(x)^{10} + 5b \cosh(x)^8 + 5(9b \cosh(x)^2 + b) \sinh(x)^8 + \\
& 40(3b \cosh(x)^3 + b \cosh(x)) \sinh(x)^7 + 10b \cosh(x)^6 + 10(21b \cosh(x)^4 + 14b \cosh(x)^2 + b) \sinh(x)^6 + 4(63b \cosh(x)^5 + 70b \cosh(x)^3 + \\
& 15b \cosh(x)) \sinh(x)^5 + 10b \cosh(x)^4 + 10(21b \cosh(x)^6 + 35b \cosh(x)^4 + 15b \cosh(x)^2 + b) \sinh(x)^4 + 40(3b \cosh(x)^7 + 7b \cosh(x)^5 + \\
& 5b \cosh(x)^3 + b \cosh(x)) \sinh(x)^3 + 5b \cosh(x)^2 + 5(9b \cosh(x)^8 + 28b \cosh(x)^6 + 30b \cosh(x)^4 + 12b \cosh(x)^2 + b) \sinh(x)^2 + 10(b \cosh(x)^9 + 4b \cosh(x)^7 + 6b \cosh(x)^5 + 4b \cosh(x)^3 + b \cosh(x)) \sinh(x) \\
& + b), -1/30(15(a b \cosh(x)^{10} + 10a b \cosh(x) \sinh(x)^9 + a b \sinh(x)^{10} + 5a b \cosh(x)^8 + 5(9a b \cosh(x)^2 + a b) \sinh(x)^8 + 10a b \cosh(x)^6 + 40(3a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)^7 + 10(21a b \cosh(x)^4 + 14 \\
& a b \cosh(x)^2 + a b) \sinh(x)^6 + 10a b \cosh(x)^4 + 4(63a b \cosh(x)^5 + 70a b \cosh(x)^3 + 15a b \cosh(x)) \sinh(x)^5 + 10(21a b \cosh(x)^6 + 35a b \cosh(x)^4 + 15a b \cosh(x)^2 + a b) \sinh(x)^4 + 5a b \cosh(x)^2 + 40(3a \\
& b \cosh(x)^7 + 7a b \cosh(x)^5 + 5a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)^3 + 5(9a b \cosh(x)^8 + 28a b \cosh(x)^6 + 30a b \cosh(x)^4 + 12a b \cosh(x)^2 + a b) \sinh(x)^2 + a b + 10(a b \cosh(x)^9 + 4a b \cosh(x)^7 + 6a b \cosh(x)^5 + 4a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}((a + b) \cosh(x)^2 + 2(a + b) \cosh(x) \sinh(x) + (a + b) \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / ((a^2 + a b) \cosh(x)^4 + 4(a^2 + a b) \cosh(x) \sinh(x)^3 + (a^2 + a b) \sinh(x)^4 + (2a^2 + 3a b) \cosh(x)^2 + (6(a^2 + a b) \cosh(x)^2 + 2a^2 + 2a^2 + 3a b) \sinh(x)^2 + a^2 + 2(2(a^2 + a b) \cosh(x)^3 + (2a^2 + 3a b) \cosh(x)) \sinh(x))) + 15(a b \cosh(x)^{10} + 10a b \cosh(x) \sinh(x)^9 + a b \sinh(x)^{10} + 5a b \cosh(x)^8 + 5(9a b \cosh(x)^2 + a b) \sinh(x)^8 + 10a b \cosh(x)^6 + 40(3a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)^7 + 10(21a b \cosh(x)^4 + 14a b \cosh(x)^2 + a b) \sinh(x)^6 + 10a b \cosh(x)^4 + 4(63a b \cosh(x)^5 + 70a b \cosh(x)^3 + 15a b \cosh(x)) \sinh(x)^5 + 10(21a b \cosh(x)^6 + 35a b \cosh(x)^4 + 15a b \cosh(x)^2 + a b) \sinh(x)^4 + 5a b \cosh(x)^2 + 40(3a b \cosh(x)^7 + 7a b \cosh(x)^5 + 5a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)^3 + 5(9a b \cosh(x)^8 + 28a b \cosh(x)^6 + 30a b \cosh(x)^4 + 12a b \cosh(x)^2 + a b) \sinh(x)^2 + a b + 10(a b \cosh(x)^9 + 4a b \cosh(x)^7 + 6a b \cosh(x)^5 + 4a b \cosh(x)^3 + a b \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x)) \sinh(x) + a)) - 2\sqrt{2}((3a^2 - 20ab) \cosh(x)^8 + 8(3a^2 - 20ab) \cosh(x) \sinh(x)^7 + (3a^2 - 20ab) \sinh(x)^8 + 4(3a^2 - 14ab - 5b^2) \cosh(x)^6 + 4(7(3a^2 - 20ab) \cosh(x)^2 + 3a^2 - 14ab - 5b^2) \sinh(x)^6 + 8(7(3a^2 - 20ab) \cosh(x)^3 + 3(3a^2 - 14ab - 5b^2) \cosh(x)) \sinh(x)^5 + 2(9a^2 - 36ab + 4b^2) \cosh(x)^4 + 2(35(3a^2 - 20ab) \cosh(x)^4 + 30(3a^2 - 14ab - 5b^2) \cosh(x)^2 + 9a^2 - 36ab + 4b^2) \sinh(x)^4 + 8(7(3a^2 - 20ab) \cosh(x)^5 + 10(3a^2 - 14ab - 5b^2) \cosh(x)^3 + (9a^2 - 36ab + 4b^2) \cosh(x)) \sinh(x)^3 + 4(3a^2 - 14ab - 5b^2) \cosh(x)^2 + 4(7(3a^2 - 20ab) \cosh(x)^6 + 15(3a^2 - 14ab - 5b^2) \cosh(x)^4 + 3(9a^2 - 36ab + 4b^2) \cosh(x)^2 + 3a^2 - 14ab - 5b^2) \sinh(x)^2 + 3a^2 - 20ab + 8((3a^2 - 20ab) \cosh(x)^7 + 3(3a^2 - 14ab - 5b^2) \cosh(x)^5 + (9a^2 - 36ab + 4b^2) \cosh(x)^3 + (3a^2 - 14ab - 5b^2) \cosh(x)) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (b \cosh(x)^{10} + 10b \cosh(x) \sinh(x)^9 + b \sinh(x)^{10} + 5b \cosh(x)^8 + 5(9b \cosh(x)^2 + b) \sinh(x)^8 + 40(3b \cosh(x)^3 + b \cosh(x)) \sinh(x)^7 + 10b \cosh(x)^6 + 10(21b \cosh(x)^4 + 14b \cosh(x)^2 + b) \sinh(x)^6 + 4(63b \cosh(x)^5 + 70b \cosh(x)^3 + 15b \cosh(x)) \sinh(x)^5 + 10b \cosh(x)^4 + 10(21b \cosh(x)^6 + 35b \cosh(x)^4 + 15b \cosh(x)^2 + b) \sinh(x)^4 + 40(3b \cosh(x)^7 + 7b \cosh(x)^5 + 5b \cosh(x)^3 + b \cosh(x)) \sinh(x)^3 + 5(9b \cosh(x)^8 + 28b \cosh(x)^6 + 30b \cosh(x)^4 + 12b \cosh(x)^2 + b) \sinh(x)^2 + 10(b \cosh(x)^9 + 4b \cosh(x)^7 + 6b \cosh(x)^5 + 4b \cosh(x)^3 + b \cosh(x)) \sinh(x) + b)
\end{aligned}$$


```
sh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.77Error: Bad Argument Type
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(x)^2)^{\frac{3}{2}} (\tanh^3(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}} \tanh(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^3 \left(a + \frac{b}{\cosh(x)^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(x))^{\frac{3}{2}} \tanh^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**3,x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**3, x)
```

3.188 $\int (a + b \operatorname{sech}^2(x))^{3/2} \tanh^2(x) dx$

Optimal. Leaf size=125

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - \frac{(3a^2 - 6ab - b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)}{8\sqrt{b}} - \frac{1}{8} (5a + b) \tanh(x) \sqrt{a - b \tanh^2(x) + b}$$

[Out] $a^{(3/2)} * \arctan(a^{(1/2)} * \tanh(x) / (a + b - b * \tanh(x)^2)^{(1/2)}) - 1/8 * (3 * a^2 - 6 * a * b - b^2) * \arctan(b^{(1/2)} * \tanh(x) / (a + b - b * \tanh(x)^2)^{(1/2)}) / b^{(1/2)} - 1/8 * (5 * a + b) * (a + b - b * \tanh(x)^2)^{(1/2)} * \tanh(x) + 1/4 * b * (a + b - b * \tanh(x)^2)^{(1/2)} * \tanh(x)^3$

Rubi [A] time = 0.36, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4141, 1975, 477, 582, 523, 217, 203, 377, 206}

$$-\frac{(3a^2 - 6ab - b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)}{8\sqrt{b}} + a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) + \frac{1}{4} b \tanh^3(x) \sqrt{a - b \tanh^2(x) + b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]

[Out] $-((3 * a^2 - 6 * a * b - b^2) * \text{ArcTan}[(\text{Sqrt}[b] * \text{Tanh}[x]) / \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]]) / (8 * \text{Sqrt}[b]) + a^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[a] * \text{Tanh}[x]) / \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]] - ((5 * a + b) * \text{Tanh}[x] * \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]) / 8 + (b * \text{Tanh}[x]^3 * \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]) / 4$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 477

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I

$$\int (e*x)^m*(a + b*x^n)^p*(c + d*x^n)^{q-2} * \text{Simp}[c*((c*b - a*d)*(m + 1) + c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n*(p + q))*x^n, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, m, p\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$$

Rule 523

$$\text{Int}[(e_ + (f_)*(x_)^(n_))/((a_ + (b_)*(x_)^(n_))*\text{Sqrt}[(c_ + (d_)*(x_)^(n_)]), x_Symbol] := \text{Dist}[f/b, \text{Int}[1/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/((a + b*x^n)*\text{Sqrt}[c + d*x^n]), x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, n\}, x]$$

Rule 582

$$\text{Int}[(g_)*(x_)^(m_)*((a_ + (b_)*(x_)^(n_))^(p_)*((c_ + (d_)*(x_)^(n_))^(q_)*((e_ + (f_)*(x_)^(n_))), x_Symbol] := \text{Simp}[(f*g^(n-1)*(g*x)^(m-n+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(b*d*(m + n*(p + q + 1) + 1)), x] - \text{Dist}[g^n/(b*d*(m + n*(p + q + 1) + 1)), \text{Int}[(g*x)^(m-n)*(a + b*x^n)^p*(c + d*x^n)^q * \text{Simp}[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, g, p, q\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1]$$

Rule 1975

$$\text{Int}[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := \text{Int}[(e*x)^m * \text{ExpandToSum}[u, x]^p * \text{ExpandToSum}[v, x]^q, x] /;$$

$$\text{FreeQ}\{e, m, p, q\}, x\} \ \&\& \ \text{BinomialQ}\{u, v\}, x\} \ \&\& \ \text{EqQ}[\text{BinomialDegree}[u, x] - \text{BinomialDegree}[v, x], 0] \ \&\& \ \text{BinomialMatchQ}\{u, v\}, x]$$

Rule 4141

$$\text{Int}[(a_ + (b_)*\text{sec}[(e_ + (f_)*(x_)]^(n_))^(p_)*((d_)*\text{tan}[(e_ + (f_)*(x_)]^(m_)), x_Symbol] := \text{With}\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, \text{Tan}[e + f*x]/ff], x] /;$$

$$\text{FreeQ}\{a, b, d, e, f, m, p\}, x\} \ \&\& \ \text{IntegerQ}[n/2] \ \&\& \ (\text{IntegerQ}[m/2] \ || \ \text{EqQ}[n, 2])$$

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^2(x))^{3/2} \tanh^2(x) dx &= \operatorname{Subst} \left(\int \frac{x^2 (a + b(1 - x^2))^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{x^2 (a + b - bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= \frac{1}{4} b \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} - \frac{1}{4} \operatorname{Subst} \left(\int \frac{x^2 (-(a + b)(4a + b) + b(5a - bx^2))^{3/2}}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\
&= -\frac{1}{8} (5a + b) \tanh(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{4} b \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} \\
&= -\frac{1}{8} (5a + b) \tanh(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{4} b \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} \\
&= -\frac{1}{8} (5a + b) \tanh(x) \sqrt{a + b - b \tanh^2(x)} + \frac{1}{4} b \tanh^3(x) \sqrt{a + b - b \tanh^2(x)} \\
&= -\frac{(3a^2 - 6ab - b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)}{8\sqrt{b}} + a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)
\end{aligned}$$

Mathematica [A] time = 0.85, size = 197, normalized size = 1.58

$$\frac{\cosh^3(x) (a + b \operatorname{sech}^2(x))^{3/2} \left(-8\sqrt{2} a^{3/2} \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + \sqrt{2} (3a^2 - 6ab - b^2) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{b} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) \right)}{4\sqrt{b} (a \cosh(2x) + a + 2b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]

[Out] -1/4*(Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*(Sqrt[2]*(3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*a^(3/2)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + (5*a - b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] + 2*b^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(Sqrt[b]*(a + 2*b + a*Cosh[2*x])^(3/2))

fricas [B] time = 1.15, size = 8582, normalized size = 68.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="fricas")

[Out] [1/16*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*

$$\begin{aligned}
& (14ab^2\cosh(x)^2 - ab^2 + b^3)\sinh(x)^6 + 4(14ab^2\cosh(x)^3 - 3(a \\
& *b^2 - b^3)\cosh(x))\sinh(x)^5 + (a^3 + 4a^2b + 9ab^2)\cosh(x)^4 + (70* \\
& ab^2\cosh(x)^4 + a^3 + 4a^2b + 9ab^2 - 30(ab^2 - b^3)\cosh(x)^2)\sin \\
& h(x)^4 + 4(14ab^2\cosh(x)^5 - 10(ab^2 - b^3)\cosh(x)^3 + (a^3 + 4a^2* \\
& b + 9ab^2)\cosh(x))\sinh(x)^3 + a^3 + 2(a^3 + 3a^2b)\cosh(x)^2 + 2(14 \\
& *ab^2\cosh(x)^6 - 15(ab^2 - b^3)\cosh(x)^4 + a^3 + 3a^2b + 3(a^3 + 4* \\
& a^2b + 9ab^2)\cosh(x)^2)\sinh(x)^2 + \sqrt{2}(b^2\cosh(x)^6 + 6b^2\cosh \\
& (x)\sinh(x)^5 + b^2\sinh(x)^6 - 3b^2\cosh(x)^4 + 3(5b^2\cosh(x)^2 - b^2) \\
& *\sinh(x)^4 + 4(5b^2\cosh(x)^3 - 3b^2\cosh(x))\sinh(x)^3 - (a^2 + 4ab)* \\
& \cosh(x)^2 + (15b^2\cosh(x)^4 - 18b^2\cosh(x)^2 - a^2 - 4ab)\sinh(x)^2 - \\
& a^2 + 2(3b^2\cosh(x)^5 - 6b^2\cosh(x)^3 - (a^2 + 4ab)\cosh(x))\sinh(x) \\
&)\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x) \\
&)\sinh(x) + \sinh(x)^2)} + 4(2ab^2\cosh(x)^7 - 3(ab^2 - b^3)\cosh(x)^5 \\
& + (a^3 + 4a^2b + 9ab^2)\cosh(x)^3 + (a^3 + 3a^2b)\cosh(x))\sinh(x)/ \\
& (\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sin \\
& h(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6)) + ((3a \\
& ^2 - 6ab - b^2)\cosh(x)^8 + 8(3a^2 - 6ab - b^2)\cosh(x)\sinh(x)^7 + (\\
& 3a^2 - 6ab - b^2)\sinh(x)^8 + 4(3a^2 - 6ab - b^2)\cosh(x)^6 + 4(7*(\\
& 3a^2 - 6ab - b^2)\cosh(x)^2 + 3a^2 - 6ab - b^2)\sinh(x)^6 + 8(7*(3a \\
& ^2 - 6ab - b^2)\cosh(x)^3 + 3(3a^2 - 6ab - b^2)\cosh(x))\sinh(x)^5 + \\
& 6(3a^2 - 6ab - b^2)\cosh(x)^4 + 2(35(3a^2 - 6ab - b^2)\cosh(x)^4 + \\
& 30(3a^2 - 6ab - b^2)\cosh(x)^2 + 9a^2 - 18ab - 3b^2)\sinh(x)^4 + 8 \\
& *(7(3a^2 - 6ab - b^2)\cosh(x)^5 + 10(3a^2 - 6ab - b^2)\cosh(x)^3 + \\
& 3(3a^2 - 6ab - b^2)\cosh(x))\sinh(x)^3 + 4(3a^2 - 6ab - b^2)\cosh(x) \\
&)^2 + 4(7(3a^2 - 6ab - b^2)\cosh(x)^6 + 15(3a^2 - 6ab - b^2)\cosh(x) \\
&)^4 + 9(3a^2 - 6ab - b^2)\cosh(x)^2 + 3a^2 - 6ab - b^2)\sinh(x)^2 + \\
& 3a^2 - 6ab - b^2 + 8((3a^2 - 6ab - b^2)\cosh(x)^7 + 3(3a^2 - 6ab \\
& b - b^2)\cosh(x)^5 + 3(3a^2 - 6ab - b^2)\cosh(x)^3 + (3a^2 - 6ab - b \\
& ^2)\cosh(x))\sinh(x))\sqrt{-b}\log(-((a - b)\cosh(x)^4 + 4(a - b)\cosh(x)* \\
& \sinh(x)^3 + (a - b)\sinh(x)^4 + 2(a + 3b)\cosh(x)^2 + 2(3(a - b)\cosh(x) \\
&)^2 + a + 3b)\sinh(x)^2 + 2\sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x) \\
&)^2 - 1)\sqrt{-b}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - \\
& 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4((a - b)\cosh(x)^3 + (a + 3b)\cosh(x)) \\
& *\sinh(x) + a - b)/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x) \\
&)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1)) + \\
& 4(ab\cosh(x)^8 + 8ab\cosh(x)\sinh(x)^7 + ab\sinh(x)^8 + 4ab\cosh(x) \\
& ^6 + 4(7ab\cosh(x)^2 + ab)\sinh(x)^6 + 6ab\cosh(x)^4 + 8(7ab\cosh(x) \\
&)^3 + 3ab\cosh(x))\sinh(x)^5 + 2(35ab\cosh(x)^4 + 30ab\cosh(x)^2 + \\
& 3ab)\sinh(x)^4 + 4ab\cosh(x)^2 + 8(7ab\cosh(x)^5 + 10ab\cosh(x)^3 \\
& + 3ab\cosh(x))\sinh(x)^3 + 4(7ab\cosh(x)^6 + 15ab\cosh(x)^4 + 9ab* \\
& \cosh(x)^2 + ab)\sinh(x)^2 + ab + 8(ab\cosh(x)^7 + 3ab\cosh(x)^5 + 3a \\
& *b\cosh(x)^3 + ab\cosh(x))\sinh(x))\sqrt{a}\log(-a\cosh(x)^4 + 4a\cosh(x) \\
&)\sinh(x)^3 + a\sinh(x)^4 + 2(a + b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + b) \\
& *\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{a} \\
&)\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) \\
&) + \sinh(x)^2)} + 4(a\cosh(x)^3 + (a + b)\cosh(x))\sinh(x) + a)/(\cosh(x)^2 \\
& + 2\cosh(x)\sinh(x) + \sinh(x)^2)) - 2\sqrt{2}((5ab - b^2)\cosh(x)^6 + 6* \\
& (5ab - b^2)\cosh(x)\sinh(x)^5 + (5ab - b^2)\sinh(x)^6 + (5ab + 7b^2) \\
& *\cosh(x)^4 + (15(5ab - b^2)\cosh(x)^2 + 5ab + 7b^2)\sinh(x)^4 + 4(5* \\
& (5ab - b^2)\cosh(x)^3 + (5ab + 7b^2)\cosh(x))\sinh(x)^3 - (5ab + 7b \\
& ^2)\cosh(x)^2 + (15(5ab - b^2)\cosh(x)^4 + 6(5ab + 7b^2)\cosh(x)^2 - \\
& 5ab - 7b^2)\sinh(x)^2 - 5ab + b^2 + 2(3(5ab - b^2)\cosh(x)^5 + 2* \\
& (5ab + 7b^2)\cosh(x)^3 - (5ab + 7b^2)\cosh(x))\sinh(x))\sqrt{(a\cosh(x) \\
&)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} \\
& / (b\cosh(x)^8 + 8b\cosh(x)\sinh(x)^7 + b\sinh(x)^8 + 4b\cosh(x)^6 + 4(7* \\
& b\cosh(x)^2 + b)\sinh(x)^6 + 8(7b\cosh(x)^3 + 3b\cosh(x))\sinh(x)^5 + 6* \\
& b\cosh(x)^4 + 2(35b\cosh(x)^4 + 30b\cosh(x)^2 + 3b)\sinh(x)^4 + 8(7b* \\
& \cosh(x)^5 + 10b\cosh(x)^3 + 3b\cosh(x))\sinh(x)^3 + 4b\cosh(x)^2 + 4(7* \\
& b\cosh(x)^6 + 15b\cosh(x)^4 + 9b\cosh(x)^2 + b)\sinh(x)^2 + 8(b\cosh(x)^
\end{aligned}$$

$$\begin{aligned}
& 7 + 3*b*\cosh(x)^5 + 3*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b), -1/8*(((3*a^2 \\
& - 6*a*b - b^2)*\cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*\cosh(x)*\sinh(x)^7 + (3*a \\
& ^2 - 6*a*b - b^2)*\sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*\cosh(x)^6 + 4*(7*(3*a \\
& ^2 - 6*a*b - b^2)*\cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*\sinh(x)^6 + 8*(7*(3*a^2 \\
& - 6*a*b - b^2)*\cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*\cosh(x))*\sinh(x)^5 + 6*(\\
& 3*a^2 - 6*a*b - b^2)*\cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*\cosh(x)^4 + 30 \\
& *(3*a^2 - 6*a*b - b^2)*\cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*\sinh(x)^4 + 8*(7 \\
& *(3*a^2 - 6*a*b - b^2)*\cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*\cosh(x)^3 + 3*(\\
& 3*a^2 - 6*a*b - b^2)*\cosh(x))*\sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*\cosh(x)^2 \\
& + 4*(7*(3*a^2 - 6*a*b - b^2)*\cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*\cosh(x)^4 \\
& + 9*(3*a^2 - 6*a*b - b^2)*\cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*\sinh(x)^2 + 3* \\
& a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*\cosh(x)^7 + 3*(3*a^2 - 6*a*b - \\
& b^2)*\cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*\cosh(x)^3 + (3*a^2 - 6*a*b - b^2) \\
& *\cosh(x))*\sinh(x))*\sqrt{b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2 - 1))*\sqrt{b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^ \\
& 2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + \\
& a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 \\
& + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - 2*(a*b*\cosh(x)^8 + 8 \\
& *a*b*\cosh(x)*\sinh(x)^7 + a*b*\sinh(x)^8 + 4*a*b*\cosh(x)^6 + 4*(7*a*b*\cosh(x) \\
& ^2 + a*b)*\sinh(x)^6 + 6*a*b*\cosh(x)^4 + 8*(7*a*b*\cosh(x)^3 + 3*a*b*\cosh(x)) \\
& *\sinh(x)^5 + 2*(35*a*b*\cosh(x)^4 + 30*a*b*\cosh(x)^2 + 3*a*b)*\sinh(x)^4 + 4* \\
& a*b*\cosh(x)^2 + 8*(7*a*b*\cosh(x)^5 + 10*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh \\
& (x)^3 + 4*(7*a*b*\cosh(x)^6 + 15*a*b*\cosh(x)^4 + 9*a*b*\cosh(x)^2 + a*b)*\sinh \\
& (x)^2 + a*b + 8*(a*b*\cosh(x)^7 + 3*a*b*\cosh(x)^5 + 3*a*b*\cosh(x)^3 + a*b*\co \\
& sh(x))*\sinh(x))*\sqrt{a}*\log((a*b^2*\cosh(x)^8 + 8*a*b^2*\cosh(x)*\sinh(x)^7 + \\
& a*b^2*\sinh(x)^8 - 2*(a*b^2 - b^3)*\cosh(x)^6 + 2*(14*a*b^2*\cosh(x)^2 - a*b^2 \\
& + b^3)*\sinh(x)^6 + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\cosh(x))*\sinh(x) \\
&)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 + a^3 + 4*a \\
& ^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*a*b^2*\cosh(x) \\
&)^5 - 10*(a*b^2 - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x) \\
& ^3 + a^3 + 2*(a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x)^6 - 15*(a*b^2 \\
& - b^3)*\cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^2) \\
& *\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh(x) \\
& ^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x) \\
& ^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (15*b^2*\cosh(x) \\
& ^4 - 18*b^2*\cosh(x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^2*\cosh(x)^5 - \\
& 6*b^2*\cosh(x)^3 - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^ \\
& 2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4 \\
& *(2*a*b^2*\cosh(x)^7 - 3*(a*b^2 - b^3)*\cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2) \\
& *\cosh(x)^3 + (a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sin \\
& h(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh \\
& (x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) - 2*(a*b*\cosh(x)^8 + 8*a*b*\cosh(x) \\
& *\sinh(x)^7 + a*b*\sinh(x)^8 + 4*a*b*\cosh(x)^6 + 4*(7*a*b*\cosh(x)^2 + a*b)*\si \\
& nh(x)^6 + 6*a*b*\cosh(x)^4 + 8*(7*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^5 + \\
& 2*(35*a*b*\cosh(x)^4 + 30*a*b*\cosh(x)^2 + 3*a*b)*\sinh(x)^4 + 4*a*b*\cosh(x)^ \\
& 2 + 8*(7*a*b*\cosh(x)^5 + 10*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^3 + 4*(7 \\
& *a*b*\cosh(x)^6 + 15*a*b*\cosh(x)^4 + 9*a*b*\cosh(x)^2 + a*b)*\sinh(x)^2 + a*b \\
& + 8*(a*b*\cosh(x)^7 + 3*a*b*\cosh(x)^5 + 3*a*b*\cosh(x)^3 + a*b*\cosh(x))*\sinh \\
& (x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a \\
& + b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + \\
& 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 \\
& + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(a*\cosh(x)^3 + \\
& (a + b)*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) \\
& + \sqrt{2}*((5*a*b - b^2)*\cosh(x)^6 + 6*(5*a*b - b^2)*\cosh(x)*\sinh(x)^5 + (\\
& 5*a*b - b^2)*\sinh(x)^6 + (5*a*b + 7*b^2)*\cosh(x)^4 + (15*(5*a*b - b^2)*\cosh \\
& (x)^2 + 5*a*b + 7*b^2)*\sinh(x)^4 + 4*(5*(5*a*b - b^2)*\cosh(x)^3 + (5*a*b + \\
& 7*b^2)*\cosh(x))*\sinh(x)^3 - (5*a*b + 7*b^2)*\cosh(x)^2 + (15*(5*a*b - b^2)*\co \\
& sh(x)^4 + 6*(5*a*b + 7*b^2)*\cosh(x)^2 - 5*a*b - 7*b^2)*\sinh(x)^2 - 5*a*b + \\
& b^2 + 2*(3*(5*a*b - b^2)*\cosh(x)^5 + 2*(5*a*b + 7*b^2)*\cosh(x)^3 - (5*a*b
\end{aligned}$$

$$\begin{aligned}
& + 7*b^2*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/16*(8*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)) + 8*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) - ((3*a^2 - 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x))^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)
\end{aligned}$$

$$\begin{aligned}
&)^2 + 8*(b*\cosh(x)^7 + 3*b*\cosh(x)^5 + 3*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + \\
&b), -1/8*(4*(a*b*\cosh(x)^8 + 8*a*b*\cosh(x)*\sinh(x)^7 + a*b*\sinh(x)^8 + 4*a \\
&*b*\cosh(x)^6 + 4*(7*a*b*\cosh(x)^2 + a*b)*\sinh(x)^6 + 6*a*b*\cosh(x)^4 + 8*(7 \\
&*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^5 + 2*(35*a*b*\cosh(x)^4 + 30*a*b*\cosh \\
&sh(x)^2 + 3*a*b)*\sinh(x)^4 + 4*a*b*\cosh(x)^2 + 8*(7*a*b*\cosh(x)^5 + 10*a*b* \\
&\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^3 + 4*(7*a*b*\cosh(x)^6 + 15*a*b*\cosh(x)^ \\
&4 + 9*a*b*\cosh(x)^2 + a*b)*\sinh(x)^2 + a*b + 8*(a*b*\cosh(x)^7 + 3*a*b*\cosh \\
&x)^5 + 3*a*b*\cosh(x)^3 + a*b*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(b*\cosh \\
&osh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + a))*\sqrt{-a}*\sqrt{(a*\cosh(x)^ \\
&2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/(a* \\
&b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 - (a^2 + 3*a*b)*\cosh \\
&x)^2 + (6*a*b*\cosh(x)^2 - a^2 - 3*a*b)*\sinh(x)^2 - a^2 + 2*(2*a*b*\cosh(x)^3 \\
&- (a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + ((3*a^2 - 6*a*b - b^2)*\cosh(x)^8 + 8* \\
&(3*a^2 - 6*a*b - b^2)*\cosh(x)*\sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*\sinh(x)^8 + \\
&4*(3*a^2 - 6*a*b - b^2)*\cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*\cosh(x)^2 + \\
&3*a^2 - 6*a*b - b^2)*\sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*\cosh(x)^3 + 3* \\
&(3*a^2 - 6*a*b - b^2)*\cosh(x))*\sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^2)*\cosh(x)^ \\
&4 + 2*(35*(3*a^2 - 6*a*b - b^2)*\cosh(x)^4 + 30*(3*a^2 - 6*a*b - b^2)*\cosh(x) \\
&)^2 + 9*a^2 - 18*a*b - 3*b^2)*\sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b - b^2)*\cosh(x) \\
&)^5 + 10*(3*a^2 - 6*a*b - b^2)*\cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*\cosh(x)) \\
&*\sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*\cosh(x)^2 + 4*(7*(3*a^2 - 6*a*b - b^2) \\
&*\cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*\cosh(x)^4 + 9*(3*a^2 - 6*a*b - b^2)*\cosh \\
&osh(x)^2 + 3*a^2 - 6*a*b - b^2)*\sinh(x)^2 + 3*a^2 - 6*a*b - b^2 + 8*((3*a^2 \\
&- 6*a*b - b^2)*\cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*\cosh(x)^5 + 3*(3*a^2 - \\
&6*a*b - b^2)*\cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{b}*\ar \\
&ctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{b}*\sqrt{((\\
&a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh \\
&x)^2))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh \\
&x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b) \\
&*\cosh(x))*\sinh(x) + a)) + 4*(a*b*\cosh(x)^8 + 8*a*b*\cosh(x)*\sinh(x)^7 + a*b* \\
&\sinh(x)^8 + 4*a*b*\cosh(x)^6 + 4*(7*a*b*\cosh(x)^2 + a*b)*\sinh(x)^6 + 6*a*b*\cosh \\
&osh(x)^4 + 8*(7*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^5 + 2*(35*a*b*\cosh(x) \\
&)^4 + 30*a*b*\cosh(x)^2 + 3*a*b)*\sinh(x)^4 + 4*a*b*\cosh(x)^2 + 8*(7*a*b*\cosh \\
&x)^5 + 10*a*b*\cosh(x)^3 + 3*a*b*\cosh(x))*\sinh(x)^3 + 4*(7*a*b*\cosh(x)^6 + \\
&15*a*b*\cosh(x)^4 + 9*a*b*\cosh(x)^2 + a*b)*\sinh(x)^2 + a*b + 8*(a*b*\cosh(x)^ \\
&7 + 3*a*b*\cosh(x)^5 + 3*a*b*\cosh(x)^3 + a*b*\cosh(x))*\sinh(x))*\sqrt{-a}*\arct \\
&an(\sqrt{2}*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - \\
&2*\cosh(x)*\sinh(x) + \sinh(x)^2))/(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh \\
&x)^2 + a)) + \sqrt{2}*((5*a*b - b^2)*\cosh(x)^6 + 6*(5*a*b - b^2)*\cosh(x)*\sinh \\
&x)^5 + (5*a*b - b^2)*\sinh(x)^6 + (5*a*b + 7*b^2)*\cosh(x)^4 + (15*(5*a*b \\
&- b^2)*\cosh(x)^2 + 5*a*b + 7*b^2)*\sinh(x)^4 + 4*(5*(5*a*b - b^2)*\cosh(x)^3 \\
&+ (5*a*b + 7*b^2)*\cosh(x))*\sinh(x)^3 - (5*a*b + 7*b^2)*\cosh(x)^2 + (15*(5* \\
&a*b - b^2)*\cosh(x)^4 + 6*(5*a*b + 7*b^2)*\cosh(x)^2 - 5*a*b - 7*b^2)*\sinh(x) \\
&^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*\cosh(x)^5 + 2*(5*a*b + 7*b^2)*\cosh(x) \\
&^3 - (5*a*b + 7*b^2)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a \\
&+ 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(b*\cosh(x)^8 + 8*b*\cosh \\
&x)*\sinh(x)^7 + b*\sinh(x)^8 + 4*b*\cosh(x)^6 + 4*(7*b*\cosh(x)^2 + b)*\sinh(x) \\
&)^6 + 8*(7*b*\cosh(x)^3 + 3*b*\cosh(x))*\sinh(x)^5 + 6*b*\cosh(x)^4 + 2*(35*b*\cosh \\
&osh(x)^4 + 30*b*\cosh(x)^2 + 3*b)*\sinh(x)^4 + 8*(7*b*\cosh(x)^5 + 10*b*\cosh(x) \\
&)^3 + 3*b*\cosh(x))*\sinh(x)^3 + 4*b*\cosh(x)^2 + 4*(7*b*\cosh(x)^6 + 15*b*\cosh \\
&x)^4 + 9*b*\cosh(x)^2 + b)*\sinh(x)^2 + 8*(b*\cosh(x)^7 + 3*b*\cosh(x)^5 + 3*b \\
&*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.46Error: Bad Argument Typ
 e

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(x)^2)^{\frac{3}{2}} (\tanh^2(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)

[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}} \tanh(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \left(a + \frac{b}{\cosh(x)^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2),x)

[Out] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(x))^{\frac{3}{2}} \tanh^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**2,x)

[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**2, x)

3.189 $\int (a + b \operatorname{sech}^2(x))^{3/2} \tanh(x) dx$

Optimal. Leaf size=57

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2}$$

[Out] $a^{(3/2)} * \operatorname{arctanh}((a + b * \operatorname{sech}(x)^2)^{(1/2)} / a^{(1/2)}) - 1/3 * (a + b * \operatorname{sech}(x)^2)^{(3/2)} - a * (a + b * \operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 266, 50, 63, 208}

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b * \operatorname{Sech}[x]^2)^{(3/2)} * \operatorname{Tanh}[x], x]$

[Out] $a^{(3/2)} * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b * \operatorname{Sech}[x]^2] / \operatorname{Sqrt}[a]] - a * \operatorname{Sqrt}[a + b * \operatorname{Sech}[x]^2] - (a + b * \operatorname{Sech}[x]^2)^{(3/2)} / 3$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p]/x, x],
```

$x, \text{Sec}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \&\& \text{IntegerQ}[(m - 1)/2] \&\& (\text{GtQ}[m, 0] \parallel \text{EqQ}[n, 2] \parallel \text{EqQ}[n, 4] \parallel \text{IGtQ}[p, 0] \parallel \text{IntegersQ}[2*n, p])$

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(x))^{3/2} \tanh(x) dx &= -\operatorname{Subst} \left(\int \frac{(a + bx^2)^{3/2}}{x} dx, x, \operatorname{sech}(x) \right) \\ &= -\left(\frac{1}{2} \operatorname{Subst} \left(\int \frac{(a + bx)^{3/2}}{x} dx, x, \operatorname{sech}^2(x) \right) \right) \\ &= -\frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, \operatorname{sech}^2(x) \right) \\ &= -a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} - \frac{1}{2} a^2 \operatorname{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \operatorname{sech}^2(x) \right) \\ &= -a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} - \frac{a^2 \operatorname{Subst} \left(\int \frac{1}{\frac{a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + bx} \right)}{b} \\ &= a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) - a \sqrt{a + b \operatorname{sech}^2(x)} - \frac{1}{3} (a + b \operatorname{sech}^2(x))^{3/2} \end{aligned}$$

Mathematica [C] time = 0.12, size = 65, normalized size = 1.14

$$\frac{2b (a + b \operatorname{sech}^2(x))^{3/2} {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}, -\frac{1}{2}, -\frac{a \cosh^2(x)}{b} \right)}{3 \sqrt{\frac{a \cosh^2(x)}{b} + 1} (a \cosh(2x) + a + 2b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x], x]

[Out] (-2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -(a*Cosh[x]^2)/b])*(a + b*Sech[x]^2)^(3/2)/(3*Sqrt[1 + (a*Cosh[x]^2)/b]*(a + 2*b + a*Cosh[2*x]))

fricas [B] time = 0.84, size = 2312, normalized size = 40.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x), x, algorithm="fricas")

[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3

$$\begin{aligned}
& + 2*a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x) \\
& ^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a \\
& ^2*b)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + 15*(2*a^3 + 5*a \\
& ^2*b + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9 \\
& *a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2*a*b + b^2)*\cosh(x)^6 + 6*(\\
& a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh(x)^6 + 3*(a \\
& ^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 + 2* \\
& a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 + 2*a*b \\
& + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 + (15*(a^2 + 2*a*b + \\
& b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^ \\
& 2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x) \\
&)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh \\
& (x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + \\
& 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^5 \\
& + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh \\
& (x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^ \\
& 3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6) + \\
& 3*(a*\cosh(x)^6 + 6*a*\cosh(x)*\sinh(x)^5 + a*\sinh(x)^6 + 3*a*\cosh(x)^4 + 3*(5 \\
& *a*\cosh(x)^2 + a)*\sinh(x)^4 + 4*(5*a*\cosh(x)^3 + 3*a*\cosh(x))*\sinh(x)^3 + 3 \\
& *a*\cosh(x)^2 + 3*(5*a*\cosh(x)^4 + 6*a*\cosh(x)^2 + a)*\sinh(x)^2 + 6*(a*\cosh \\
& (x)^5 + 2*a*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a)*\sqrt{a}*\log(-(a*\cosh(x)^4 + \\
& 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b) \\
& *\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a} \\
&)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)} + 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\co \\
& sh(x)*\sinh(x) + \sinh(x)^2)) - 16*\sqrt{2}*(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x) \\
& ^3 + a*\sinh(x)^4 + (2*a + b)*\cosh(x)^2 + (6*a*\cosh(x)^2 + 2*a + b)*\sinh(x)^ \\
& 2 + 2*(2*a*\cosh(x)^3 + (2*a + b)*\cosh(x))*\sinh(x) + a)*\sqrt{(a*\cosh(x)^2 + \\
& a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh \\
& (x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 + 1)*\sinh(x)^4 + 3* \\
& \cosh(x)^4 + 4*(5*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 + 6*\cosh \\
& (x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 + 2*\cosh(x)^3 + \cosh(x))* \\
& \sinh(x) + 1), -1/6*(3*(a*\cosh(x)^6 + 6*a*\cosh(x)*\sinh(x)^5 + a*\sinh(x)^6 + \\
& 3*a*\cosh(x)^4 + 3*(5*a*\cosh(x)^2 + a)*\sinh(x)^4 + 4*(5*a*\cosh(x)^3 + 3*a*\co \\
& sh(x))*\sinh(x)^3 + 3*a*\cosh(x)^2 + 3*(5*a*\cosh(x)^4 + 6*a*\cosh(x)^2 + a)*\si \\
& nh(x)^2 + 6*(a*\cosh(x)^5 + 2*a*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a)*\sqrt{-a} \\
& *\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\si \\
& nh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 \\
& - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\c \\
& osh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(\\
& a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\co \\
& sh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + 3*(a*\cosh(x)^6 + 6*a*\cosh(x) \\
& *\sinh(x)^5 + a*\sinh(x)^6 + 3*a*\cosh(x)^4 + 3*(5*a*\cosh(x)^2 + a)*\sinh(x)^4 \\
& + 4*(5*a*\cosh(x)^3 + 3*a*\cosh(x))*\sinh(x)^3 + 3*a*\cosh(x)^2 + 3*(5*a*\cosh \\
& (x))^4 + 6*a*\cosh(x)^2 + a)*\sinh(x)^2 + 6*(a*\cosh(x)^5 + 2*a*\cosh(x)^3 + a*\cos \\
& h(x))*\sinh(x) + a)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2 - 1))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x) \\
&)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 \\
& + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x) \\
& ^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) + 8*\sqrt{2}*(a*\cosh \\
& (x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + (2*a + b)*\cosh(x)^2 + (6*a*\cos \\
& h(x)^2 + 2*a + b)*\sinh(x)^2 + 2*(2*a*\cosh(x)^3 + (2*a + b)*\cosh(x))*\sinh(x) \\
& + a)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sin \\
& h(x) + \sinh(x)^2)))/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cos \\
& h(x)^2 + 1)*\sinh(x)^4 + 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 \\
& + 3*(5*\cosh(x)^4 + 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 \\
& + 2*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [A] time = 0.05, size = 56, normalized size = 0.98

$$-\frac{(a+b\operatorname{sech}(x)^2)^{\frac{3}{2}}}{3} + \ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)a^{\frac{3}{2}} - a\sqrt{a+b\operatorname{sech}(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(3/2)*tanh(x),x)

[Out] -1/3*(a+b*sech(x)^2)^(3/2)+ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))*a^(3/2)-a*(a+b*sech(x)^2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)

mupad [B] time = 3.36, size = 45, normalized size = 0.79

$$a^{3/2} \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right) - \frac{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}{3} - a\sqrt{a + \frac{b}{\cosh(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b/cosh(x)^2)^(3/2),x)

[Out] a^(3/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(3/2)/3 - a*(a + b/cosh(x)^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(x))^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x),x)

[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x), x)

3.190 $\int (a + b \operatorname{sech}^2(x))^{3/2} dx$

Optimal. Leaf size=88

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) + \frac{1}{2} b \tanh(x) \sqrt{a - b \tanh^2(x) + b} + \frac{1}{2} \sqrt{b} (3a + b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)$$

[Out] $a^{(3/2)} * \operatorname{arctanh}(a^{(1/2)} * \tanh(x) / (a + b - b * \tanh(x)^2)^{(1/2)}) + 1/2 * (3 * a + b) * \operatorname{arctan}(b^{(1/2)} * \tanh(x) / (a + b - b * \tanh(x)^2)^{(1/2)}) * b^{(1/2)} + 1/2 * b * (a + b - b * \tanh(x)^2)^{(1/2)} * \tanh(x)$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4128, 416, 523, 217, 203, 377, 206}

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) + \frac{1}{2} b \tanh(x) \sqrt{a - b \tanh^2(x) + b} + \frac{1}{2} \sqrt{b} (3a + b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b * \operatorname{Sech}[x]^2)^{(3/2)}, x]$

[Out] $(\operatorname{Sqrt}[b] * (3 * a + b) * \operatorname{ArcTan}[(\operatorname{Sqrt}[b] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[x]^2]]) / 2 + a^{(3/2)} * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[x]^2]] + (b * \operatorname{Tanh}[x] * \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[x]^2]) / 2$

Rule 203

$\operatorname{Int}[(a + (b * x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTan}[(\operatorname{Rt}[b, 2] * x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

$\operatorname{Int}[(a + (b * x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] * x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\operatorname{Int}[1 / \operatorname{Sqrt}[(a + (b * x^2)^{-1})], x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1 / (1 - b * x^2), x], x, x / \operatorname{Sqrt}[a + b * x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

$\operatorname{Int}[(a + (b * x^{(n)})^{(p)}) / ((c + (d * x^{(n)})^{(q)})), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1 / (c - (b * c - a * d) * x^n), x], x, x / (a + b * x^n)^{(1/n)}] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b * c - a * d, 0] && EqQ[n * p + 1, 0] && IntegerQ[n]

Rule 416

$\operatorname{Int}[(a + (b * x^{(n)})^{(p)}) * ((c + (d * x^{(n)})^{(q)})), x_Symbol] \rightarrow \operatorname{Simp}[(d * x * (a + b * x^n)^{(p+1)} * (c + d * x^n)^{(q-1)}) / (b * (n * (p+q) + 1)), x] + \operatorname{Dist}[1 / (b * (n * (p+q) + 1)), \operatorname{Int}[(a + b * x^n)^p * (c + d * x^n)^{(q-2)} * \operatorname{Simp}[c * (b * c * (n * (p+q) + 1) - a * d) + d * (b * c * (n * (p+2 * q - 1) + 1) - a * d * (n * (q-1) + 1)) * x^n, x], x] /;$ FreeQ[{a, b, c, d, n, p}, x] && NeQ[b * c - a * d, 0] && GtQ[q, 1] && NeQ[n * (p+q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] & & NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^2(x))^{3/2} dx &= \operatorname{Subst} \left(\int \frac{(a + b - bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} b \tanh(x) \sqrt{a + b - b \tanh^2(x)} - \frac{1}{2} \operatorname{Subst} \left(\int \frac{-(a + b)(2a + b) + b(3a + b)x^2}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} b \tanh(x) \sqrt{a + b - b \tanh^2(x)} + a^2 \operatorname{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} b \tanh(x) \sqrt{a + b - b \tanh^2(x)} + a^2 \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) \\ &= \frac{1}{2} \sqrt{b} (3a + b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) + a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) + \end{aligned}$$

Mathematica [A] time = 0.27, size = 152, normalized size = 1.73

$$\frac{\operatorname{sech}(x) (a \cosh^2(x) + b) \sqrt{a + b \operatorname{sech}^2(x)} \left(2\sqrt{2} a^{3/2} \cosh^2(x) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + b \sinh(x) \sqrt{a \cosh(2x) + a + 2b} \right)}{(a \cosh(2x) + a + 2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2), x]
```

```
[Out] ((b + a*Cosh[x]^2)*Sech[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Cosh[x]^2 + 2*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Cosh[x]^2 + b*Sqrt[a + 2*b + a*Cosh[2*x]]*Sinh[x]))/(a + 2*b + a*Cosh[2*x])^(3/2)
```

fricas [B] time = 1.85, size = 4140, normalized size = 47.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sq
rt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cos
h(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
- (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)
^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(
x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)
^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(
x) + 3*a + b)*sqrt(-b)*log(-(a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^
3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a
+ 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1
)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x)
+ a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)
^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x)
)*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), 1/4*(2*((3*a + b)*cosh(x)^4 +
4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2
+ 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 +
(3*a + b)*cosh(x))*sinh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*
cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
+ a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a
*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)
*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(
14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
```



```
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a) - sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(x)^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(x)^2)^(3/2),x)

[Out] int((a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cosh(x)^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(x)^2)^(3/2),x)

[Out] int((a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)**2)**(3/2),x)

[Out] Integral((a + b*sech(x)**2)**(3/2), x)

3.191 $\int \coth(x) \left(a + b \operatorname{sech}^2(x) \right)^{3/2} dx$

Optimal. Leaf size=70

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) + b \sqrt{a + b \operatorname{sech}^2(x)} - (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a + b}} \right)$$

[Out] $a^{(3/2)} * \operatorname{arctanh}((a + b * \operatorname{sech}(x)^2)^{(1/2)} / a^{(1/2)}) - (a + b)^{(3/2)} * \operatorname{arctanh}((a + b * \operatorname{sech}(x)^2)^{(1/2)} / (a + b)^{(1/2)}) + b * (a + b * \operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4139, 446, 84, 156, 63, 208}

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right) + b \sqrt{a + b \operatorname{sech}^2(x)} - (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]*(a + b*Sech[x]^2)^(3/2), x]`

[Out] $a^{(3/2)} * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b * \operatorname{Sech}[x]^2] / \operatorname{Sqrt}[a]] - (a + b)^{(3/2)} * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b * \operatorname{Sech}[x]^2] / \operatorname{Sqrt}[a + b]] + b * \operatorname{Sqrt}[a + b * \operatorname{Sech}[x]^2]$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 84

`Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), Int[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x*(e + f*x)^(p - 2))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]`

Rule 156

`Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

$$\begin{aligned}
& 2) * \cosh(x)^2 * \sinh(x)^6 + 4 * (14 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + 3 * (2 * a^3 \\
& + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)) * \sinh(x)^5 + (6 * a^3 + 14 * a^2 * b + 9 * a * b \\
& ^2) * \cosh(x)^4 + (70 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 6 * a^3 + 14 * a^2 * b + \\
& 9 * a * b^2 + 30 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^4 + 4 * (14 \\
& * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^5 + 10 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)^3 \\
& + (6 * a^3 + 14 * a^2 * b + 9 * a * b^2) * \cosh(x)) * \sinh(x)^3 + a^3 + 2 * (2 * a^3 \\
& + 3 * a^2 * b) * \cosh(x)^2 + 2 * (14 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^6 + 15 * (2 * a^3 \\
& + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)^4 + 2 * a^3 + 3 * a^2 * b + 3 * (6 * a^3 + 14 * a^2 \\
& * b + 9 * a * b^2) * \cosh(x)^2) * \sinh(x)^2 + \sqrt{2} * ((a^2 + 2 * a * b + b^2) * \cosh(x)^6 \\
& + 6 * (a^2 + 2 * a * b + b^2) * \cosh(x) * \sinh(x)^5 + (a^2 + 2 * a * b + b^2) * \sinh(x)^6 \\
& + 3 * (a^2 + 2 * a * b + b^2) * \cosh(x)^4 + 3 * (5 * (a^2 + 2 * a * b + b^2) * \cosh(x)^2 + a^2 \\
& + 2 * a * b + b^2) * \sinh(x)^4 + 4 * (5 * (a^2 + 2 * a * b + b^2) * \cosh(x)^3 + 3 * (a^2 + \\
& 2 * a * b + b^2) * \cosh(x)) * \sinh(x)^3 + (3 * a^2 + 4 * a * b) * \cosh(x)^2 + (15 * (a^2 + 2 * \\
& a * b + b^2) * \cosh(x)^4 + 18 * (a^2 + 2 * a * b + b^2) * \cosh(x)^2 + 3 * a^2 + 4 * a * b) * \sinh \\
& (x)^2 + a^2 + 2 * (3 * (a^2 + 2 * a * b + b^2) * \cosh(x)^5 + 6 * (a^2 + 2 * a * b + b^2) * \\
& \cosh(x)^3 + (3 * a^2 + 4 * a * b) * \cosh(x)) * \sinh(x)) * \sqrt{a} * \sqrt{(a * \cosh(x)^2 + a \\
& * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4 * (2 * (\\
& a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^7 + 3 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh \\
& (x)^5 + (6 * a^3 + 14 * a^2 * b + 9 * a * b^2) * \cosh(x)^3 + (2 * a^3 + 3 * a^2 * b) * \cosh(x)) \\
& * \sinh(x) / (\cosh(x)^6 + 6 * \cosh(x)^5 * \sinh(x) + 15 * \cosh(x)^4 * \sinh(x)^2 + 20 * \cosh \\
& (x)^3 * \sinh(x)^3 + 15 * \cosh(x)^2 * \sinh(x)^4 + 6 * \cosh(x) * \sinh(x)^5 + \sinh(x)^6) \\
& + 2 * ((a + b) * \cosh(x)^2 + 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 \\
& + a + b) * \sqrt{a + b} * \log(((2 * a + b) * \cosh(x)^4 + 4 * (2 * a + b) * \cosh(x) * \sinh(x) \\
& ^3 + (2 * a + b) * \sinh(x)^4 + 2 * (2 * a + 3 * b) * \cosh(x)^2 + 2 * (3 * (2 * a + b) * \cosh(x) \\
& ^2 + 2 * a + 3 * b) * \sinh(x)^2 - 2 * \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh \\
& (x)^2 + 1) * \sqrt{a + b} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 \\
& - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4 * ((2 * a + b) * \cosh(x)^3 + (2 * a + 3 * b) * \\
& \cosh(x)) * \sinh(x) + 2 * a + b) / (\cosh(x)^4 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 + \\
& 2 * (3 * \cosh(x)^2 - 1) * \sinh(x)^2 - 2 * \cosh(x)^2 + 4 * (\cosh(x)^3 - \cosh(x)) * \sinh(x) \\
& + 1)) + (a * \cosh(x)^2 + 2 * a * \cosh(x) * \sinh(x) + a * \sinh(x)^2 + a) * \sqrt{a} * \log \\
& (- (a * \cosh(x)^4 + 4 * a * \cosh(x) * \sinh(x)^3 + a * \sinh(x)^4 + 2 * b * \cosh(x)^2 + 2 * (\\
& 3 * a * \cosh(x)^2 + b) * \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh \\
& (x)^2 - 1) * \sqrt{a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - \\
& 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4 * (a * \cosh(x)^3 + b * \cosh(x)) * \sinh(x) + a) \\
& / (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4 * \sqrt{2} * b * \sqrt{(a * \cosh(x) \\
& ^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / (\\
& \cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1), 1/4 * (4 * ((a + b) * \cosh(x)^2 + \\
& 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 + a + b) * \sqrt{-a - b} * \arctan \\
& (\sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{-a - b} * \sqrt{(\\
& a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh \\
& (x)^2)) / (a * \cosh(x)^4 + 4 * a * \cosh(x) * \sinh(x)^3 + a * \sinh(x)^4 + 2 * (a + 2 * b) * \cosh \\
& (x)^2 + 2 * (3 * a * \cosh(x)^2 + a + 2 * b) * \sinh(x)^2 + 4 * (a * \cosh(x)^3 + (a + 2 * b) \\
&) * \cosh(x)) * \sinh(x) + a)) + (a * \cosh(x)^2 + 2 * a * \cosh(x) * \sinh(x) + a * \sinh(x)^2 \\
& + a) * \sqrt{a} * \log(((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^8 + 8 * (a^3 + 2 * a^2 * b + a \\
& * b^2) * \cosh(x) * \sinh(x)^7 + (a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^8 + 2 * (2 * a^3 + 5 * \\
& a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)^6 + 2 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3 + 14 * \\
& (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x)^6 + 4 * (14 * (a^3 + 2 * a^2 * b + a * b^2) \\
&) * \cosh(x)^3 + 3 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)) * \sinh(x)^5 + (6 * a \\
& ^3 + 14 * a^2 * b + 9 * a * b^2) * \cosh(x)^4 + (70 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 \\
& + 6 * a^3 + 14 * a^2 * b + 9 * a * b^2 + 30 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x) \\
& ^2) * \sinh(x)^4 + 4 * (14 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^5 + 10 * (2 * a^3 + 5 * a^2 \\
& * b + 4 * a * b^2 + b^3) * \cosh(x)^3 + (6 * a^3 + 14 * a^2 * b + 9 * a * b^2) * \cosh(x)) * \sinh(x) \\
& ^3 + a^3 + 2 * (2 * a^3 + 3 * a^2 * b) * \cosh(x)^2 + 2 * (14 * (a^3 + 2 * a^2 * b + a * b^2) * \\
& \cosh(x)^6 + 15 * (2 * a^3 + 5 * a^2 * b + 4 * a * b^2 + b^3) * \cosh(x)^4 + 2 * a^3 + 3 * a^2 * b \\
& + 3 * (6 * a^3 + 14 * a^2 * b + 9 * a * b^2) * \cosh(x)^2) * \sinh(x)^2 + \sqrt{2} * ((a^2 + 2 \\
& * a * b + b^2) * \cosh(x)^6 + 6 * (a^2 + 2 * a * b + b^2) * \cosh(x) * \sinh(x)^5 + (a^2 + 2 * \\
& a * b + b^2) * \sinh(x)^6 + 3 * (a^2 + 2 * a * b + b^2) * \cosh(x)^4 + 3 * (5 * (a^2 + 2 * a * b \\
& + b^2) * \cosh(x)^2 + a^2 + 2 * a * b + b^2) * \sinh(x)^4 + 4 * (5 * (a^2 + 2 * a * b + b^2) * \\
& \cosh(x)^3 + 3 * (a^2 + 2 * a * b + b^2) * \cosh(x)) * \sinh(x)^3 + (3 * a^2 + 4 * a * b) * \cosh
\end{aligned}$$

$$\begin{aligned}
& (x)^2 + (15*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + (a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*\sqrt{2}*b*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1), -1/2*((a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + (a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - ((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a + b)*\sqrt{a + b}*\log(((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 + 2*(2*a + 3*b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a + 3*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a + b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*((2*a + b)*\cosh(x)^3 + (2*a + 3*b)*\cosh(x))*\sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) - 2*\sqrt{2}*b*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1), -1/2*((a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + (a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - 2*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a + b)*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a - b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - 2*\sqrt{2}*b*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)]
\end{aligned}$$

giac [B] time = 0.47, size = 134, normalized size = 1.91

$$\frac{4 \left(\left(\sqrt{a} e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a} \right) b^2 - \sqrt{a} b^2 \right)}{\left(\sqrt{a} e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a} \right)^2 + 2 \left(\sqrt{a} e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a} \right) \sqrt{a} + a + 4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] -4*((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*b^2 - sqrt(a)*b^2)/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))^2 + 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*sqrt(a) + a + 4*b)

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \coth(x) (a + b \operatorname{sech}(x)^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)

[Out] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x) \left(a + \frac{b}{\cosh(x)^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a + b/cosh(x)^2)^(3/2),x)

[Out] int(coth(x)*(a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(x))^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sech(x)**2)**(3/2),x)

[Out] Integral((a + b*sech(x)**2)**(3/2)*coth(x), x)

3.192 $\int \coth^2(x) (a + b \operatorname{sech}^2(x))^{3/2} dx$

Optimal. Leaf size=81

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - b^{3/2} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - (a+b) \coth(x) \sqrt{a - b \tanh^2(x) + b}$$

[Out] $-b^{(3/2)} * \arctan(b^{(1/2)} * \tanh(x) / (a+b-b*\tanh(x)^2)^{(1/2)}) + a^{(3/2)} * \operatorname{arctanh}(a^{(1/2)} * \tanh(x) / (a+b-b*\tanh(x)^2)^{(1/2)}) - (a+b) * \coth(x) * (a+b-b*\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4141, 1975, 474, 523, 217, 203, 377, 206}

$$a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - b^{3/2} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b \tanh^2(x) + b}} \right) - (a+b) \coth(x) \sqrt{a - b \tanh^2(x) + b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x]^2 * (a + b * \text{Sech}[x]^2)^{(3/2)}, x]$

[Out] $-(b^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[b] * \text{Tanh}[x]) / \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]]) + a^{(3/2)} * \text{ArcTanh}[(\text{Sqrt}[a] * \text{Tanh}[x]) / \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]] - (a + b) * \text{Coth}[x] * \text{Sqrt}[a + b - b * \text{Tanh}[x]^2]$

Rule 203

$\text{Int}[(a_ + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTan}[(\text{Rt}[b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

$\text{Int}[(a_ + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\text{Int}[1 / \text{Sqrt}[(a_ + (b_.) * (x_)^2)], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1 / (1 - b * x^2), x], x, x / \text{Sqrt}[a + b * x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

$\text{Int}[(a_ + (b_.) * (x_)^{n_})^{p_} / ((c_ + (d_.) * (x_)^{n_})], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1 / (c - (b * c - a * d) * x^n), x], x, x / (a + b * x^n)^{(1/n)}] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b * c - a * d, 0] && EqQ[n * p + 1, 0] && IntegerQ[n]

Rule 474

$\text{Int}[(e_.) * (x_)^{m_} * ((a_ + (b_.) * (x_)^{n_})^{p_} * ((c_ + (d_.) * (x_)^{n_})^{q_})], x_Symbol] \rightarrow \text{Simp}[(c * (e * x)^{(m+1)} * (a + b * x^n)^{(p+1)} * (c + d * x^n)^{(q-1)}) / (a * e * (m+1)), x] - \text{Dist}[1 / (a * e^n * (m+1)), \text{Int}[(e * x)^{(m+n)} * (a + b * x^n)^p * (c + d * x^n)^{(q-2)} * \text{Simp}[c * (c * b - a * d) * (m+1) + c * n * (b * c * (p+1) + a * d * (q-1)) + d * ((c * b - a * d) * (m+1) + c * b * n * (p+q)) * x^n, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[b * c - a * d, 0] && IGtQ[n, 0] && GtQ[q,

1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned}
 \int \coth^2(x) (a + b \operatorname{sech}^2(x))^{3/2} dx &= \operatorname{Subst} \left(\int \frac{(a + b(1 - x^2))^{3/2}}{x^2(1 - x^2)} dx, x, \tanh(x) \right) \\
 &= \operatorname{Subst} \left(\int \frac{(a + b - bx^2)^{3/2}}{x^2(1 - x^2)} dx, x, \tanh(x) \right) \\
 &= -(a + b) \coth(x) \sqrt{a + b - b \tanh^2(x)} + \operatorname{Subst} \left(\int \frac{a^2 - b^2 + b^2 x^2}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \frac{1}{\sqrt{a + b - bx^2}} \right) \\
 &= -(a + b) \coth(x) \sqrt{a + b - b \tanh^2(x)} + a^2 \operatorname{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + b - bx^2}} dx, x, \frac{1}{\sqrt{a + b - bx^2}} \right) \\
 &= -(a + b) \coth(x) \sqrt{a + b - b \tanh^2(x)} + a^2 \operatorname{Subst} \left(\int \frac{1}{1 - ax^2} dx, x, \frac{1}{\sqrt{a + b - bx^2}} \right) \\
 &= -b^{3/2} \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right) + a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.35, size = 144, normalized size = 1.78

$$\frac{2(a \cosh^2(x) + b) \sqrt{a + b \operatorname{sech}^2(x)} \left(-\sqrt{2} a^{3/2} \cosh(x) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + \sqrt{2} b^{3/2} \cosh(x) \tanh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) \right)}{(a \cosh(2x) + a + 2b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2*(a + b*Sech[x]^2)^(3/2), x]

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Cosh[x] - Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Cosh[x] + (a + b)*Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[x])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])^(3/2)
```

fricas [B] time = 4.04, size = 3349, normalized size = 41.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/4*(4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)
```

$$\begin{aligned}
&)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 \\
& + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 + 4ab) \cosh(x)^2 \\
& + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 - 4ab) \sinh(x)^2 - a^2 + 2(\\
& 3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 + 4ab) \cosh(x)) \sinh(x) \sqrt{a} \\
& \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} + 4(2ab^2 \cosh(x)^7 - 3(ab^2 - b^3) \cosh(x)^5 + (a^3 + 4 \\
& a^2b + 9ab^2) \cosh(x)^3 + (a^3 + 3a^2b) \cosh(x)) \sinh(x) / (\cosh(x)^6 \\
& + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 1 \\
& 5 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) - (a \cosh(x)^2 + \\
& 2a \cosh(x) \sinh(x) + a \sinh(x)^2 - a) \sqrt{a} \log(-a \cosh(x)^4 + 4a \cosh \\
& (x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + \\
& b) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{ \\
& (a) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} + 4(a \cosh(x)^3 + (a + b) \cosh(x)) \sinh(x) + a) / (\cosh(x)^ \\
& 2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 4 \sqrt{2} (a + b) \sqrt{(a \cosh(x)^2 + \\
& a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (\cosh \\
& (x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1), -1/2((a \cosh(x)^2 + 2a \cosh(x) \\
&) \sinh(x) + a \sinh(x)^2 - a) \sqrt{-a} \arctan(\sqrt{2} (b \cosh(x)^2 + 2b \cosh \\
& (x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + \\
& a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (ab \cosh(x)^4 + 4ab \\
& b \cosh(x) \sinh(x)^3 + ab \sinh(x)^4 - (a^2 + 3ab) \cosh(x)^2 + (6ab \cosh \\
& (x)^2 - a^2 - 3ab) \sinh(x)^2 - a^2 + 2(2ab \cosh(x)^3 - (a^2 + 3ab) \c \\
& osh(x)) \sinh(x)) + (a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 - a) \sqrt{ \\
& -a} \arctan(\sqrt{2} \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / \\
& (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a \cosh(x)^2 + 2a \cosh(x) \sin \\
& h(x) + a \sinh(x)^2 + a)) - (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 \\
& - b) \sqrt{-b} \log(-((a - b) \cosh(x)^4 + 4(a - b) \cosh(x) \sinh(x)^3 + (a - \\
& b) \sinh(x)^4 + 2(a + 3b) \cosh(x)^2 + 2(3(a - b) \cosh(x)^2 + a + 3b) \s \\
& inh(x)^2 + 2 \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{(- \\
& b) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} + 4((a - b) \cosh(x)^3 + (a + 3b) \cosh(x)) \sinh(x) + a - b \\
&) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x) \\
&)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1)) + 2 \sqrt{2} (a + \\
& b) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1), -1/2((a \\
& \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 - a) \sqrt{-a} \arctan(\sqrt{2} \\
& (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh \\
& (x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} \\
& / (ab \cosh(x)^4 + 4ab \cosh(x) \sinh(x)^3 + ab \sinh(x)^4 - (a^2 + 3ab) \c \\
& osh(x)^2 + (6ab \cosh(x)^2 - a^2 - 3ab) \sinh(x)^2 - a^2 + 2(2ab \cosh(x) \\
&)^3 - (a^2 + 3ab) \cosh(x)) \sinh(x)) + 2(b \cosh(x)^2 + 2b \cosh(x) \sinh \\
& (x) + b \sinh(x)^2 - b) \sqrt{b} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2 - 1) \sqrt{b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh \\
& (x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^ \\
& 3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x) \\
&)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x)) \sinh(x) + a) + (a \cosh(x)^2 + 2 \\
& a \cosh(x) \sinh(x) + a \sinh(x)^2 - a) \sqrt{-a} \arctan(\sqrt{2} \sqrt{-a} \sqrt{ \\
& ((a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sin \\
& h(x)^2))} / (a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 + a)) + 2 \sqrt{2} \\
& (a + b) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \s \\
& sinh(x) + \sinh(x)^2))} / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1)]
\end{aligned}$$

giac [B] time = 0.52, size = 222, normalized size = 2.74

$$\frac{4 \left(\left(\sqrt{a} e^{(2x)} - \sqrt{ae^{(4x)} + 2ae^{(2x)} + 4be^{(2x)} + a} \right) a^2 + 2 \left(\sqrt{a} e^{(2x)} - \sqrt{ae^{(4x)} + 2ae^{(2x)} + 4be^{(2x)} + a} \right) ab + \left(\sqrt{a} e^{(2x)} - \sqrt{ae^{(4x)} + 2ae^{(2x)} + 4be^{(2x)} + a} \right) a^2 \right)}{\left(\sqrt{a} e^{(2x)} - \sqrt{ae^{(4x)} + 2ae^{(2x)} + 4be^{(2x)} + a} \right)^2 - 2 \left(\sqrt{a} e^{(2x)} - \sqrt{ae^{(4x)} + 2ae^{(2x)} + 4be^{(2x)} + a} \right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] $4*((\sqrt{a})e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a})*a^2 + 2*(\sqrt{a})e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a})*ab + (\sqrt{a})e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a})*b^2 + a^{5/2} + 2a^{3/2}*b + \sqrt{a}*b^2)/((\sqrt{a})e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a})^2 - 2*(\sqrt{a})e^{2x} - \sqrt{ae^{4x} + 2ae^{2x} + 4be^{2x} + a})*\sqrt{a} - 3a - 4b)$

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (\coth^2(x)) (a + b\operatorname{sech}(x)^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)

[Out] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^2 \left(a + \frac{b}{\cosh(x)^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2),x)

[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(3/2),x)

[Out] Timed out

3.193 $\int (a + b \operatorname{sech}^2(c + dx))^{5/2} dx$

Optimal. Leaf size=170

$$\frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(c+dx)}{\sqrt{a-b \tanh^2(c+dx)+b}}\right)}{d} + \frac{\sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a-b \tanh^2(c+dx)+b}}\right)}{8d} + \frac{b \tanh(c + dx) (a - b \tanh^2(c + dx))^{3/2}}{4d}$$

[Out] $a^{5/2} \operatorname{arctanh}(a^{1/2} \tanh(dx+c) / (a+b-b \tanh(dx+c)^2)^{1/2}) / d + 1/8 * (15 * a^2 + 10 * a * b + 3 * b^2) * \operatorname{arctan}(b^{1/2} \tanh(dx+c) / (a+b-b \tanh(dx+c)^2)^{1/2}) * b^{1/2} / d + 1/8 * b * (7 * a + 3 * b) * (a+b-b \tanh(dx+c)^2)^{1/2} * \tanh(dx+c) / d + 1/4 * b * \tanh(dx+c) * (a+b-b \tanh(dx+c)^2)^{3/2} / d$

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4128, 416, 528, 523, 217, 203, 377, 206}

$$\frac{\sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a-b \tanh^2(c+dx)+b}}\right)}{8d} + \frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(c+dx)}{\sqrt{a-b \tanh^2(c+dx)+b}}\right)}{d} + \frac{b \tanh(c + dx) (a - b \tanh^2(c + dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(5/2), x]

[Out] $(\operatorname{Sqrt}[b] * (15 * a^2 + 10 * a * b + 3 * b^2) * \operatorname{ArcTan}[(\operatorname{Sqrt}[b] * \operatorname{Tanh}[c + d * x]) / \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[c + d * x]^2]]) / (8 * d) + (a^{5/2} * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Tanh}[c + d * x]) / \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[c + d * x]^2]]) / d + (b * (7 * a + 3 * b) * \operatorname{Tanh}[c + d * x] * \operatorname{Sqrt}[a + b - b * \operatorname{Tanh}[c + d * x]^2]) / (8 * d) + (b * \operatorname{Tanh}[c + d * x] * (a + b - b * \operatorname{Tanh}[c + d * x]^2)^{3/2}) / (4 * d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 416

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(b*(n*(p+q) + 1)), x] + Dist[1/(b*(n*(p+q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q-2)*Simp

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] & NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a + b \operatorname{sech}^2(c + dx))^{5/2} dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^{5/2}}{1-x^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^{3/2}}{4d} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+b-bx^2} ((a+b)(b-4(a+b))+b)}{1-x^2} dx, x, \tanh(c + dx)\right)}{4d}$$

$$= \frac{b(7a + 3b) \tanh(c + dx) \sqrt{a + b - b \tanh^2(c + dx)}}{8d} + \frac{b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^{3/2}}{4d}$$

$$= \frac{b(7a + 3b) \tanh(c + dx) \sqrt{a + b - b \tanh^2(c + dx)}}{8d} + \frac{b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^{3/2}}{4d}$$

$$= \frac{b(7a + 3b) \tanh(c + dx) \sqrt{a + b - b \tanh^2(c + dx)}}{8d} + \frac{b \tanh(c + dx) (a + b - b \tanh^2(c + dx))^{3/2}}{4d}$$

$$= \frac{\sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a+b-b \tanh^2(c+dx)}}\right)}{8d} + \frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(c+dx)}{\sqrt{a+b-b \tanh^2(c+dx)}}\right)}{d}$$

Mathematica [A] time = 9.28, size = 280, normalized size = 1.65

$$\frac{\cosh^5(c + dx) (a + b \operatorname{sech}^2(c + dx))^{5/2} \left(8a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \sinh(c+dx)}{\sqrt{a \sinh^2(c+dx)+a+b}}\right) + \sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a \sinh^2(c+dx)+a+b}}\right) \right)}{\sqrt{2} d (a \cosh(2c + 2dx) + a + 2b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^(5/2), x]

[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a + b + a*Sinh[c + d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b + a*Sinh[c + d*x]^2]])*Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2))/(Sqrt[2]*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^(5/2)) + (Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2)*((b^2*Sech[c]*Sech[c + d*x]^4*Sinh[d*x])/d + (3*Sech[c]*Sech[c + d*x]^2*(3*a*b*Sinh[d*x] + b^2*Sinh[d*x]))/(2*d) + (3*b*(3*a + b)*Sech[c + d*x]*Tanh[c])/(2*d) + (b^2*Sech[c + d*x]^3*Tanh[c])/d))/(a + 2*b + a*Cosh[2*c + 2*d*x])^2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.13Error: Bad Argument Type

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(dx + c)^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^(5/2), x)

[Out] int((a+b*sech(d*x+c)^2)^(5/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx + c)^2 + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^(5/2), x, algorithm="maxima")

[Out] integrate((b*sech(d*x + c)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cosh(c + dx)^2} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x)^2)^(5/2), x)`

[Out] `int((a + b/cosh(c + d*x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}^2(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sech(d*x+c)**2)**(5/2), x)`

[Out] `Integral((a + b*sech(c + d*x)**2)**(5/2), x)`

$$3.194 \quad \int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=66

$$-\frac{(a+b\operatorname{sech}^2(x))^{3/2}}{3b^2} + \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] $-1/3*(a+b*\operatorname{sech}(x)^2)^{(3/2)}/b^2+\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(1/2)}+(a+2*b)*(a+b*\operatorname{sech}(x)^2)^{(1/2)}/b^2$

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4139, 446, 88, 63, 208}

$$-\frac{(a+b\operatorname{sech}^2(x))^{3/2}}{3b^2} + \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[x]^5/Sqrt[a + b*Sech[x]^2], x]`

[Out] `ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + ((a + 2*b)*Sqrt[a + b*Sech[x]^2])/b^2 - (a + b*Sech[x]^2)^(3/2)/(3*b^2)`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 4139

`Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)^(m_.)], x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di`

st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegerQ[2*n, p])

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^5(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx &= -\operatorname{Subst}\left(\int \frac{(-1 + x^2)^2}{x\sqrt{a + bx^2}} dx, x, \operatorname{sech}(x)\right) \\
 &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{(-1 + x)^2}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x)\right)\right) \\
 &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \left(\frac{-a - 2b}{b\sqrt{a + bx}} + \frac{1}{x\sqrt{a + bx}} + \frac{\sqrt{a + bx}}{b}\right) dx, x, \operatorname{sech}^2(x)\right)\right) \\
 &= \frac{(a + 2b)\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} - \frac{(a + b\operatorname{sech}^2(x))^{3/2}}{3b^2} - \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x)\right) \\
 &= \frac{(a + 2b)\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} - \frac{(a + b\operatorname{sech}^2(x))^{3/2}}{3b^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{b} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{a + b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(a + 2b)\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} - \frac{(a + b\operatorname{sech}^2(x))^{3/2}}{3b^2}
 \end{aligned}$$

Mathematica [A] time = 0.52, size = 109, normalized size = 1.65

$$\frac{\operatorname{sech}(x) \left(\frac{\operatorname{sech}(x)(a \cosh(2x) + a + 2b)(2a - b\operatorname{sech}^2(x) + 6b)}{3b^2} + \frac{\sqrt{2} \sqrt{a \cosh(2x) + a + 2b} \log(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x))}{\sqrt{a}} \right)}{2\sqrt{a + b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/Sqrt[a + b*Sech[x]^2], x]

[Out] (Sech[x]*((Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])/Sqrt[a] + ((a + 2*b + a*Cosh[2*x])*Sech[x]*(2*a + 6*b - b*Sech[x]^2))/(3*b^2)))/(2*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.61, size = 2678, normalized size = 40.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3

$x)^4 + 2*(a^2 + 2*a*b)*\cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*\cosh(x)^2 + a^2 + 2*a*b)*\sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*\cosh(x)^3 + (a^2 + 2*a*b)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*b^2*\cosh(x)^6 + 6*a*b^2*\cosh(x)*\sinh(x)^5 + a*b^2*\sinh(x)^6 + 3*a*b^2*\cosh(x)^4 + 3*a*b^2*\cosh(x)^2 + 3*(5*a*b^2*\cosh(x)^2 + a*b^2)*\sinh(x)^4 + 4*(5*a*b^2*\cosh(x)^3 + 3*a*b^2*\cosh(x))*\sinh(x)^3 + a*b^2 + 3*(5*a*b^2*\cosh(x)^4 + 6*a*b^2*\cosh(x)^2 + a*b^2)*\sinh(x)^2 + 6*(a*b^2*\cosh(x)^5 + 2*a*b^2*\cosh(x)^3 + a*b^2*\cosh(x))*\sinh(x))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)

[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^5}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(tanh(x)**5/sqrt(a + b*sech(x)**2), x)

$$3.195 \quad \int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=90

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}} + \frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

[Out] $-1/2*(a+3*b)*\arctan(b^{(1/2)*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/b^{(3/2)}+\arctan(h(a^{(1/2)*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/a^{(1/2)}+1/2*(a+b-b*\tanh(x)^2)^{(1/2)*\tanh(x)/b}$

Rubi [A] time = 0.23, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4141, 1975, 479, 523, 217, 203, 377, 206}

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}} + \frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/Sqrt[a + b*Sech[x]^2], x]

[Out] $-((a+3*b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[x])/\text{Sqrt}[a+b-b*\text{Tanh}[x]^2]])/(2*b^{(3/2)}) + \text{ArcTanh}[(\text{Sqrt}[a]*\text{Tanh}[x])/\text{Sqrt}[a+b-b*\text{Tanh}[x]^2]]/\text{Sqrt}[a] + (\text{Tanh}[x]*\text{Sqrt}[a+b-b*\text{Tanh}[x]^2])/(2*b)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 479

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f
_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx = \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)\sqrt{a+b(1-x^2)}} dx, x, \tanh(x) \right)$$

$$= \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)$$

$$= \frac{\tanh(x)\sqrt{a+b-b \tanh^2(x)}}{2b} - \frac{\operatorname{Subst} \left(\int \frac{a+b+(-a-3b)x^2}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{2b}$$

$$= \frac{\tanh(x)\sqrt{a+b-b \tanh^2(x)}}{2b} - \frac{(a+3b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{2b} + \operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)$$

$$= \frac{\tanh(x)\sqrt{a+b-b \tanh^2(x)}}{2b} - \frac{(a+3b) \operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{2b} + \operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)$$

$$= -\frac{(a+3b) \tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{2b^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{\sqrt{a}} + \frac{\tanh(x)\sqrt{a+b-b \tanh^2(x)}}{2b}$$

Mathematica [A] time = 0.53, size = 169, normalized size = 1.88

$$\frac{\operatorname{sech}(x) \left(2\sqrt{2} b^{3/2} \sqrt{a \cosh(2x) + a + 2b} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) + \sqrt{a} \left(\sqrt{b} \tanh(x) \operatorname{sech}(x) (a \cosh(2x) + a + 2b) \right) \right)}{4\sqrt{a} b^{3/2} \sqrt{a + b \operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Sech[x]^2], x]
```

```
[Out] (Sech[x]*(2*Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*(-(Sqrt[2]*(a + 3*b)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*Sqrt[a + 2*b + a*Cosh[2*x]]) + Sqrt[b]*(a + 2*b + a*Cosh[2*x])*Sech[x]*Tanh[x]))/(4*Sqrt[a]*b^(3/2)*Sqrt[a + b*Sech[x]^2])
```

```
fricas [B] time = 0.66, size = 4569, normalized size = 50.77
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x))^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x))^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x))^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x))^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/4*(2*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 +
```

$$\begin{aligned}
& 2*(3*(a^2 + 3*a*b)*\cosh(x)^2 + a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 3*a*b + 4*((a \\
& ^2 + 3*a*b)*\cosh(x)^3 + (a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{b}*\arctan(\sqrt{b} \\
& *(2*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{b}*\sqrt{(a*\cosh(x) \\
& ^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a \\
& *\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + \\
& 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x)) \\
& *\sinh(x) + a)) - (b^2*\cosh(x)^4 + 4*b^2*\cosh(x)*\sinh(x)^3 + b^2*\sinh(x)^4 + \\
& 2*b^2*\cosh(x)^2 + 2*(3*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 4*(b^2*\cosh(x) \\
& ^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{a}*\log((a*b^2*\cosh(x)^8 + 8*a*b^2*\cosh(x) \\
& *\sinh(x)^7 + a*b^2*\sinh(x)^8 - 2*(a*b^2 - b^3)*\cosh(x)^6 + 2*(14*a*b^2*\cosh \\
& (x)^2 - a*b^2 + b^3)*\sinh(x)^6 + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\co \\
& sh(x))*\sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 \\
& + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14 \\
& *a*b^2*\cosh(x)^5 - 10*(a*b^2 - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\c \\
& osh(x))*\sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x) \\
& ^6 - 15*(a*b^2 - b^3)*\cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^ \\
& 2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 \\
& + b^2*\sinh(x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4 \\
& *(5*b^2*\cosh(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (1 \\
& 5*b^2*\cosh(x)^4 - 18*b^2*\cosh(x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^ \\
& 2*\cosh(x)^5 - 6*b^2*\cosh(x)^3 - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{ \\
& t((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \si \\
& nh(x)^2)) + 4*(2*a*b^2*\cosh(x)^7 - 3*(a*b^2 - b^3)*\cosh(x)^5 + (a^3 + 4*a^2 \\
& *b + 9*a*b^2)*\cosh(x)^3 + (a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6* \\
& \cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\co \\
& sh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) - (b^2*\cosh(x)^4 + 4* \\
& b^2*\cosh(x)*\sinh(x)^3 + b^2*\sinh(x)^4 + 2*b^2*\cosh(x)^2 + 2*(3*b^2*\cosh(x)^ \\
& 2 + b^2)*\sinh(x)^2 + b^2 + 4*(b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{a} \\
& *\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + b)*\cosh(x) \\
&)^2 + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)* \\
& \sinh(x) + \sinh(x)^2 + 1)*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b) \\
& /(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(a*\cosh(x)^3 + (a + b)*\co \\
& sh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\sqrt{2} \\
& *(a*b*\cosh(x)^2 + 2*a*b*\cosh(x)*\sinh(x) + a*b*\sinh(x)^2 - a*b)*\sqrt{(a*\cos \\
& h(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2) \\
&))/(a*b^2*\cosh(x)^4 + 4*a*b^2*\cosh(x)*\sinh(x)^3 + a*b^2*\sinh(x)^4 + 2*a*b^2 \\
& *\cosh(x)^2 + a*b^2 + 2*(3*a*b^2*\cosh(x)^2 + a*b^2)*\sinh(x)^2 + 4*(a*b^2*\cos \\
& h(x)^3 + a*b^2*\cosh(x))*\sinh(x)), -1/4*(2*(b^2*\cosh(x)^4 + 4*b^2*\cosh(x)*\si \\
& nh(x)^3 + b^2*\sinh(x)^4 + 2*b^2*\cosh(x)^2 + 2*(3*b^2*\cosh(x)^2 + b^2)*\sinh \\
& (x)^2 + b^2 + 4*(b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{ \\
& 2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\c \\
& osh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^ \\
& 2)))/(a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 - (a^2 + 3*a*b) \\
&)*\cosh(x)^2 + (6*a*b*\cosh(x)^2 - a^2 - 3*a*b)*\sinh(x)^2 - a^2 + 2*(2*a*b*\co \\
& sh(x)^3 - (a^2 + 3*a*b)*\cosh(x))*\sinh(x)) + 2*(b^2*\cosh(x)^4 + 4*b^2*\cosh \\
& (x)*\sinh(x)^3 + b^2*\sinh(x)^4 + 2*b^2*\cosh(x)^2 + 2*(3*b^2*\cosh(x)^2 + b^2)* \\
& \sinh(x)^2 + b^2 + 4*(b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\\
& \sqrt{2}*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2* \\
& \cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh \\
& (x)^2 + a)) + ((a^2 + 3*a*b)*\cosh(x)^4 + 4*(a^2 + 3*a*b)*\cosh(x)*\sinh(x)^3 + \\
& (a^2 + 3*a*b)*\sinh(x)^4 + 2*(a^2 + 3*a*b)*\cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*\co \\
& sh(x)^2 + a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*\cosh(x)^3 \\
& + (a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{-b}*\log(-((a - b)*\cosh(x)^4 + 4*(a \\
& - b)*\cosh(x)*\sinh(x)^3 + (a - b)*\sinh(x)^4 + 2*(a + 3*b)*\cosh(x)^2 + 2*(3*(\\
& a - b)*\cosh(x)^2 + a + 3*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\si \\
& nh(x) + \sinh(x)^2 - 1)*\sqrt{-b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/ \\
& (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*((a - b)*\cosh(x)^3 + (a + \\
& 3*b)*\cosh(x))*\sinh(x) + a - b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 \\
& + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\si
\end{aligned}$$


```

nh(x) + 1)) - 2*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)
)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b
^2*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*si
nh(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/2*((b^2*cosh(x)^
4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*co
sh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a
)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh
(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2
- a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)) + ((a^2 + 3*
a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^
4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)
*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(
x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)
*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a)) - sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^
2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2
*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh
(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2), x)

[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(1/2), x)

[Out] Integral(tanh(x)**4/sqrt(a + b*sech(x)**2), x)

$$3.196 \quad \int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=42

$$\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(1/2)}+(a+b*\operatorname{sech}(x)^2)^{(1/2)}/b$

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4139, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^3/\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2], x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/\operatorname{Sqrt}[a] + \operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/b$

Rule 63

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 80

$\operatorname{Int}[(a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/(d*f*(n+p+2)), x] + \operatorname{Dist}[(a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1)))/(d*f*(n+p+2)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \operatorname{NeQ}[n+p+2, 0]$

Rule 208

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 4139

$\operatorname{Int}[(a_.) + (b_.)*((c_.)*\operatorname{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)})^{(p_.)}*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/f, \operatorname{Subst}[\operatorname{Int}[((-1 + ff^2*x^2)^{((m-1)/2)}*(a + b*(c*ff*x)^n)^p/x, x],$

$x, \text{Sec}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \&\& \text{IntegerQ}[(m - 1)/2] \&\& (\text{GtQ}[m, 0] \parallel \text{EqQ}[n, 2] \parallel \text{EqQ}[n, 4] \parallel \text{IGtQ}[p, 0] \parallel \text{IntegersQ}[2*n, p])$

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(x)}{\sqrt{a + b\text{sech}^2(x)}} dx &= \text{Subst} \left(\int \frac{-1 + x^2}{x\sqrt{a + bx^2}} dx, x, \text{sech}(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{-1 + x}{x\sqrt{a + bx}} dx, x, \text{sech}^2(x) \right) \\ &= \frac{\sqrt{a + b\text{sech}^2(x)}}{b} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \text{sech}^2(x) \right) \\ &= \frac{\sqrt{a + b\text{sech}^2(x)}}{b} - \frac{\text{Subst} \left(\int \frac{1}{\frac{-a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\text{sech}^2(x)} \right)}{b} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a + b\text{sech}^2(x)}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{\sqrt{a + b\text{sech}^2(x)}}{b} \end{aligned}$$

Mathematica [B] time = 0.22, size = 105, normalized size = 2.50

$$\frac{\text{sech}^2(x)(a \cosh(2x) + a + 2b)}{2b\sqrt{a + b\text{sech}^2(x)}} + \frac{\text{sech}(x)\sqrt{a \cosh(2x) + a + 2b} \log(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2}\sqrt{a} \cosh(x))}{\sqrt{2}\sqrt{a}\sqrt{a + b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3/Sqrt[a + b*Sech[x]^2], x]

[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2]) + ((a + 2*b + a*Cosh[2*x])*Sech[x]^2)/(2*b*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.52, size = 1650, normalized size = 39.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6

```

+ 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(-(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*a*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b), -1/2*((b*cosh(x)^2
+ 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*
a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)
*cosh(x))*sinh(x))) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)
*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*
(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^
3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(
x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(tanh(x)**3/sqrt(a + b*sech(x)**2), x)

$$3.197 \quad \int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{1/2}-\operatorname{arctan}(b^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/b^{1/2}$

Rubi [A] time = 0.19, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4141, 1975, 483, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/Sqrt[a + b*Sech[x]^2], x]

[Out] $-(\operatorname{ArcTan}[(\operatorname{Sqrt}[b]\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b - b\operatorname{Tanh}[x]^2]]/\operatorname{Sqrt}[b]) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a]\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b - b\operatorname{Tanh}[x]^2]]/\operatorname{Sqrt}[a]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 483

Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m-n)*(c + d*x^n)^q, x], x] - Dist[(a*e^n)/b, Int[((e*x)^(m-n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^2(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx &= \operatorname{Subst} \left(\int \frac{x^2}{(1-x^2) \sqrt{a+b(1-x^2)}} dx, x, \tanh(x) \right) \\ &= \operatorname{Subst} \left(\int \frac{x^2}{(1-x^2) \sqrt{a+b-bx^2}} dx, x, \tanh(x) \right) \\ &= -\operatorname{Subst} \left(\int \frac{1}{\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right) + \operatorname{Subst} \left(\int \frac{1}{(1-x^2) \sqrt{a+b-bx^2}} dx, x, \tanh(x) \right) \\ &= \operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right) - \operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right) \\ &= -\frac{\tan^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{\sqrt{b}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 1.78

$$\frac{\operatorname{sech}(x) \sqrt{a \cosh(2x) + a + 2b} \left(\frac{\tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right)}{\sqrt{a}} - \frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{b} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right)}{\sqrt{b}} \right)}{\sqrt{2} \sqrt{a + b \operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Sech[x]^2], x]
```

```
[Out] ((-(ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])]/Sqrt[b]) + ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])]/Sqrt[a])*Sqrt[a + 2*b + a*Cosh[2*x])*Sech[x])/(Sqrt[2]*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.56, size = 2856, normalized size = 47.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")
```



```
[Out] [1/4*(sqrt(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*si
nh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*
sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9
*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10
*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)
*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*
b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*c
osh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^
2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)
^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*a*sqrt(-b)*log(-(a - b)*cosh(x)^4 + 4
*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*
(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (
a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x)
)*sinh(x) + 1)) + sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b), -1/4*(4*a*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt
(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
+ 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 3
0*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4
+ a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(
2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)
^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(
x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)
)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
(a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^
7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
+ 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) - sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
)*sinh(x) + sinh(x)^2)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(
```

```
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
+ 3*a*b)*cosh(x))*sinh(x)) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + a*sqrt(-b)*log
(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*
(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqrt(
2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a
*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*
cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)) + 2*a*sqrt(b)*arctan(sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh
(x) + a)) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/(a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)

[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2), x)`

[Out] `int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**2/(a+b*sech(x)**2)**(1/2), x)`

[Out] `Integral(tanh(x)**2/sqrt(a + b*sech(x)**2), x)`

$$3.198 \quad \int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4139, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Sech[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4139

Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/f, Subst[Int[((-1 + ff^2*x^2)^(m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx &= -\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a + bx^2}} dx, x, \operatorname{sech}(x)\right) \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x)\right)\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{-a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{b} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a + b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 70, normalized size = 2.80

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x) + a + 2b} \log\left(\sqrt{a\cosh(2x) + a + 2b} + \sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{2}\sqrt{a}\sqrt{a + b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/Sqrt[a + b*Sech[x]^2], x]

[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.46, size = 1430, normalized size = 57.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)

```
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2))) / a, -1/2*(sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2
+ 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 +
a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sq
rt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
(a + 2*b)*cosh(x))*sinh(x) + a)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 30, normalized size = 1.20

$$\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b \operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

mupad [B] time = 1.67, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(1/2),x)
```

[Out] $\operatorname{atanh}\left(\frac{a + b/\cosh(x)^2}{a}\right)^{1/2}/a^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*sech(x)**2)**(1/2), x)`

[Out] `Integral(tanh(x)/sqrt(a + b*sech(x)**2), x)`

$$3.199 \quad \int \frac{1}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{1/2}$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4128, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2], x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[x]^2]]/\operatorname{Sqrt}[a]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 377

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}/((c_ + (d_)*(x_)^{(n_)})), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[n*p + 1, 0] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 4128

$\operatorname{Int}[(a_ + (b_)*\sec[(e_ + (f_)*(x_))]^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{With}\{\operatorname{ff} = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[\operatorname{ff}/f, \operatorname{Subst}[\operatorname{Int}[(a + b + b*\operatorname{ff}^2*x^2)^p/(1 + \operatorname{ff}^2*x^2), x], x, \operatorname{Tan}[e + f*x]/\operatorname{ff}], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x \ \& \ \operatorname{NeQ}[a + b, 0] \ \&\& \ \operatorname{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+b\operatorname{sech}^2(x)}} dx &= \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x)\right) \\ &= \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [B] time = 0.04, size = 62, normalized size = 2.14

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}}\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sech[x]^2], x]

[Out] (ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.45, size = 1059, normalized size = 36.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)) + sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/a]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(x)^2)^(1/2),x)

[Out] int(1/(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(1/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*sech(x)**2), x)

$$3.200 \quad \int \frac{\coth(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(1/2)} - \operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/(a+b)^{(1/2)})/(a+b)^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]/Sqrt[a + b*Sech[x]^2], x]`

[Out] `ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]]/Sqrt[a + b]`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 86

`Int[((e_.) + (f_.)*(x_))^(p_/)((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 4139

`Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],`

$x, \text{Sec}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ (\text{GtQ}[m, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ \text{IGtQ}[p, 0] \ || \ \text{IntegersQ}[2*n, p])$

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x(-1+x^2)\sqrt{a+bx^2}} dx, x, \operatorname{sech}(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1+x)x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1+x)\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b \operatorname{sech}^2(x)} \right)}{b} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b \operatorname{sech}^2(x)} \right)}{b} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)}{\sqrt{a}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} \end{aligned}$$

Mathematica [B] time = 0.26, size = 124, normalized size = 2.21

$$\frac{\operatorname{sech}(x)\sqrt{a \cosh(2x) + a + 2b} \left(\sqrt{a+b} \log \left(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x) \right) - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a+b} \cosh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) \right)}{\sqrt{2} \sqrt{a} \sqrt{a+b} \sqrt{a + b \operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/Sqrt[a + b*Sech[x]^2], x]

[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]) + Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.57, size = 3663, normalized size = 65.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3

$$\begin{aligned}
& + 3a^2b + 3(6a^3 + 14a^2b + 9ab^2) \cosh(x)^2 \sinh(x)^2 + \sqrt{2} \cdot \\
& (a^2 + 2ab + b^2) \cosh(x)^6 + 6(a^2 + 2ab + b^2) \cosh(x) \sinh(x)^5 + (\\
& a^2 + 2ab + b^2) \sinh(x)^6 + 3(a^2 + 2ab + b^2) \cosh(x)^4 + 3(5(a^2 \\
& + 2ab + b^2) \cosh(x)^2 + a^2 + 2ab + b^2) \sinh(x)^4 + 4(5(a^2 + 2ab \\
& + b^2) \cosh(x)^3 + 3(a^2 + 2ab + b^2) \cosh(x)) \sinh(x)^3 + (3a^2 + 4a \\
& *b) \cosh(x)^2 + (15(a^2 + 2ab + b^2) \cosh(x)^4 + 18(a^2 + 2ab + b^2) * \\
& \cosh(x)^2 + 3a^2 + 4ab) \sinh(x)^2 + a^2 + 2(3(a^2 + 2ab + b^2) \cosh(\\
& x)^5 + 6(a^2 + 2ab + b^2) \cosh(x)^3 + (3a^2 + 4ab) \cosh(x)) \sinh(x)) * \\
& \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) * \sinh(x) \\
& + \sinh(x)^2))} + 4(2(a^3 + 2a^2b + ab^2) \cosh(x)^7 + 3(2a^3 + \\
& 5a^2b + 4ab^2 + b^3) \cosh(x)^5 + (6a^3 + 14a^2b + 9ab^2) \cosh(x)^3 \\
& + (2a^3 + 3a^2b) \cosh(x)) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 1 \\
& 5 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \\
& * \cosh(x) \sinh(x)^5 + \sinh(x)^6)) + 2 \sqrt{a+b} * a * \log(((2a+b) \cosh(x)^4 \\
& + 4(2a+b) \cosh(x) \sinh(x)^3 + (2a+b) \sinh(x)^4 + 2(2a+3b) \cosh \\
& (x)^2 + 2(3(2a+b) \cosh(x)^2 + 2a+3b) \sinh(x)^2 - 2 \sqrt{2} * (\cosh(x) \\
&)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) * \sqrt{a+b} * \sqrt{(a \cosh(x)^2 + a \\
& \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4((2a \\
& + b) \cosh(x)^3 + (2a+3b) \cosh(x)) \sinh(x) + 2a+b) / (\cosh(x)^4 + 4 \cosh \\
& (x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + \\
& 4(\cosh(x)^3 - \cosh(x)) \sinh(x) + 1)) + (a+b) \sqrt{a} * \log(-(a \cosh(x)^4 \\
& + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2b \cosh(x)^2 + 2(3a \cosh(x)^2 + \\
& b) \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) * \sqrt{ \\
& (a) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(\\
& x) + \sinh(x)^2))} + 4(a \cosh(x)^3 + b \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 + 2 * \\
& \cosh(x) \sinh(x) + \sinh(x)^2))) / (a^2 + ab), 1/4 * (4a * \sqrt{-a-b} * \arctan(\sqrt{ \\
& 2} * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) * \sqrt{-a-b} * \sqrt{(a \\
& \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x) \\
& ^2))} / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+2b) \cosh(\\
& x)^2 + 2(3a \cosh(x)^2 + a+2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a+2b) \c \\
& osh(x)) \sinh(x) + a)) + (a+b) \sqrt{a} * \log(((a^3 + 2a^2b + ab^2) \cosh(x) \\
&)^8 + 8(a^3 + 2a^2b + ab^2) \cosh(x) \sinh(x)^7 + (a^3 + 2a^2b + ab^2) \\
& * \sinh(x)^8 + 2(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^6 + 2(2a^3 + 5a \\
& ^2b + 4ab^2 + b^3 + 14(a^3 + 2a^2b + ab^2) \cosh(x)^2) \sinh(x)^6 + 4 * \\
& (14(a^3 + 2a^2b + ab^2) \cosh(x)^3 + 3(2a^3 + 5a^2b + 4ab^2 + b^3) \\
& * \cosh(x)) \sinh(x)^5 + (6a^3 + 14a^2b + 9ab^2) \cosh(x)^4 + (70(a^3 + 2 \\
& *a^2b + ab^2) \cosh(x)^4 + 6a^3 + 14a^2b + 9ab^2 + 30(2a^3 + 5a^2 * \\
& b + 4ab^2 + b^3) \cosh(x)^2) \sinh(x)^4 + 4(14(a^3 + 2a^2b + ab^2) \cosh \\
& (x)^5 + 10(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^3 + (6a^3 + 14a^2b \\
& + 9ab^2) \cosh(x)) \sinh(x)^3 + a^3 + 2(2a^3 + 3a^2b) \cosh(x)^2 + 2(1 \\
& 4(a^3 + 2a^2b + ab^2) \cosh(x)^6 + 15(2a^3 + 5a^2b + 4ab^2 + b^3) * \\
& \cosh(x)^4 + 2a^3 + 3a^2b + 3(6a^3 + 14a^2b + 9ab^2) \cosh(x)^2) \sinh \\
& (x)^2 + \sqrt{2} * ((a^2 + 2ab + b^2) \cosh(x)^6 + 6(a^2 + 2ab + b^2) \cosh \\
& (x) \sinh(x)^5 + (a^2 + 2ab + b^2) \sinh(x)^6 + 3(a^2 + 2ab + b^2) \cosh \\
& (x)^4 + 3(5(a^2 + 2ab + b^2) \cosh(x)^2 + a^2 + 2ab + b^2) \sinh(x)^4 + \\
& 4(5(a^2 + 2ab + b^2) \cosh(x)^3 + 3(a^2 + 2ab + b^2) \cosh(x)) \sinh(x) \\
&)^3 + (3a^2 + 4ab) \cosh(x)^2 + (15(a^2 + 2ab + b^2) \cosh(x)^4 + 18(a \\
& ^2 + 2ab + b^2) \cosh(x)^2 + 3a^2 + 4ab) \sinh(x)^2 + a^2 + 2(3(a^2 + \\
& 2ab + b^2) \cosh(x)^5 + 6(a^2 + 2ab + b^2) \cosh(x)^3 + (3a^2 + 4ab) * \\
& \cosh(x)) \sinh(x)) * \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(\\
& x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4(2(a^3 + 2a^2b + ab^2) \cosh(\\
& x)^7 + 3(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^5 + (6a^3 + 14a^2b + \\
& 9ab^2) \cosh(x)^3 + (2a^3 + 3a^2b) \cosh(x)) \sinh(x)) / (\cosh(x)^6 + 6 \cosh \\
& (x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(\\
& x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6)) + (a+b) \sqrt{a} * \log(-(\\
& a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2b \cosh(x)^2 + 2(3a \\
& \cosh(x)^2 + b) \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x) \\
& ^2 - 1) * \sqrt{a} * \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 * \\
& \cosh(x) \sinh(x) + \sinh(x)^2))} + 4(a \cosh(x)^3 + b \cosh(x)) \sinh(x) + a) / (co
\end{aligned}$$

```

sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a +
b)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)
*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)
)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (
6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)
*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh
(x))*sinh(x) + a)) - sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*
a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + 4*((2*a + b)*cosh(x)^3
+ (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a + b)*arctan(sqrt(2)*
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
+ (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(
x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2
+ 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*
a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^
2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a))
- 2*a*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)))/(a^2 + a*b)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)

[Out] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(coth(x)/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(coth(x)/sqrt(a + b*sech(x)**2), x)

$$3.201 \quad \int \frac{\coth^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}} - \frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{1/2}-\coth(x)*(a+b-b\tanh(x)^2)^{1/2}/(a+b)$

Rubi [A] time = 0.19, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4141, 1975, 480, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}} - \frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]^2/Sqrt[a + b*Sech[x]^2], x]`

[Out] `ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(a + b)`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 377

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]`

Rule 480

`Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]`

Rule 1975

`Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi`

alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^(m*(a + b*(1 + ff^2*x^2)^(n/2)))^p)/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx &= \operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b(1-x^2)}} dx, x, \tanh(x) \right) \\ &= \operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \frac{\operatorname{Subst} \left(\int \frac{a+b}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a+b} \\ &= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{\sqrt{a}} - \frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} \end{aligned}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 1.77

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b} \left((a+b)\tanh^{-1} \left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}} \right) - \sqrt{a}\operatorname{csch}(x)\sqrt{a\sinh^2(x)+a+b} \right)}{\sqrt{2}\sqrt{a}(a+b)\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/Sqrt[a + b*Sech[x]^2], x]

[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*((a + b)*ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]] - Sqrt[a]*Csch[x]*Sqrt[a + b + a*Sinh[x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*Sqrt[a + b*Sech[x]^2])

fricas [B] time = 0.51, size = 1365, normalized size = 25.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2), x, algorithm="fricas")

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sin
h(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*s
inh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^
3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*
a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^
3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*
cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^
2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*
b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b
^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*co
sh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2
*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15
*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*s
inh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*co
sh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b
), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
- a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b
)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)
^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\sqrt{b \operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)^2}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b \operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(coth(x)**2/sqrt(a + b*sech(x)**2), x)

$$3.202 \quad \int \frac{\coth^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}} dx$$

Optimal. Leaf size=90

$$-\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

[Out] $-1/2*(2*a+3*b)*\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)/(a+b)^{(1/2)})/(a+b)^{(3/2)}+\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)/a^{(1/2)})/a^{(1/2)}-1/2*\coth(x)^2*(a+b*\operatorname{sech}(x)^2)^{(1/2)/(a+b))}$

Rubi [A] time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 103, 156, 63, 208}

$$-\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/Sqrt[a + b*Sech[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)) - (Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(2*(a + b))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (
f_)*(x_)^(m_)], x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^3(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx &= -\operatorname{Subst}\left(\int \frac{1}{x(-1+x^2)^2\sqrt{a+bx^2}} dx, x, \operatorname{sech}(x)\right) \\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= -\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} + \frac{\operatorname{Subst}\left(\int \frac{a+b+\frac{bx}{(-1+x)x\sqrt{a+bx}}}{(-1+x)x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2(a+b)} \\ &= -\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right) + \frac{(2a+3b)\operatorname{Su}}{2(a+b)} \\ &= -\frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b} + \frac{(2a+3b)\operatorname{S}}{2(a+b)} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}} - \frac{\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{2(a+b)} \end{aligned}$$

Mathematica [A] time = 0.85, size = 159, normalized size = 1.77

$$\frac{\sqrt{2}\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(2(a+b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a}\sqrt{a+b}} - \operatorname{csch}^2(x)(a\co$$

$$4(a+b)\sqrt{a+b\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Sech[x]^2], x]
```

```
[Out] (-(a + 2*b + a*Cosh[2*x])*Csch[x]^2 + (Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]
])*(-(Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
+ a*Cosh[2*x]]]) + 2*(a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*(a + b)*Sqrt[a + b*
Sech[x]^2])
```

fricas [B] time = 0.73, size = 6475, normalized size = 71.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x) \\ &)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 2*(\\ & 3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a* \\ & b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x) \\ &)*\sqrt{a}*\log(((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2*b + \\ & a*b^2)*\cosh(x)*\sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^8 + 2*(2*a^3 + \\ & 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1 \\ & 4*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b \\ & ^2)*\cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6 \\ & *a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 \\ & + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^4 \\ & + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^3 \\ & + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*\cosh(x)^2 \\ & + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^4 \\ & + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + \\ & 2*a*b + b^2)*\cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + \\ & 2*a*b + b^2)*\sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a* \\ & b + b^2)*\cosh(x)^2 + a^2 + 2*a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2) \\ &)*\cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 \\ & + (15*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^2 \\ & + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x) \\ &)*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\ & + \sinh(x)^2))} + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b \\ & b + 4*a*b^2 + b^3)*\cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh(x) \\ &)/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 \\ & + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + ((2*a^2 + 3*a*b)*\cosh(x)^4 \\ & + 4*(2*a^2 + 3*a*b)*\cosh(x)*\sinh(x)^3 + (2*a^2 + 3*a*b)*\sinh(x)^4 - 2*(2*a^2 + 3*a*b)*\cosh(x)^2 \\ & + 2*(3*(2*a^2 + 3*a*b)*\cosh(x)^2 - 2*a^2 - 3*a*b)*\sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*\cosh(x)^3 \\ & - (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{(a + b)*\log(((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 \\ & + (2*a + b)*\sinh(x)^4 + 2*(2*a + 3*b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a + 3*b) \\ &)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{(a + b)*\sqrt{(a*\cosh(x)^2 \\ & + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*((2*a + b)*\cosh(x)^3 \\ & + (2*a + 3*b)*\cosh(x))*\sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 \\ & - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) + ((a^2 + 2*a*b + b^2)*\cosh(x)^4 \\ & + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 \\ & + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 \\ & - (a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 \\ & + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 \\ & - 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} \\ & + 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 \\ & + 2*(a^2 + a*b)*\cosh(x)*\sinh(x) + (a^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} \\ &)/((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)*\sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^4 + a^3 + 2* \end{aligned}$$

$$\begin{aligned}
& a^2b + ab^2 - 2*(a^3 + 2*a^2b + ab^2)*\cosh(x)^2 - 2*(a^3 + 2*a^2b + a \\
& b^2 - 3*(a^3 + 2*a^2b + ab^2)*\cosh(x)^2*\sinh(x)^2 + 4*((a^3 + 2*a^2b + \\
& ab^2)*\cosh(x)^3 - (a^3 + 2*a^2b + ab^2)*\cosh(x))*\sinh(x)), 1/4*(2*((2*a^ \\
& 2 + 3*a*b)*\cosh(x)^4 + 4*(2*a^2 + 3*a*b)*\cosh(x)*\sinh(x)^3 + (2*a^2 + 3*a*b \\
&)*\sinh(x)^4 - 2*(2*a^2 + 3*a*b)*\cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*\cosh(x)^2 \\
& - 2*a^2 - 3*a*b)*\sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*\cosh(x)^3 - \\
& (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + \\
& 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a - b}*\sqrt{(a*\cosh(x)^2 + a*\sinh \\
& (x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/(a*\cosh(x)^4 \\
& + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh \\
& (x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + \\
& a)) + ((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x) \\
&)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 2*(\\
& 3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a* \\
& b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*a*b + b^2)*\cosh(x))*\s \\
& \sinh(x))*\sqrt{a}*\log(((a^3 + 2*a^2b + ab^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2b + \\
& ab^2)*\cosh(x)*\sinh(x)^7 + (a^3 + 2*a^2b + ab^2)*\sinh(x)^8 + 2*(2*a^3 + \\
& 5*a^2b + 4*a*b^2 + b^3)*\cosh(x)^6 + 2*(2*a^3 + 5*a^2b + 4*a*b^2 + b^3 + 1 \\
& 4*(a^3 + 2*a^2b + ab^2)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2b + ab \\
& ^2)*\cosh(x)^3 + 3*(2*a^3 + 5*a^2b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6 \\
& *a^3 + 14*a^2b + 9*a*b^2)*\cosh(x)^4 + (70*(a^3 + 2*a^2b + ab^2)*\cosh(x)^ \\
& 4 + 6*a^3 + 14*a^2b + 9*a*b^2 + 30*(2*a^3 + 5*a^2b + 4*a*b^2 + b^3)*\cosh(\\
& x)^2)*\sinh(x)^4 + 4*(14*(a^3 + 2*a^2b + ab^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a \\
& ^2b + 4*a*b^2 + b^3)*\cosh(x)^3 + (6*a^3 + 14*a^2b + 9*a*b^2)*\cosh(x))*\s \\
& \sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2b)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2b + ab^2 \\
&)*\cosh(x)^6 + 15*(2*a^3 + 5*a^2b + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^ \\
& 2b + 3*(6*a^3 + 14*a^2b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + \\
& 2*a*b + b^2)*\cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + \\
& 2*a*b + b^2)*\sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a* \\
& b + b^2)*\cosh(x)^2 + a^2 + 2*a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2 \\
&)*\cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\co \\
& sh(x)^2 + (15*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x) \\
&)^2 + 3*a^2 + 4*a*b)*\sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + \\
& 6*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a} \\
&)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)) + 4*(2*(a^3 + 2*a^2b + ab^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2* \\
& b + 4*a*b^2 + b^3)*\cosh(x)^5 + (6*a^3 + 14*a^2b + 9*a*b^2)*\cosh(x)^3 + (2* \\
& a^3 + 3*a^2b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh \\
& (x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(\\
& x)*\sinh(x)^5 + \sinh(x)^6)) + ((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a* \\
& b + b^2)*\cosh(x)*\sinh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b \\
& + b^2)*\cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2) \\
&)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + \\
& 2*a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh \\
& (x)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqr \\
& t(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a}*\sqrt{(a*\cosh(x) \\
&)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + \\
& 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2)) - 2*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sinh \\
& (x) + (a^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + \\
& a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^3 + 2*a^2b + a* \\
& b^2)*\cosh(x)^4 + 4*(a^3 + 2*a^2b + ab^2)*\cosh(x)*\sinh(x)^3 + (a^3 + 2*a^2 \\
& *b + ab^2)*\sinh(x)^4 + a^3 + 2*a^2b + ab^2 - 2*(a^3 + 2*a^2b + ab^2)*\c \\
& osh(x)^2 - 2*(a^3 + 2*a^2b + ab^2 - 3*(a^3 + 2*a^2b + ab^2)*\cosh(x)^2)* \\
& \sinh(x)^2 + 4*((a^3 + 2*a^2b + ab^2)*\cosh(x)^3 - (a^3 + 2*a^2b + ab^2)* \\
& \cosh(x))*\sinh(x)), -1/4*(2*((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b \\
& + b^2)*\cosh(x)*\sinh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + \\
& b^2)*\cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\s \\
& \sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2 \\
& *(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a \\
& ^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x) \\
& ^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)* \\
& \sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sin \\
& h(x))) + 2*((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*s \\
& inh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 \\
& + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + \\
& 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*a*b + b^2)*\cosh(x) \\
&)*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x) \\
& ^2 - 1)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - ((2*a^2 + 3*a*b)*\cosh(x)^4 + 4*(2*a^2 + 3*a*b)*\cosh(x)*\sinh(x)^3 + (2*a^2 + 3*a*b)*\sinh(x)^4 - 2*(2*a^2 + 3*a*b)*\cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*\cosh(x)^2 - 2*a^2 - 3*a*b)*\sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*\cosh(x)^3 - (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 + 2*(2*a + 3*b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a + 3*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{a + b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*((2*a + b)*\cosh(x)^3 + (2*a + 3*b)*\cosh(x))*\sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) + 2*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sinh(x) + (a^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)*\sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*\cosh(x))^2*\sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*\cosh(x))*\sinh(x)), -1/2*((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + ((a^2 + 2*a*b + b^2)*\cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a^2 + 2*a*b + b^2)*\sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a^2 - 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(x)^3 - (a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - ((2*a^2 + 3*a*b)*\cosh(x)^4 + 4*(2*a^2 + 3*a*b)*\cosh(x)*\sinh(x)^3 + (2*a^2 + 3*a*b)*\sinh(x)^4 - 2*(2*a^2 + 3*a*b)*\cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*\cosh(x)^2 - 2*a^2 - 3*a*b)*\sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*\cosh(x)^3 - (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a - b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) + \sqrt{2}*((a
\end{aligned}$$

$$\begin{aligned} &^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sinh(x) + (a^2 + a*b)*\sinh(x)^2 \\ &+ a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 4*(a^3 + 2 \\ &*a^2*b + a*b^2)*\cosh(x)*\sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^4 + a^3 \\ &+ 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2 - 2*(a^3 + 2*a^2*b \\ &+ a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + 2*a^2 \\ &*b + a*b^2)*\cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*\cosh(x))*\sinh(x))] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b\operatorname{sech}(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)

[Out] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^3}{\sqrt{b\operatorname{sech}(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{a + \frac{b}{\cosh(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2),x)

[Out] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b\operatorname{sech}^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(a+b*sech(x)**2)**(1/2),x)

[Out] Integral(coth(x)**3/sqrt(a + b*sech(x)**2), x)

$$3.203 \quad \int \frac{\tanh^5(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}$$

[Out] $\operatorname{arctanh}\left(\frac{(a+b\operatorname{sech}(x)^2)^{1/2}/a^{1/2}}{a^{3/2}}\right) - (a+b)^2/a/b^2/(a+b\operatorname{sech}(x)^2)^{1/2} - (a+b\operatorname{sech}(x)^2)^{1/2}/b^2$

Rubi [A] time = 0.15, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4139, 446, 87, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2), x]`

[Out] `ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)^2/(a*b^2*Sqrt[a + b*Sech[x]^2]) - Sqrt[a + b*Sech[x]^2]/b^2`

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.))/((a_.) + (b_.)*(
x_)), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_.))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])

Rubi steps

$$\begin{aligned} \int \frac{\tanh^5(x)}{(a + b\operatorname{sech}^2(x))^{3/2}} dx &= -\operatorname{Subst}\left(\int \frac{(-1 + x^2)^2}{x(a + bx^2)^{3/2}} dx, x, \operatorname{sech}(x)\right) \\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1 + x)^2}{x(a + bx)^{3/2}} dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(a + b)^2}{ab(a + bx)^{3/2}} + \frac{1}{b\sqrt{a + bx}} + \frac{1}{ax\sqrt{a + bx}}\right) dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= -\frac{(a + b)^2}{ab^2\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a} \\ &= -\frac{(a + b)^2}{ab^2\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{ab} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a + b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{(a + b)^2}{ab^2\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\sqrt{a + b\operatorname{sech}^2(x)}}{b^2} \end{aligned}$$

Mathematica [A] time = 0.53, size = 129, normalized size = 1.90

$$\frac{\operatorname{sech}^3(x) \left(\frac{\sqrt{2}(a \cosh(2x) + a + 2b)^{3/2} \log(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x))}{a^{3/2}} - \frac{\operatorname{sech}(x)(a \cosh(2x) + a + 2b)((2a^2 + 2ab + b^2) \cosh(2x) + 2a^2 + 4ab)}{ab^2} \right)}{4(a + b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2), x]

[Out] (Sech[x]^3*((Sqrt[2]*(a + 2*b + a*Cosh[2*x]))^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(2*a^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cosh[2*x])*Sech[x])/(a*b^2))/(4*(a + b*Sech[x]^2)^(3/2))

fricas [B] time = 0.68, size = 3360, normalized size = 49.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*((a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^2

$$\begin{aligned}
& 3) * \cosh(x)^3 + (3*a*b^2 + 4*b^3) * \cosh(x) * \sinh(x) * \sqrt{a} * \log((a^3 + 2*a^2*b + a*b^2) * \cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2) * \cosh(x) * \sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2) * \sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^2) * \sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)) * \sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2) * \cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)^2) * \sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2) * \cosh(x)) * \sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b) * \cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2) * \cosh(x)^2) * \sinh(x)^2 + \sqrt{2} * ((a^2 + 2*a*b + b^2) * \cosh(x)^6 + 6*(a^2 + 2*a*b + b^2) * \cosh(x) * \sinh(x)^5 + (a^2 + 2*a*b + b^2) * \sinh(x)^6 + 3*(a^2 + 2*a*b + b^2) * \cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2) * \cosh(x)^2 + a^2 + 2*a*b + b^2) * \sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2) * \cosh(x)^3 + 3*(a^2 + 2*a*b + b^2) * \cosh(x)) * \sinh(x)^3 + (3*a^2 + 4*a*b) * \cosh(x)^2 + (15*(a^2 + 2*a*b + b^2) * \cosh(x)^4 + 18*(a^2 + 2*a*b + b^2) * \cosh(x)^2 + 3*a^2 + 4*a*b) * \sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2) * \cosh(x)^5 + 6*(a^2 + 2*a*b + b^2) * \cosh(x)^3 + (3*a^2 + 4*a*b) * \cosh(x)) * \sinh(x) * \sqrt{a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2*b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2) * \cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3) * \cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2) * \cosh(x)^3 + (2*a^3 + 3*a^2*b) * \cosh(x)) * \sinh(x) / (\cosh(x)^6 + 6 * \cosh(x)^5 * \sinh(x) + 15 * \cosh(x)^4 * \sinh(x)^2 + 20 * \cosh(x)^3 * \sinh(x)^3 + 15 * \cosh(x)^2 * \sinh(x)^4 + 6 * \cosh(x) * \sinh(x)^5 + \sinh(x)^6)) + (a * b^2 * \cosh(x)^6 + 6 * a * b^2 * \cosh(x) * \sinh(x)^5 + a * b^2 * \sinh(x)^6 + (3 * a * b^2 + 4 * b^3) * \cosh(x)^4 + (15 * a * b^2 * \cosh(x)^2 + 3 * a * b^2 + 4 * b^3) * \sinh(x)^4 + 4 * (5 * a * b^2 * \cosh(x)^3 + (3 * a * b^2 + 4 * b^3) * \cosh(x)) * \sinh(x)^3 + a * b^2 + (3 * a * b^2 + 4 * b^3) * \cosh(x)^2 + (15 * a * b^2 * \cosh(x)^4 + 3 * a * b^2 + 4 * b^3 + 6 * (3 * a * b^2 + 4 * b^3) * \cosh(x)^2) * \sinh(x)^2 + 2 * (3 * a * b^2 * \cosh(x)^5 + 2 * (3 * a * b^2 + 4 * b^3) * \cosh(x))^3 + (3 * a * b^2 + 4 * b^3) * \cosh(x) * \sinh(x) * \sqrt{a} * \log(- (a * \cosh(x)^4 + 4 * a * \cosh(x) * \sinh(x)^3 + a * \sinh(x)^4 + 2 * b * \cosh(x)^2 + 2 * (3 * a * \cosh(x)^2 + b) * \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 4 * (a * \cosh(x)^3 + b * \cosh(x)) * \sinh(x) + a) / (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) - 4 * \sqrt{2} * ((2 * a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 4 * (2 * a^3 + 2 * a^2 * b + a * b^2) * \cosh(x) * \sinh(x)^3 + (2 * a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^4 + 2 * a^3 + 2 * a^2 * b + a * b^2 + 2 * (2 * a^3 + 4 * a^2 * b + a * b^2) * \cosh(x)^2 + 2 * (2 * a^3 + 4 * a^2 * b + a * b^2 + 3 * (2 * a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x))^2 + 4 * ((2 * a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + (2 * a^3 + 4 * a^2 * b + a * b^2) * \cosh(x)) * \sinh(x) * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / (a^3 * b^2 * \cosh(x)^6 + 6 * a^3 * b^2 * \cosh(x) * \sinh(x)^5 + a^3 * b^2 * \sinh(x)^6 + a^3 * b^2 + (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x)^4 + (15 * a^3 * b^2 * \cosh(x)^2 + 3 * a^3 * b^2 + 4 * a^2 * b^3) * \sinh(x)^4 + 4 * (5 * a^3 * b^2 * \cosh(x))^3 + (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x)) * \sinh(x)^3 + (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x)^2 + (15 * a^3 * b^2 * \cosh(x)^4 + 3 * a^3 * b^2 + 4 * a^2 * b^3 + 6 * (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x)^2) * \sinh(x)^2 + 2 * (3 * a^3 * b^2 * \cosh(x)^5 + 2 * (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x))^3 + (3 * a^3 * b^2 + 4 * a^2 * b^3) * \cosh(x)) * \sinh(x)), -1/2 * ((a * b^2 * \cosh(x)^6 + 6 * a * b^2 * \cosh(x) * \sinh(x)^5 + a * b^2 * \sinh(x)^6 + (3 * a * b^2 + 4 * b^3) * \cosh(x)^4 + (15 * a * b^2 * \cosh(x)^2 + 3 * a * b^2 + 4 * b^3) * \sinh(x)^4 + 4 * (5 * a * b^2 * \cosh(x)^3 + (3 * a * b^2 + 4 * b^3) * \cosh(x)) * \sinh(x)^3 + a * b^2 + (3 * a * b^2 + 4 * b^3) * \cosh(x)^2 + (15 * a * b^2 * \cosh(x)^4 + 3 * a * b^2 + 4 * b^3 + 6 * (3 * a * b^2 + 4 * b^3) * \cosh(x)^2) * \sinh(x)^2 + 2 * (3 * a * b^2 * \cosh(x)^5 + 2 * (3 * a * b^2 + 4 * b^3) * \cosh(x))^3 + (3 * a * b^2 + 4 * b^3) * \cosh(x)) * \sinh(x) * \sqrt{-a} * \arctan(\sqrt{2} * ((a + b) * \cosh(x)^2 + 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 + a) * \sqrt{-a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) / ((a^2 + a * b) * \cosh(x)^4 + 4 * (a^2 + a * b) * \cosh(x) * \sinh(x)^3 + (a^2 + a * b) * \sinh(x)^4 + (2 * a^2 + 3 * a * b) * \cosh(x)^2 + (6 * (a^2 + a * b) * \cosh(x)^2 + 2 * a^2 + 3 * a * b) * \sinh(x)^2 + a^2 + 2 * (2 * (a^2 + a * b) * \cosh(x)^3 + (2 * a^2 + 3 * a * b) *
\end{aligned}$$

```

cosh(x))*sinh(x))) + (a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*s
inh(x)^6 + (3*a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*
b^3)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^
3 + a*b^2 + (3*a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4
*b^3 + 6*(3*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*
(3*a*b^2 + 4*b^3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*
cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2
*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 4*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2
)*sinh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)
^2 + 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*si
nh(x)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2
)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*si
nh(x)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 +
(15*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*co
sh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b
^3)*cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^
2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2
+ 4*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b \operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{(b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^5}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2), x)`

[Out] `int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b \operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**5/(a+b*sech(x)**2)**(3/2), x)`

[Out] `Integral(tanh(x)**5/(a + b*sech(x)**2)**(3/2), x)`

$$3.204 \quad \int \frac{\tanh^4(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}} - \frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

[Out] $\arctan(b^{(1/2)}*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/b^{(3/2)}+\operatorname{arctanh}(a^{(1/2)}*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/a^{(3/2)}-(a+b)*\tanh(x)/a/b/(a+b-b*\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4141, 1975, 470, 523, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}} - \frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2), x]`

[Out] `ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(3/2) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - ((a + b)*Tanh[x])/(a*b*Sqrt[a + b - b*Tanh[x]^2])`

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 377

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]`

Rule 470

`Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(`

$b*n*(b*c - a*d)*(p + 1)$, Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] :> Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 4141

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)])^(n_)^(p_)*((d_)*tan[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(x)}{(a + b \operatorname{sech}^2(x))^{3/2}} dx &= \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^{3/2}} dx, x, \tanh(x) \right) \\ &= \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{a+b-ax^2}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{ab} \\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a} + \frac{\operatorname{Subst} \left(\int \frac{1}{\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a} \\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a} + \frac{\operatorname{Subst} \left(\int \frac{1}{1+bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a} \\ &= \frac{\tan^{-1} \left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{b^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a^{3/2}} - \frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.41, size = 169, normalized size = 1.97

$$\frac{\operatorname{sech}^3(x)(a \cosh(2x) + a + 2b) \left(\sqrt{a} \left(2\sqrt{b}(a+b) \sinh(x) - \sqrt{2a} \sqrt{a \cosh(2x) + a + 2b} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{b} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}} \right) \right) \right)}{4a^{3/2}b^{3/2} (a + b \operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]

[Out] -1/4*((a + 2*b + a*Cosh[2*x])*Sech[x]^3*(-(Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + Sqrt[a]*(-(Sqrt[2]*a*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + 2*Sqrt[b]*(a + b)*Sinh[x])))/(a^(3/2)*b^(3/2)*(a + b*Sech[x]^2)^(3/2))

fricas [B] time = 0.70, size = 5170, normalized size = 60.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)

$$\begin{aligned}
& * \sinh(x) + \sinh(x)^2) + 4\sqrt{2}*(a^2*b + a*b^2 - (a^2*b + a*b^2)*\cosh(x) \\
& ^2 - 2*(a^2*b + a*b^2)*\cosh(x)*\sinh(x) - (a^2*b + a*b^2)*\sinh(x)^2)*\sqrt{((a \\
& *\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x) \\
&)^2)))/(a^3*b^2*\cosh(x)^4 + 4*a^3*b^2*\cosh(x)*\sinh(x)^3 + a^3*b^2*\sinh(x)^4 \\
& + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*\cosh(x)^2 + 2*(3*a^3*b^2*\cosh(x)^2 + a \\
& ^3*b^2 + 2*a^2*b^3)*\sinh(x)^2 + 4*(a^3*b^2*\cosh(x)^3 + (a^3*b^2 + 2*a^2*b^3 \\
&)*\cosh(x))*\sinh(x)), 1/4*(4*(a^3*\cosh(x)^4 + 4*a^3*\cosh(x)*\sinh(x)^3 + a^3* \\
& \sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*\cosh(x)^2 + 2*(3*a^3*\cosh(x)^2 + a^3 + \\
& 2*a^2*b)*\sinh(x)^2 + 4*(a^3*\cosh(x)^3 + (a^3 + 2*a^2*b)*\cosh(x))*\sinh(x))* \\
& \sqrt{b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{ \\
& b}*\sqrt{((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
&) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + \\
& 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + \\
& (a + 2*b)*\cosh(x))*\sinh(x) + a) + (a*b^2*\cosh(x)^4 + 4*a*b^2*\cosh(x)*\sinh(x) \\
&)^3 + a*b^2*\sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*\cosh(x)^2 + 2*(3*a*b^2*c \\
& \cosh(x)^2 + a*b^2 + 2*b^3)*\sinh(x)^2 + 4*(a*b^2*\cosh(x)^3 + (a*b^2 + 2*b^3)* \\
& \cosh(x))*\sinh(x))*\sqrt{a}*\log((a*b^2*\cosh(x)^8 + 8*a*b^2*\cosh(x)*\sinh(x)^7 \\
& + a*b^2*\sinh(x)^8 - 2*(a*b^2 - b^3)*\cosh(x)^6 + 2*(14*a*b^2*\cosh(x)^2 - a*b \\
& ^2 + b^3)*\sinh(x)^6 + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\cosh(x))*\sinh \\
& (x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 + a^3 + 4 \\
& *a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*a*b^2*\cosh \\
& (x)^5 - 10*(a*b^2 - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x))*\sin \\
& h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x)^6 - 15*(a* \\
& b^2 - b^3)*\cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^ \\
& 2)*\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh \\
& (x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*cos \\
& h(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (15*b^2*\cosh \\
& (x)^4 - 18*b^2*\cosh(x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^2*\cosh(x)^5 \\
& - 6*b^2*\cosh(x)^3 - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{((a*\cosh(x) \\
&)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + \\
& 4*(2*a*b^2*\cosh(x)^7 - 3*(a*b^2 - b^3)*\cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^ \\
& 2)*\cosh(x)^3 + (a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*s \\
& \sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sin \\
& h(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + (a*b^2*\cosh(x)^4 + 4*a*b^2*cos \\
& h(x)*\sinh(x)^3 + a*b^2*\sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*\cosh(x)^2 + 2* \\
& (3*a*b^2*\cosh(x)^2 + a*b^2 + 2*b^3)*\sinh(x)^2 + 4*(a*b^2*\cosh(x)^3 + (a*b^2 \\
& + 2*b^3)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x) \\
&)^3 + a*\sinh(x)^4 + 2*(a + b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^ \\
& 2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{a}*\sqrt{((a \\
& *\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x) \\
&)^2)) + 4*(a*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh \\
& (x)*\sinh(x) + \sinh(x)^2)) + 4\sqrt{2}*(a^2*b + a*b^2 - (a^2*b + a*b^2)*\cosh(x) \\
&)^2 - 2*(a^2*b + a*b^2)*\cosh(x)*\sinh(x) - (a^2*b + a*b^2)*\sinh(x)^2)*\sqrt{ \\
& (a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh \\
& (x)^2)))/(a^3*b^2*\cosh(x)^4 + 4*a^3*b^2*\cosh(x)*\sinh(x)^3 + a^3*b^2*\sinh(x) \\
& ^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*\cosh(x)^2 + 2*(3*a^3*b^2*\cosh(x)^2 + \\
& a^3*b^2 + 2*a^2*b^3)*\sinh(x)^2 + 4*(a^3*b^2*\cosh(x)^3 + (a^3*b^2 + 2*a^2*b \\
& ^3)*\cosh(x))*\sinh(x)), -1/2*((a*b^2*\cosh(x)^4 + 4*a*b^2*\cosh(x)*\sinh(x)^3 + \\
& a*b^2*\sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*\cosh(x)^2 + 2*(3*a*b^2*\cosh(x) \\
& ^2 + a*b^2 + 2*b^3)*\sinh(x)^2 + 4*(a*b^2*\cosh(x)^3 + (a*b^2 + 2*b^3)*\cosh(x) \\
&))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b* \\
& \sinh(x)^2 + a)*\sqrt{-a}*\sqrt{((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x) \\
&)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x) \\
&)^3 + a*b*\sinh(x)^4 - (a^2 + 3*a*b)*\cosh(x)^2 + (6*a*b*\cosh(x)^2 - a^2 - 3*a \\
& *b)*\sinh(x)^2 - a^2 + 2*(2*a*b*\cosh(x)^3 - (a^2 + 3*a*b)*\cosh(x))*\sinh(x)) \\
& + (a*b^2*\cosh(x)^4 + 4*a*b^2*\cosh(x)*\sinh(x)^3 + a*b^2*\sinh(x)^4 + a*b^2 + \\
& 2*(a*b^2 + 2*b^3)*\cosh(x)^2 + 2*(3*a*b^2*\cosh(x)^2 + a*b^2 + 2*b^3)*\sinh(x) \\
&)^2 + 4*(a*b^2*\cosh(x)^3 + (a*b^2 + 2*b^3)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arcta \\
& n(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a}*\sqrt{(a*
\end{aligned}$$

$$\frac{\cosh(x)^2 + a \sinh(x)^2 + a + 2b}{(\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} \cdot \frac{1}{(a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x) \sinh(x) + a)) + (a^3 \cosh(x)^4 + 4a^3 \cosh(x) \sinh(x)^3 + a^3 \sinh(x)^4 + a^3 + 2(a^3 + 2a^2b) \cosh(x)^2 + 2(3a^3 \cosh(x)^2 + a^3 + 2a^2b) \sinh(x)^2 + 4(a^3 \cosh(x)^3 + (a^3 + 2a^2b) \cosh(x) \sinh(x)) \sqrt{-b} \log(-((a - b) \cosh(x)^4 + 4(a - b) \cosh(x) \sinh(x)^3 + (a - b) \sinh(x)^4 + 2(a + 3b) \cosh(x)^2 + 2(3(a - b) \cosh(x)^2 + a + 3b) \sinh(x)^2 + 2 \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 4((a - b) \cosh(x)^3 + (a + 3b) \cosh(x) \sinh(x) + a - b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x) \sinh(x) + 1)) - 2 \sqrt{2} (a^2b + a^2b - (a^2b + a^2b) \cosh(x)^2 - 2(a^2b + a^2b) \cosh(x) \sinh(x) - (a^2b + a^2b) \sinh(x)^2) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a^3b^2 \cosh(x)^4 + 4a^3b^2 \cosh(x) \sinh(x)^3 + a^3b^2 \sinh(x)^4 + a^3b^2 + 2(a^3b^2 + 2a^2b^3) \cosh(x)^2 + 2(3a^3b^2 \cosh(x)^2 + a^3b^2 + 2a^2b^3) \sinh(x)^2 + 4(a^3b^2 \cosh(x)^3 + (a^3b^2 + 2a^2b^3) \cosh(x) \sinh(x)), -1/2((a^2b^2 \cosh(x)^4 + 4a^2b^2 \cosh(x) \sinh(x)^3 + a^2b^2 \sinh(x)^4 + a^2b^2 + 2(a^2b^2 + 2b^3) \cosh(x)^2 + 2(3a^2b^2 \cosh(x)^2 + a^2b^2 + 2b^3) \sinh(x)^2 + 4(a^2b^2 \cosh(x)^3 + (a^2b^2 + 2b^3) \cosh(x) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2} (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a^2b \cosh(x)^4 + 4a^2b \cosh(x) \sinh(x)^3 + a^2b \sinh(x)^4 - (a^2 + 3ab) \cosh(x)^2 + (6a^2b \cosh(x)^2 - a^2 - 3ab) \sinh(x)^2 - a^2 + 2(2a^2b \cosh(x)^3 - (a^2 + 3ab) \cosh(x) \sinh(x))) + (a^2b^2 \cosh(x)^4 + 4a^2b^2 \cosh(x) \sinh(x)^3 + a^2b^2 \sinh(x)^4 + a^2b^2 + 2(a^2b^2 + 2b^3) \cosh(x)^2 + 2(3a^2b^2 \cosh(x)^2 + a^2b^2 + 2b^3) \sinh(x)^2 + 4(a^2b^2 \cosh(x)^3 + (a^2b^2 + 2b^3) \cosh(x) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x) \sinh(x) + a)) - 2(a^3 \cosh(x)^4 + 4a^3 \cosh(x) \sinh(x)^3 + a^3 \sinh(x)^4 + a^3 + 2(a^3 + 2a^2b) \cosh(x)^2 + 2(3a^3 \cosh(x)^2 + a^3 + 2a^2b) \sinh(x)^2 + 4(a^3 \cosh(x)^3 + (a^3 + 2a^2b) \cosh(x) \sinh(x)) \sqrt{b} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x) \sinh(x) + a)) - 2 \sqrt{2} (a^2b + a^2b - (a^2b + a^2b) \cosh(x)^2 - 2(a^2b + a^2b) \cosh(x) \sinh(x) - (a^2b + a^2b) \sinh(x)^2) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (a^3b^2 \cosh(x)^4 + 4a^3b^2 \cosh(x) \sinh(x)^3 + a^3b^2 \sinh(x)^4 + a^3b^2 + 2(a^3b^2 + 2a^2b^3) \cosh(x)^2 + 2(3a^3b^2 \cosh(x)^2 + a^3b^2 + 2a^2b^3) \sinh(x)^2 + 4(a^3b^2 \cosh(x)^3 + (a^3b^2 + 2a^2b^3) \cosh(x) \sinh(x))}]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b \operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)`

[Out] `int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{(b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2),x)`

[Out] `int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b \operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**4/(a+b*sech(x)**2)**(3/2),x)`

[Out] `Integral(tanh(x)**4/(a + b*sech(x)**2)**(3/2), x)`

$$3.205 \quad \int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(3/2)}+(-a-b)/a/b/(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4139, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^3/(a + b*\operatorname{Sech}[x]^2)^{(3/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(3/2)} - (a + b)/(a*b*\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2])$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 78

$\operatorname{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*e - a*f)*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)}/(f*(p+1)*(c*f - d*e)), x] - \operatorname{Dist}[(a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1))]/(f*(p+1)*(c*f - d*e)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& (!\operatorname{LtQ}[n, -1] || \operatorname{IntegerQ}[p] || !(\operatorname{IntegerQ}[n] || !(\operatorname{EqQ}[e, 0] || !(\operatorname{EqQ}[c, 0] || \operatorname{LtQ}[p, n]))))$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)*((c_.) + (d_.)*(x_.)^{(n_.))^{(q_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p]/x, x],
x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(x)}{(a + b \operatorname{sech}^2(x))^{3/2}} dx &= \operatorname{Subst} \left(\int \frac{-1 + x^2}{x(a + bx^2)^{3/2}} dx, x, \operatorname{sech}(x) \right) \\ &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{-1 + x}{x(a + bx)^{3/2}} dx, x, \operatorname{sech}^2(x) \right) \\ &= -\frac{a + b}{ab\sqrt{a + b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right)}{2a} \\ &= -\frac{a + b}{ab\sqrt{a + b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \operatorname{sech}^2(x)} \right)}{ab} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{a + b}{ab\sqrt{a + b \operatorname{sech}^2(x)}} \end{aligned}$$

Mathematica [B] time = 0.23, size = 103, normalized size = 2.10

$$\frac{\operatorname{sech}^3(x) \left(\sqrt{2} (a \cosh(2x) + a + 2b)^{3/2} \log \left(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x) \right) - \frac{2\sqrt{a}(a+b) \cosh(x)(a \cosh(2x) + a + 2b)}{b} \right)}{4a^{3/2} (a + b \operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2), x]
```

```
[Out] (((-2*Sqrt[a]*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x]))/b + Sqrt[2]*(a + 2*b
+ a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2
*x]]])*Sech[x]^3)/(4*a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.54, size = 2194, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2
*b^2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a
*b*cosh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2
a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 +
2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b +
```

$$\begin{aligned}
& 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^4 \\
& + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(\\
& 2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + 2*a^2* \\
& b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^3 + (6* \\
& a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*\co \\
& sh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4 \\
& *a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)* \\
& \cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2*a*b + b^2)*\cosh(x)^6 + 6*(a^2 + 2* \\
& a*b + b^2)*\cosh(x))*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh(x)^6 + 3*(a^2 + 2*a \\
& *b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 + 2*a*b + b^ \\
& 2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*\c \\
& osh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*\cos \\
& h(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + 4*a*b)*\sinh(x)^2 + a^2 \\
& + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + (3 \\
& *a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a \\
& + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(2*(a^3 + 2*a^2*b \\
& + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^5 + (6*a^3 \\
& + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cos \\
& h(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x) \\
&)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + (a*b*\cos \\
& h(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 + 2*(a*b + 2*b^2)*\cosh(x)^ \\
& 2 + 2*(3*a*b*\cosh(x)^2 + a*b + 2*b^2)*\sinh(x)^2 + a*b + 4*(a*b*\cosh(x)^3 + \\
& (a*b + 2*b^2)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sin \\
& h(x)^3 + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqrt{ \\
& 2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a}*\sqrt{(a*\cosh(\\
& x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} \\
& + 4*(a*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2)) - 4*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sin \\
& h(x) + (a^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + \\
& a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^3*b*\cosh(x)^4 + \\
& 4*a^3*b*\cosh(x)*\sinh(x)^3 + a^3*b*\sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2) \\
& *\cosh(x)^2 + 2*(3*a^3*b*\cosh(x)^2 + a^3*b + 2*a^2*b^2)*\sinh(x)^2 + 4*(a^3*b \\
& *\cosh(x)^3 + (a^3*b + 2*a^2*b^2)*\cosh(x))*\sinh(x)), -1/2*((a*b*\cosh(x)^4 + \\
& 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 + 2*(a*b + 2*b^2)*\cosh(x)^2 + 2*(3* \\
& a*b*\cosh(x)^2 + a*b + 2*b^2)*\sinh(x)^2 + a*b + 4*(a*b*\cosh(x)^3 + (a*b + 2* \\
& b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + \\
& b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a* \\
& \sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^2 + a \\
& *b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (\\
& 2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x) \\
&)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x)) \\
& + (a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 + 2*(a*b + 2*b^ \\
& 2)*\cosh(x)^2 + 2*(3*a*b*\cosh(x)^2 + a*b + 2*b^2)*\sinh(x)^2 + a*b + 4*(a*b*c \\
& osh(x)^3 + (a*b + 2*b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x) \\
& ^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh \\
& (x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 \\
& + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh \\
& (x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + \\
& a)) + 2*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sinh(x) + (a \\
& ^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b) \\
& /(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^3*b*\cosh(x)^4 + 4*a^3*b*c \\
& osh(x)*\sinh(x)^3 + a^3*b*\sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)*\cosh(x)^ \\
& 2 + 2*(3*a^3*b*\cosh(x)^2 + a^3*b + 2*a^2*b^2)*\sinh(x)^2 + 4*(a^3*b*\cosh(x)^ \\
& 3 + (a^3*b + 2*a^2*b^2)*\cosh(x))*\sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by `u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
 tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
 titution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)

[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2),x)

[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(3/2),x)

[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(3/2), x)

$$3.206 \quad \int \frac{\tanh^2(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

[Out] $\operatorname{arctanh}(a^{(1/2)}*\tanh(x)/(a+b-b*\tanh(x)^2)^{(1/2)})/a^{(3/2)}-\tanh(x)/a/(a+b-b*\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4141, 1975, 471, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^2/(a + b*\operatorname{Sech}[x]^2)^{(3/2)}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/(\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[x]^2])]/a^{(3/2)} - \operatorname{Tanh}[x]/(a*\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[x]^2])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 377

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}/((c_ + (d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[n*p + 1, 0] \ \&\& \operatorname{IntegerQ}[n]$

Rule 471

$\operatorname{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}))^{(q_)}), x_Symbol] \rightarrow \operatorname{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(n*(b*c - a*d)*(p+1)), x] - \operatorname{Dist}[e^n/(n*(b*c - a*d)*(p+1)), \operatorname{Int}[(e*x)^{(m-n)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\operatorname{Simp}[c*(m-n+1) + d*(m+n*(p+q+1)+1]*x^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, q\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{GeQ}[n, m-n+1] \ \&\& \operatorname{GtQ}[m-n+1, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 1975

$\operatorname{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_)^{(m_)}), x_Symbol] \rightarrow \operatorname{Int}[(e*x)^m*\operatorname{ExpandToSum}[u, x]^p*\operatorname{ExpandToSum}[v, x]^q, x] /; \operatorname{FreeQ}\{e, m, p, q\}, x] \ \&\& \operatorname{BinomialQ}\{u, v\}, x] \ \&\& \operatorname{EqQ}[\operatorname{BinomialDegree}[u, x] - \operatorname{BinomialDegree}[v, x], 0] \ \&\& ! \operatorname{BinomialMatchQ}\{u, v\}, x]$

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(a + b\operatorname{sech}^2(x))^{3/2}} dx = \operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^{3/2}} dx, x, \tanh(x)\right)$$

$$= \operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x)\right)}{a}$$

$$= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [B] time = 0.76, size = 128, normalized size = 2.51

$$\frac{\operatorname{sech}^2(x)(a \cosh(2x) + a + 2b) \left(\operatorname{sech}(x) \sinh^{-1}\left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a+b}}\right) (a \cosh(2x) + a + 2b) - 2\sqrt{a} \sqrt{a+b} \tanh(x) \sqrt{\frac{a \sinh^2(x)}{a+b}} \right)}{4a^{3/2} \sqrt{a+b} \sqrt{\frac{a \sinh^2(x) + a + b}{a+b}} (a + b \operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2), x]
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^2*(ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*(a + 2*b + a*Cosh[2*x])*Sech[x] - 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]*Tanh[x]))/(4*a^(3/2)*Sqrt[a + b]*(a + b*Sech[x]^2)^(3/2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

fricas [B] time = 0.54, size = 1733, normalized size = 33.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
```

$$\begin{aligned}
& a*b^2 + b^3)*\sinh(x)^6 + 4*(14*a*b^2*\cosh(x)^3 - 3*(a*b^2 - b^3)*\cosh(x))* \\
& \sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*a*b^2*\cosh(x)^4 + a^3 \\
& + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*a*b^2* \\
& \cosh(x)^5 - 10*(a*b^2 - b^3)*\cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*\cosh(x)) \\
& *\sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*a*b^2*\cosh(x)^6 - 15 \\
& *(a*b^2 - b^3)*\cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*\cosh \\
& (x)^2)*\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*s \\
& \sinh(x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2 \\
& *\cosh(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 + 4*a*b)*\cosh(x)^2 + (15*b^2*c \\
& osh(x)^4 - 18*b^2*\cosh(x)^2 - a^2 - 4*a*b)*\sinh(x)^2 - a^2 + 2*(3*b^2*\cosh \\
& (x)^5 - 6*b^2*\cosh(x)^3 - (a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*c \\
& osh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2 \\
&)) + 4*(2*a*b^2*\cosh(x)^7 - 3*(a*b^2 - b^3)*\cosh(x)^5 + (a^3 + 4*a^2*b + 9* \\
& a*b^2)*\cosh(x)^3 + (a^3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x) \\
& ^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2 \\
& *\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + (a*\cosh(x)^4 + 4*a*\cosh(x) \\
& *\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2 \\
& *b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)*\sqrt{a}*lo \\
& g(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + b)*\cosh(x)^2 \\
& + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sin \\
& h(x) + \sinh(x)^2 + 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(c \\
& osh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(a*\cosh(x)^3 + (a + b)*\cosh \\
& (x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 4*\sqrt{2}*(\\
& a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 - a)*\sqrt{(a*\cosh(x)^2 + a* \\
& \sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^3*cos \\
& h(x)^4 + 4*a^3*\cosh(x)*\sinh(x)^3 + a^3*\sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)* \\
& \cosh(x)^2 + 2*(3*a^3*\cosh(x)^2 + a^3 + 2*a^2*b)*\sinh(x)^2 + 4*(a^3*\cosh(x)^ \\
& 3 + (a^3 + 2*a^2*b)*\cosh(x))*\sinh(x)), -1/2*((a*\cosh(x)^4 + 4*a*\cosh(x)*\sin \\
& h(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)* \\
& \sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)*\sqrt{-a}*arcta \\
& n(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + a)*\sqrt{-a}*sq \\
& rt((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + s \\
& \sinh(x)^2)))/(a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 - (a^2 \\
& + 3*a*b)*\cosh(x)^2 + (6*a*b*\cosh(x)^2 - a^2 - 3*a*b)*\sinh(x)^2 - a^2 + 2*(2 \\
& *a*b*\cosh(x)^3 - (a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + (a*\cosh(x)^4 + 4*a*\cosh \\
& (x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a \\
& + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)*\sqrt{-a} \\
& *arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a}*s \\
& qrt((a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b) \\
&)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + \\
& 2*b)*\cosh(x))*\sinh(x) + a)) + 2*\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) \\
& + a*\sinh(x)^2 - a)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - \\
& 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^3*\cosh(x)^4 + 4*a^3*\cosh(x)*\sinh(x)^3 \\
& + a^3*\sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*\cosh(x)^2 + 2*(3*a^3*\cosh(x)^2 + \\
& a^3 + 2*a^2*b)*\sinh(x)^2 + 4*(a^3*\cosh(x)^3 + (a^3 + 2*a^2*b)*\cosh(x))*\sinh \\
& (x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)`

[Out] `int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2),x)`

[Out] `int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b\operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**2/(a+b*sech(x)**2)**(3/2),x)`

[Out] `Integral(tanh(x)**2/(a + b*sech(x)**2)**(3/2), x)`

$$3.207 \quad \int \frac{\tanh(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}$$

[Out] $\operatorname{arctanh}((a+b\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(3/2)}-1/a/(a+b\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 266, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]/(a + b\operatorname{Sech}[x]^2)^{(3/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(3/2)} - 1/(a\operatorname{Sqrt}[a + b\operatorname{Sech}[x]^2])$

Rule 51

$\operatorname{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}(c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1)), x] - \operatorname{Dist}[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), \operatorname{Int}[(a + b*x)^{(m + 1)}(c + d*x)^n, x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{LtQ}[n, -1] \&\& (\operatorname{EqQ}[a, 0] \mid\mid (\operatorname{NeQ}[c, 0] \&\& \operatorname{LtQ}[m - n, 0] \&\& \operatorname{IntegerQ}[n]))) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 63

$\operatorname{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\operatorname{Int}[(a_. + (b_.)(x_.)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 266

$\operatorname{Int}[(x_.)^{(m_.)}((a_.) + (b_.)(x_.)^n)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)}(a + b*x)^p, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 4139

$\operatorname{Int}[(a_. + (b_.)((c_.)\operatorname{sec}[(e_.) + (f_.)(x_.)])^{(n_.)})^{(p_.)}\operatorname{tan}[(e_.) + (f_.)(x_.)]^{(m_.)}, x_Symbol] \rightarrow \operatorname{With}[\{\operatorname{ff} = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Di}$

st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegerQ[2*n, p])

Rubi steps

$$\begin{aligned} \int \frac{\tanh(x)}{(a + b\operatorname{sech}^2(x))^{3/2}} dx &= -\operatorname{Subst}\left(\int \frac{1}{x(a + bx^2)^{3/2}} dx, x, \operatorname{sech}(x)\right) \\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a + bx)^{3/2}} dx, x, \operatorname{sech}^2(x)\right)\right) \\ &= -\frac{1}{a\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a} \\ &= -\frac{1}{a\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{ab} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a + b\operatorname{sech}^2(x)}} \end{aligned}$$

Mathematica [B] time = 0.40, size = 98, normalized size = 2.28

$$\frac{\operatorname{sech}^3(x)(a \cosh(2x) + a + 2b) \left(2\sqrt{a} \cosh(x) - \sqrt{2} \sqrt{a \cosh(2x) + a + 2b} \log\left(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a}\right)\right)}{4a^{3/2} (a + b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(3/2), x]

[Out] -1/4*((a + 2*b + a*Cosh[2*x])*(2*Sqrt[a]*Cosh[x] - Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]])*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])*Sech[x]^3)/(a^(3/2)*(a + b*Sech[x]^2)^(3/2))

fricas [B] time = 0.52, size = 2034, normalized size = 47.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2

$$\begin{aligned}
& 2 + b^3) \cosh(x)^2 \sinh(x)^4 + 4 \cdot (14 \cdot (a^3 + 2 \cdot a^2 \cdot b + a \cdot b^2) \cosh(x)^5 + 10 \cdot (2 \cdot a^3 + 5 \cdot a^2 \cdot b + 4 \cdot a \cdot b^2 + b^3) \cosh(x)^3 + (6 \cdot a^3 + 14 \cdot a^2 \cdot b + 9 \cdot a \cdot b^2) \\
&) \cosh(x)) \sinh(x)^3 + a^3 + 2 \cdot (2 \cdot a^3 + 3 \cdot a^2 \cdot b) \cosh(x)^2 + 2 \cdot (14 \cdot (a^3 + 2 \cdot a^2 \cdot b + a \cdot b^2) \cosh(x)^6 + 15 \cdot (2 \cdot a^3 + 5 \cdot a^2 \cdot b + 4 \cdot a \cdot b^2 + b^3) \cosh(x)^4 \\
& + 2 \cdot a^3 + 3 \cdot a^2 \cdot b + 3 \cdot (6 \cdot a^3 + 14 \cdot a^2 \cdot b + 9 \cdot a \cdot b^2) \cosh(x)^2) \sinh(x)^2 + \sqrt{2} \cdot ((a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^6 + 6 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x) \sinh(x)^5 \\
& + (a^2 + 2 \cdot a \cdot b + b^2) \sinh(x)^6 + 3 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^4 + 3 \cdot (5 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^2 + a^2 + 2 \cdot a \cdot b + b^2) \sinh(x)^4 + 4 \cdot (5 \cdot (a^2 \\
& + 2 \cdot a \cdot b + b^2) \cosh(x)^3 + 3 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)) \sinh(x)^3 + (3 \cdot a^2 + 4 \cdot a \cdot b) \cosh(x)^2 + (15 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^4 + 18 \cdot (a^2 + 2 \cdot a \cdot b \\
& + b^2) \cosh(x)^2 + 3 \cdot a^2 + 4 \cdot a \cdot b) \sinh(x)^2 + a^2 + 2 \cdot (3 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^5 + 6 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cosh(x)^3 + (3 \cdot a^2 + 4 \cdot a \cdot b) \cosh(x)) \sinh(x) \\
&) \sqrt{a} \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)} + 4 \cdot (2 \cdot (a^3 + 2 \cdot a^2 \cdot b + a \cdot b^2) \cosh(x)^7 + 3 \cdot (2 \cdot a^3 + 5 \cdot a^2 \cdot b + 4 \cdot a \cdot b^2 + b^3) \cosh(x)^5 + (6 \cdot a^3 + 14 \cdot a^2 \cdot b + 9 \cdot a \cdot b^2) \cosh(x)^3 + (2 \cdot a^3 + 3 \cdot a^2 \cdot b) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cdot \cosh(x)^5 \sinh(x) + 15 \cdot \cosh(x)^4 \sinh(x)^2 + 20 \cdot \cosh(x)^3 \sinh(x)^3 + 15 \cdot \cosh(x)^2 \sinh(x)^4 + 6 \cdot \cosh(x) \sinh(x)^5 + \sinh(x)^6) + (a \cdot \cosh(x)^4 + 4 \cdot a \cdot \cosh(x) \sinh(x)^3 + a \cdot \sinh(x)^4 + 2 \cdot (a + 2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a \cdot \cosh(x)^2 + a + 2 \cdot b) \sinh(x)^2 + 4 \cdot (a \cdot \cosh(x)^3 + (a + 2 \cdot b) \cosh(x)) \sinh(x) + a) \sqrt{a} \log(-(a \cdot \cosh(x)^4 + 4 \cdot a \cdot \cosh(x) \sinh(x)^3 + a \cdot \sinh(x)^4 + 2 \cdot b \cdot \cosh(x)^2 + 2 \cdot (3 \cdot a \cdot \cosh(x)^2 + b) \sinh(x)^2 + \sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \sqrt{a} \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)} + 4 \cdot (a \cdot \cosh(x)^3 + b \cdot \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) - 4 \cdot \sqrt{2} \cdot (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a) \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) / (a^3 \cosh(x)^4 + 4 \cdot a^3 \cosh(x) \cdot \sinh(x)^3 + a^3 \sinh(x)^4 + a^3 + 2 \cdot (a^3 + 2 \cdot a^2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a^3 \cosh(x)^2 + a^3 + 2 \cdot a^2 \cdot b) \sinh(x)^2 + 4 \cdot (a^3 \cosh(x)^3 + (a^3 + 2 \cdot a^2 \cdot b) \cosh(x)) \sinh(x)), -1/2 \cdot ((a \cdot \cosh(x)^4 + 4 \cdot a \cdot \cosh(x) \sinh(x)^3 + a \cdot \sinh(x)^4 + 2 \cdot (a + 2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a \cdot \cosh(x)^2 + a + 2 \cdot b) \sinh(x)^2 + 4 \cdot (a \cdot \cosh(x)^3 + (a + 2 \cdot b) \cosh(x)) \sinh(x) + a) \sqrt{-a} \arctan(\sqrt{2} \cdot ((a + b) \cosh(x)^2 + 2 \cdot (a + b) \cosh(x) \sinh(x) + (a + b) \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) / ((a^2 + a \cdot b) \cosh(x)^4 + 4 \cdot (a^2 + a \cdot b) \cosh(x) \sinh(x)^3 + (a^2 + a \cdot b) \sinh(x)^4 + (2 \cdot a^2 + 3 \cdot a \cdot b) \cosh(x)^2 + (6 \cdot (a^2 + a \cdot b) \cosh(x)^2 + 2 \cdot a^2 + 3 \cdot a \cdot b) \sinh(x)^2 + a^2 + 2 \cdot (2 \cdot (a^2 + a \cdot b) \cosh(x)^3 + (2 \cdot a^2 + 3 \cdot a \cdot b) \cosh(x)) \sinh(x))) + (a \cdot \cosh(x)^4 + 4 \cdot a \cdot \cosh(x) \sinh(x)^3 + a \cdot \sinh(x)^4 + 2 \cdot (a + 2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a \cdot \cosh(x)^2 + a + 2 \cdot b) \sinh(x)^2 + 4 \cdot (a \cdot \cosh(x)^3 + (a + 2 \cdot b) \cosh(x)) \sinh(x) + a) \sqrt{-a} \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a} \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) / (a \cdot \cosh(x)^4 + 4 \cdot a \cdot \cosh(x) \sinh(x)^3 + a \cdot \sinh(x)^4 + 2 \cdot (a + 2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a \cdot \cosh(x)^2 + a + 2 \cdot b) \sinh(x)^2 + 4 \cdot (a \cdot \cosh(x)^3 + (a + 2 \cdot b) \cosh(x)) \sinh(x) + a)) + 2 \cdot \sqrt{2} \cdot (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \sinh(x) + a \cdot \sinh(x)^2 + a) \sqrt{(a \cdot \cosh(x)^2 + a \cdot \sinh(x)^2 + a + 2 \cdot b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) / (a^3 \cosh(x)^4 + 4 \cdot a^3 \cosh(x) \sinh(x)^3 + a^3 \sinh(x)^4 + a^3 + 2 \cdot (a^3 + 2 \cdot a^2 \cdot b) \cosh(x)^2 + 2 \cdot (3 \cdot a^3 \cosh(x)^2 + a^3 + 2 \cdot a^2 \cdot b) \sinh(x)^2 + 4 \cdot (a^3 \cosh(x)^3 + (a^3 + 2 \cdot a^2 \cdot b) \cosh(x)) \sinh(x))]]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu

tion variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Error: Bad Argument Type

maple [A] time = 0.10, size = 46, normalized size = 1.07

$$-\frac{1}{a\sqrt{a+b\operatorname{sech}(x)^2}} + \frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a+b*sech(x)^2)^(3/2),x)`

[Out] `-1/a/(a+b*sech(x)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)`

mupad [B] time = 1.74, size = 35, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+\frac{b}{\cosh(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a + b/cosh(x)^2)^(3/2),x)`

[Out] `atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/cosh(x)^2)^(1/2))`

sympy [A] time = 5.93, size = 44, normalized size = 1.02

$$-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*sech(x)**2)**(3/2),x)`

[Out] `-1/(a*sqrt(a + b*sech(x)**2)) - atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a*sqrt(-a))`

$$3.208 \quad \int \frac{1}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{3/2}-b\tanh(x)/a/(a+b)/(a+b-b\tanh(x)^2)^{1/2}$

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4128, 382, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b\operatorname{Sech}[x]^2)^{-3/2}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b - b\operatorname{Tanh}[x]^2]]/a^{3/2} - (b\operatorname{Tanh}[x])/(a*(a + b)*\operatorname{Sqrt}[a + b - b\operatorname{Tanh}[x]^2])$

Rule 206

$\operatorname{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$

Rule 377

$\operatorname{Int}[(a_ + (b_ .)*(x_)^n)^p/((c_) + (d_ .)*(x_)^n), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] \text{ ; FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[n*p + 1, 0] \ \&\& \operatorname{IntegerQ}[n]$

Rule 382

$\operatorname{Int}[(a_ + (b_ .)*(x_)^n)^p*((c_) + (d_ .)*(x_)^n)^q, x_Symbol] \rightarrow -\operatorname{Simp}[(b*x*(a + b*x^n)^{p+1}*(c + d*x^n)^{q+1})/(a*n*(p+1)*(b*c - a*d)), x] + \operatorname{Dist}[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), \operatorname{Int}[(a + b*x^n)^{p+1}*(c + d*x^n)^q, x], x] \text{ ; FreeQ}\{a, b, c, d, n, q\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[n*(p+q+2)+1, 0] \ \&\& (\operatorname{LtQ}[p, -1] \ \|\ \operatorname{!LtQ}[q, -1]) \ \&\& \operatorname{NeQ}[p, -1]$

Rule 4128

$\operatorname{Int}[(a_ + (b_ .)*\operatorname{sec}[(e_ .) + (f_ .)*(x_)]^2)^p, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(a + b + b*ff^2*x^2)^p/(1 + ff^2*x^2), x], x, \operatorname{Tan}[e + f*x]/ff], x]\} \text{ ; FreeQ}\{a, b, e, f, p\}, x \ \&\& \operatorname{NeQ}[a + b, 0] \ \&\& \operatorname{NeQ}[p, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \operatorname{sech}^2(x))^{3/2}} dx &= \operatorname{Subst} \left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= -\frac{b \tanh(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a} \\
&= -\frac{b \tanh(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{a} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{a^{3/2}} - \frac{b \tanh(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.84, size = 107, normalized size = 1.88

$$\frac{\operatorname{sech}^3(x)(a \cosh(2x) + a + 2b) \left((a+b)^{3/2} \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a+b}} \right) \sqrt{\frac{a \cosh(2x) + a + 2b}{a+b}} - \sqrt{2} \sqrt{a} b \sinh(x) \right)}{2\sqrt{2} a^{3/2} (a+b) (a + b \operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[x]^2)^(-3/2), x]

[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^3*((a + b)^(3/2)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)] - Sqrt[2]*Sqrt[a]*b*Sinh[x]))/(2*Sqrt[2]*a^(3/2)*(a + b)*(a + b*Sech[x]^2)^(3/2))

fricas [B] time = 0.55, size = 2095, normalized size = 36.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 +

$$\frac{(a^3 + 4a^2b + 9ab^2)\cosh(x)^3 + (a^3 + 3a^2b)\cosh(x)\sinh(x)}{(\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6)} + \frac{((a^2 + ab)\cosh(x)^4 + 4(a^2 + ab)\cosh(x)\sinh(x)^3 + (a^2 + ab)\sinh(x)^4 + 2(a^2 + 3ab + 2b^2)\cosh(x)^2 + 2(3(a^2 + ab)\cosh(x)^2 + a^2 + 3ab + 2b^2)\sinh(x)^2 + a^2 + ab + 4((a^2 + ab)\cosh(x)^3 + (a^2 + 3ab + 2b^2)\cosh(x)\sinh(x))\sqrt{a}\log(-(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2(a + b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + b)\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1))\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)) + 4(a\cosh(x)^3 + (a + b)\cosh(x)\sinh(x) + a)/(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2)) - 4\sqrt{2}(a*b\cosh(x)^2 + 2*a*b\cosh(x)\sinh(x) + a*b\sinh(x)^2 - a*b)\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/((a^4 + a^3b)\cosh(x)^4 + 4(a^4 + a^3b)\cosh(x)\sinh(x)^3 + (a^4 + a^3b)\sinh(x)^4 + a^4 + a^3b + 2(a^4 + 3a^3b + 2a^2b^2)\cosh(x)^2 + 2(a^4 + 3a^3b + 2a^2b^2 + 3(a^4 + a^3b)\cosh(x)^2)\sinh(x)^2 + 4((a^4 + a^3b)\cosh(x)^3 + (a^4 + 3a^3b + 2a^2b^2)\cosh(x)\sinh(x)), -1/2(((a^2 + ab)\cosh(x)^4 + 4(a^2 + ab)\cosh(x)\sinh(x)^3 + (a^2 + ab)\sinh(x)^4 + 2(a^2 + 3ab + 2b^2)\cosh(x)^2 + 2(3(a^2 + ab)\cosh(x)^2 + a^2 + 3ab + 2b^2)\sinh(x)^2 + a^2 + ab + 4((a^2 + ab)\cosh(x)^3 + (a^2 + 3ab + 2b^2)\cosh(x)\sinh(x))\sqrt{-a}\arctan(\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + a)\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(a*b\cosh(x)^4 + 4*a*b\cosh(x)\sinh(x)^3 + a*b\sinh(x)^4 - (a^2 + 3*a*b)\cosh(x)^2 + (6*a*b\cosh(x)^2 - a^2 - 3*a*b)\sinh(x)^2 - a^2 + 2*(2*a*b\cosh(x)^3 - (a^2 + 3*a*b)\cosh(x))\sinh(x))) + ((a^2 + ab)\cosh(x)^4 + 4(a^2 + ab)\cosh(x)\sinh(x)^3 + (a^2 + ab)\sinh(x)^4 + 2(a^2 + 3ab + 2b^2)\cosh(x)^2 + 2(3(a^2 + ab)\cosh(x)^2 + a^2 + 3ab + 2b^2)\sinh(x)^2 + a^2 + ab + 4((a^2 + ab)\cosh(x)^3 + (a^2 + 3ab + 2b^2)\cosh(x)\sinh(x))\sqrt{-a}\arctan(\sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2(a + 2b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + 2b)\sinh(x)^2 + 4(a\cosh(x)^3 + (a + 2b)\cosh(x)\sinh(x) + a)) + 2\sqrt{2}(a*b\cosh(x)^2 + 2*a*b\cosh(x)\sinh(x) + a*b\sinh(x)^2 - a*b)\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/((a^4 + a^3b)\cosh(x)^4 + 4(a^4 + a^3b)\cosh(x)\sinh(x)^3 + (a^4 + a^3b)\sinh(x)^4 + a^4 + a^3b + 2(a^4 + 3a^3b + 2a^2b^2)\cosh(x)^2 + 2(a^4 + 3a^3b + 2a^2b^2 + 3(a^4 + a^3b)\cosh(x)^2)\sinh(x)^2 + 4((a^4 + a^3b)\cosh(x)^3 + (a^4 + 3a^3b + 2a^2b^2)\cosh(x)\sinh(x))]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sech(x)^2)^(3/2),x)`

[Out] `int(1/(a+b*sech(x)^2)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sech(x)^2 + a)^(-3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b/cosh(x)^2)^(3/2),x)`

[Out] `int(1/(a + b/cosh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(x)**2)**(3/2),x)`

[Out] `Integral((a + b*sech(x)**2)**(-3/2), x)`

$$3.209 \quad \int \frac{\coth(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{b}{a(a+b)\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(3/2)} - \operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)})/(a+b)^{(1/2)})/(a+b)^{(3/2)} - b/a/(a+b)/(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4139, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{b}{a(a+b)\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[x]/(a + b*\operatorname{Sech}[x]^2)^{(3/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(3/2)} - \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a + b]]/(a + b)^{(3/2)} - b/(a*(a + b)*\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2])$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 85

$\operatorname{Int}[(e_. + (f_.)*(x_.))^{(p_.)}/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))], x_Symbol] \rightarrow \operatorname{Simp}[(f*(e + f*x)^{(p+1)})/((p+1)*(b*e - a*f)*(d*e - c*f)), x] + \operatorname{Dist}[1/((b*e - a*f)*(d*e - c*f)), \operatorname{Int}[(b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^{(p+1)}/((a + b*x)*(c + d*x)), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{LtQ}[p, -1]$

Rule 156

$\operatorname{Int}[(e_. + (f_.)*(x_.))^{(p_.)*((g_.) + (h_.)*(x_.))}/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))], x_Symbol] \rightarrow \operatorname{Dist}[(b*g - a*h)/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(a + b*x), x], x] - \operatorname{Dist}[(d*g - c*h)/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(c + d*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h, x\}$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (
f_)*(x_)^(m_), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^(m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\coth(x)}{(a + b\operatorname{sech}^2(x))^{3/2}} dx = \operatorname{Subst}\left(\int \frac{1}{x(-1 + x^2)(a + bx^2)^{3/2}} dx, x, \operatorname{sech}(x)\right)$$

$$= \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{(-1 + x)x(a + bx)^{3/2}} dx, x, \operatorname{sech}^2(x)\right)$$

$$= -\frac{b}{a(a + b)\sqrt{a + b\operatorname{sech}^2(x)}} + \frac{\operatorname{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a(a + b)}$$

$$= -\frac{b}{a(a + b)\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2(a + b)}$$

$$= -\frac{b}{a(a + b)\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{ab} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{2(a + b)}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a + b)^{3/2}} - \frac{b}{a(a + b)\sqrt{a + b\operatorname{sech}^2(x)}}$$

Mathematica [A] time = 0.69, size = 155, normalized size = 1.96

$$\frac{\operatorname{sech}^2(x) \left(\frac{\sqrt{2} \operatorname{sech}(x)(a \cosh(2x) + a + 2b)^{3/2} \left((a+b)^{3/2} \log(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x)) - a^{3/2} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{a+b} \cosh(x)}{\sqrt{a \cosh(2x) + a + 2b}}\right) \right)}{\sqrt{a} \sqrt{a+b}} - 2b(a \cosh(2x) + a + 2b)^{3/2} \right)}{4a(a + b)(a + b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(3/2), x]
[Out] (Sech[x]^2*(-2*b*(a + 2*b + a*Cosh[2*x]) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])
^(3/2)*(-a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]) + (a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*C
osh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*a*(a + b)*(a + b*Sech[x]^2
^(3/2))
```

fricas [B] time = 0.74, size = 6939, normalized size = 87.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left((a^3 + 2a^2b + ab^2) \cosh(x)^4 + 4(a^3 + 2a^2b + ab^2) \cosh(x) \sinh(x)^3 + (a^3 + 2a^2b + ab^2) \sinh(x)^4 + a^3 + 2a^2b + ab^2 + 2(a^3 + 4a^2b + 5ab^2 + 2b^3) \cosh(x)^2 + 2(a^3 + 4a^2b + 5ab^2 + 2b^3 + 3(a^3 + 2a^2b + ab^2) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 2a^2b + ab^2) \cosh(x)^3 + (a^3 + 4a^2b + 5ab^2 + 2b^3) \cosh(x)) \sinh(x) \right) \sqrt{a} \log \left((a^3 + 2a^2b + ab^2) \cosh(x)^8 + 8(a^3 + 2a^2b + ab^2) \cosh(x) \sinh(x)^7 + (a^3 + 2a^2b + ab^2) \sinh(x)^8 + 2(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^6 + 2(2a^3 + 5a^2b + 4ab^2 + b^3 + 14(a^3 + 2a^2b + ab^2) \cosh(x)^2) \sinh(x)^6 + 4(14(a^3 + 2a^2b + ab^2) \cosh(x)^3 + 3(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)) \sinh(x)^5 + (6a^3 + 14a^2b + 9ab^2) \cosh(x)^4 + (70(a^3 + 2a^2b + ab^2) \cosh(x)^4 + 6a^3 + 14a^2b + 9ab^2 + 30(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^2) \sinh(x)^4 + 4(14(a^3 + 2a^2b + ab^2) \cosh(x)^5 + 10(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^3 + (6a^3 + 14a^2b + 9ab^2) \cosh(x)) \sinh(x)^3 + a^3 + 2(2a^3 + 3a^2b) \cosh(x)^2 + 2(14(a^3 + 2a^2b + ab^2) \cosh(x)^6 + 15(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^4 + 2a^3 + 3a^2b + 3(6a^3 + 14a^2b + 9ab^2) \cosh(x)^2) \sinh(x)^2 + \sqrt{2}((a^2 + 2ab + b^2) \cosh(x)^6 + 6(a^2 + 2ab + b^2) \cosh(x) \sinh(x)^5 + (a^2 + 2ab + b^2) \sinh(x)^6 + 3(a^2 + 2ab + b^2) \cosh(x)^4 + 3(5(a^2 + 2ab + b^2) \cosh(x)^2 + a^2 + 2ab + b^2) \sinh(x)^4 + 4(5(a^2 + 2ab + b^2) \cosh(x)^3 + 3(a^2 + 2ab + b^2) \cosh(x)) \sinh(x)^3 + (3a^2 + 4ab) \cosh(x)^2 + (15(a^2 + 2ab + b^2) \cosh(x)^4 + 18(a^2 + 2ab + b^2) \cosh(x)^2 + 3a^2 + 4ab) \sinh(x)^2 + a^2 + 2(3(a^2 + 2ab + b^2) \cosh(x)^5 + 6(a^2 + 2ab + b^2) \cosh(x)^3 + (3a^2 + 4ab) \cosh(x)) \sinh(x) \right) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4(2(a^3 + 2a^2b + ab^2) \cosh(x)^7 + 3(2a^3 + 5a^2b + 4ab^2 + b^3) \cosh(x)^5 + (6a^3 + 14a^2b + 9ab^2) \cosh(x)^3 + (2a^3 + 3a^2b) \cosh(x) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 2(a^3 \cosh(x)^4 + 4a^3 \cosh(x) \sinh(x)^3 + a^3 \sinh(x)^4 + a^3 + 2(a^3 + 2a^2b) \cosh(x)^2 + 2(3a^3 \cosh(x)^2 + a^3 + 2a^2b) \sinh(x)^2 + 4(a^3 \cosh(x)^3 + (a^3 + 2a^2b) \cosh(x)) \sinh(x)) \sqrt{a} \log \left((2a + b) \cosh(x)^4 + 4(2a + b) \cosh(x) \sinh(x)^3 + (2a + b) \sinh(x)^4 + 2(2a + 3b) \cosh(x)^2 + 2(3(2a + b) \cosh(x)^2 + 2a + 3b) \sinh(x)^2 - 2 \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a + b} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((2a + b) \cosh(x)^3 + (2a + 3b) \cosh(x)) \sinh(x) + 2a + b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x)) \sinh(x) + 1) + ((a^3 + 2a^2b + ab^2) \cosh(x)^4 + 4(a^3 + 2a^2b + ab^2) \cosh(x) \sinh(x)^3 + (a^3 + 2a^2b + ab^2) \sinh(x)^4 + a^3 + 2a^2b + ab^2 + 2(a^3 + 4a^2b + 5ab^2 + 2b^3) \cosh(x)^2 + 2(a^3 + 4a^2b + 5ab^2 + 2b^3 + 3(a^3 + 2a^2b + ab^2) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 2a^2b + ab^2) \cosh(x)^3 + (a^3 + 4a^2b + 5ab^2 + 2b^3) \cosh(x)) \sinh(x) \right) \sqrt{a} \log \left(-a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2b \cosh(x)^2 + 2(3a \cosh(x)^2 + b) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4(a \cosh(x)^3 + b \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) - 4 \sqrt{2}(a^2b + ab^2 + (a^2b + ab^2) \cosh(x)^2 + 2(a^2b + ab^2) \cosh(x) \sinh(x) + (a^2b + ab^2) \sinh(x)^2) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} \right) / (a^5 + 2a^4b + a^3b^2 + (a^5 + 2a^4b + a^3$$

$$\begin{aligned}
& *b^2)*\cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)*\sinh(x)^3 + (a^5 + 2* \\
& a^4*b + a^3*b^2)*\sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*\cosh \\
& (x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b \\
& ^2)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^3 + (a^5 + \\
& 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*\cosh(x))*\sinh(x)), 1/4*(4*(a^3*\cosh(x)^4 + \\
& 4*a^3*\cosh(x)*\sinh(x)^3 + a^3*\sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*\cosh(x)^ \\
& 2 + 2*(3*a^3*\cosh(x)^2 + a^3 + 2*a^2*b)*\sinh(x)^2 + 4*(a^3*\cosh(x)^3 + (a^3 \\
& + 2*a^2*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh \\
& (x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a - b}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 \\
& + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/(a*\cosh(x)^4 + 4*a \\
& *\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 \\
& + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a) + \\
& ((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)*\sinh \\
& (x)^3 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 \\
& + 4*a^2*b + 5*a*b^2 + 2*b^3)*\cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^ \\
& 3 + 3*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + 2*a^2*b + a* \\
& b^2)*\cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*\cosh(x))*\sinh(x))*\sqrt{a} \\
&)*\log(((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*\cosh(x) \\
&)*\sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a* \\
& b^2 + b^3)*\cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2 \\
& *b + a*b^2)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^3 \\
& + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 14*a^2* \\
& b + 9*a*b^2)*\cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 6*a^3 + 14 \\
& *a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^ \\
& 4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 \\
& + b^3)*\cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + \\
& 2*(2*a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^6 + \\
& 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 \\
& + 14*a^2*b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2*a*b + b^2)* \\
& \cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh \\
& (x)^6 + 3*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x) \\
&)^2 + a^2 + 2*a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + \\
& 3*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x)^2 + (15* \\
& (a^2 + 2*a*b + b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + 3*a^2 + \\
& 4*a*b)*\sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6*(a^2 + 2*a* \\
& b + b^2)*\cosh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh \\
& (x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} \\
& + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + \\
& b^3)*\cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^3 + 3*a^2*b) \\
& *\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^ \\
& 2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \\
& \sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^ \\
& 2)*\cosh(x)*\sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^4 + a^3 + 2*a^2*b + \\
& a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*\cosh(x)^2 + 2*(a^3 + 4*a^2*b + \\
& 5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 \\
& + 2*a^2*b + a*b^2)*\cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*\cosh(x))*\sinh \\
& (x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2 \\
& *b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh \\
& (x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + \\
& 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(a*\cosh(x)^3 + b*\cosh \\
& (x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2) - 4*\sqrt{2} \\
& *(a^2*b + a*b^2 + (a^2*b + a*b^2)*\cosh(x)^2 + 2*(a^2*b + a*b^2)*\cosh(x)*\sinh \\
& (x) + (a^2*b + a*b^2)*\sinh(x)^2)*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b) \\
&)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + \\
& (a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)*\sinh \\
& (x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*\sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b \\
& ^2 + 2*a^2*b^3)*\cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a \\
& ^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2) \\
& *\cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*\cosh(x))*\sinh(x)), -1/
\end{aligned}$$

$$\begin{aligned}
& 2 * (((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 4 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x) * \sinh(x)^3 + (a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^4 + a^3 + 2 * a^2 * b + a * b^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3 + 3 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * ((a + b) * \cosh(x)^2 + 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 + a) * \sqrt{-a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) / ((a^2 + a * b) * \cosh(x)^4 + 4 * (a^2 + a * b) * \cosh(x) * \sinh(x)^3 + (a^2 + a * b) * \sinh(x)^4 + (2 * a^2 + 3 * a * b) * \cosh(x)^2 + (6 * (a^2 + a * b) * \cosh(x)^2 + 2 * a^2 + 3 * a * b) * \sinh(x)^2 + a^2 + 2 * (2 * (a^2 + a * b) * \cosh(x)^3 + (2 * a^2 + 3 * a * b) * \cosh(x)) * \sinh(x))) + ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 4 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x) * \sinh(x)^3 + (a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^4 + a^3 + 2 * a^2 * b + a * b^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3 + 3 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{-a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) / (a * \cosh(x)^4 + 4 * a * \cosh(x) * \sinh(x)^3 + a * \sinh(x)^4 + 2 * (a + 2 * b) * \cosh(x)^2 + 2 * (3 * a * \cosh(x)^2 + a + 2 * b) * \sinh(x)^2 + 4 * (a * \cosh(x)^3 + (a + 2 * b) * \cosh(x)) * \sinh(x) + a) - (a^3 * \cosh(x)^4 + 4 * a^3 * \cosh(x) * \sinh(x)^3 + a^3 * \sinh(x)^4 + a^3 + 2 * (a^3 + 2 * a^2 * b) * \cosh(x)^2 + 2 * (3 * a^3 * \cosh(x)^2 + a^3 + 2 * a^2 * b) * \sinh(x)^2 + 4 * (a^3 * \cosh(x)^3 + (a^3 + 2 * a^2 * b) * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \log(((2 * a + b) * \cosh(x)^4 + 4 * (2 * a + b) * \cosh(x) * \sinh(x)^3 + (2 * a + b) * \sinh(x)^4 + 2 * (2 * a + 3 * b) * \cosh(x)^2 + 2 * (3 * (2 * a + b) * \cosh(x)^2 + 2 * a + 3 * b) * \sinh(x)^2 - 2 * \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{a + b} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) + 4 * ((2 * a + b) * \cosh(x)^3 + (2 * a + 3 * b) * \cosh(x)) * \sinh(x) + 2 * a + b) / (\cosh(x)^4 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4 + 2 * (3 * \cosh(x)^2 - 1) * \sinh(x)^2 - 2 * \cosh(x)^2 + 4 * (\cosh(x)^3 - \cosh(x)) * \sinh(x) + 1)) + 2 * \sqrt{2} * (a^2 * b + a * b^2 + (a^2 * b + a * b^2) * \cosh(x)^2 + 2 * (a^2 * b + a * b^2) * \cosh(x) * \sinh(x) + (a^2 * b + a * b^2) * \sinh(x)^2) * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) / (a^5 + 2 * a^4 * b + a^3 * b^2 + (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(x)^4 + 4 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(x) * \sinh(x)^3 + (a^5 + 2 * a^4 * b + a^3 * b^2) * \sinh(x)^4 + 2 * (a^5 + 4 * a^4 * b + 5 * a^3 * b^2 + 2 * a^2 * b^3) * \cosh(x)^2 + 2 * (a^5 + 4 * a^4 * b + 5 * a^3 * b^2 + 2 * a^2 * b^3 + 3 * (a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^5 + 2 * a^4 * b + a^3 * b^2) * \cosh(x)^3 + (a^5 + 4 * a^4 * b + 5 * a^3 * b^2 + 2 * a^2 * b^3) * \cosh(x)) * \sinh(x)), -1/2 * (((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 4 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x) * \sinh(x)^3 + (a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^4 + a^3 + 2 * a^2 * b + a * b^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3 + 3 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * ((a + b) * \cosh(x)^2 + 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 + a) * \sqrt{-a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) / ((a^2 + a * b) * \cosh(x)^4 + 4 * (a^2 + a * b) * \cosh(x) * \sinh(x)^3 + (a^2 + a * b) * \sinh(x)^4 + (2 * a^2 + 3 * a * b) * \cosh(x)^2 + (6 * (a^2 + a * b) * \cosh(x)^2 + 2 * a^2 + 3 * a * b) * \sinh(x)^2 + a^2 + 2 * (2 * (a^2 + a * b) * \cosh(x)^3 + (2 * a^2 + 3 * a * b) * \cosh(x)) * \sinh(x))) + ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^4 + 4 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x) * \sinh(x)^3 + (a^3 + 2 * a^2 * b + a * b^2) * \sinh(x)^4 + a^3 + 2 * a^2 * b + a * b^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)^2 + 2 * (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3 + 3 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 + 2 * a^2 * b + a * b^2) * \cosh(x)^3 + (a^3 + 4 * a^2 * b + 5 * a * b^2 + 2 * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{-a} * \sqrt{(a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b)} / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))) / (a * \cosh(x)^4 + 4 * a * \cosh(x) * \sinh(x)^3 + a * \sinh(x)^4 + 2 * (a + 2 * b) * \cosh(x)^2 + 2 * (3 * a * \cosh(x)^2 + a + 2 * b) * \sinh(x)^2 + 4 * (a * \cosh(x)^3 + (a + 2 * b) * \cosh(x)) * \sinh(x) + a) - 2 * (a^3 * \cosh(x)^4 + 4 * a^3 * \cosh(x) * \sinh(x)^3 + a^3 * \sinh(x)^4 + a^3 + 2 * (a^3 + 2 * a
\end{aligned}$$

```

^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT>Error: Bad Argument Type

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)

[Out] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a + b/cosh(x)^2)^(3/2),x)

[Out] int(coth(x)/(a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{coth}(x)}{(a + b \operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)**2)**(3/2), x)

[Out] Integral(coth(x)/(a + b*sech(x)**2)**(3/2), x)

$$3.210 \quad \int \frac{\coth^2(x)}{(a+b\operatorname{sech}^2(x))^{3/2}} dx$$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2} - \frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

[Out] $\operatorname{arctanh}(a^{(1/2)}\tanh(x)/(a+b-b\tanh(x)^2)^{(1/2)})/a^{(3/2)}-b\coth(x)/a/(a+b)/$
 $(a+b-b\tanh(x)^2)^{(1/2)}-(a-b)\coth(x)*(a+b-b\tanh(x)^2)^{(1/2)}/a/(a+b)^2$

Rubi [A] time = 0.27, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4141, 1975, 472, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}} - \frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2} - \frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]`

[Out] `ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Coth[x])/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]) - ((a - b)*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(a*(a + b)^2)`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 377

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]`

Rule 472

`Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]`

Rule 583

```

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

```

Rule 1975

```

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

```

Rule 4141

```

Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])

```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{(a + b \operatorname{sech}^2(x))^{3/2}} dx &= \operatorname{Subst} \left(\int \frac{1}{x^2 (1-x^2) (a+b(1-x^2))^{3/2}} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{1}{x^2 (1-x^2) (a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= -\frac{b \coth(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{-a+b-2bx^2}{x^2(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a(a+b)} \\
&= -\frac{b \coth(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{(a-b) \coth(x) \sqrt{a+b-b \tanh^2(x)}}{a(a+b)^2} + \frac{\operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a(a+b)} \\
&= -\frac{b \coth(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{(a-b) \coth(x) \sqrt{a+b-b \tanh^2(x)}}{a(a+b)^2} + \frac{\operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a(a+b)} \\
&= -\frac{b \coth(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{(a-b) \coth(x) \sqrt{a+b-b \tanh^2(x)}}{a(a+b)^2} + \frac{\operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a(a+b)} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{a^{3/2}} - \frac{b \coth(x)}{a(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{(a-b) \coth(x) \sqrt{a+b-b \tanh^2(x)}}{a(a+b)^2}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 120, normalized size = 1.36

$$\frac{\operatorname{sech}^3(x) \left(\frac{\sqrt{2}(a \cosh(2x) + a + 2b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x) + a + 2b}}\right)}{a^{3/2}} - \frac{(a \cosh(2x) + a + 2b)(\operatorname{acsch}(x)(a \cosh(2x) + a + 2b) + 2b^2 \sinh(x))}{a(a+b)^2} \right)}{4(a + b \operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(3/2), x]

[Out] (Sech[x]^3*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*(a + 2*b + a*Cosh[2*x])^(3/2))/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(a*(a + 2*b + a*Cosh[2*x])*Csch[x] + 2*b^2*Sinh[x]))/(a*(a + b)^2))/(4*(a + b*Sech[x]^2)^(3/2))

fricas [B] time = 0.67, size = 3941, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x))^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(

$$\begin{aligned}
& -(a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a+b) \cosh(x)^2 + \\
& 2(3a \cosh(x)^2 + a+b) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 4(a \cosh(x)^3 + (a+b) \cosh(x) \sinh(x) + a) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) - 4 \sqrt{2}((a^3 + a^2 b^2) \cosh(x)^4 + 4(a^3 + a^2 b^2) \cosh(x) \sinh(x)^3 + (a^3 + a^2 b^2) \sinh(x)^4 + a^3 + a^2 b^2 + 2(a^3 + 2a^2 b - a^2 b^2) \cosh(x)^2 + 2(a^3 + 2a^2 b - a^2 b^2 + 3(a^3 + a^2 b^2) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + a^2 b^2) \cosh(x)^3 + (a^3 + 2a^2 b - a^2 b^2) \cosh(x) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))) / ((a^5 + 2a^4 b + a^3 b^2) \cosh(x)^6 + 6(a^5 + 2a^4 b + a^3 b^2) \cosh(x) \sinh(x)^5 + (a^5 + 2a^4 b + a^3 b^2) \sinh(x)^6 - a^5 - 2a^4 b - a^3 b^2 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^4 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3 + 15(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^2) \sinh(x)^4 + 4(5(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^3 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x) \sinh(x))^3 - (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^2 - (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3 - 15(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^4 - 6(a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^2) \sinh(x)^2 + 2(3(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^5 + 2(a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^3 - (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)) \sinh(x)), -1/2((a^3 + 2a^2 b + a^2 b^2) \cosh(x)^6 + 6(a^3 + 2a^2 b + a^2 b^2) \cosh(x) \sinh(x)^5 + (a^3 + 2a^2 b + a^2 b^2) \sinh(x)^6 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^4 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3 + 15(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^2) \sinh(x)^4 + 4(5(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^3 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x) \sinh(x))^3 - a^3 - 2a^2 b - a^2 b^2 - (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^2 + (15(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^4 - a^3 - 6a^2 b - 9a^2 b^2 - 4b^3 + 6(a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^2) \sinh(x)^2 + 2(3(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^5 + 2(a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^3 - (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (a^2 b \cosh(x)^4 + 4a^2 b \cosh(x) \sinh(x)^3 + a^2 b \sinh(x)^4 - (a^2 + 3a^2 b) \cosh(x)^2 + (6a^2 b \cosh(x)^2 - a^2 - 3a^2 b) \sinh(x)^2 - a^2 + 2(2a^2 b \cosh(x)^3 - (a^2 + 3a^2 b) \cosh(x)) \sinh(x))) + ((a^3 + 2a^2 b + a^2 b^2) \cosh(x)^6 + 6(a^3 + 2a^2 b + a^2 b^2) \cosh(x) \sinh(x)^5 + (a^3 + 2a^2 b + a^2 b^2) \sinh(x)^6 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^4 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3 + 15(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^2) \sinh(x)^4 + 4(5(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^3 + (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x) \sinh(x))^3 - a^3 - 2a^2 b - a^2 b^2 - (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^2 + (15(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^4 - a^3 - 6a^2 b - 9a^2 b^2 - 4b^3 + 6(a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^2) \sinh(x)^2 + 2(3(a^3 + 2a^2 b + a^2 b^2) \cosh(x)^5 + 2(a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)^3 - (a^3 + 6a^2 b + 9a^2 b^2 + 4b^3) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x) \sinh(x) + a)) + 2 \sqrt{2}((a^3 + a^2 b^2) \cosh(x)^4 + 4(a^3 + a^2 b^2) \cosh(x) \sinh(x)^3 + (a^3 + a^2 b^2) \sinh(x)^4 + a^3 + a^2 b^2 + 2(a^3 + 2a^2 b - a^2 b^2) \cosh(x)^2 + 2(a^3 + 2a^2 b - a^2 b^2 + 3(a^3 + a^2 b^2) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + a^2 b^2) \cosh(x)^3 + (a^3 + 2a^2 b - a^2 b^2) \cosh(x) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b)} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))) / ((a^5 + 2a^4 b + a^3 b^2) \cosh(x)^6 + 6(a^5 + 2a^4 b + a^3 b^2) \cosh(x) \sinh(x)^5 + (a^5 + 2a^4 b + a^3 b^2) \sinh(x)^6 - a^5 - 2a^4 b - a^3 b^2 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^4 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3 + 15(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^2) \sinh(x)^4 + 4(5(a^5 + 2a^4 b + a^3 b^2) \cosh(x)^3 + (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x) \sinh(x))^3 - (a^5 + 6a^4 b + 9a^3 b^2 + 4a^2 b^3) \cosh(x)^2 - (a^5 + 6a^4 b
\end{aligned}$$

$*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^4 - 6*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x)^2*\sinh(x)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x))*\sinh(x)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT>Error: Bad Argument Type

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)

[Out] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{(b\operatorname{sech}(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2),x)

[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b\operatorname{sech}^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(3/2),x)

[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(3/2), x)

$$3.211 \quad \int \frac{\tanh^6(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab(a-b\tanh^2(x)+b)^{3/2}}$$

[Out] $-\arctan(b^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/b^{5/2} + \operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{5/2} - (1/a^2 - 1/b^2)\tanh(x)/(a+b-b\tanh(x)^2)^{1/2} - 1/3*(a+b)*\tanh(x)^3/a/b/(a+b-b\tanh(x)^2)^{3/2}$

Rubi [A] time = 0.34, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4141, 1975, 470, 578, 523, 217, 203, 377, 206}

$$-\frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab(a-b\tanh^2(x)+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2), x]

[Out] $-(\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2]])/b^{5/2}) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2]]/a^{5/2} - ((a+b)*\operatorname{Tanh}[x]^3)/(3*a*b*(a+b-b*\operatorname{Tanh}[x]^2)^{3/2}) - ((a^{-2}) - b^{-2})*\operatorname{Tanh}[x]/\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p_))

```
(p + 1)*(c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)
^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^q*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_)*sec[(e_) + (f_)*(x_)^(n_)])^(p_)*((d_)*tan[(e_) + (f
_)*(x_)^(n_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^6(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{x^6}{(1-x^2)(a+b(1-x^2))^{5/2}} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{x^6}{(1-x^2)(a+b-bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= -\frac{(a+b)\tanh^3(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{\operatorname{Subst} \left(\int \frac{x^2(3(a+b)-3ax^2)}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3ab} \\
&= -\frac{(a+b)\tanh^3(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} - \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{3(a^2-b^2)-3a^2x^2}{(1-x^2)\sqrt{a+b-bx^2}} dx \right)}{3a^2b^2} \\
&= -\frac{(a+b)\tanh^3(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} - \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx \right)}{a^2} \\
&= -\frac{(a+b)\tanh^3(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} - \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a^2} \\
&= -\frac{\tan^{-1} \left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{b^{5/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab(a+b-b\tanh^2(x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.75, size = 178, normalized size = 1.51

$$\frac{\operatorname{sech}^5(x) \left(\frac{\sqrt{2}(a \cosh(2x)+a+2b)^{5/2} \left(b^{5/2} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a}\cosh(2x)+a+2b} \right) - a^{5/2} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a}\cosh(2x)+a+2b} \right) \right)}{a^{5/2}b^{5/2}} + \frac{2(a+b)\sinh(x)(3a^2+a(3a-4b)\cosh(2x)-3a^2b^2)}{3a^2b^2} \right)}{8(a+b\operatorname{sech}^2(x))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2), x]

[Out] (Sech[x]^5*((Sqrt[2]*(-(a^(5/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])) + b^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]])*(a + 2*b + a*Cosh[2*x])^(5/2))/(a^(5/2)*b^(5/2)) + (2*(a + b)*(a + 2*b + a*Cosh[2*x])*(3*a^2 + 4*a*b - 6*b^2 + a*(3*a - 4*b)*Cosh[2*x])*Sinh[x])/(3*a^2*b^2))/(8*(a + b*Sech[x]^2)^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)

[Out] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^6}{(b\operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^6}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2),x)

[Out] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{(a + b\operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**6/(a+b*sech(x)**2)**(5/2),x)

[Out] Integral(tanh(x)**6/(a + b*sech(x)**2)**(5/2), x)

$$3.212 \quad \int \frac{\tanh^5(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{(a+b)^2}{3ab^2(a+b\operatorname{sech}^2(x))^{3/2}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(5/2)}-1/3*(a+b)^2/a/b^2/(a+b*\operatorname{sech}(x)^2)^{(3/2)}+(-1/a^2+1/b^2)/(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4139, 446, 87, 63, 208}

$$-\frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{a+b\operatorname{sech}^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{(a+b)^2}{3ab^2(a+b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^5/(a+b*\operatorname{Sech}[x]^2)^{(5/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(5/2)} - (a+b)^2/(3*a*b^2*(a+b*\operatorname{Sech}[x]^2)^{(3/2)}) - (a^{(-2)} - b^{(-2)})/\operatorname{Sqrt}[a+b*\operatorname{Sech}[x]^2]$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a+b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 87

$\operatorname{Int}[(c_. + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)} / ((a_.) + (b_.)*(x_.)), x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(e + f*x)^{\operatorname{FractionalPart}[p]}, ((c + d*x)^n*(e + f*x)^{\operatorname{IntegerPart}[p]} / (a + b*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{FractionQ}[p]$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.))^{(q_.)}, x_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a+b*x)^p*(c+d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 4139

$\operatorname{Int}[(a_. + (b_.)*((c_.)*\operatorname{sec}[(e_.) + (f_.)*(x_.)])^{(n_.))^{(p_.)}*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] := \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Di}$

st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^5(x)}{(a + b\operatorname{sech}^2(x))^{5/2}} dx &= -\operatorname{Subst}\left(\int \frac{(-1 + x^2)^2}{x(a + bx^2)^{5/2}} dx, x, \operatorname{sech}(x)\right) \\
 &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1 + x)^2}{x(a + bx)^{5/2}} dx, x, \operatorname{sech}^2(x)\right)\right) \\
 &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(a + b)^2}{ab(a + bx)^{5/2}} + \frac{a^2 - b^2}{a^2b(a + bx)^{3/2}} + \frac{1}{a^2x\sqrt{a + bx}}\right) dx, x, \operatorname{sech}^2(x)\right)\right) \\
 &= -\frac{(a + b)^2}{3ab^2(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a^2} \\
 &= -\frac{(a + b)^2}{3ab^2(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{a^2b} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{a + b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{(a + b)^2}{3ab^2(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{a + b\operatorname{sech}^2(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.56, size = 126, normalized size = 1.66

$$\frac{\operatorname{sech}^5(x) \left(\frac{\sqrt{2}(a \cosh(2x) + a + 2b)^{5/2} \log(\sqrt{a \cosh(2x) + a + 2b} + \sqrt{2} \sqrt{a} \cosh(x))}{a^{5/2}} + \frac{4(a + b) \cosh(x)(a^2 + a(a - 2b) \cosh(2x) + ab - 3b^2)(a \cosh(2x) + a + 2b)}{3a^2b^2} \right)}{8(a + b\operatorname{sech}^2(x))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2), x]

[Out] (((4*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(a^2 + a*b - 3*b^2 + a*(a - 2*b)*Cosh[2*x]))/(3*a^2*b^2) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])/a^(5/2))*Sech[x]^5)/(8*(a + b*Sech[x]^2)^(5/2))

fricas [B] time = 0.76, size = 5184, normalized size = 68.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin

$$\begin{aligned}
& h(x)^4 + a^2b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^3 \\
& + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3) \\
&) *cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a \\
& ^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8 \\
& *(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 \\
& + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((\\
& a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x) \\
&)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 \\
&) *cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b \\
& ^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a \\
& ^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a* \\
& b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + \\
& 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(1 \\
& 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)* \\
& cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^ \\
& 3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^ \\
& 3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^ \\
& 2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^ \\
& 6 + 6*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 \\
& + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a \\
& ^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + \\
& 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2 \\
& *a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*s \\
& inh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2) \\
&) *cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + \\
& a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2* \\
& (a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cos \\
& h(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x) \\
&) *sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*c \\
& osh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x) \\
& ^6) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x))*sinh(x)^7 + a^2*b^2*sinh(x) \\
& ^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2 \\
& *a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x) \\
&) *sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh \\
& (x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin \\
& h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^ \\
& 3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3) \\
&) *cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a \\
& ^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8 \\
& *(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 \\
& + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(- \\
& (a*cosh(x)^4 + 4*a*cosh(x))*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a* \\
& cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x) \\
& ^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c \\
& osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co \\
& sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8*sqrt(2)*((a^4 - a^3*b - 2*a^2 \\
& *b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x))*sinh(x)^5 + (a^4 - a^ \\
& 3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^ \\
& 4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a^2*b^2)*cosh \\
& (x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b - 2*a^2*b^2) \\
&) *cosh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x)^3 + 3*(\\
& a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*b - 2*a^2*b^ \\
& 2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3*b - 2*a^2*b \\
& ^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^2)*cosh(x)^5 \\
& + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3*b - 2*a^2*b \\
& ^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/ \\
& (cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^8 + 8*a^5*b^ \\
& 2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2 + 2*a^4*b^3) \\
&) *cosh(x)^6 + 4*(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh(x)^6 + 8*(7
\end{aligned}$$

$$\begin{aligned}
& *a^5*b^2*\cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^4 + 2*(35*a^5*b^2*\cosh(x)^4 + 3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^5*b^2*\cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^2 + 4*(7*a^5*b^2*\cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^5*b^2*\cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^5 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*\cosh(x))*\sinh(x)), - \\
& 1/6*(3*(a^2*b^2*\cosh(x)^8 + 8*a^2*b^2*\cosh(x))*\sinh(x)^7 + a^2*b^2*\sinh(x)^8 + 4*(a^2*b^2 + 2*a*b^3)*\cosh(x)^6 + 4*(7*a^2*b^2*\cosh(x)^2 + a^2*b^2 + 2*a*b^3)*\sinh(x)^6 + 8*(7*a^2*b^2*\cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^4 + 2*(35*a^2*b^2*\cosh(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*\cosh(x)^2)*\sinh(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*\cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*\cosh(x)^3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*\cosh(x)^2 + 4*(7*a^2*b^2*\cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*\cosh(x)^4 + a^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^2*b^2*\cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*\cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x))*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + 3*(a^2*b^2*\cosh(x)^8 + 8*a^2*b^2*\cosh(x))*\sinh(x)^7 + a^2*b^2*\sinh(x)^8 + 4*(a^2*b^2 + 2*a*b^3)*\cosh(x)^6 + 4*(7*a^2*b^2*\cosh(x)^2 + a^2*b^2 + 2*a*b^3)*\sinh(x)^6 + 8*(7*a^2*b^2*\cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^4 + 2*(35*a^2*b^2*\cosh(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*\cosh(x)^2)*\sinh(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*\cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*\cosh(x)^3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*\cosh(x)^2 + 4*(7*a^2*b^2*\cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*\cosh(x)^4 + a^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^2*b^2*\cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*\cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*\cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x))*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) - 4*\sqrt{2}*((a^4 - a^3*b - 2*a^2*b^2)*\cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*\cosh(x))*\sinh(x)^5 + (a^4 - a^3*b - 2*a^2*b^2)*\sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x)^4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a^2*b^2)*\cosh(x)^2)*\sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b - 2*a^2*b^2)*\cosh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x))*\sinh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x)^2 + 3*(5*(a^4 - a^3*b - 2*a^2*b^2)*\cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x)^2)*\sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^2)*\cosh(x)^5 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x)^3 + (a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^5*b^2*\cosh(x)^8 + 8*a^5*b^2*\cosh(x))*\sinh(x)^7 + a^5*b^2*\sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^6 + 4*(7*a^5*b^2*\cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*\sinh(x)^6 + 8*(7*a^5*b^2*\cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^4 + 2*(35*a^5*b^2*\cosh(x)^4 + 3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^5*b^2*\cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*\cosh(x)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^5*b^2 + 2*a^4*b^3)
\end{aligned}$$


```
*b^3)*cosh(x)^2 + 4*(7*a^5*b^2*cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*(a^5*b^2*cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{(b\operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^5}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b\operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(5/2), x)
```

$$3.213 \quad \int \frac{\tanh^4(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}} - \frac{(a+b)\tanh(x)}{3ab(a-b\tanh^2(x)+b)^{3/2}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b\tanh(x)^2)^{1/2})/a^{5/2}+1/3*(a-3*b)*\tanh(x)/a^2/b/(a+b\tanh(x)^2)^{1/2}-1/3*(a+b)*\tanh(x)/a/b/(a+b\tanh(x)^2)^{3/2}$

Rubi [A] time = 0.26, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4141, 1975, 470, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}} - \frac{(a+b)\tanh(x)}{3ab(a-b\tanh^2(x)+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^4/(a+b*\operatorname{Sech}[x]^2)^{5/2}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2]]/a^{5/2}-((a+b)*\operatorname{Tanh}[x])/(3*a*b*(a+b-b*\operatorname{Tanh}[x]^2)^{3/2})+((a-3*b)*\operatorname{Tanh}[x])/(3*a^2*b*\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_*)+(b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \operatorname{||} \operatorname{LtQ}[b, 0])$

Rule 377

$\operatorname{Int}[(a_*)+(b_.)*(x_)^{(n_)}]^{(p_)} / ((c_*)+(d_.)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c-(b*c-a*d)*x^n), x], x, x/(a+b*x^n)^{1/n}] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{EqQ}[n*p+1, 0] \&\& \operatorname{IntegerQ}[n]$

Rule 470

$\operatorname{Int}[(e_.)*(x_)^{(m_.)}*((a_*)+(b_.)*(x_)^{(n_)}]^{(p_)}*((c_*)+(d_.)*(x_)^{(n_)}])^{(q_)}, x_Symbol] \rightarrow -\operatorname{Simp}[(a*e^{(2*n-1)}*(e*x)^{(m-2*n+1)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(b*n*(b*c-a*d)*(p+1)), x] + \operatorname{Dist}[e^{(2*n)} / (b*n*(b*c-a*d)*(p+1)), \operatorname{Int}[(e*x)^{(m-2*n)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^q * \operatorname{Simp}[a*c*(m-2*n+1)+(a*d*(m-n+n*q+1)+b*c*n*(p+1)]*x^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, q\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{GtQ}[m-n+1, n] \&\& \operatorname{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_.)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_)*((d_.)*tan[(e_.) + (f
_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^(m*(a + b*(1 + ff^2*x^2)^(n/2)))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^{5/2}} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= -\frac{(a+b)\tanh(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{\operatorname{Subst} \left(\int \frac{a+b+(-a+2b)x^2}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3ab} \\
&= -\frac{(a+b)\tanh(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} - \frac{\operatorname{Subst} \left(\int -\frac{3b(a+b)}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{3a^2b(a+b)} \\
&= -\frac{(a+b)\tanh(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a^2} \\
&= -\frac{(a+b)\tanh(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, -\frac{\tanh(x)}{a} \right)}{a^2} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} \right)}{a^{5/2}} - \frac{(a+b)\tanh(x)}{3ab(a+b-b\tanh^2(x))^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}}
\end{aligned}$$

Mathematica [B] time = 2.17, size = 290, normalized size = 3.22

$$\text{sech}^4(x) \left(\frac{\sqrt{2} \text{csch}(x) \text{sech}(x)}{\left(\frac{16 \left(a \sinh^2(x) + a + b \right) \left(\frac{a \sinh^2(x)}{a+b} + 1 \right)}{\left(a \sinh^2(x) + a + b \right)^2} + \frac{3a(a+b) \sinh^2(x)}{a \sinh^2(x) + a + b} - \frac{3\sqrt{a} \sqrt{a+b} \sinh(x) \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a+b}} \right)}{\sqrt{\frac{a \sinh^2(x) + a + b}{a+b}}} \right)}{a^3} + \frac{12 \sinh^4(x)}{a+b} \right) + \frac{\text{sech}^4(x)}{(a \sinh^2(x) + a + b)^{3/2}}$$

384 (a

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2), x]

[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a + b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)^(3/2) + (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a + b)^2 - (12*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))

fricas [B] time = 0.70, size = 3559, normalized size = 39.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")

[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos

$$\begin{aligned}
& h(x)^3 - (a^2 + 4ab)\cosh(x)\sinh(x)\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4(2ab^2\cosh(x)^7 - 3(ab^2 - b^3)\cosh(x)^5 + (a^3 + 4a^2b + 9ab^2)\cosh(x)^3 + (a^3 + 3a^2b)\cosh(x)\sinh(x))/(\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6) + 3(a^2\cosh(x)^8 + 8a^2\cosh(x)\sinh(x)^7 + a^2\sinh(x)^8 + 4(a^2 + 2ab)\cosh(x)^6 + 4(7a^2\cosh(x)^2 + a^2 + 2ab)\sinh(x)^6 + 8(7a^2\cosh(x)^3 + 3(a^2 + 2ab)\cosh(x))\sinh(x)^5 + 2(3a^2 + 8ab + 8b^2)\cosh(x)^4 + 2(35a^2\cosh(x)^4 + 30(a^2 + 2ab)b)\cosh(x)^2 + 3a^2 + 8ab + 8b^2)\sinh(x)^4 + 8(7a^2\cosh(x)^5 + 10(a^2 + 2ab)\cosh(x)^3 + (3a^2 + 8ab + 8b^2)\cosh(x))\sinh(x)^3 + 4(a^2 + 2ab)\cosh(x)^2 + 4(7a^2\cosh(x)^6 + 15(a^2 + 2ab)\cosh(x)^4 + 3(3a^2 + 8ab + 8b^2)\cosh(x)^2 + a^2 + 2ab)\sinh(x)^2 + a^2 + 8(a^2\cosh(x)^7 + 3(a^2 + 2ab)\cosh(x)^5 + (3a^2 + 8ab + 8b^2)\cosh(x)^3 + (a^2 + 2ab)\cosh(x))\sinh(x))\sqrt{a}\log(-(a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2(a + b)\cosh(x)^2 + 2(3a\cosh(x)^2 + a + b)\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1))\sqrt{a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4(a\cosh(x)^3 + (a + b)\cosh(x))\sinh(x) + a)/(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2)) - 16\sqrt{2}(a^2\cosh(x)^6 + 6a^2\cosh(x)\sinh(x)^5 + a^2\sinh(x)^6 + 3ab\cosh(x)^4 + 3(5a^2\cosh(x)^2 + ab)\sinh(x)^4 - 3ab\cosh(x)^2 + 4(5a^2\cosh(x)^3 + 3ab\cosh(x))\sinh(x)^3 + 3(5a^2\cosh(x)^4 + 6ab\cosh(x)^2 - ab)\sinh(x)^2 - a^2 + 6(a^2\cosh(x)^5 + 2ab\cosh(x)^3 - ab\cosh(x))\sinh(x))\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(a^5\cosh(x)^8 + 8a^5\cosh(x)\sinh(x)^7 + a^5\sinh(x)^8 + 4(a^5 + 2a^4b)\cosh(x)^6 + 4(7a^5\cosh(x)^2 + a^5 + 2a^4b)\sinh(x)^6 + 8(7a^5\cosh(x)^3 + 3(a^5 + 2a^4b)\cosh(x))\sinh(x)^5 + a^5 + 2(3a^5 + 8a^4b + 8a^3b^2)\cosh(x)^4 + 2(35a^5\cosh(x)^4 + 3a^5 + 8a^4b + 8a^3b^2 + 30(a^5 + 2a^4b)\cosh(x)^2)\sinh(x)^4 + 8(7a^5\cosh(x)^5 + 10(a^5 + 2a^4b)\cosh(x)^3 + (3a^5 + 8a^4b + 8a^3b^2)\cosh(x))\sinh(x)^3 + 4(a^5 + 2a^4b)\cosh(x)^2 + 4(7a^5\cosh(x)^6 + a^5 + 2a^4b + 15(a^5 + 2a^4b)\cosh(x)^4 + 3(3a^5 + 8a^4b + 8a^3b^2)\cosh(x)^2)\sinh(x)^2 + 8(a^5\cosh(x)^7 + 3(a^5 + 2a^4b)\cosh(x)^5 + (3a^5 + 8a^4b + 8a^3b^2)\cosh(x)^3 + (a^5 + 2a^4b)\cosh(x))\sinh(x)), -1/6(3(a^2\cosh(x)^8 + 8a^2\cosh(x)\sinh(x)^7 + a^2\sinh(x)^8 + 4(a^2 + 2ab)\cosh(x)^6 + 4(7a^2\cosh(x)^2 + a^2 + 2ab)\sinh(x)^6 + 8(7a^2\cosh(x)^3 + 3(a^2 + 2ab)\cosh(x))\sinh(x)^5 + 2(3a^2 + 8ab + 8b^2)\cosh(x)^4 + 2(35a^2\cosh(x)^4 + 30(a^2 + 2ab)b)\cosh(x)^2 + 3a^2 + 8ab + 8b^2)\sinh(x)^4 + 8(7a^2\cosh(x)^5 + 10(a^2 + 2ab)\cosh(x)^3 + (3a^2 + 8ab + 8b^2)\cosh(x))\sinh(x)^3 + 4(a^2 + 2ab)\cosh(x)^2 + 4(7a^2\cosh(x)^6 + 15(a^2 + 2ab)\cosh(x)^4 + 3(3a^2 + 8ab + 8b^2)\cosh(x)^2 + a^2 + 2ab)\sinh(x)^2 + a^2 + 8(a^2\cosh(x)^7 + 3(a^2 + 2ab)\cosh(x)^5 + (3a^2 + 8ab + 8b^2)\cosh(x)^3 + (a^2 + 2ab)\cosh(x))\sinh(x))\sqrt{-a}\arctan(\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + a))\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(ab\cosh(x)^4 + 4ab\cosh(x)\sinh(x)^3 + ab\sinh(x)^4 - (a^2 + 3ab)\cosh(x)^2 + (6ab\cosh(x)^2 - a^2 - 3ab)\sinh(x)^2 - a^2 + 2(2ab\cosh(x)^3 - (a^2 + 3ab)\cosh(x))\sinh(x)) + 3(a^2\cosh(x)^8 + 8a^2\cosh(x)\sinh(x)^7 + a^2\sinh(x)^8 + 4(a^2 + 2ab)\cosh(x)^6 + 4(7a^2\cosh(x)^2 + a^2 + 2ab)\sinh(x)^6 + 8(7a^2\cosh(x)^3 + 3(a^2 + 2ab)\cosh(x))\sinh(x)^5 + 2(3a^2 + 8ab + 8b^2)\cosh(x)^4 + 2(35a^2\cosh(x)^4 + 30(a^2 + 2ab)b)\cosh(x)^2 + 3a^2 + 8ab + 8b^2)\sinh(x)^4 + 8(7a^2\cosh(x)^5 + 10(a^2 + 2ab)\cosh(x)^3 + (3a^2 + 8ab + 8b^2)\cosh(x))\sinh(x)^3 + 4(a^2 + 2ab)\cosh(x)^2 + 4(7a^2\cosh(x)^6 + 15(a^2 + 2ab)\cosh(x)^4 + 3(3a^2 + 8ab + 8b^2)\cosh(x)^2 + a^2 + 2ab)\sinh(x)^2 + a^2 + 8(a^2\cosh(x)^7 + 3(a^2 + 2ab)\cosh(x)^5 + (3a^2 + 8ab + 8b^2)\cosh(x)^3 + (a^2 + 2ab)\cosh(x))\sinh(x))\sqrt{-a}\arctan(\sqrt{2}\sqrt{-a}\sqrt{(a\cosh(x)^2 + a\sinh(x)^2 + a + 2b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/(a^
\end{aligned}$$

$$\begin{aligned} & \cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a) + 8*\sqrt{2}*(a^2*\cosh(x) \\ &)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a*b*\cosh(x)^4 + 3*(5*a^2* \\ & \cosh(x)^2 + a*b)*\sinh(x)^4 - 3*a*b*\cosh(x)^2 + 4*(5*a^2*\cosh(x)^3 + 3*a*b*c \\ & \cosh(x))*\sinh(x)^3 + 3*(5*a^2*\cosh(x)^4 + 6*a*b*\cosh(x)^2 - a*b)*\sinh(x)^2 - \\ & a^2 + 6*(a^2*\cosh(x)^5 + 2*a*b*\cosh(x)^3 - a*b*\cosh(x))*\sinh(x))*\sqrt{(a*c \\ & \cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^ \\ & 2)))/(a^5*\cosh(x)^8 + 8*a^5*\cosh(x)*\sinh(x)^7 + a^5*\sinh(x)^8 + 4*(a^5 + 2* \\ & a^4*b)*\cosh(x)^6 + 4*(7*a^5*\cosh(x)^2 + a^5 + 2*a^4*b)*\sinh(x)^6 + 8*(7*a^5 \\ & *\cosh(x)^3 + 3*(a^5 + 2*a^4*b)*\cosh(x))*\sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4* \\ & b + 8*a^3*b^2)*\cosh(x)^4 + 2*(35*a^5*\cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^ \\ & 2 + 30*(a^5 + 2*a^4*b)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^5*\cosh(x)^5 + 10*(a^5 \\ & + 2*a^4*b)*\cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(x))*\sinh(x)^3 + 4 \\ & *(a^5 + 2*a^4*b)*\cosh(x)^2 + 4*(7*a^5*\cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 + \\ & 2*a^4*b)*\cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(x)^2)*\sinh(x)^2 \\ & + 8*(a^5*\cosh(x)^7 + 3*(a^5 + 2*a^4*b)*\cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3 \\ & *b^2)*\cosh(x)^3 + (a^5 + 2*a^4*b)*\cosh(x))*\sinh(x))] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)

[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{(b\operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2), x)`

[Out] `int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b \operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**4/(a+b*sech(x)**2)**(5/2), x)`

[Out] `Integral(tanh(x)**4/(a + b*sech(x)**2)**(5/2), x)`

$$3.214 \quad \int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{a+b}{3ab(a+b\operatorname{sech}^2(x))^{3/2}}$$

[Out] $\operatorname{arctanh}\left(\frac{(a+b\operatorname{sech}(x)^2)^{1/2}/a^{1/2}}{a^{5/2}+1/3*(-a-b)/a/b/(a+b\operatorname{sech}(x)^2)^{3/2}}\right)-1/a^2/(a+b\operatorname{sech}(x)^2)^{1/2}$

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4139, 446, 78, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{a+b}{3ab(a+b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^3/(a+b\operatorname{Sech}[x]^2)^{5/2}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{5/2} - (a+b)/(3*a*b*(a+b\operatorname{Sech}[x]^2)^{3/2}) - 1/(a^2*\operatorname{Sqrt}[a+b\operatorname{Sech}[x]^2])$

Rule 51

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}]/((b*c - a*d)*(m+1)), x] - \operatorname{Dist}[(d*(m+n+2))/((b*c - a*d)*(m+1)), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^n, x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{LtQ}[n, -1] \&\& (\operatorname{EqQ}[a, 0] \mid\mid (\operatorname{NeQ}[c, 0] \&\& \operatorname{LtQ}[m-n, 0] \&\& \operatorname{IntegerQ}[n]))) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 78

$\operatorname{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*e - a*f)*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)}]/(f*(p+1)*(c*f - d*e)), x] - \operatorname{Dist}[(a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1))]/(f*(p+1)*(c*f - d*e)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^{(p+1)}, x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& (!\operatorname{LtQ}[n, -1] \mid\mid \operatorname{IntegerQ}[p] \mid\mid !(\operatorname{IntegerQ}[n] \mid\mid !(\operatorname{EqQ}[e, 0] \mid\mid !(\operatorname{EqQ}[c, 0] \mid\mid \operatorname{LtQ}[p, n])))$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_ .), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4139

Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^3(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{-1 + x^2}{x(a + bx^2)^{5/2}} dx, x, \operatorname{sech}(x) \right) \\
 &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{-1 + x}{x(a + bx)^{5/2}} dx, x, \operatorname{sech}^2(x) \right) \\
 &= -\frac{a + b}{3ab(a + b \operatorname{sech}^2(x))^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \operatorname{sech}^2(x) \right)}{2a} \\
 &= -\frac{a + b}{3ab(a + b \operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2 \sqrt{a + b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right)}{2a^2} \\
 &= -\frac{a + b}{3ab(a + b \operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2 \sqrt{a + b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \operatorname{sech}^2(x)} \right)}{a^2 b} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a + b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)}{a^{5/2}} - \frac{a + b}{3ab(a + b \operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2 \sqrt{a + b \operatorname{sech}^2(x)}}
 \end{aligned}$$

Mathematica [A] time = 1.00, size = 124, normalized size = 1.82

$$\frac{\operatorname{sech}^5(x)(a \cosh(2x) + a + 2b)(a^{3/2}(a + 4b) \cosh(3x) + 3\sqrt{a}(a + 2b)^2 \cosh(x) - 3\sqrt{2}b(a \cosh(2x) + a + 2b))}{24a^{5/2}b(a + b \operatorname{sech}^2(x))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2), x]

[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(3*Sqrt[a]*(a + 2*b)^2*Cosh[x] + a^(3/2)*(a + 4*b)*Cosh[3*x] - 3*Sqrt[2]*b*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]])*Sech[x]^5)/(a^(5/2)*b*(a + b*Sech[x]^2)^(5/2))

fricas [B] time = 0.73, size = 4644, normalized size = 68.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/12*(3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4 \\ & *(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh \\ & (x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(\\ & 3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8* \\ & a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh \\ & (x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x) \\ &))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 \\ & + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 \\ & + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*co \\ & sh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x) \\ &)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b \\ & b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 \\ & + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 \\ & + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + \\ & a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + \\ & (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(\\ & x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co \\ & sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + \\ & 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))* \\ & sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a* \\ & b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3 \\ & *a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^ \\ & 2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 \\ & + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2 \\ & *a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + \\ & b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b) \\ & *cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos \\ & h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^ \\ & 5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sq \\ & rt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh \\ & (x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a \\ & ^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + \\ & (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c \\ & osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co \\ & sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x) \\ &)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^ \\ & 2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2) \\ & *cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b \\ & *cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*si \\ & nh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b \\ & + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x) \\ & ^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^ \\ & 2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 \\ & + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + \\ & (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x) \\ & *sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 \\ & + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))*sqrt(a)*sqrt((a*c \\ & osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^ \\ & 2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh \\ & (x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*b) \\ & *cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2) \\ &)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*sin \\ & h(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(\\ & x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(\\ & 5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b + \\ & 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a \\ & \end{aligned}$$

$$\begin{aligned}
& ^2*b + 4*a*b^2)*\cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x))*\sinh(x))*\sqrt[4]{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/(a^5*b*\cosh(x)^8 + 8*a^5*b*\cosh(x)*\sinh(x)^7 + a^5*b*\sinh(x)^8 + 4*(a^5*b + 2*a^4*b^2)*\cosh(x)^6 + 4*(7*a^5*b*\cosh(x)^2 + a^5*b + 2*a^4*b^2)*\sinh(x)^6 + a^5*b + 8*(7*a^5*b*\cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x)^4 + 2*(35*a^5*b*\cosh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^5*b*\cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*\cosh(x)^3 + (3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x))*\sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2)*\cosh(x)^2 + 4*(7*a^5*b*\cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*b^2)*\cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^5*b*\cosh(x)^7 + 3*(a^5*b + 2*a^4*b^2)*\cosh(x)^5 + (3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x)^3 + (a^5*b + 2*a^4*b^2)*\cosh(x))*\sinh(x)), -1/6*(3*(a^2*b*\cosh(x)^8 + 8*a^2*b*\cosh(x)*\sinh(x)^7 + a^2*b*\sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*\cosh(x)^6 + 4*(7*a^2*b*\cosh(x)^2 + a^2*b + 2*a*b^2)*\sinh(x)^6 + 8*(7*a^2*b*\cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^4 + 2*(35*a^2*b*\cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^2*b*\cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*\cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x))*\sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*\cosh(x)^2 + 4*(7*a^2*b*\cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*\cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^2*b*\cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*\cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^3 + (a^2*b + 2*a*b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + 3*a*b)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + 3*a*b)*\sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + 3*(a^2*b*\cosh(x)^8 + 8*a^2*b*\cosh(x)*\sinh(x)^7 + a^2*b*\sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*\cosh(x)^6 + 4*(7*a^2*b*\cosh(x)^2 + a^2*b + 2*a*b^2)*\sinh(x)^6 + 8*(7*a^2*b*\cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^4 + 2*(35*a^2*b*\cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^2*b*\cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*\cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x))*\sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*\cosh(x)^2 + 4*(7*a^2*b*\cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*\cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^2)*\sinh(x)^2 + 8*(a^2*b*\cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*\cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*\cosh(x)^3 + (a^2*b + 2*a*b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*(a*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a)) + 2*\sqrt{2}*((a^3 + 4*a^2*b)*\cosh(x)^6 + 6*(a^3 + 4*a^2*b)*\cosh(x)*\sinh(x)^5 + (a^3 + 4*a^2*b)*\sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(5*(a^3 + 4*a^2*b)*\cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x))*\sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x)^2 + 3*(5*(a^3 + 4*a^2*b)*\cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + 6*((a^3 + 4*a^2*b)*\cosh(x)^5 + 2*(a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*\cosh(x))*\sinh(x))*\sqrt[4]{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/(a^5*b*\cosh(x)^8 + 8*a^5*b*\cosh(x)*\sinh(x)^7 + a^5*b*\sinh(x)^8 + 4*(a^5*b + 2*a^4*b^2)*\cosh(x)^6 + 4*(7*a^5*b*\cosh(x)^2 + a^5*b + 2*a^4*b^2)*\sinh(x)^6 + a^5*b + 8*(7*a^5*b*\cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x)^4 + 2*(35*a^5*b*\cosh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*a^5*b*\cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*\cosh(x)^3 + (3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*\cosh(x))*\sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2)
\end{aligned}$$

) $\cosh(x)^2 + 4(7a^5b\cosh(x)^6 + a^5b + 2a^4b^2 + 15(a^5b + 2a^4b^2)b^2)\cosh(x)^4 + 3(3a^5b + 8a^4b^2 + 8a^3b^3)\cosh(x)^2\sinh(x)^2 + 8(a^5b\cosh(x)^7 + 3(a^5b + 2a^4b^2)\cosh(x)^5 + (3a^5b + 8a^4b^2 + 8a^3b^3)\cosh(x)^3 + (a^5b + 2a^4b^2)\cosh(x))\sinh(x)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)

[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{(b\operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^3}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2),x)

[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(5/2),x)

[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(5/2), x)

$$3.215 \quad \int \frac{\tanh^2(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}} - \frac{\tanh(x)}{3a(a-b\tanh^2(x)+b)^{3/2}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{5/2}-1/3*(2*a+3*b)*\tanh(x)/a^2/(a+b)/(a+b-b\tanh(x)^2)^{1/2}-1/3*\tanh(x)/a/(a+b-b\tanh(x)^2)^{3/2}$

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4141, 1975, 471, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}} - \frac{\tanh(x)}{3a(a-b\tanh^2(x)+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]^2/(a+b*\operatorname{Sech}[x]^2)^{5/2}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2]]/a^{5/2}-\operatorname{Tanh}[x]/(3*a*(a+b-b*\operatorname{Tanh}[x]^2)^{3/2})-((2*a+3*b)*\operatorname{Tanh}[x])/(3*a^2*(a+b)*\operatorname{Sqrt}[a+b-b*\operatorname{Tanh}[x]^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_*)(x_*)^2^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt}Q[a, 0] \parallel \operatorname{Lt}Q[b, 0])$

Rule 377

$\operatorname{Int}[(a_*)(x_*)^{(n_*)}^{(p_*)}/((c_*)(x_*)^{(n_*)}), x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(c-(b*c-a*d)*x^n), x], x, x/(a+b*x^n)^{1/n}] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{EqQ}[n*p+1, 0] \&\& \operatorname{IntegerQ}[n]$

Rule 471

$\operatorname{Int}[(e_*)(x_*)^{(m_*)}((a_*)(x_*)^{(n_*)}^{(p_*)}((c_*)(x_*)^{(n_*)}^{(q_*)}), x_Symbol] \rightarrow \operatorname{Simp}[(e^{(n-1)}*(e*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(n*(b*c-a*d)*(p+1)), x] - \operatorname{Dist}[e^n/(n*(b*c-a*d)*(p+1)), \operatorname{Int}[(e*x)^{(m-n)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^q*\operatorname{Simp}[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, q\}, x \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{GeQ}[n, m-n+1] \&\& \operatorname{GtQ}[m-n+1, 0] \&\& \operatorname{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^{5/2}} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= -\frac{\tanh(x)}{3a(a+b-b \tanh^2(x))^{3/2}} + \frac{\operatorname{Subst} \left(\int \frac{1+2x^2}{(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3a} \\
&= -\frac{\tanh(x)}{3a(a+b-b \tanh^2(x))^{3/2}} - \frac{(2a+3b) \tanh(x)}{3a^2(a+b)\sqrt{a+b-b \tanh^2(x)}} - \frac{\operatorname{Subst} \left(\int -\frac{3(a+b)}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{3a^2(a+b)} \\
&= -\frac{\tanh(x)}{3a(a+b-b \tanh^2(x))^{3/2}} - \frac{(2a+3b) \tanh(x)}{3a^2(a+b)\sqrt{a+b-b \tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \tanh(x) \right)}{a^2} \\
&= -\frac{\tanh(x)}{3a(a+b-b \tanh^2(x))^{3/2}} - \frac{(2a+3b) \tanh(x)}{3a^2(a+b)\sqrt{a+b-b \tanh^2(x)}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-ax^2} dx, x, \tanh(x) \right)}{a^2} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{a^{5/2}} - \frac{\tanh(x)}{3a(a+b-b \tanh^2(x))^{3/2}} - \frac{(2a+3b) \tanh(x)}{3a^2(a+b)\sqrt{a+b-b \tanh^2(x)}}
\end{aligned}$$

Mathematica [B] time = 1.20, size = 290, normalized size = 3.30

$$\text{sech}^4(x) \left(\frac{\sqrt{2} \text{csch}(x) \text{sech}(x) \left(\frac{16(a \sinh^2(x) + a + b) \left(\frac{a \sinh^2(x)}{a + b} + 1 \right) \left(\frac{a^2(a + b) \sinh^4(x)}{(a \sinh^2(x) + a + b)^2} + \frac{3a(a + b) \sinh^2(x)}{a \sinh^2(x) + a + b} - \frac{3\sqrt{a} \sqrt{a + b} \sinh(x) \sinh^{-1} \left(\frac{\sqrt{a} \sinh(x)}{\sqrt{a + b}} \right)}{\sqrt{\frac{a \sinh^2(x) + a + b}{a + b}}} \right)}{a^3} + \frac{12 \sinh^4(x)}{a + b} \right)}{(a \sinh^2(x) + a + b)^{3/2}} \right)$$

384

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2), x]

[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a + b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b])])/a^3))/(a + b + a*Sinh[x]^2)^(3/2) - (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a + b)^2 + (4*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))

fricas [B] time = 0.77, size = 4989, normalized size = 56.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")

[Out] [1/12*(3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*

$$\begin{aligned}
& b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + \\
& 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) * \\
& \sinh(x)^3 - (a^2 + 4ab) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 \\
& - a^2 - 4ab) \sinh(x)^2 - a^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 \\
& + 4ab) \cosh(x)) \sinh(x) * \sqrt{a} * \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + \\
& 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4(2ab^2 \cosh(x)^7 - \\
& 3(ab^2 - b^3) \cosh(x)^5 + (a^3 + 4a^2b + 9ab^2) \cosh(x)^3 + (a^3 + 3a^2b) \\
& \cosh(x) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 \\
& + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 \\
& + \sinh(x)^6) + 3((a^3 + a^2b) \cosh(x)^8 + 8(a^3 + a^2b) \cosh(x) \sinh(x)^7 \\
& + (a^3 + a^2b) \sinh(x)^8 + 4(a^3 + 3a^2b + 2ab^2) \cosh(x)^6 \\
& + 4(a^3 + 3a^2b + 2ab^2 + 7(a^3 + a^2b) \cosh(x)^2) \sinh(x)^6 + 8(7(a^3 + a^2b) \\
& \cosh(x)^3 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x)^5 + \\
& 2(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^4 + 2(35(a^3 + a^2b) \cosh(x)^4 \\
& + 3a^3 + 11a^2b + 16ab^2 + 8b^3 + 30(a^3 + 3a^2b + 2ab^2) \\
& \cosh(x)^2) \sinh(x)^4 + 8(7(a^3 + a^2b) \cosh(x)^5 + 10(a^3 + 3a^2b + \\
& 2ab^2) \cosh(x)^3 + (3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)) \sinh(x)^3 \\
& + a^3 + a^2b + 4(a^3 + 3a^2b + 2ab^2) \cosh(x)^2 + 4(7(a^3 + a^2b) \\
& \cosh(x)^6 + 15(a^3 + 3a^2b + 2ab^2) \cosh(x)^4 + a^3 + 3a^2b + 2ab^2 \\
& + 3(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^2) \sinh(x)^2 + 8((a^3 + a^2b) \\
& \cosh(x)^7 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)^5 + (3a^3 + 11a^2b + 16ab^2 \\
& + 8b^3) \cosh(x)^3 + (a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x) * \sqrt{a} * \log(-a \cosh(x)^4 \\
& + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + b) \cosh(x)^2 + 2(3a \cosh(x)^2 \\
& + a + b) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \\
& * \sqrt{a} * \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} + 4(a \cosh(x)^3 + (a + b) \cosh(x)) \sinh(x) + a) / (\cosh(x)^2 \\
& + 2 \cosh(x) \sinh(x) + \sinh(x)^2) - 4 \sqrt{2} ((3a^3 + 4a^2b) \cosh(x)^6 + 6(3a^3 + 4a^2b) \\
& \cosh(x) \sinh(x)^5 + (3a^3 + 4a^2b) \sinh(x)^6 + 3(a^3 + 4a^2b + 4ab^2) \cosh(x)^4 \\
& + 3(a^3 + 4a^2b + 4ab^2 + 5(3a^3 + 4a^2b) \cosh(x)^2) \sinh(x)^4 + 4(5(3a^3 + 4a^2b) \\
& \cosh(x)^3 + 3(a^3 + 4a^2b + 4ab^2) \cosh(x)) \sinh(x)^3 - 3a^3 - 4a^2b - 3(a^3 + 4a^2b \\
& + 4ab^2) \cosh(x)^2 + 3(5(3a^3 + 4a^2b) \cosh(x)^4 - a^3 - 4a^2b - 4ab^2 + 6(a^3 + 4a^2b \\
& + 4ab^2) \cosh(x)^2) \sinh(x)^2 + 6((3a^3 + 4a^2b) \cosh(x)^5 + 2(a^3 + 4a^2b \\
& + 4ab^2) \cosh(x)^3 - (a^3 + 4a^2b + 4ab^2) \cosh(x)) \sinh(x) * \sqrt{(a \cosh(x)^2 \\
& + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^6 \\
& + a^5b) \cosh(x)^8 + 8(a^6 + a^5b) \cosh(x) \sinh(x)^7 + (a^6 + a^5b) \sinh(x)^8 \\
& + 4(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^6 + 4(a^6 + 3a^5b + 2a^4b^2 + 7(a^6 + a^5b) \\
& \cosh(x)^2) \sinh(x)^6 + a^6 + a^5b + 8(7(a^6 + a^5b) \cosh(x)^3 + 3(a^6 + 3a^5b \\
& + 2a^4b^2) \cosh(x)) \sinh(x)^5 + 2(3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x)^4 \\
& + 2(3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3 + 35(a^6 + a^5b) \cosh(x)^4 + 30(a^6 + 3a^5b \\
& + 2a^4b^2) \cosh(x)^2) \sinh(x)^4 + 8(7(a^6 + a^5b) \cosh(x))^5 + 10(a^6 + 3a^5b \\
& + 2a^4b^2) \cosh(x)^3 + (3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x) \sinh(x)^3 \\
& + 4(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^2 + 4(7(a^6 + a^5b) \cosh(x)^6 + a^6 + 3a^5b \\
& + 2a^4b^2 + 15(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^4 + 3(3a^6 + 11a^5b + 16a^4b^2 \\
& + 8a^3b^3) \cosh(x)^2) \sinh(x)^2 + 8((a^6 + a^5b) \cosh(x)^7 + 3(a^6 + 3a^5b \\
& + 2a^4b^2) \cosh(x)^5 + (3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x)^3 \\
& + (a^6 + 3a^5b + 2a^4b^2) \cosh(x)) \sinh(x), -1/6(3((a^3 + a^2b) \cosh(x)^8 \\
& + 8(a^3 + a^2b) \cosh(x) \sinh(x)^7 + (a^3 + a^2b) \sinh(x)^8 + 4(a^3 + 3a^2b \\
& + 2ab^2) \cosh(x)^6 + 4(a^3 + 3a^2b + 2ab^2 + 7(a^3 + a^2b) \cosh(x)^2) \sinh(x)^6 \\
& + 8(7(a^3 + a^2b) \cosh(x)^3 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x)^5 \\
& + 2(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^4 + 2(35(a^3 + a^2b) \cosh(x)^4 \\
& + 3a^3 + 11a^2b + 16ab^2 + 8b^3 + 30(a^3 + 3a^2b + 2ab^2) \cosh(x)^2) \sinh(x)^4 \\
& + 8(7(a^3 + a^2b) \cosh(x)^5 + 10(a^3 + 3a^2b + 2ab^2) \cosh(x)^3 + (3a^3 + 11a^2b \\
& + 16ab^2 + 8b^3) \cosh(x)) \sinh(x)^3 + a^3 + a^2b + 4(a^3 + 3a^2b + 2ab^2) \\
& \cosh(x)^2 + 4(7(a^3 + a^2b) \cosh(x)^6 + 15(a^3 + 3a^2b + 2ab^2) \cosh(x)^4 + 3(a^3 + 3a^2b \\
& + 2ab^2) \cosh(x)) \sinh(x) * \sqrt{a} * \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))}
\end{aligned}$$

$$\begin{aligned}
& b^2 \cosh(x)^4 + a^3 + 3a^2b + 2ab^2 + 3(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^2 \sinh(x)^2 + 8((a^3 + a^2b) \cosh(x)^7 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)^5 + (3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^3 + (a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x) \sqrt{-a} \arctan(\sqrt{2}) (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (ab \cosh(x)^4 + 4ab \cosh(x) \sinh(x)^3 + ab \sinh(x)^4 - (a^2 + 3ab) \cosh(x)^2 + (6ab \cosh(x)^2 - a^2 - 3ab) \sinh(x)^2 - a^2 + 2(2ab \cosh(x)^3 - (a^2 + 3ab) \cosh(x)) \sinh(x)) + 3((a^3 + a^2b) \cosh(x)^8 + 8(a^3 + a^2b) \cosh(x) \sinh(x)^7 + (a^3 + a^2b) \sinh(x)^8 + 4(a^3 + 3a^2b + 2ab^2) \cosh(x)^6 + 4(a^3 + 3a^2b + 2ab^2 + 7(a^3 + a^2b) \cosh(x)^2) \sinh(x)^6 + 8(7(a^3 + a^2b) \cosh(x)^3 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x)^5 + 2(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^4 + 2(35(a^3 + a^2b) \cosh(x)^4 + 3a^3 + 11a^2b + 16ab^2 + 8b^3 + 30(a^3 + 3a^2b + 2ab^2) \cosh(x)^2) \sinh(x)^4 + 8(7(a^3 + a^2b) \cosh(x)^5 + 10(a^3 + 3a^2b + 2ab^2) \cosh(x)^3 + (3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)) \sinh(x)^3 + a^3 + a^2b + 4(a^3 + 3a^2b + 2ab^2) \cosh(x)^2 + 4(7(a^3 + a^2b) \cosh(x)^6 + 15(a^3 + 3a^2b + 2ab^2) \cosh(x)^4 + a^3 + 3a^2b + 2ab^2 + 3(3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^2) \sinh(x)^2 + 8((a^3 + a^2b) \cosh(x)^7 + 3(a^3 + 3a^2b + 2ab^2) \cosh(x)^5 + (3a^3 + 11a^2b + 16ab^2 + 8b^3) \cosh(x)^3 + (a^3 + 3a^2b + 2ab^2) \cosh(x)) \sinh(x) \sqrt{-a} \arctan(\sqrt{2}) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 + a)) + 2\sqrt{2} ((3a^3 + 4a^2b) \cosh(x)^6 + 6(3a^3 + 4a^2b) \cosh(x) \sinh(x)^5 + (3a^3 + 4a^2b) \sinh(x)^6 + 3(a^3 + 4a^2b + 4ab^2) \cosh(x)^4 + 3(a^3 + 4a^2b + 4ab^2 + 5(3a^3 + 4a^2b) \cosh(x)^2) \sinh(x)^4 + 4(5(3a^3 + 4a^2b) \cosh(x)^3 + 3(a^3 + 4a^2b + 4ab^2) \cosh(x)) \sinh(x)^3 - 3a^3 - 4a^2b - 3(a^3 + 4a^2b + 4ab^2) \cosh(x)^2 + 3(5(3a^3 + 4a^2b) \cosh(x)^4 - a^3 - 4a^2b - 4ab^2 + 6(a^3 + 4a^2b + 4ab^2) \cosh(x)^2) \sinh(x)^2 + 6((3a^3 + 4a^2b) \cosh(x)^5 + 2(a^3 + 4a^2b + 4ab^2) \cosh(x)^3 - (a^3 + 4a^2b + 4ab^2) \cosh(x)) \sinh(x) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / ((a^6 + a^5b) \cosh(x)^8 + 8(a^6 + a^5b) \cosh(x) \sinh(x)^7 + (a^6 + a^5b) \sinh(x)^8 + 4(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^6 + 4(a^6 + 3a^5b + 2a^4b^2 + 7(a^6 + a^5b) \cosh(x)^2) \sinh(x)^6 + a^6 + a^5b + 8(7(a^6 + a^5b) \cosh(x)^3 + 3(a^6 + 3a^5b + 2a^4b^2) \cosh(x)) \sinh(x)^5 + 2(3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3 + 35(a^6 + a^5b) \cosh(x)^4 + 30(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^2) \sinh(x)^4 + 8(7(a^6 + a^5b) \cosh(x)^5 + 10(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^3 + (3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x)) \sinh(x)^3 + 4(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^2 + 4(7(a^6 + a^5b) \cosh(x)^6 + a^6 + 3a^5b + 2a^4b^2 + 15(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^4 + 3(3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x)^2) \sinh(x)^2 + 8((a^6 + a^5b) \cosh(x)^7 + 3(a^6 + 3a^5b + 2a^4b^2) \cosh(x)^5 + (3a^6 + 11a^5b + 16a^4b^2 + 8a^3b^3) \cosh(x)^3 + (a^6 + 3a^5b + 2a^4b^2) \cosh(x)) \sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b \operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)`

[Out] `int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{(b \operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2),x)`

[Out] `int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b \operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**2/(a+b*sech(x)**2)**(5/2),x)`

[Out] `Integral(tanh(x)**2/(a + b*sech(x)**2)**(5/2), x)`

$$3.216 \quad \int \frac{\tanh(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{1}{3a(a+b\operatorname{sech}^2(x))^{3/2}}$$

[Out] $\operatorname{arctanh}((a+b*\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(5/2)}-1/3/a/(a+b*\operatorname{sech}(x)^2)^{(3/2)}-1/a^2/(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4139, 266, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a(a+b\operatorname{sech}^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[x]/(a+b*\operatorname{Sech}[x]^2)^{(5/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(5/2)}-1/(3*a*(a+b*\operatorname{Sech}[x]^2)^{(3/2)})-1/(a^2*\operatorname{Sqrt}[a+b*\operatorname{Sech}[x]^2])$

Rule 51

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}]/((b*c - a*d)*(m + 1)), x] - \operatorname{Dist}[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), \operatorname{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{LtQ}[n, -1] \&\& (\operatorname{EqQ}[a, 0] \mid\mid (\operatorname{NeQ}[c, 0] \&\& \operatorname{LtQ}[m - n, 0] \&\& \operatorname{IntegerQ}[n]))) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /;$ $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b]$

Rule 266

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, m, n, p\}, x \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/ff, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p]/x, x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{(a + b\operatorname{sech}^2(x))^{5/2}} dx &= -\operatorname{Subst}\left(\int \frac{1}{x(a + bx^2)^{5/2}} dx, x, \operatorname{sech}(x)\right) \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a + bx)^{5/2}} dx, x, \operatorname{sech}^2(x)\right)\right) \\
&= -\frac{1}{3a(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \operatorname{sech}^2(x)\right)}{2a} \\
&= -\frac{1}{3a(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \operatorname{sech}^2(x)\right)}{2a^2} \\
&= -\frac{1}{3a(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2\sqrt{a + b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{-a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b\operatorname{sech}^2(x)}\right)}{a^2b} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a(a + b\operatorname{sech}^2(x))^{3/2}} - \frac{1}{a^2\sqrt{a + b\operatorname{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.72, size = 112, normalized size = 1.81

$$\frac{\operatorname{sech}^5(x)(a \cosh(2x) + a + 2b)(4a^{3/2} \cosh(3x) + 12\sqrt{a}(a + b) \cosh(x) - 3\sqrt{2}(a \cosh(2x) + a + 2b)^{3/2} \log(\sqrt{a \cosh(2x) + a + 2b}))}{24a^{5/2}(a + b\operatorname{sech}^2(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(5/2), x]
```

```
[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(12*Sqrt[a]*(a + b)*Cosh[x] + 4*a^(3/2)*Cosh[3*x] - 3*Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*(a + b*Sech[x]^2)^(5/2))
```

fricas [B] time = 0.70, size = 3994, normalized size = 64.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
```

$$\begin{aligned}
& * \cosh(x)^4 + 2*(35*a^2*\cosh(x)^4 + 30*(a^2 + 2*a*b)*\cosh(x)^2 + 3*a^2 + 8*a \\
& * b + 8*b^2)*\sinh(x)^4 + 8*(7*a^2*\cosh(x)^5 + 10*(a^2 + 2*a*b)*\cosh(x)^3 + (\\
& 3*a^2 + 8*a*b + 8*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 + 2*a*b)*\cosh(x)^2 + 4*(\\
& 7*a^2*\cosh(x)^6 + 15*(a^2 + 2*a*b)*\cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*\co \\
& sh(x)^2 + a^2 + 2*a*b)*\sinh(x)^2 + a^2 + 8*(a^2*\cosh(x)^7 + 3*(a^2 + 2*a*b) \\
& *\cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*\cosh(x)^3 + (a^2 + 2*a*b)*\cosh(x))*\sin \\
& h(x))*\sqrt{a}*\log(((a^3 + 2*a^2*b + a*b^2)*\cosh(x)^8 + 8*(a^3 + 2*a^2*b + a \\
& *b^2)*\cosh(x)*\sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*\sinh(x)^8 + 2*(2*a^3 + 5* \\
& a^2*b + 4*a*b^2 + b^3)*\cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14* \\
& (a^3 + 2*a^2*b + a*b^2)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2 \\
&)*\cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x))*\sinh(x)^5 + (6*a \\
& ^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^4 \\
& + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x) \\
& ^2)*\sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^5 + 10*(2*a^3 + 5*a^2 \\
& *b + 4*a*b^2 + b^3)*\cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x))*\sinh(\\
& x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*\cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)* \\
& \cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*\cosh(x)^4 + 2*a^3 + 3*a^2* \\
& b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*((a^2 + 2 \\
& *a*b + b^2)*\cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2* \\
& a*b + b^2)*\sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b \\
& + b^2)*\cosh(x)^2 + a^2 + 2*a*b + b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)* \\
& \cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^3 + (3*a^2 + 4*a*b)*\cosh \\
& (x)^2 + (15*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*\cosh(x)^ \\
& 2 + 3*a^2 + 4*a*b)*\sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 6 \\
& *(a^2 + 2*a*b + b^2)*\cosh(x)^3 + (3*a^2 + 4*a*b)*\cosh(x))*\sinh(x))*\sqrt{a}* \\
& \sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + \\
& sinh(x)^2)} + 4*(2*(a^3 + 2*a^2*b + a*b^2)*\cosh(x)^7 + 3*(2*a^3 + 5*a^2*b \\
& + 4*a*b^2 + b^3)*\cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*\cosh(x)^3 + (2*a^ \\
& 3 + 3*a^2*b)*\cosh(x))*\sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x) \\
&)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x) \\
& *sinh(x)^5 + sinh(x)^6)) + 3*(a^2*\cosh(x)^8 + 8*a^2*\cosh(x)*sinh(x)^7 + a^2 \\
& *sinh(x)^8 + 4*(a^2 + 2*a*b)*\cosh(x)^6 + 4*(7*a^2*\cosh(x)^2 + a^2 + 2*a*b)* \\
& sinh(x)^6 + 8*(7*a^2*\cosh(x)^3 + 3*(a^2 + 2*a*b)*\cosh(x))*sinh(x)^5 + 2*(3* \\
& a^2 + 8*a*b + 8*b^2)*\cosh(x)^4 + 2*(35*a^2*\cosh(x)^4 + 30*(a^2 + 2*a*b)*\cos \\
& h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*\cosh(x)^5 + 10*(a^2 + \\
& 2*a*b)*\cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*\cosh(x))*sinh(x)^3 + 4*(a^2 + 2* \\
& a*b)*\cosh(x)^2 + 4*(7*a^2*\cosh(x)^6 + 15*(a^2 + 2*a*b)*\cosh(x)^4 + 3*(3*a^2 \\
& + 8*a*b + 8*b^2)*\cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*\cosh(x) \\
& ^7 + 3*(a^2 + 2*a*b)*\cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*\cosh(x)^3 + (a^2 + \\
& 2*a*b)*\cosh(x))*sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*sinh(x)^3 \\
& + a*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + b)*sinh(x)^2 + \sqrt{2}* \\
& (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + \\
& a*\sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)} + 4*(a \\
& *\cosh(x)^3 + b*\cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(\\
& x)^2)) - 16*\sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*sinh(x)^5 + a^2*sinh(x)^ \\
& 6 + 3*(a^2 + a*b)*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2 + a*b)*sinh(x)^4 + 4 \\
& *(5*a^2*\cosh(x)^3 + 3*(a^2 + a*b)*\cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*\cosh(x) \\
&)^2 + 3*(5*a^2*\cosh(x)^4 + 6*(a^2 + a*b)*\cosh(x)^2 + a^2 + a*b)*sinh(x)^2 + \\
& a^2 + 6*(a^2*\cosh(x)^5 + 2*(a^2 + a*b)*\cosh(x)^3 + (a^2 + a*b)*\cosh(x))*si \\
& nh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si \\
& nh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x) \\
& ^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh \\
& (x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*\cosh(x))*sinh(x)^5 + a^5 + 2 \\
& *(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8* \\
& a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*\cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh \\
& (x)^5 + 10*(a^5 + 2*a^4*b)*\cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(x) \\
&))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*\cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a \\
& ^4*b + 15*(a^5 + 2*a^4*b)*\cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*\cosh(\\
& x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*\cosh(x)^5 + (3*a^5 +
\end{aligned}$$

$$8a^4b + 8a^3b^2) \cosh(x)^3 + (a^5 + 2a^4b) \cosh(x) \sinh(x)), -1/6(3(a^2 \cosh(x)^8 + 8a^2 \cosh(x) \sinh(x)^7 + a^2 \sinh(x)^8 + 4(a^2 + 2ab) \cosh(x)^6 + 4(7a^2 \cosh(x)^2 + a^2 + 2ab) \sinh(x)^6 + 8(7a^2 \cosh(x)^3 + 3(a^2 + 2ab) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 8ab + 8b^2) \cosh(x)^4 + 2(35a^2 \cosh(x)^4 + 30(a^2 + 2ab) \cosh(x)^2 + 3a^2 + 8ab + 8b^2) \sinh(x)^4 + 8(7a^2 \cosh(x)^5 + 10(a^2 + 2ab) \cosh(x)^3 + (3a^2 + 8ab + 8b^2) \cosh(x)) \sinh(x)^3 + 4(a^2 + 2ab) \cosh(x)^2 + 4(7a^2 \cosh(x)^6 + 15(a^2 + 2ab) \cosh(x)^4 + 3(3a^2 + 8ab + 8b^2) \cosh(x)^2 + a^2 + 2ab) \sinh(x)^2 + a^2 + 8(a^2 \cosh(x)^7 + 3(a^2 + 2ab) \cosh(x)^5 + (3a^2 + 8ab + 8b^2) \cosh(x)^3 + (a^2 + 2ab) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + a) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^2 + ab) \cosh(x)^4 + 4(a^2 + ab) \cosh(x) \sinh(x)^3 + (a^2 + ab) \sinh(x)^4 + (2a^2 + 3ab) \cosh(x)^2 + (6(a^2 + ab) \cosh(x)^2 + 2a^2 + 3ab) \sinh(x)^2 + a^2 + 2(2(a^2 + ab) \cosh(x)^3 + (2a^2 + 3ab) \cosh(x)) \sinh(x))) + 3(a^2 \cosh(x)^8 + 8a^2 \cosh(x) \sinh(x)^7 + a^2 \sinh(x)^8 + 4(a^2 + 2ab) \cosh(x)^6 + 4(7a^2 \cosh(x)^2 + a^2 + 2ab) \sinh(x)^6 + 8(7a^2 \cosh(x)^3 + 3(a^2 + 2ab) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 8ab + 8b^2) \cosh(x)^4 + 2(35a^2 \cosh(x)^4 + 30(a^2 + 2ab) \cosh(x)^2 + 3a^2 + 8ab + 8b^2) \sinh(x)^4 + 8(7a^2 \cosh(x)^5 + 10(a^2 + 2ab) \cosh(x)^3 + (3a^2 + 8ab + 8b^2) \cosh(x)) \sinh(x)^3 + 4(a^2 + 2ab) \cosh(x)^2 + 4(7a^2 \cosh(x)^6 + 15(a^2 + 2ab) \cosh(x)^4 + 3(3a^2 + 8ab + 8b^2) \cosh(x)^2 + a^2 + 2ab) \sinh(x)^2 + a^2 + 8(a^2 \cosh(x)^7 + 3(a^2 + 2ab) \cosh(x)^5 + (3a^2 + 8ab + 8b^2) \cosh(x)^3 + (a^2 + 2ab) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^4 + 4a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2(a + 2b) \cosh(x)^2 + 2(3a \cosh(x)^2 + a + 2b) \sinh(x)^2 + 4(a \cosh(x)^3 + (a + 2b) \cosh(x)) \sinh(x) + a)) + 8 \sqrt{2} (a^2 \cosh(x)^6 + 6a^2 \cosh(x) \sinh(x)^5 + a^2 \sinh(x)^6 + 3(a^2 + ab) \cosh(x)^4 + 3(5a^2 \cosh(x)^2 + a^2 + ab) \sinh(x)^4 + 4(5a^2 \cosh(x)^3 + 3(a^2 + ab) \cosh(x)) \sinh(x)^3 + 3(a^2 + ab) \cosh(x)^2 + 3(5a^2 \cosh(x)^4 + 6(a^2 + ab) \cosh(x)^2 + a^2 + ab) \sinh(x)^2 + a^2 + 6(a^2 \cosh(x)^5 + 2(a^2 + ab) \cosh(x)^3 + (a^2 + ab) \cosh(x)) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a^5 \cosh(x)^8 + 8a^5 \cosh(x) \sinh(x)^7 + a^5 \sinh(x)^8 + 4(a^5 + 2a^4b) \cosh(x)^6 + 4(7a^5 \cosh(x)^2 + a^5 + 2a^4b) \sinh(x)^6 + 8(7a^5 \cosh(x)^3 + 3(a^5 + 2a^4b) \cosh(x)) \sinh(x)^5 + a^5 + 2(3a^5 + 8a^4b + 8a^3b^2) \cosh(x)^4 + 2(35a^5 \cosh(x)^4 + 3a^5 + 8a^4b + 8a^3b^2 + 30(a^5 + 2a^4b) \cosh(x)^2) \sinh(x)^4 + 8(7a^5 \cosh(x)^5 + 10(a^5 + 2a^4b) \cosh(x)^3 + (3a^5 + 8a^4b + 8a^3b^2) \cosh(x)) \sinh(x)^3 + 4(a^5 + 2a^4b) \cosh(x)^2 + 4(7a^5 \cosh(x)^6 + a^5 + 2a^4b + 15(a^5 + 2a^4b) \cosh(x)^4 + 3(3a^5 + 8a^4b + 8a^3b^2) \cosh(x)^2) \sinh(x)^2 + 8(a^5 \cosh(x)^7 + 3(a^5 + 2a^4b) \cosh(x)^5 + (3a^5 + 8a^4b + 8a^3b^2) \cosh(x)^3 + (a^5 + 2a^4b) \cosh(x)) \sinh(x))]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Warning, replacing 0 by `u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
 tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
 titution variable should perhaps be purged.Error: Bad Argument Type

maple [A] time = 0.11, size = 61, normalized size = 0.98

$$-\frac{1}{3a(a+b\operatorname{sech}(x)^2)^{\frac{3}{2}}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}(x)^2}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a+b*sech(x)^2)^(5/2), x)

[Out] -1/3/a/(a+b*sech(x)^2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)+1/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)

mupad [B] time = 3.07, size = 50, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3a}+\frac{a+\frac{b}{\cosh(x)^2}}{a^2}}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a + b/cosh(x)^2)^(5/2), x)

[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(5/2) - (1/(3*a) + (a + b/cosh(x)^2)/a^2)/(a + b/cosh(x)^2)^(3/2)

sympy [A] time = 12.68, size = 65, normalized size = 1.05

$$-\frac{1}{3a(a+b\operatorname{sech}^2(x))^{\frac{3}{2}}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*sech(x)**2)**(5/2), x)

[Out] -1/(3*a*(a + b*sech(x)**2)**(3/2)) - 1/(a**2*sqrt(a + b*sech(x)**2)) - atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a**2*sqrt(-a))

$$3.217 \quad \int \frac{1}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=95

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}} - \frac{b\tanh(x)}{3a(a+b)(a-b\tanh^2(x)+b)^{3/2}}$$

[Out] $\operatorname{arctanh}(a^{1/2}\tanh(x)/(a+b-b\tanh(x)^2)^{1/2})/a^{5/2}-1/3*b*(5*a+3*b)*\tanh(x)/a^2/(a+b)^2/(a+b-b\tanh(x)^2)^{1/2}-1/3*b*\tanh(x)/a/(a+b)/(a+b-b\tanh(x)^2)^{3/2}$

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4128, 414, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}} - \frac{b\tanh(x)}{3a(a+b)(a-b\tanh^2(x)+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sech}[x]^2)^{-5/2}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[x]^2]]/a^{5/2} - (b*\operatorname{Tanh}[x])/(3*a*(a + b)*(a + b - b*\operatorname{Tanh}[x]^2)^{3/2}) - (b*(5*a + 3*b)*\operatorname{Tanh}[x])/(3*a^2*(a + b)^2*\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[x]^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_*) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 377

$\operatorname{Int}[(a_*) + (b_.)*(x_)^{(n_)})^{(p_)} / ((c_*) + (d_.)*(x_)^{(n_)}), x_Symbol] := \operatorname{Subst}[\operatorname{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[n*p + 1, 0] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 414

$\operatorname{Int}[(a_*) + (b_.)*(x_)^{(n_)})^{(p_)} * ((c_*) + (d_.)*(x_)^{(n_)})^{(q_)}, x_Symbol] := -\operatorname{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c - a*d)), x] + \operatorname{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \operatorname{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q * \operatorname{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n, q\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ !(\ !\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[q] \ \&\& \ \operatorname{LtQ}[q, -1]) \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 527


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^(p/
(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{1}{(1 - x^2)(a + b - bx^2)^{5/2}} dx, x, \tanh(x) \right) \\ &= -\frac{b \tanh(x)}{3a(a + b)(a + b - b \tanh^2(x))^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{-3a - b - 2bx^2}{(1 - x^2)(a + b - bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3a(a + b)} \\ &= -\frac{b \tanh(x)}{3a(a + b)(a + b - b \tanh^2(x))^{3/2}} - \frac{b(5a + 3b) \tanh(x)}{3a^2(a + b)^2 \sqrt{a + b - b \tanh^2(x)}} + \operatorname{Subst} \left(\int \frac{1}{(1 - x^2)(a + b - bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= -\frac{b \tanh(x)}{3a(a + b)(a + b - b \tanh^2(x))^{3/2}} - \frac{b(5a + 3b) \tanh(x)}{3a^2(a + b)^2 \sqrt{a + b - b \tanh^2(x)}} + \operatorname{Subst} \left(\int \frac{1}{(1 - x^2)(a + b - bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= -\frac{b \tanh(x)}{3a(a + b)(a + b - b \tanh^2(x))^{3/2}} - \frac{b(5a + 3b) \tanh(x)}{3a^2(a + b)^2 \sqrt{a + b - b \tanh^2(x)}} + \operatorname{Subst} \left(\int \frac{1}{(1 - x^2)(a + b - bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a + b - b \tanh^2(x)}} \right)}{a^{5/2}} - \frac{b \tanh(x)}{3a(a + b)(a + b - b \tanh^2(x))^{3/2}} - \frac{b(5a + 3b) \tanh(x)}{3a^2(a + b)^2 \sqrt{a + b - b \tanh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.45, size = 130, normalized size = 1.37

$$\frac{\operatorname{sech}^5(x) \left(\frac{\sqrt{2}(a \cosh(2x) + a + 2b)^{5/2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a} \cosh(2x) + a + 2b} \right)}{a^{5/2}} - \frac{4b \sinh(x)(a \cosh(2x) + a + 2b)(3a^2 + a(3a + 2b) \cosh(2x) + 7ab + 3b^2)}{3a^2(a + b)^2} \right)}{8(a + b \operatorname{sech}^2(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-5/2), x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]])*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - (4*b*(a + 2*b + a*Cosh[2*
```


$$\begin{aligned}
& *b^2 + 24*a*b^3 + 8*b^4)*\cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)* \\
& \cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 + 2*(a + b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^2 + \sqrt{2} \\
&)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4* \\
& (a*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2) - 8*\sqrt{2}*((3*a^3*b + 2*a^2*b^2)*\cosh(x)^6 + 6*(3*a^3*b + \\
& 2*a^2*b^2)*\cosh(x)*\sinh(x)^5 + (3*a^3*b + 2*a^2*b^2)*\sinh(x)^6 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3 \\
& *b + 2*a^2*b^2)*\cosh(x)^2)*\sinh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b + 2*a^2*b^2)*\cosh(x)^3 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x)^3 \\
& - 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*\cosh(x)^4 - a^3*b - 4*a^2*b^2 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x)^2)*\sinh(x)^2 + 6*((3*a^3*b + 2*a^2*b^2)*\cosh(x)^5 + 2*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x)^3 - (a^3*b + 4*a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x))*\sqrt{a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)*\sinh(x)^7 + (a^7 + 2*a^6*b + a^5*b^2)*\sinh(x)^8 + a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^4 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4 + 35*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^5 + 10*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^7 + 2*a^6*b + a^5*b^2)*\cosh(x)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*\cosh(x)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*\cosh(x))*\sinh(x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)*\sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^2)*\sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*\cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + a))*\sqrt{-a}*\sqrt{(a*\cosh(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*b*\cosh(x)^4 + 4*a*b*\cosh(x)*\sinh(x)^3 + a*b*\sinh(x)^4 - (a^2 + 3*a*b)*\cosh(x)^2 + (6*a*b*\cosh(x)^2 - a^2 - 3*a*b)*\sinh(x)^2 - a^2 + 2*(2*a*b*\cosh(x)^3 - (a^2 + 3*a*b)*\cosh(x))*\sinh(x))) + 3*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)*\sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*\cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^
\end{aligned}$$

$$\begin{aligned}
& 2) * \sinh(x)^6 + 8 * (7 * (a^4 + 2 * a^3 * b + a^2 * b^2) * \cosh(x)^3 + 3 * (a^4 + 4 * a^3 * b \\
& + 5 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^4 + 14 * a^3 * b + 27 * a^2 * b^2 \\
& + 24 * a * b^3 + 8 * b^4) * \cosh(x)^4 + 2 * (35 * (a^4 + 2 * a^3 * b + a^2 * b^2) * \cosh(x)^4 \\
& + 3 * a^4 + 14 * a^3 * b + 27 * a^2 * b^2 + 24 * a * b^3 + 8 * b^4 + 30 * (a^4 + 4 * a^3 * b + 5 \\
& * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^2) * \sinh(x)^4 + a^4 + 2 * a^3 * b + a^2 * b^2 + 8 * (7 * (\\
& a^4 + 2 * a^3 * b + a^2 * b^2) * \cosh(x)^5 + 10 * (a^4 + 4 * a^3 * b + 5 * a^2 * b^2 + 2 * a * b^3) \\
& * \cosh(x)^3 + (3 * a^4 + 14 * a^3 * b + 27 * a^2 * b^2 + 24 * a * b^3 + 8 * b^4) * \cosh(x)) * \\
& \sinh(x)^3 + 4 * (a^4 + 4 * a^3 * b + 5 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^2 + 4 * (7 * (a^4 + \\
& 2 * a^3 * b + a^2 * b^2) * \cosh(x)^6 + 15 * (a^4 + 4 * a^3 * b + 5 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^4 \\
& + a^4 + 4 * a^3 * b + 5 * a^2 * b^2 + 2 * a * b^3 + 3 * (3 * a^4 + 14 * a^3 * b + 27 * a^2 * b^2 \\
& + 24 * a * b^3 + 8 * b^4) * \cosh(x)^2) * \sinh(x)^2 + 8 * ((a^4 + 2 * a^3 * b + a^2 * b^2) * \cosh(x)^7 \\
& + 3 * (a^4 + 4 * a^3 * b + 5 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^5 + (3 * a^4 + \\
& 14 * a^3 * b + 27 * a^2 * b^2 + 24 * a * b^3 + 8 * b^4) * \cosh(x)^3 + (a^4 + 4 * a^3 * b + 5 * a^2 * b^2 \\
& + 2 * a * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * \sqrt{-a} * \sqrt{(\\
& a * \cosh(x)^2 + a * \sinh(x)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) \\
& / (a * \cosh(x)^2 + 2 * a * \cosh(x) * \sinh(x) + a * \sinh(x)^2 + a)) + 4 * \sqrt{2} * (\\
& (3 * a^3 * b + 2 * a^2 * b^2) * \cosh(x)^6 + 6 * (3 * a^3 * b + 2 * a^2 * b^2) * \cosh(x) * \sinh(x)^5 \\
& + (3 * a^3 * b + 2 * a^2 * b^2) * \sinh(x)^6 + 3 * (a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3) * \cosh(x) \\
&)^4 + 3 * (a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3 + 5 * (3 * a^3 * b + 2 * a^2 * b^2) * \cosh(x)^2) * \sinh(x)^4 \\
& - 3 * a^3 * b - 2 * a^2 * b^2 + 4 * (5 * (3 * a^3 * b + 2 * a^2 * b^2) * \cosh(x)^3 + 3 * (\\
& a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)) * \sinh(x)^3 - 3 * (a^3 * b + 4 * a^2 * b^2 + 2 * \\
& a * b^3) * \cosh(x)^2 + 3 * (5 * (3 * a^3 * b + 2 * a^2 * b^2) * \cosh(x)^4 - a^3 * b - 4 * a^2 * b^2 \\
& - 2 * a * b^3 + 6 * (a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^2) * \sinh(x)^2 + 6 * ((3 * a^3 * b \\
& + 2 * a^2 * b^2) * \cosh(x)^5 + 2 * (a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)^3 - (\\
& a^3 * b + 4 * a^2 * b^2 + 2 * a * b^3) * \cosh(x)) * \sinh(x)) * \sqrt{(a * \cosh(x)^2 + a * \sinh(x) \\
&)^2 + a + 2 * b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) / ((a^7 + 2 * a^6 * b \\
& + a^5 * b^2) * \cosh(x)^8 + 8 * (a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x) * \sinh(x)^7 + (a^7 \\
& + 2 * a^6 * b + a^5 * b^2) * \sinh(x)^8 + a^7 + 2 * a^6 * b + a^5 * b^2 + 4 * (a^7 + 4 * a^6 * b \\
& + 5 * a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x)^6 + 4 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3 \\
& + 7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x)^2) * \sinh(x)^6 + 8 * (7 * (a^7 + 2 * a^6 * b \\
& + a^5 * b^2) * \cosh(x)^3 + 3 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x) \\
&) * \sinh(x)^5 + 2 * (3 * a^7 + 14 * a^6 * b + 27 * a^5 * b^2 + 24 * a^4 * b^3 + 8 * a^3 * b^4) * \\
& \cosh(x)^4 + 2 * (3 * a^7 + 14 * a^6 * b + 27 * a^5 * b^2 + 24 * a^4 * b^3 + 8 * a^3 * b^4 + 35 * \\
& (a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x)^4 + 30 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * \\
& * b^3) * \cosh(x)^2) * \sinh(x)^4 + 8 * (7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x)^5 + 10 * \\
& (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x)^3 + (3 * a^7 + 14 * a^6 * b + 27 * \\
& a^5 * b^2 + 24 * a^4 * b^3 + 8 * a^3 * b^4) * \cosh(x)) * \sinh(x)^3 + 4 * (a^7 + 4 * a^6 * b + 5 * \\
& a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x)^2 + 4 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3 \\
& + 7 * (a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x)^6 + 15 * (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * \\
& a^4 * b^3) * \cosh(x)^4 + 3 * (3 * a^7 + 14 * a^6 * b + 27 * a^5 * b^2 + 24 * a^4 * b^3 + 8 * a^3 * \\
& * b^4) * \cosh(x)^2) * \sinh(x)^2 + 8 * ((a^7 + 2 * a^6 * b + a^5 * b^2) * \cosh(x)^7 + 3 * (a^7 \\
& + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x)^5 + (3 * a^7 + 14 * a^6 * b + 27 * a^5 * \\
& * b^2 + 24 * a^4 * b^3 + 8 * a^3 * b^4) * \cosh(x)^3 + (a^7 + 4 * a^6 * b + 5 * a^5 * b^2 + 2 * a^4 * b^3) * \cosh(x)) * \sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(x)^2)^(5/2),x)

[Out] int(1/(a+b*sech(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sech(x)^2 + a)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(x)^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cosh(x)^2)^(5/2),x)

[Out] int(1/(a + b/cosh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(x)**2)**(5/2),x)

[Out] Integral((a + b*sech(x)**2)**(-5/2), x)

$$3.218 \quad \int \frac{\coth(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=109

$$\frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{b}{3a(a+b)(a+b\operatorname{sech}^2(x))^{3/2}} - \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

[Out] $\operatorname{arctanh}((a+b\operatorname{sech}(x)^2)^{(1/2)}/a^{(1/2)})/a^{(5/2)} - \operatorname{arctanh}((a+b\operatorname{sech}(x)^2)^{(1/2)})/(a+b)^{(1/2)})/(a+b)^{(5/2)} - 1/3*b/a/(a+b)/(a+b*\operatorname{sech}(x)^2)^{(3/2)} - b*(2*a+b)/a^2/(a+b)^2/(a+b*\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4139, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\operatorname{sech}^2(x)}} + \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{b}{3a(a+b)(a+b\operatorname{sech}^2(x))^{3/2}} - \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[x]/(a + b*\operatorname{Sech}[x]^2)^{(5/2)}, x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a]]/a^{(5/2)} - \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2]/\operatorname{Sqrt}[a + b]]/(a + b)^{(5/2)} - b/(3*a*(a + b)*(a + b*\operatorname{Sech}[x]^2)^{(3/2)}) - (b*(2*a + b))/(a^2*(a + b)^2*\operatorname{Sqrt}[a + b*\operatorname{Sech}[x]^2])$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 85

$\operatorname{Int}[(e_. + (f_.)*(x_.))^{(p_.)}/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))], x_Symbol] \rightarrow \operatorname{Simp}[(f*(e + f*x)^{(p+1)})/((p+1)*(b*e - a*f)*(d*e - c*f)), x] + \operatorname{Dist}[1/((b*e - a*f)*(d*e - c*f)), \operatorname{Int}[(b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^{(p+1)})/((a + b*x)*(c + d*x)), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{LtQ}[p, -1]$

Rule 152

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.)), x_Symbol] \rightarrow \operatorname{Simp}[(b*g - a*h)*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/((m+1)*(b*c - a*d)*(b*e - a*f)), x] + \operatorname{Dist}[1/((m+1)*(b*c - a*d)*(b*e - a*f)), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m+1) - (b*g - a*h)*(d*e*(n+1) + c*f*(p+1)) - d*f*(b*g - a*h)*(m+n+p+3)*x, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, g, h, n, p\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegersQ}[2*m, 2*n, 2*p]$

Rule 156

Int[(((e_) + (f_)*(x_))^(p_)*((g_) + (h_)*(x_)))/(((a_) + (b_)*(x_))*
((c_) + (d_)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_ .
) , x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4139

Int[((a_) + (b_)*((c_)*sec[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (
f_)*(x_)^(m_)] , x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p/x, x],
x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
- 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])

Rubi steps

$$\begin{aligned}
 \int \frac{\coth(x)}{(a + b \operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{5/2}} dx, x, \operatorname{sech}(x) \right) \\
 &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}} dx, x, \operatorname{sech}^2(x) \right) \\
 &= -\frac{b}{3a(a+b)(a+b \operatorname{sech}^2(x))^{3/2}} + \frac{\operatorname{Subst} \left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}} dx, x, \operatorname{sech}^2(x) \right)}{2a(a+b)} \\
 &= -\frac{b}{3a(a+b)(a+b \operatorname{sech}^2(x))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{-\frac{1}{2}(a+b)^2 + \frac{1}{2}}{(-1+x)x} dx, x, \operatorname{sech}^2(x) \right)}{a^2} \\
 &= -\frac{b}{3a(a+b)(a+b \operatorname{sech}^2(x))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \operatorname{sech}^2(x) \right)}{2a^2} \\
 &= -\frac{b}{3a(a+b)(a+b \operatorname{sech}^2(x))^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \operatorname{sech}^2(x)}} - \frac{\operatorname{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \operatorname{sech}^2(x) \right)}{a^2} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \operatorname{sech}^2(x)}}{\sqrt{a}} \right)}{a^{5/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \operatorname{sech}^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} - \frac{b}{3a(a+b)(a+b \operatorname{sech}^2(x))^{3/2}} - \frac{b}{a^2}
 \end{aligned}$$

Mathematica [B] time = 1.11, size = 242, normalized size = 2.22

$$\operatorname{sech}^5(x) \left(\frac{2b \cosh(x)(7a^2 + a(7a+4b) \cosh(2x) + 16ab + 6b^2)(a \cosh(2x) + a + 2b)}{3a^2(a+b)^2} - \frac{(a \cosh(2x) + a + 2b)^{5/2} \left(\sqrt{a} (a^2 - 2ab - b^2) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a+b} \cosh(x)}{\sqrt{a} \cosh(2x) + a + 2b} \right) \right)}{8(a + b \operatorname{sech}^2(x))^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(5/2), x]

[Out] ((((-2*b*Cosh[x]*(a + 2*b + a*Cosh[2*x]))*(7*a^2 + 16*a*b + 6*b^2 + a*(7*a + 4*b)*Cosh[2*x]))/(3*a^2*(a + b)^2) - ((a + 2*b + a*Cosh[2*x])^(5/2)*(Sqrt[a]*(a^2 - 2*a*b - b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])] + (a + b)^2*(Sqrt[a]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])] - 2*Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])))/(Sqrt[2]*a^(5/2)*(a + b)^(5/2))*Sech[x]^5/(8*(a + b*Sech[x]^2)^(5/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x); OUTPUT:Error: Bad Argument Type

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a + b \operatorname{sech}(x)^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*sech(x)^2)^(5/2), x)

[Out] int(coth(x)/(a+b*sech(x)^2)^(5/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(b \operatorname{sech}(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a + b/cosh(x)^2)^(5/2), x)

[Out] int(coth(x)/(a + b/cosh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a + b \operatorname{sech}^2(x)\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sech(x)**2)**(5/2), x)

[Out] Integral(coth(x)/(a + b*sech(x)**2)**(5/2), x)

$$3.219 \quad \int \frac{\coth^2(x)}{(a+b\operatorname{sech}^2(x))^{5/2}} dx$$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}} - \frac{b}{3a(a+b)}$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b*b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(7*a+3*b)*coth(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a-3*b)*(3*a+b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/a^2/(a+b)^3-1/3*b*coth(x)/a/(a+b)/(a+b-b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.38, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}} - \frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}} - \frac{b}{3a(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Coth[x])/(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(7*a + 3*b)*Coth[x])/(3*a^2*(a + b)^2*Sqrt[a + b - b*Tanh[x]^2]) - ((a - 3*b)*(3*a + b)*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*a^2*(a + b)^3)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[m] && !ntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p]/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{(a + b\operatorname{sech}^2(x))^{5/2}} dx &= \operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))^{5/2}} dx, x, \tanh(x) \right) \\
&= \operatorname{Subst} \left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{-3a+b-4bx^2}{x^2(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3a(a+b)} \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} + \operatorname{Subst} \left(\int \frac{(a-b)}{x^2(1-x^2)(a+b-bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} - \frac{(a-3b)(3a+b)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} - \frac{(a-3b)(3a+b)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} - \frac{(a-3b)(3a+b)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} \\
&= -\frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} - \frac{(a-3b)(3a+b)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b-b \tanh^2(x)}} \right)}{a^{5/2}} - \frac{b \coth(x)}{3a(a+b)(a+b-b \tanh^2(x))^{3/2}} - \frac{b(7a+3b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b-b \tanh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.83, size = 155, normalized size = 1.17

$$\operatorname{sech}^5(x) \left(\frac{\sqrt{2}(a \cosh(2x)+a+2b)^{5/2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{a} \sinh(x)}{\sqrt{a \cosh(2x)+a+2b}} \right)}{a^{5/2}} - \frac{(a \cosh(2x)+a+2b)(3a^2 \operatorname{csch}(x)(a \cosh(2x)+a+2b)^2 - 4b^3(a+b) \sinh(x) + 2b^2(9a+4b))}{3a^2(a+b)^3} \right)$$

$$8(a + b\operatorname{sech}^2(x))^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(5/2), x]

[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]])*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - ((a + 2*b + a*Cosh[2*x])*(3*a^2*(a + 2*b + a*Cosh[2*x])^2*Csch[x] - 4*b^3*(a + b)*Sinh[x] + 2*b^2*(9*a + 4*b)*(a + 2*b + a*Cosh[2*x])*Sinh[x]))/(3*a^2*(a + b)^3))/(8*(a + b*Sech[x]^2)^(5/2))

fricas [B] time = 4.18, size = 11205, normalized size = 84.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3

$$\begin{aligned}
& *a^3*b^2 + a^2*b^3)*\cosh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2* \\
& b^3 + 8*a*b^4)*\cosh(x)^2*\sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^ \\
& 2*b^3)*\cosh(x)^5 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 \\
&)*\cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 \\
&)*\cosh(x))*\sinh(x)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4 \\
& *b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*\cosh(x)^4 + 2*(105*(a^5 + \\
& 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37* \\
& a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 \\
& + 8*a*b^4)*\cosh(x)^4 + 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a* \\
& b^4 + 8*b^5)*\cosh(x)^2*\sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2* \\
& b^3)*\cosh(x)^7 + 7*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*c \\
& osh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*c \\
& osh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cos \\
& h(x))*\sinh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*co \\
& sh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^8 + 28*(3*a^5 + \\
& 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*\cosh(x)^6 - 3*a^5 - 17*a^4*b \\
& - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37* \\
& a^2*b^3 + 28*a*b^4 + 8*b^5)*\cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37 \\
& *a^2*b^3 + 28*a*b^4 + 8*b^5)*\cosh(x)^2)*\sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3 \\
& *a^3*b^2 + a^2*b^3)*\cosh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b \\
& ^3 + 8*a*b^4)*\cosh(x)^7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a \\
& *b^4 + 8*b^5)*\cosh(x)^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a \\
& *b^4 + 8*b^5)*\cosh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a \\
& *b^4)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-(a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + \\
& a*\sinh(x)^4 + 2*(a + b)*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 + a + b)*\sinh(x)^2 + \\
& \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a}*\sqrt{(a*cos \\
& h(x)^2 + a*\sinh(x)^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2) \\
&) + 4*(a*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a)/(\cosh(x)^2 + 2*\cosh(x)*s \\
& inh(x) + \sinh(x)^2)) - 4*\sqrt{2}*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x)^8 \\
& + 8*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x)*\sinh(x)^7 + (3*a^5 + 9*a^3*b^2 \\
& + 4*a^2*b^3)*\sinh(x)^8 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x) \\
& ^6 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 7*(3*a^5 + 9*a^3*b^2 + 4*a^ \\
& 2*b^3)*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x) \\
& ^3 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x))*\sinh(x)^5 + 3*a^5 + \\
& 9*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b \\
& ^4)*\cosh(x)^4 + 2*(9*a^5 + 24*a^4*b + 15*a^3*b^2 - 36*a^2*b^3 - 12*a*b^4 + \\
& 35*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*\cosh(x)^4 + 30*(3*a^5 + 6*a^4*b + 8*a^2* \\
& b^3 + 3*a*b^4)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)* \\
& cosh(x)^5 + 10*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x)^3 + 3*(3*a^5 \\
& + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*\cosh(x))*\sinh(x)^3 + 4*(3*a^ \\
& 5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x)^2 + 4*(7*(3*a^5 + 9*a^3*b^2 + 4* \\
& a^2*b^3)*\cosh(x)^6 + 3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 15*(3*a^5 + 6* \\
& a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x)^4 + 9*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 1 \\
& 2*a^2*b^3 - 4*a*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b \\
& ^3)*\cosh(x)^7 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x)^5 + 3*(3* \\
& a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*\cosh(x)^3 + (3*a^5 + 6*a^ \\
& 4*b + 8*a^2*b^3 + 3*a*b^4)*\cosh(x))*\sinh(x))*\sqrt{(a*\cosh(x)^2 + a*\sinh(x) \\
& ^2 + a + 2*b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^8 + 3*a^7*b \\
& + 3*a^6*b^2 + a^5*b^3)*\cosh(x)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 \\
&)*\cosh(x)*\sinh(x)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*\sinh(x)^10 + (3 \\
& *a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*\cosh(x)^8 + (3*a^8 + \\
& 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4 + 45*(a^8 + 3*a^7*b + 3*a^6 \\
& *b^2 + a^5*b^3)*\cosh(x)^2)*\sinh(x)^8 - a^8 - 3*a^7*b - 3*a^6*b^2 - a^5*b^3 \\
& + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*\cosh(x)^3 + (3*a^8 + 17*a^7*b \\
& + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*\cosh(x))*\sinh(x)^7 + 2*(a^8 + 7*a^7 \\
& *b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*\cosh(x)^6 + 2*(a^8 + \\
& 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 + 105*(a^8 + 3* \\
& a^7*b + 3*a^6*b^2 + a^5*b^3)*\cosh(x)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 \\
& + 27*a^5*b^3 + 8*a^4*b^4)*\cosh(x)^2)*\sinh(x)^6 + 4*(63*(a^8 + 3*a^7*b + 3*a
\end{aligned}$$

$$\begin{aligned}
& ^6b^2 + a^5b^3) \cosh(x)^5 + 14(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^3 + 3(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x) \sinh(x)^5 - 2(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^4 - 2(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^6 - 35(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^4 - 15(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^2 \sinh(x)^4 + 8(15(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \cosh(x)^7 + 7(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^5 + 5(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^3 - (a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x) \sinh(x)^3 - (3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^2 + (45(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \cosh(x))^8 - 3a^8 - 17a^7b - 33a^6b^2 - 27a^5b^3 - 8a^4b^4 + 28(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^6 + 30(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^4 - 12(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^2 \sinh(x)^2 + 2(5(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \cosh(x))^9 + 4(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^7 + 6(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^5 - 4(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^3 - (3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x) \sinh(x)), - \\
& 1/6(3((a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^10 + 10(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x) \sinh(x))^9 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \sinh(x))^10 + (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^8 + (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4 + 45(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^2) \sinh(x))^8 + 8(15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^3 + (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x) \sinh(x))^7 + 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^6 + 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5 + 105(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^4 + 14(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^2) \sinh(x))^6 + 4(63(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^5 + 14(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^3 + 3(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x) \sinh(x))^5 - a^5 - 3a^4b - 3a^3b^2 - a^2b^3 - 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^4 + 2(105(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^6 - a^5 - 7a^4b - 23a^3b^2 - 37a^2b^3 - 28a^1b^4 - 8b^5 + 35(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^4 + 15(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^2) \sinh(x))^4 + 8(15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^7 + 7(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^5 + 5(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^3 - (a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x) \sinh(x))^3 - (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^2 + (45(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^8 + 28(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^6 - 3a^5 - 17a^4b - 33a^3b^2 - 27a^2b^3 - 8a^1b^4 + 30(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^4 - 12(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^2 \sinh(x)^2 + 2(5(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x))^9 + 4(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x)^7 + 6(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^5 - 4(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a^1b^4 + 8b^5) \cosh(x)^3 - (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8a^1b^4) \cosh(x) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(b \cosh(x))^2 + 2b \cosh(x) \sinh(x) + b \sinh(x))^2 + a) \sqrt{-a} \sqrt{(a \cosh(x))^2 + a \sinh(x)^2 + a + 2b} / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / (a^2b \cosh(x)^4 + 4a^2b \cosh(x) \sinh(x)^3 + a^2b \sinh(x)^4 - (a^2 + 3a^2b) \cosh(x)^2 + (6a^2b \cosh(x)^2 - a^2 - 3a^2b) \sinh(x)^2 - a^2 + 2(2a^2b \cosh(x))^3 - (a^2 + 3a^2b) \cosh(x) \sinh(x))) + 3((a^5 +
\end{aligned}$$

$$\begin{aligned}
& 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^{10} + 10(a^5 + 3a^4b + 3a^3b^2 \\
& + a^2b^3) \cosh(x) \sinh(x)^9 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \sinh(x) \\
&)^{10} + (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^8 + (\\
& 3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4 + 45(a^5 + 3a^4b + \\
& 3a^3b^2 + a^2b^3) \cosh(x)^2) \sinh(x)^8 + 8(15(a^5 + 3a^4b + 3a^3b^2 \\
& + a^2b^3) \cosh(x)^3 + (3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab \\
& b^4) \cosh(x)) \sinh(x)^7 + 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a \\
& *b^4 + 8b^5) \cosh(x)^6 + 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28a \\
& *b^4 + 8b^5 + 105(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^4 + 14(3 \\
& a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^2) \sinh(x)^6 + \\
& 4(63(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^5 + 14(3a^5 + 17a^4b \\
& b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^3 + 3(a^5 + 7a^4b + 23a^ \\
& 3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)) \sinh(x)^5 - a^5 - 3a^4b - \\
& 3a^3b^2 - a^2b^3 - 2(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 \\
& + 8b^5) \cosh(x)^4 + 2(105(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x) \\
&)^6 - a^5 - 7a^4b - 23a^3b^2 - 37a^2b^3 - 28ab^4 - 8b^5 + 35(3a^5 \\
& + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^4 + 15(a^5 + 7a^ \\
& 4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^2) \sinh(x)^4 + 8 \\
& (15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^7 + 7(3a^5 + 17a^4b + \\
& 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^5 + 5(a^5 + 7a^4b + 23a^3b \\
& ^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^3 - (a^5 + 7a^4b + 23a^3b^2 \\
& + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)) \sinh(x)^3 - (3a^5 + 17a^4b + \\
& 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^2 + (45(a^5 + 3a^4b + 3a^3b \\
& ^2 + a^2b^3) \cosh(x)^8 + 28(3a^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + \\
& 8ab^4) \cosh(x)^6 - 3a^5 - 17a^4b - 33a^3b^2 - 27a^2b^3 - 8ab^4 + \\
& 30(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^4 \\
& - 12(a^5 + 7a^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^2 \\
&) \sinh(x)^2 + 2(5(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^9 + 4(3a \\
& ^5 + 17a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)^7 + 6(a^5 + 7a \\
& ^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^5 - 4(a^5 + 7a \\
& ^4b + 23a^3b^2 + 37a^2b^3 + 28ab^4 + 8b^5) \cosh(x)^3 - (3a^5 + 17 \\
& a^4b + 33a^3b^2 + 27a^2b^3 + 8ab^4) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan \\
& (\sqrt{2} \sqrt{-a} \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2b) / (\cosh(x)^2 - \\
& 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh \\
& (x)^2 + a)) + 2 \sqrt{2} * ((3a^5 + 9a^3b^2 + 4a^2b^3) \cosh(x)^8 + 8(3a \\
& ^5 + 9a^3b^2 + 4a^2b^3) \cosh(x) \sinh(x)^7 + (3a^5 + 9a^3b^2 + 4a^2b \\
& ^3) \sinh(x)^8 + 4(3a^5 + 6a^4b + 8a^2b^3 + 3ab^4) \cosh(x)^6 + 4(3 \\
& *a^5 + 6a^4b + 8a^2b^3 + 3ab^4 + 7(3a^5 + 9a^3b^2 + 4a^2b^3) \cosh \\
& (x)^2) \sinh(x)^6 + 8(7(3a^5 + 9a^3b^2 + 4a^2b^3) \cosh(x)^3 + 3(3 \\
& a^5 + 6a^4b + 8a^2b^3 + 3ab^4) \cosh(x)) \sinh(x)^5 + 3a^5 + 9a^3b^2 \\
& + 4a^2b^3 + 6(3a^5 + 8a^4b + 5a^3b^2 - 12a^2b^3 - 4ab^4) \cosh(x) \\
&)^4 + 2(9a^5 + 24a^4b + 15a^3b^2 - 36a^2b^3 - 12ab^4 + 35(3a^5 \\
& + 9a^3b^2 + 4a^2b^3) \cosh(x)^4 + 30(3a^5 + 6a^4b + 8a^2b^3 + 3a \\
& *b^4) \cosh(x)^2) \sinh(x)^4 + 8(7(3a^5 + 9a^3b^2 + 4a^2b^3) \cosh(x)^5 \\
& + 10(3a^5 + 6a^4b + 8a^2b^3 + 3ab^4) \cosh(x)^3 + 3(3a^5 + 8a^4b \\
& b + 5a^3b^2 - 12a^2b^3 - 4ab^4) \cosh(x)) \sinh(x)^3 + 4(3a^5 + 6a^4 \\
& *b + 8a^2b^3 + 3ab^4) \cosh(x)^2 + 4(7(3a^5 + 9a^3b^2 + 4a^2b^3) \cosh \\
& (x)^6 + 3a^5 + 6a^4b + 8a^2b^3 + 3ab^4 + 15(3a^5 + 6a^4b + 8 \\
& a^2b^3 + 3ab^4) \cosh(x)^4 + 9(3a^5 + 8a^4b + 5a^3b^2 - 12a^2b^3 \\
& - 4ab^4) \cosh(x)^2) \sinh(x)^2 + 8(((3a^5 + 9a^3b^2 + 4a^2b^3) \cosh(x) \\
&)^7 + 3(3a^5 + 6a^4b + 8a^2b^3 + 3ab^4) \cosh(x)^5 + 3(3a^5 + 8a \\
& ^4b + 5a^3b^2 - 12a^2b^3 - 4ab^4) \cosh(x)^3 + (3a^5 + 6a^4b + 8a \\
& ^2b^3 + 3ab^4) \cosh(x)) \sinh(x)) \sqrt{(a \cosh(x)^2 + a \sinh(x)^2 + a + 2 \\
& *b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^8 + 3a^7b + 3a^6b \\
& ^2 + a^5b^3) \cosh(x)^{10} + 10(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \cosh(x) \\
&) \sinh(x)^9 + (a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \sinh(x)^{10} + (3a^8 + 17 \\
& *a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x)^8 + (3a^8 + 17a^7b \\
& + 33a^6b^2 + 27a^5b^3 + 8a^4b^4 + 45(a^8 + 3a^7b + 3a^6b^2 + a^ \\
& 5b^3) \cosh(x)^2) \sinh(x)^8 - a^8 - 3a^7b - 3a^6b^2 - a^5b^3 + 8(15(
\end{aligned}$$

$$\begin{aligned}
& a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \cosh(x)^3 + (3a^8 + 17a^7b + 33a^6 \\
& *b^2 + 27a^5b^3 + 8a^4b^4) \cosh(x) * \sinh(x)^7 + 2*(a^8 + 7a^7b + 23a \\
& ^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) \cosh(x)^6 + 2*(a^8 + 7a^7b \\
& + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5 + 105*(a^8 + 3a^7b + 3 \\
& *a^6b^2 + a^5b^3) \cosh(x)^4 + 14*(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b \\
& ^3 + 8a^4b^4) \cosh(x)^2) * \sinh(x)^6 + 4*(63*(a^8 + 3a^7b + 3a^6b^2 + \\
& a^5b^3) \cosh(x)^5 + 14*(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4 \\
& *b^4) \cosh(x)^3 + 3*(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + \\
& 8a^3b^5) \cosh(x) * \sinh(x)^5 - 2*(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 \\
& + 28a^4b^4 + 8a^3b^5) \cosh(x)^4 - 2*(a^8 + 7a^7b + 23a^6b^2 + 37a \\
& ^5b^3 + 28a^4b^4 + 8a^3b^5 - 105*(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) \\
& * \cosh(x)^6 - 35*(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * \co \\
& sh(x)^4 - 15*(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3* \\
& b^5) \cosh(x)^2) * \sinh(x)^4 + 8*(15*(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * \cos \\
& h(x)^7 + 7*(3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * \cosh(x) \\
& ^5 + 5*(a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * \c \\
& osh(x)^3 - (a^8 + 7a^7b + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^ \\
& 5) * \cosh(x) * \sinh(x)^3 - (3a^8 + 17a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4 \\
& *b^4) * \cosh(x)^2 + (45*(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * \cosh(x)^8 - 3a \\
& ^8 - 17a^7b - 33a^6b^2 - 27a^5b^3 - 8a^4b^4 + 28*(3a^8 + 17a^7b \\
& + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * \cosh(x)^6 + 30*(a^8 + 7a^7b + 23a \\
& ^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * \cosh(x)^4 - 12*(a^8 + 7a^7b \\
& + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * \cosh(x)^2) * \sinh(x)^2 + \\
& 2*(5*(a^8 + 3a^7b + 3a^6b^2 + a^5b^3) * \cosh(x)^9 + 4*(3a^8 + 17a^7b \\
& + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * \cosh(x)^7 + 6*(a^8 + 7a^7b + 23a \\
& ^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * \cosh(x)^5 - 4*(a^8 + 7a^7b \\
& + 23a^6b^2 + 37a^5b^3 + 28a^4b^4 + 8a^3b^5) * \cosh(x)^3 - (3a^8 + 17 \\
& *a^7b + 33a^6b^2 + 27a^5b^3 + 8a^4b^4) * \cosh(x) * \sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.51Error: Bad Argument Typ
e

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b \operatorname{sech}(x)^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)

[Out] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{(b \operatorname{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2), x)

[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a + b \operatorname{sech}^2(x)\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(5/2), x)

[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(5/2), x)

$$3.220 \quad \int \frac{1}{(a+b\operatorname{sech}^2(c+dx))^{7/2}} dx$$

Optimal. Leaf size=183

$$\frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a} \operatorname{tanh}(c+dx)}{\sqrt{a-b \operatorname{tanh}^2(c+dx)+b}}\right)}{a^{7/2}d} - \frac{b(9a+5b) \operatorname{tanh}(c+dx)}{15a^2d(a+b)^2(a-b \operatorname{tanh}^2(c+dx)+b)^{3/2}} - \frac{b(33a^2+40ab+15b^2) \operatorname{tanh}(c+dx)}{15a^3d(a+b)^3\sqrt{a-b \operatorname{tanh}^2(c+dx)+b}}$$

[Out] $\operatorname{arctanh}(a^{1/2} \operatorname{tanh}(d*x+c) / (a+b-b*\operatorname{tanh}(d*x+c)^2)^{1/2}) / a^{7/2} / d - 1/15*b*(33*a^2+40*a*b+15*b^2)*\operatorname{tanh}(d*x+c) / a^3 / (a+b)^3 / d / (a+b-b*\operatorname{tanh}(d*x+c)^2)^{1/2} - 1/5*b*\operatorname{tanh}(d*x+c) / a / (a+b) / d / (a+b-b*\operatorname{tanh}(d*x+c)^2)^{5/2} - 1/15*b*(9*a+5*b)*\operatorname{tanh}(d*x+c) / a^2 / (a+b)^2 / d / (a+b-b*\operatorname{tanh}(d*x+c)^2)^{3/2}$

Rubi [A] time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4128, 414, 527, 12, 377, 206}

$$\frac{b(33a^2+40ab+15b^2) \operatorname{tanh}(c+dx)}{15a^3d(a+b)^3\sqrt{a-b \operatorname{tanh}^2(c+dx)+b}} + \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a} \operatorname{tanh}(c+dx)}{\sqrt{a-b \operatorname{tanh}^2(c+dx)+b}}\right)}{a^{7/2}d} - \frac{b(9a+5b) \operatorname{tanh}(c+dx)}{15a^2d(a+b)^2(a-b \operatorname{tanh}^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*x]^2)^{-7/2}, x]$

[Out] $\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[c + d*x]^2]] / (a^{7/2}*d) - (b*\operatorname{Tanh}[c + d*x]) / (5*a*(a + b)*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^{5/2}) - (b*(9*a + 5*b)*\operatorname{Tanh}[c + d*x]) / (15*a^2*(a + b)^2*d*(a + b - b*\operatorname{Tanh}[c + d*x]^2)^{3/2}) - (b*(33*a^2 + 40*a*b + 15*b^2)*\operatorname{Tanh}[c + d*x]) / (15*a^3*(a + b)^3*d*\operatorname{Sqrt}[a + b - b*\operatorname{Tanh}[c + d*x]^2])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 377

$\operatorname{Int}[(a_*) + (b_*)(x_)^{(n_*)})^{(p_*)} / ((c_*) + (d_*)(x_)^{(n_*)}), x_Symbol] := \operatorname{Subst}[\operatorname{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[n*p + 1, 0] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 414

$\operatorname{Int}[(a_*) + (b_*)(x_)^{(n_*)})^{(p_*)} * ((c_*) + (d_*)(x_)^{(n_*)})^{(q_*)}, x_Symbol] := -\operatorname{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}) / (a*n*(p+1)*(b*c - a*d)), x] + \operatorname{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \operatorname{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q * \operatorname{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n, q\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{Lt}Q[p, -1] \ \&\& \ !(\ !\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[q] \ \&\& \ \operatorname{Lt}Q[q, -1]) \ \&\& \ \operatorname{IntBinomial}Q[a, b, c,$

d, n, p, q, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^{7/2}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{7/2}} dx, x, \tanh(c + dx)\right)}{d} \\
 &= -\frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{-5a-b-4bx^2}{(1-x^2)(a+b-bx^2)^{5/2}} dx, x, \tanh(c + dx)\right)}{5a(a+b)d} \\
 &= -\frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{b(9a+5b) \tanh(c + dx)}{15a^2(a+b)^2d(a+b-b \tanh^2(c + dx))^{3/2}} \\
 &= -\frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{b(9a+5b) \tanh(c + dx)}{15a^2(a+b)^2d(a+b-b \tanh^2(c + dx))^{3/2}} \\
 &= -\frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{b(9a+5b) \tanh(c + dx)}{15a^2(a+b)^2d(a+b-b \tanh^2(c + dx))^{3/2}} \\
 &= -\frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{b(9a+5b) \tanh(c + dx)}{15a^2(a+b)^2d(a+b-b \tanh^2(c + dx))^{3/2}} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{a} \tanh(c+dx)}{\sqrt{a+b-b \tanh^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b \tanh(c + dx)}{5a(a+b)d(a+b-b \tanh^2(c + dx))^{5/2}} - \frac{b(9a+5b) \tanh(c + dx)}{15a^2(a+b)^2d(a+b-b \tanh^2(c + dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 7.98, size = 330, normalized size = 1.80

$$\operatorname{sech}^7(c+dx) \left(\frac{15}{4} e^{-7(c+dx)} \left(a \left(e^{2(c+dx)} + 1 \right)^2 + 4be^{2(c+dx)} \right)^{7/2} \left(\tanh^{-1} \left(\frac{ae^{2(c+dx)} + a + 2b}{\sqrt{a} \sqrt{a \left(e^{2(c+dx)} + 1 \right)^2 + 4be^{2(c+dx)}}} \right) - \tanh^{-1} \left(\frac{a}{\sqrt{a} \sqrt{a \left(e^{2(c+dx)} + 1 \right)^2 + 4be^{2(c+dx)}}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^(-7/2), x]

[Out] (Sech[c + d*x]^7*((15*(4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x))))^2)^(7/2)*(ArcTanh[(a + 2*b + a*E^(2*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))] + a*(1 + E^(2*(c + d*x)))] - ArcTanh[(a + a*E^(2*(c + d*x)) + 2*b*E^(2*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))]])/(4*E^(7*(c + d*x)) - (4*Sqrt[a]*b*(a + 2*b + a*Cosh[2*(c + d*x)])*(135*a^4 + 480*a^3*b + 709*a^2*b^2 + 460*a*b^3 + 120*b^4 + 4*a*(45*a^3 + 135*a^2*b + 117*a*b^2 + 35*b^3)*Cosh[2*(c + d*x)] + a^2*(45*a^2 + 60*a*b + 23*b^2)*Cosh[4*(c + d*x)])*Sinh[c + d*x])/(a + b)^3)/(960*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^(7/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 1.44Error: Bad Argument Type

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}(dx + c)^2)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c)^2)^(7/2), x)

[Out] int(1/(a+b*sech(d*x+c)^2)^(7/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(dx + c)^2 + a)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="maxima")

[Out] integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(c+dx)^2}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/cosh(c + d*x)^2)^(7/2),x)

[Out] int(1/(a + b/cosh(c + d*x)^2)^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{sech}^2(c + dx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c)**2)**(7/2),x)

[Out] Integral((a + b*sech(c + d*x)**2)**(-7/2), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```



```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,````)
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+``) or
    type(expn,``*``)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```



```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```